Laser: Fundamentals and Applications
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Lecture – 11
Threshold Condition

Hello and welcome back, today is the first day of third week of this lecture series. So, in the previous week we learnt about the fundamental issues involving the laser construction and we learnt about how we can amplify the stimulated emission, to get laser. At the end of the class in the previous week we started talking about the threshold condition. So, we can understand that there are you know there are two things; one is the amplification of the stimulated emission which is the gain and there are several losses that we will talk about today.

So, there must be an optimum value of this gain, above which we can see laser output otherwise we will not see so called quote unquote laser output. So, we need to learn about the threshold conditions.

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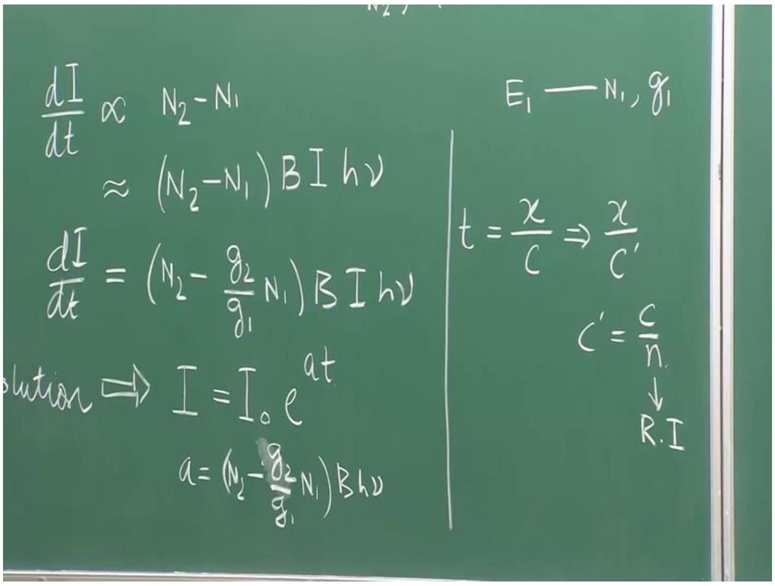

So, we will start from there today, so we have learned about population inversion already, and we also know about the stimulated emission. So, knowing the population inversion and stimulated emission what we can say is the intensity of stimulated radiation stimulated photons. The change in that intensity will be proportional to the difference in the population in the two levels involving the laser action.

So, if I write the populations of the upper and lower states as N 2 and N 1 respectively, hum and the intensity of the stimulated photons as I, then the change in the intensity I with time that is the rate of increase of stimulated photon, we can write as dI dt and you can see this is the this is positive. So, it signifies that there is an increase, in case of a decay we write it as negative we know that. So, this is proportional to the population difference ok.

So, we can write this in a very similar way as we have already learnt in the you know first week of this lecture series, with a rate constant which is for stimulated emission into the initial intensity like this. So, this is for a case where I have considered only two states and having no degeneracy. Degeneracy meaning that there are several such states having the same energy in the same properties. So, if I say for the state having population N 1 has a degeneracy of g 1, that is g 1 number of states have the energy of this state which is say I write E 1 and then correspondingly this is g 2.

So, it is better to consider these degeneracies because in real life there are several states having degeneracies. So, if I consider this degeneracy then it will be. So, up to these we should not have much problem because we have seen very similar kind of formalism. So, now, we are only thing is that we are talking in terms of the intensities of the photon, and the rate of increase of intensity. Now this is a differential equation and what will be the solution. So, solution is I equals to I 0 at a time t equal to 0, e to the power a t, so alright.

Now, this is expression of intensity of stimulated photons at any given time t, now we know that time is related to distance and time is related to then the speed of and we can correlate the time with the distance using the speed of light, because we are dealing with the properties of light only. So, t is equals to x where x is my distance whatever that be by speed of light correct. So, in real systems we may encounter you know a medium which is different from say vacuum. So, in that case my expression will be modified to C prime, where C prime is nothing, but taking into account of the refractive index of the medium right. So, this is my refractive index of the medium so fine.

So, since I can convert t into this, I can express this equation in terms of this one also. So, one thing I forgot to mention. So, this is essentially nothing, but N 2 minus N 1 with the degeneracies are maybe, so now if we express this in terms of the distance x.

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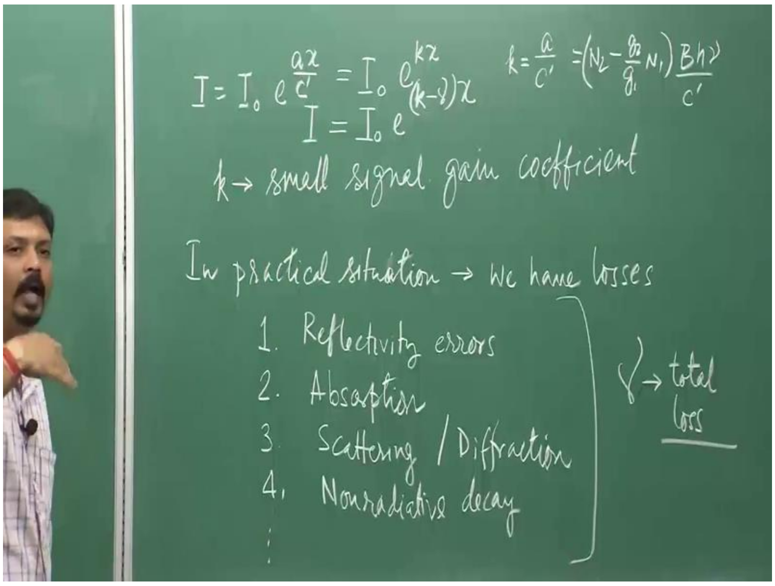

