Laser: Fundamentals and Applications
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Lecture - 15
Some Numerical problem

Hello and welcome. So, in the last class we learnt about a few terms related to lasers like average peak and peak power, a pulse energy repetition rate and also time bandwidth etcetera etcetera and we were supposed to talk about different pulsing techniques that are used in a building a laser particularly pulse laser. I just thought of you know discussing a few problems you know trying to solve certain problems based on whatever we learnt over the last couple of weeks will be quite helpful before we you know move on. So, today we are going to look at certain problems and trying to solve it. So, let us take the first problem.

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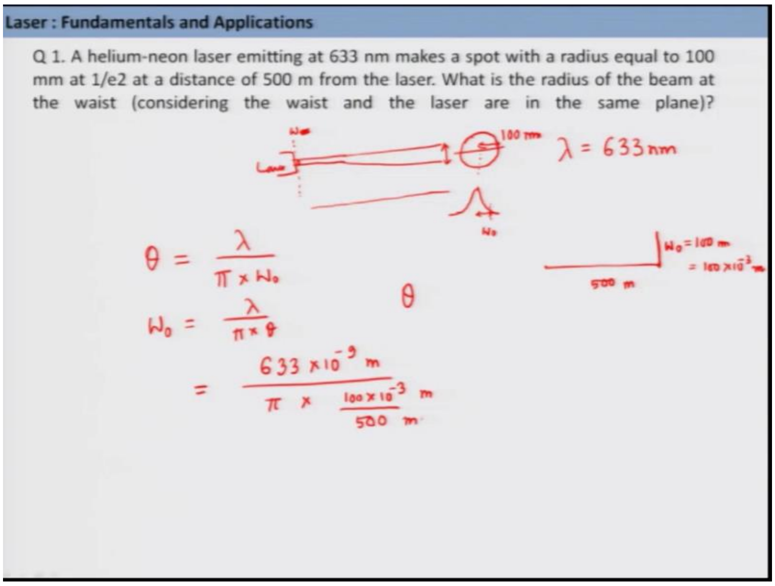

So, the problem states a helium neon laser emitting at 633 nanometer makes a spot with a radius equal to 100 millimeter at 1 upon e square at a distance of 500 meter from the laser.

The question is what is the radius of the beam at the waist. So, essentially the question you know that has been asked is to find out what is the beam waist. So, you are starting, essentially the problem if we just draw it. So, you have a laser source. So, this is a laser and a beam is coming like this.

Now if you go at far distance, so beam will slightly diverge right and if you go really really you know really large distance like 500 meters as the question suggests so slowly the beam will you know grew to a larger spot size. So, you at the beginning at this particular point it would have certain you know value for the beam waist well here also it will have certain diameter of the beam.

So, we are supposed to find out what is the beam waist now here you see what are the information that we already have in the question. So, that is the most important thing whenever we come across equation we have to understand what are the information that has been provided in the question and the thing that has been given is that the wavelength lambda is equals to 633 nanometer could you say sincerely red light and at 500 meter distance. So, you know whatever the distance is there the beam will look like this right. Now the beam will not be you know equally intense all over the place. So, if I take a cross section of the beam it is probably going to look like this if it is say tm 0 0 mode or it is like a Gaussian. So, when the intensity decreases from this center point, from highest intensity at the central point it decays down as you move toward the outer side of this circle a spot.

When you reach the value of e square 1 upon e square then you stop there and you collect that particular distance do you measure that distance and that becomes your radius. So, suppose this is the place where it is 1 upon a square, so this distance is my beam radius correct is that we have learnt earlier. So, now, we have to find out what is that value of omega 0. Now this is something various it is a just omega how we can do that. So, we have also learnt the relation between the beam west and the beam divergence in one of the previous classes, and what was that relation? The relation was theta which measure gives you a measure of the divergence of the beam when theta is in radian and that is related to the wavelength of light and beam waist as this pi omega 0 is the beam waist that we are talking about. So, how do we know what is theta.

So, let me first rearrange this one. So, we need to find out this omega 0 that is equal to this. Now, what is theta? So, theta is in radian, now, if you look at this problem. So, this problem if you just you know draw it in a very nice way. So, essentially this is your starting point of laser and the beam goes up to a distance 500, so this is 500 meter and I have certain you know radius of the beam and that is this one. So, I have this radius of the spot diameter here as 100 millimeter.

