**Implementation Aspects of Quantum Computing**
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**Lecture – 07**
**Quantum Parallelism: DJ Algorithm and Implementation Aspects**

In the last class we discussed about Teleportation; in this class we will talk about Quantum Parallelism.

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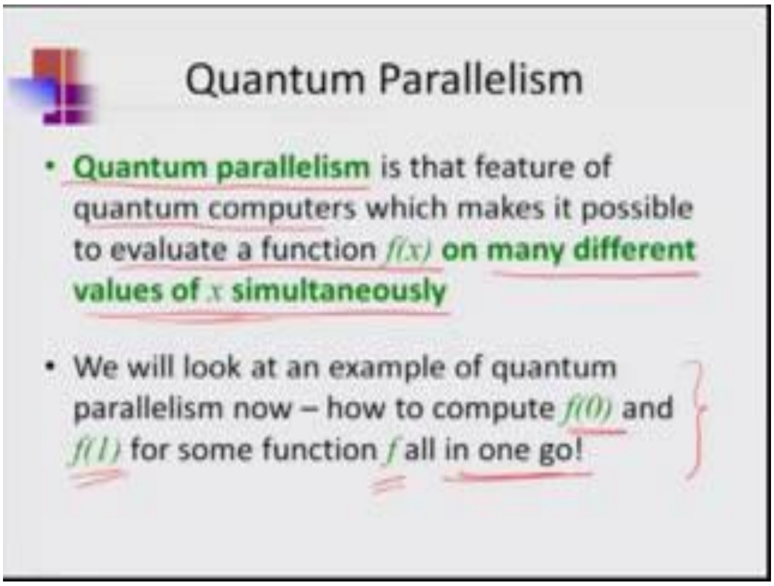


Quantum Parallelism is the feature of quantum computers which makes it possible to evaluate a function on many different values of x simultaneously. We look at an example of Quantum Parallelism now; how to compute say f of 0 and f of 1 for some function f all in one go. So, the main thing is to be able to do this entire computation of all possible functional values in one step. This is the basic idea of using Quantum Parallelism to solve a problem where we are going to finds out the solutions simultaneously.

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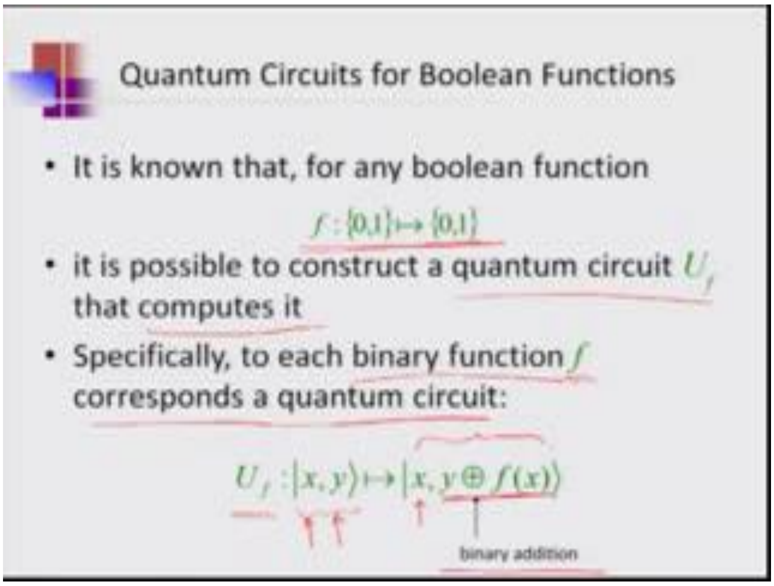


So, how do we do a Quantum Circuit for Boolean Functions? It is known that for any Boolean Function we can write a circuit which will look like this. The function corresponds to a Boolean 0, 1 going to it is corollary 0, 1. It is possible to construct a Quantum Circuit with a unitary function U of f that computes this. Specifically to each binary function f corresponds to a Quantum Circuit.

So, for example, if we take this unitary function f; then the set x and y forming our original function; if there exist a function f then we will be reproducing this Binary Addition as a result of this operation. So, this is the principle behind this particular kind of Boolean Functional Operation. So, this is generally the description of the problem that we are looking at. Now we have to construct a circuit which will essentially do this Boolean Function.