Then how we can write that we can write as, I equals to I 0 into e to the power a and x by C prime fine. So, if I club this a by C prime then I can rewrite this one as like this where my k is equal to a by C prime which is this ok.

So, this is my expression for I for particular distance that my photon may be traveling. Now this k is known as small signal gain coefficient. So, we will remember this one that k is a gain coefficient. So, we will utilize this one in a later step. So, in actual condition in practical word or in practical situation, we have several losses in the cavity losses which we have several sources of losses what are they which comes from the error in the reflectivity.

So, reflectivity errors; what does it mean that I know that one of the mirror should have 100 percent reflectivity, another one should be partially transmitted. So, say has 98 percent reflectivity, but if in you know in reality if it is not 98 percent, if it is a 97.5 percent and the other one is says 99 percent, then it is different from what we expect and in that condition I will have losses. So, this is one second, there are several other things in a medium. So, suppose you know I need to cool down my I need to cool down my you know flash lamp because that will create a lot of heat or the electric discharge system.

So, there will be you know liquid media and all. So, there are chances of absorption of photons and this absorption is not necessarily going to take place within the medium itself, but it will be you know it can be absorbed in several other parts of the cavity as well. So, the moment there is an absorption of the photon then we are getting an output which is less than what it should be there can be scattering right there can be the diffraction, there can be non radiative decays, which means that I want lazing I want stimulated emission from they here to here. So, these two states should be radiatively coupled.

But instead of you know a radiation emission, radiative emission if there is a non radiative emission that is the molecule sitting here relaxes back to this step without emitting photon that is non radiative decay we have learned about that. So, if there are you know certain amount of non radiative decay, then also we are actually loosing the amount of photon. So, there are several such you know losses are present within the cavity. So, we need to you know take care of this losses then only we can get sufficient amount of you know laser output.

So, all this losses there are you know few more losses that we can take into account. So, if we consider a particular type of cavity. So, we can figure out what are the different types of losses are there, and what are the total amount of losses and if we club all of them together and put all the loss by this gamma. So, gamma refers to total loss within the cavity, then I need to modify this. So, what how should I modify this one? I should modify this as I 0 into e to the power k minus gamma why because k is my gain coefficient. So, the losses will take out amount from my again correct. So, this is what I will get. So, I equals to this.

Now, we need to consider another thing, we got this expression thinking about the active medium the process involving population inversion and the stimulated emission, and all the losses. One thing we did not consider which is by design of a laser cavity it is always there what is there.

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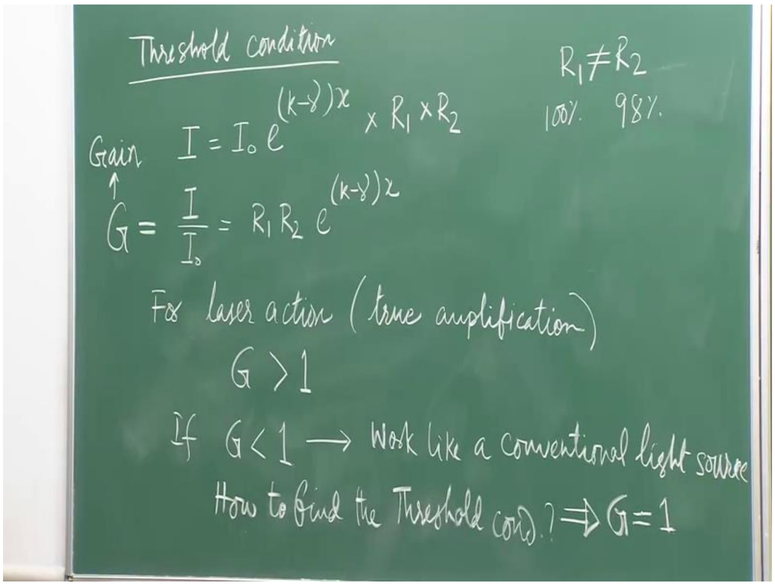

Because I know for a laser cavity R1 is not equal to R 2 where R 1 and R 2 are the reflectivities of 2 meters. So, one is 100 percent I will assume 100 percent here, because I am already taking care of the losses which is known to me. So, we should not doubly count it and this R 2 say 98 percent.