So, here this distance this radius is 100 mm and this have grown from the laser spot size here and I have to find out whatever that spot size right. So, essentially this omega 0 here is essentially 100 millimeter. So, 100 millimeter means 100 into 10 power minus 3 meter and this is 500 meter. So, what will be while theta in radian? It will be very easily, so lambda is 633 nanometer so; that means, 10 to the power minus 9 meter you have pi and your theta in radian will be given by 100 into 10 to power minus 3 meter divided by here it is 500 meter ok.

What we used? We used our knowledge regarding the beam divergence and its relation with the beam waist. So, let us move to the second problem.

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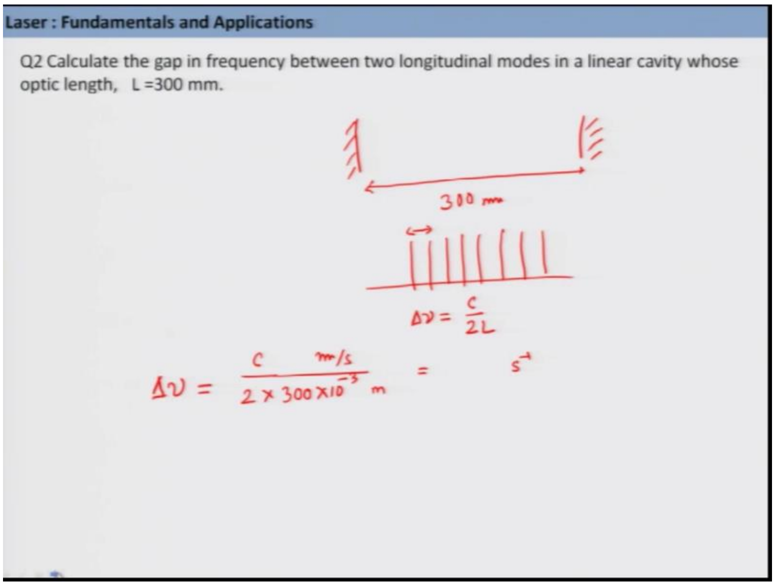

So, we are supposed to calculate the gap in frequency between two longitudinal modes in a linear cavity whose optic length optical length is 300 millimeter. So, essentially you have a cavity and it says linear cavity. So, that is what we have discussed we did not discuss any other type of cavities that are possible. So, essentially this distance is 300 millimeter.

Now there are several longitudinal modes that are possible fine now given this cavity length what are the modes you know and particularly what will be the gap between two successive longitudinal modes, and that we have to find in frequency unit. So, we know that if there are several modes that are possible what is the gap between this two that we have already learned we if we write it as delta nu then this is C over 2 L where L is my length of the cavity correct. So, therefore, for this one my delta nu will be equals to C over 2 into 300 into 10 to the power minus 3 ok.

So, this will be meter per second and this will be meter. So, this will be some value in per second or hertz. So, this you can easily calculate in this way.

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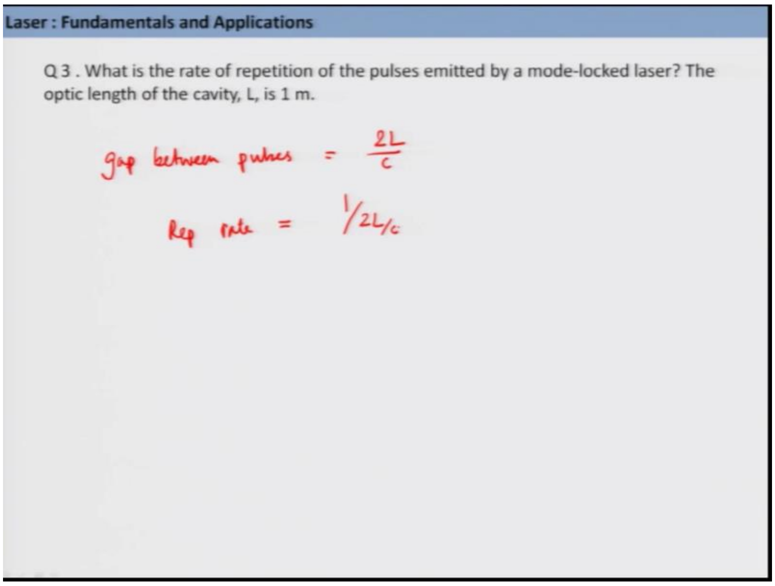

So, let us move to the next problem. So, the question is what is the rate of repetition or in other word what is the repetition rate of the pulses emitted by a mode lock laser, the optic length of the cavity L is 1 meter. So, now, we have not talked about different types of lasers like in terms of their you know pulsing techniques, so right now do not worry about this term mode locked we will be talking about it in the following class itself.