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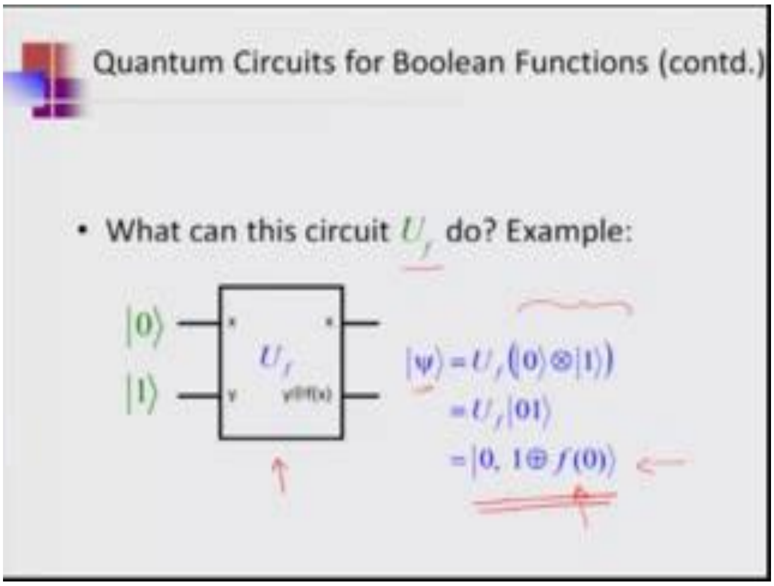


So, what can this circuit do? Here is an example; we have the function; we have the states. Let us take this example where we have the input wave functions 0 and 1; which goes through this circuit using the unitary operator U of f. What it will achieve is the idea that we had done which is that they have final wave function psi is going to have the unitary function operate on the two function simultaneously 0 and 1 which will produce the final result where the function has been added to the first bit and not the 0th bit.

So, this is the basic idea behind this. Here the Binary Addition is achieved on a particular bit out of the two bit is that have been put in. In our particular case these are the qubit is that we are talking about the x and the y out of which x remain the same while the function y is undergoing the Binary Addition. If we do the same thing in terms of this example that we just did when we have the 2 qubit is 0 and 1; this particular circuit involving the unitary operator U of f does the Binary Addition only on the qubit 1 while leaving the other qubit 0. So, the entire qubit set is operated at once to give rise to this solution.

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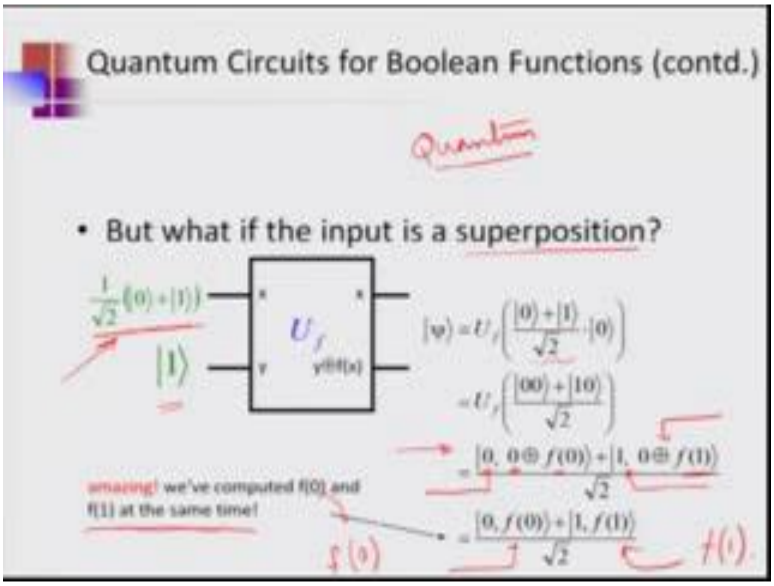


What if the input is a Superposition? Earlier what we were looking at was pure states; 0 and 1 had come in; it is like the Boolean condition of the classical case, but if we put in a Superposition states say the Superposition between 0 and 1 and the other state can be 0 or 1 we have chosen 1. Then what happens? When we do this what we find out that the function is going to operate on the two in the way that we will have to get the values of 0 and 1. So, that is the basic idea here; we have the unitary function working on these Superposition function and that gives rise to the two different kind of conditions where the 0 bit is having the addition with respect to f of 0 where the other one under the condition that our first qubit is 0 and when our first qubit is 1; then the Boolean addition is going to be with respect to f of 1.