So, you can see that this is not 100 percent we know that this is my design of the laser. So, this 2 percent always goes go out, I need to consider this here also because that will lead to that overall you know intensity or the photon density within the cavity correct. So, this expression has to be modified as I equals to I 0 k minus gamma x multiplied by R 1 multiplied by R 2 this is my, you know real life situation right.

So, if I now rewrite this one as I by I 0 equals to R 1 R 2 into exponential k minus gamma x. This term I by I 0 is known as gain, where G is my gain. We have been talking about gain and all. So, this is the mathematical form of the gain. So, if we if we really need to have laser amplification. So, for laser action; that means a true amplification right. So, true amplification what we need? We need this quantity to be greater than one ok.

So, when gain is greater than 1 that is more than 100 percent. So, I need G to be greater than 1, if G is not greater than 1. So, if say G is less than 1 what do I have? I do not have any amplification. So, essentially my the photons which is coming out of my device if I call it laser or whatever when G is less than 1, the photon characteristics are very similar to the spontaneous photon. So, at this condition the characteristic is very similar to spontaneous emission. So, the device will work just like it will work like a conventional light source. We do not have any problem in understanding this one. So, if there is no amplification. So, basically it will be spontaneous emission which is you know dominant and it will be just like a chaotic system, what is chaotic system we will talk in a while.

So, we definitely want this G value to be greater than one so; that means, there is a threshold. We started saying that there must be a threshold and this is how it looks mathematically. So, how do I know what is the threshold condition. So, how to find out find the threshold condition? It is very easy right it has to be greater than 1. So, if I find out what is the condition on say this gain coefficient k when G is equal to1; that means, if I just increase go above that even by the slightest amount it will place it will have positive gain right.

So, in order to find this out I need to equate G. So, the answer is I will find out by equating G to unity, let us do that and find it out. So, I will keep this part.

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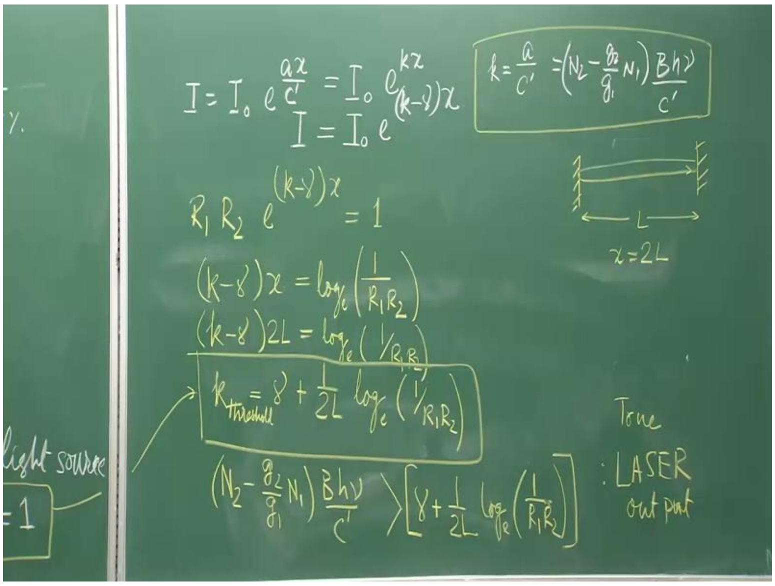

So, my G is I am equating G 2 the unity. So, what I can write is R 1 R 2 multiplied by this exponential term equals to 1 right. So, what I can do is equals to if I take logarithm e waste. So, log E 1 by R 1 R 2.

Now, what is this x actually? We have been writing about the distance this x is essentially my cavity length for a round trip. So, this length is L rate we have learned this in last few classes. So, one round trip means is going and coming back which means x is essentially 2L. If you look at this expression where we introduced this R 1 in R 2 it should tell you clearly that we are considering a round trip without a round trip I cannot you know complete one wavelength even, ok.

So, for that we are having this x equals to 2L. So, what I will do I will get into 2L equals to this quantity. So, what do I need to find out? I need to find out this k. So, my k is equal to gamma plus 1 upon twice L, now this k is a special key why because we obtained it by putting this threshold condition that is G equals to 1. So, this actually led to this k value right.

So, I will specifically mention this one as k threshold, now what do I need to do. So, that I get finally, a laser output what I need to do is to do something so that this k which is my small signal gain coefficient should be greater than this one right. So, what I need to have here is N 2 minus g 2 by g 1, C prime should be greater than this. So, this is the condition for achieving laser amplification ok.

So, this is for actually getting laser output a true laser output if I have this value that is the gain coefficient less than or equal to this threshold value I will not see any laser output only it goes above this value I get a laser output. So, this is how we get the condition for getting laser the threshold condition for getting laser.

So, today we will stop here and in the next class we will look at certain properties of laser which you have already heard of we will see what are the basic reason for having those properties say for example, directionality you know a beam, diameter beam weighed coherence and monochromatic city intensity etcetera, etcetera. And we will also relate certain applications to these properties. So, we will see you again in the next class.

Thank you very much.