So, just take it like a its a type of laser and for this you know particular type of laser what we can have is the gap between two successive pulses is given by 2 L by C and, this is essentially there are several you know pulses coming right. So, this is that you know gap between two pulses correct. So, if I know this gap then I will definitely know that in one second time how many times this amount of gap we appear and that will give me the total number of repetition or this will give me the repetition rate. So, essentially the rep rate will be 1 upon this quantity. So, you can easily solve this one.

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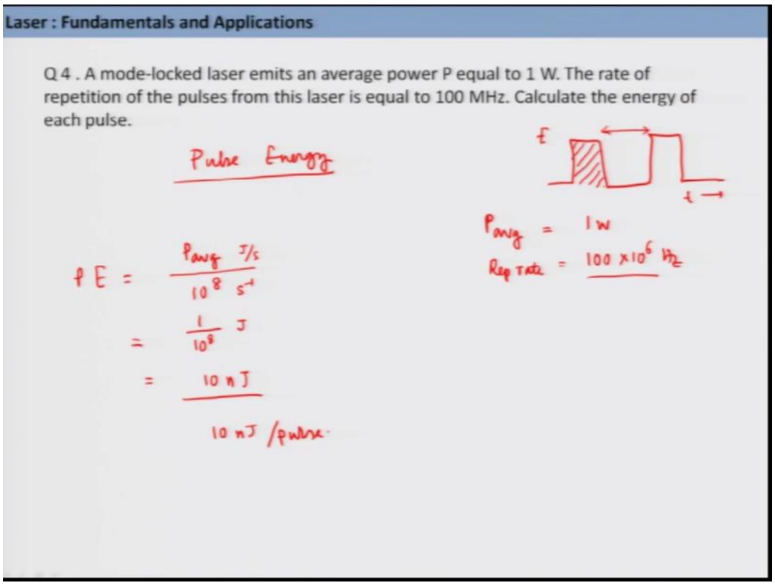

Now the next question also it talks about a mode lock laser. So, you can you can ignore this mode lock thing here. So, now, the question is that if the this laser emits an average power P which is equal to 1 what the rate of repetition of the pulses from this laser is equals to 100 megahertz calculate the energy of each pulse. So, essentially this question asks about pulse energy.

So, we know what pulse energy is. So, pulse energy is essentially if these are the pulses as a function of time and this is the energy. So, what is the energy of this pulse now what are the information that we have been given we have been given P average which is equals to 1 watt all right and it has been given a repetition rate of 100 mega hertz right. So, now, in 1 second there are such 100 into 10 to the power 6 number of pulses will be there and when the pulse is not there will be no light that is this gap for between each two pulses correct. Now if I measure the inner energy over 1 second period of time I will get essentially the energy of each pulse multiplied by 10 to the power 8 because there are 10 to the power 8 number of pulses in a second therefore, that will be my average power because I am measuring it over a second. If I know my average power if I know how many pulses are there it is very easy to find out the pulse energy correct. So, if it is P average divide 10 to the power 8 right. So, this is in second inverse and this is in Joule per second correct. So, this is my pulse energy. So, this will be 1 by 10 to the power 8 correct Joule equals to essentially 10 nano Joule because 10 to power minus 8 is essentially 10 into the power minus 9 nano Joule.

So, this is the energy. So, most of the time it is written as 10 nano Joule per pulse whenever you see this you will know that this is you know pulse energy that we are talking about.

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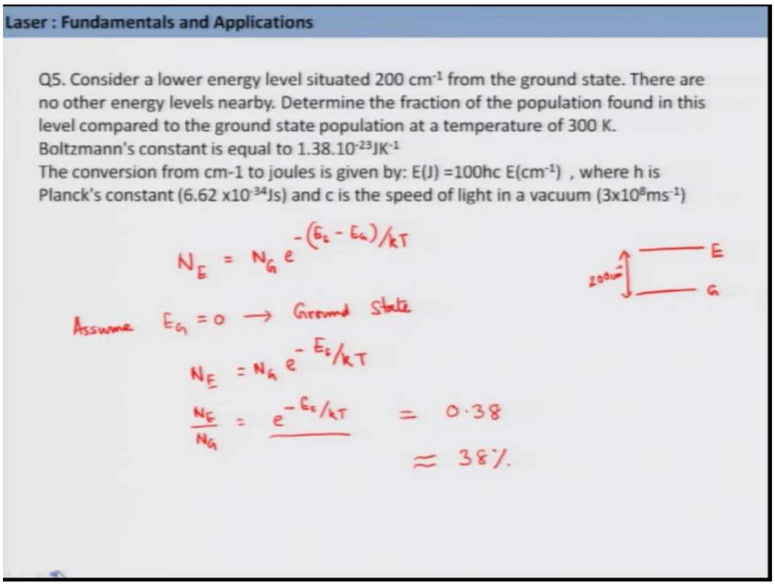

So, let us move on to the next problem. So, the next question says consider a low energy level situated 200 centimeter inverse from the ground state. So, let us keep drawing. So, this is my ground state and then a low energy excited state and this gap is 200 centimeter inverse there are no other energy levels nearby.

So, this is fairly a two level system determine the fraction of population found in this level compared to the ground state population at a temperature of 300 k which is essentially room temperature. And then the value of Boltzmann constant is given and how to convert centimeter inverse to Joule that is also given and a Planck Constant and the speed of light is also provided.

So, we are simply asked to calculate the you know population fraction of the population at level e. So, suppose I want to calculate population of any given state how do I calculate we simply use the Boltzmann distribution at under equilibrium. So, for any two level, like you know if I write N E as excited state equals 2 N G into e to the power minus E E minus E G over kT correct this is a very well known. Now in our case if I want to find out just a fraction then if I assume that the ground state you know the lowest state to be just the ground state and the energy is 0, just for simplicity if you take any given energy there is no harm, but just for simplicity if I make E Gequals to 0 assume.

So, essentially my equation becomes tau and this is for ground state and I reiterate if you do not assume it no problem only thing is that you need to know the energy of that state. So, my equation becomes N G into E to the power minus E E by kT.

So, ultimately what I have to find out in E by N G correct. So, this is I have all the information. So, if I take this energy of down state to be 0 then the excited state has energy of 200 centimeter inverse with the given you know values you can easily convert it to Joule and ultimately you find out this quantity over here and if you do that you will find this is approximately this or in other word it is 38 prevent population that lie in the excited state.

So, you know this problem can be also given with another level and given the lifetime of the respective states and you should be able to find out what will be the population or in a fraction of population in one of these states. So, for example, like a 3 level system.

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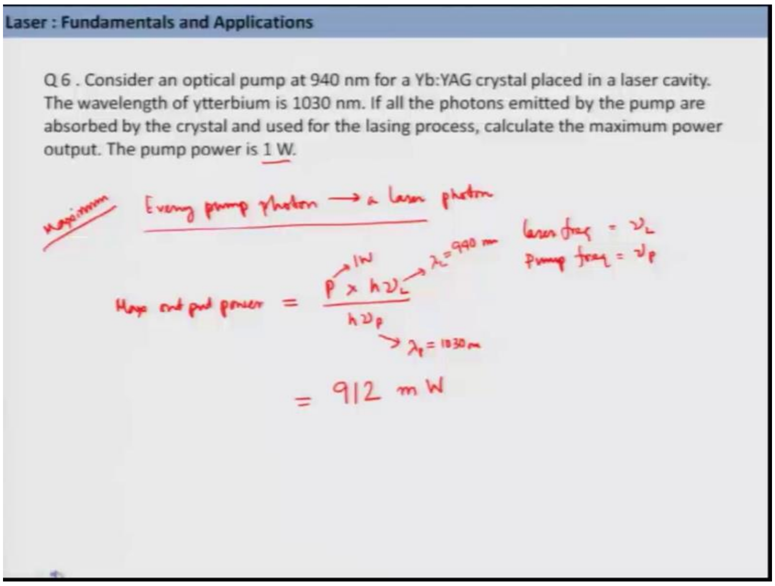

So, let us look at this next question. So, consider an optical pump act 940 nanometer for ytterbium YAG crystal placed in a laser cavity the wave length of ytterbium is 1030 nanometer which means that you know when it fluoresce the stimulated fluorescence has a wavelength of 1030 nanometer. If all the photons emitted by pump are absorbed by the crystal and used for lazing processes calculate the maximum power of the output and the pump power is given right 1 1.

So, what best efficiency we can get for every photon given you are getting output. So, essentially what we say is every pump photon will lead to a laser photon that is the best that we can have we cannot have more than that. So, in that case if I define the laser frequency as nu L and the pump frequency as nu P then I can easily find out that the maximum output power of the laser maximum based on this condition. So, this is the maximum possible output power. So, that will be given by the power of the pump. So, multiplied by the energy of the laser divided by the energy of the pump that is used. So, here you have been given information about all of them only thing is that you will need to convert wavelengths given to frequency. So, power is pump power is given as one watt. So, this is essentially one watt and the frequencies of the lasers. So, this corresponds to lambda P which is 1030 nanometer and this corresponds to lambda L 40.

So, every information is there only thing is that you have to convert two particular you know unit and then just get the calculator number. And if you do that you will find it use I am just writing the final answer is 912milliwatt.

So, in this problem if you notice clearly it asks for your concept right. So, whenever someone is asking the you know maximum efficiency. So, maximum efficiency can be only when you have 100 percent conversion. So, every pump photon is ultimately leading to one laser photon so that is the maxima possibility otherwise what do you know like there are the thousands of photons are coming and some of them are interacting and giving rise to you know even less number of laser photon most of them are getting lost, but if we assume that there is you know like you know.

If this is a process at pump photons heating and the laser photon is coming out the quantum efficiency of this process if it is 100 percent; that means, for each pump photon there will be one laser photon coming out. So, if you have this concept clear then the rest of the problem is really easy.

So, rest of the thing is just the conservation of energy correct. Now we will consider a couple of more problems before we conclude the lecture today.

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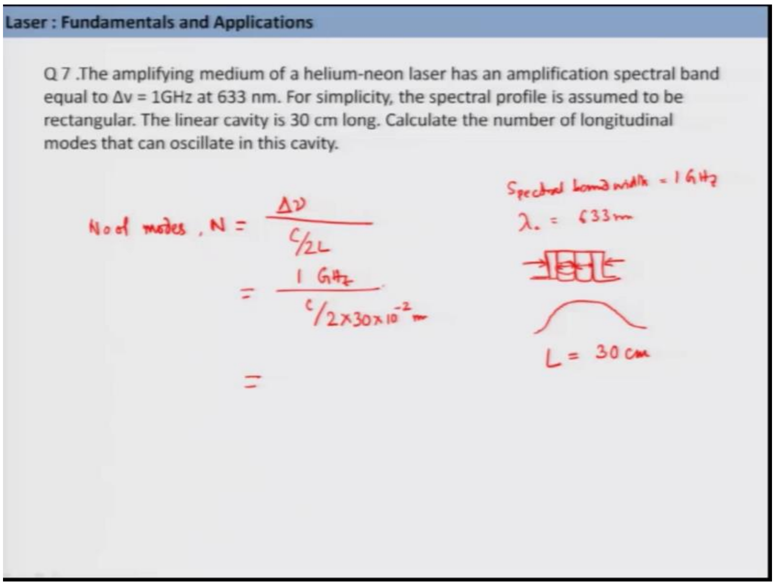

So, the question next question is the amplifying medium of a helium neon laser has an amplification spectral band equal to delta nu because to 1 gigahertz.

So, essentially we have been given the spectral bandwidth. So, the bandwidth is equals to one gigahertz and the central frequency is 633 nanometer and also the question says that the profile of the you know spectral and the spectral profile is rectangular. So, you have like you know bandwidth is like this, most of the time we showed like you know the bandwidths like something like this. So, here it is just like you know like rectangle. So, this makes our life much much simpler. And then the cavity length is 30 centimeter, now we have to calculate the number of longitudinal modes that can oscillate in this cavity. So, this problem we have seen during the class.

So, the number of modes that can be supported by the cavity if I call it n then essentially is the you know number of times or in other what let me tell it in a different way like if there are say number of modes within this cavity. So, how do I find out the number of that mode because we know this gap right between two longitudinal modes and also we know this bandwidth that is you know that is something which we have to which is already provided that is 1 gigahertz right. So, now, if we know this gap then I need I will find out that how many modes are you know with how many modes we can fit in within this envelope. So, the bandwidth is given as delta nu and this gap between two modes is by C by twice L correct. So, therefore, I have one gigahertz and C by 30 centimeter so; that means 10 to the power minus 2 meter. So, we can easily find out how many modes will be there you know which will satisfy the standing wave conditions.

So, that is the crack of this problem that we need to find out how many modes we will satisfy the standing mode condition and if you know the expression which connects the length of the cavity and the frequency of light and then you know that what is the gap between two such modes. So, you know nu, you know how nu is related to the cavity link and then what is the difference between say nu 1 and nu 2. So, knowing that gap and knowing the overall bandwidth spectral bandwidth of your laser active medium it is extremely easy to find out how many such modes will be there, so that is all we did.

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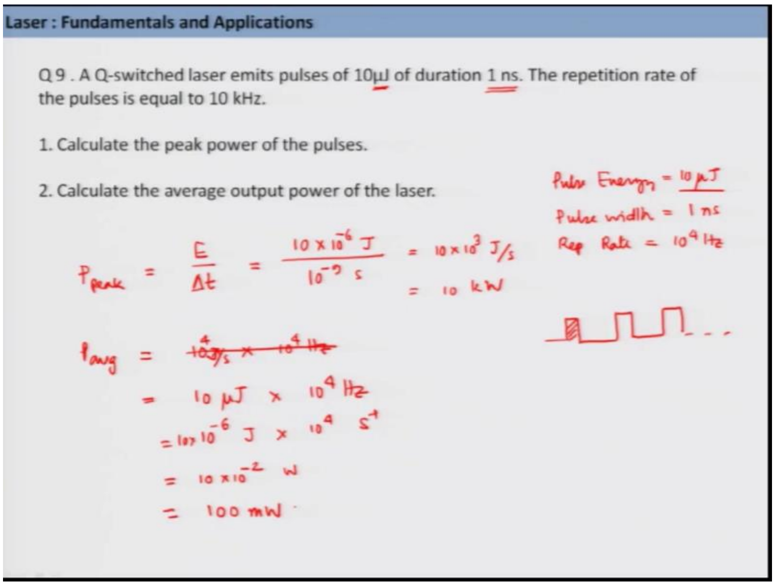

So, here last question that we will look at is if we take a Q-switched laser that emits pulses of 10 micro Joule of duration 1 nanosecond and the repetition rate of the pulses is equal to 10 kilohertz. So, what I have we have pulse energy you see you have to notice the unit rate, so this is 10 micro Joule. So, it is pulse energy that is equals to 10 micro Joule and pulse with the duration is this.

So, pulse width is one nano second and the repetition rate is equals to 10 kilohertz, 10 to the power 4 hertz. So, first thing we have to calculate the peak power. So, peak power is given by the energy of a pulse divided by its pulse width. So, essentially peak power is equal to energy of a pulse divided by the pulse width. So, here what we have? We have 10 micro Joule right. So, divided by pulse width is, 1 nanosecond, so 10 to the power minus 9 second. So, you can easily calculate what will be the peak power and if you do that you will see it is 10 into 10 to the power 3 Joule and you can you know see how much is the peak power look right. So, this is sorry per second. So, this is essentially 10 kilowatt.

Now what is the average power? Now if you know the peak power you are if you know the pulse energy. So, you essentially have the system. So, you have you know this energy now you know how many times such pulses will be there.

So, you know the repetition rate so that if you multiply the repetition rate with the pulse energy you get essentially the average power. So, P average is equals to pulse energy that is 10 to, 10 to the power 4 Joule per second multiplied by the repetition rate that is 10 to the power 4 hertz then. So, we have to take correct pulse energy which is given here sorry for the mistake. So, essentially is 10 micro Joule multiplied by 10 rep rate. So, 10 power minus 10 into 10 to the power minus 6 Joule into 10 to the power 4. So, this gives rise to 10 into 10 to the minus 2 watt or in other word this is 100 milliwatt. So, if I know the very basic concept about peak power and average power then we can always find out what are the values of these quantities. We definitely would know in that case how these powers are related to several other quantities like repetition rate or pulse energy, pulse width etcetera etcetera.

So, today we tried to look at certain problems which are very relevant to whatever we have learnt in last couple of weeks and we solved them. So, I hope this will you know help you in understanding this overall topic in a much better way as well as it will help you to prepare for your exams. So, we will come back with a new topic that is the pulsing techniques for pulse lasers in the next week.

Thank you very much.