So, when we have this situation; we end up having a condition where our both f of 0 and f of 1 are simultaneously being done as a result of applying the Superposition state in the input condition. That is the uniqueness of having the Superposition state which is the Quantum condition versus the Boolean condition which was more classical in the earlier case. When we put in only the pure states as we had discussed earlier Boolean addition principle works on one of them; however, if you put in the Superposition condition then the Boolean condition essentially results in producing the two conditions of the function which is both 0 and 1. So, both of the conditions are satisfied.

As a result we have computed f of 0 and f of 1 at the same time. So, that was the basic idea behind this entire exercise where we wanted to show that we can actually produce both the solutions simultaneously or in other words we are achieving Quantum Parallelism as we talk about. Once more let us look at this carefully. The basic idea is to utilize the principle of Quantum Parallelism which is a particular feature of quantum computers that makes it possible to evaluate a function on many different values of x simultaneously. So, the inputs many inputs the functional outputs are given in one time. We wanted to see how this works. Here is the example case where we want to actually achieve for the inputs of 0 and 1; how the function is evaluated simultaneously.

In order to do that; we set up the problem first assuming that we are essentially going to have a function which achieves Binary Addition; so, that was our idea of constructing a Quantum Circuit which would have achieved the binary functional response of Binary Addition. When we did that on just the pure qubit is coming in 0 and 1; then we saw that the principle of Binary Addition works on one of the qubit is as the set up was done that was how we are done it. In this particular case for the qubit 0 remained where as the Binary Addition worked on qubit 1 because we were using and which essentially result it in a functional evolution of f of 0.

Now, instead of doing that if we send in; the input function as a Superposition; we find that what we finally achieve are the two functions functional values of f of 1 and f of 0 simultaneously which is the same idea that we had started off with to compute both the values of the function at the same time. So, that is the principle of producing Quantum Parallelism which is been used here to show that it works. Now that we have shown that it works in this particular principle let us go forward and see how it happens.

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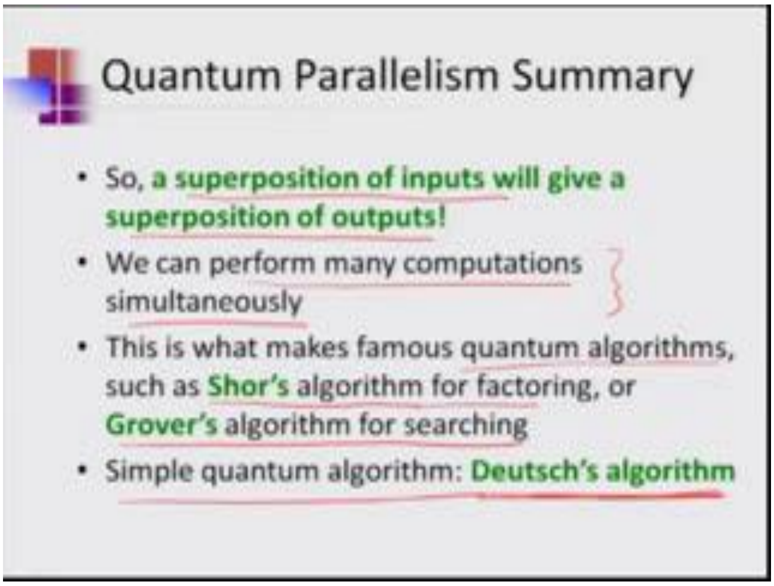


In summary; a Superposition of Inputs will give a Superposition of Outputs; which is how it is. We can perform many computations simultaneously that is one of the most important things. As a result of all these; what can we do? We can do some of the famous algorithms that we have mentioned in passing many times, but we have actually not gone ahead and computed them yet; for example, the Shor's Algorithm for factorizing or Grover's Algorithm for searching. But in the first case what we would like to do is the Simple Quantum Algorithm of the Deutsch's Algorithm where we are going to simultaneously find solutions by using Quantum Parallelism. Let us now look at a particular case which implies a Quantum Algorithm to give rise to Deutsch's Algorithm.

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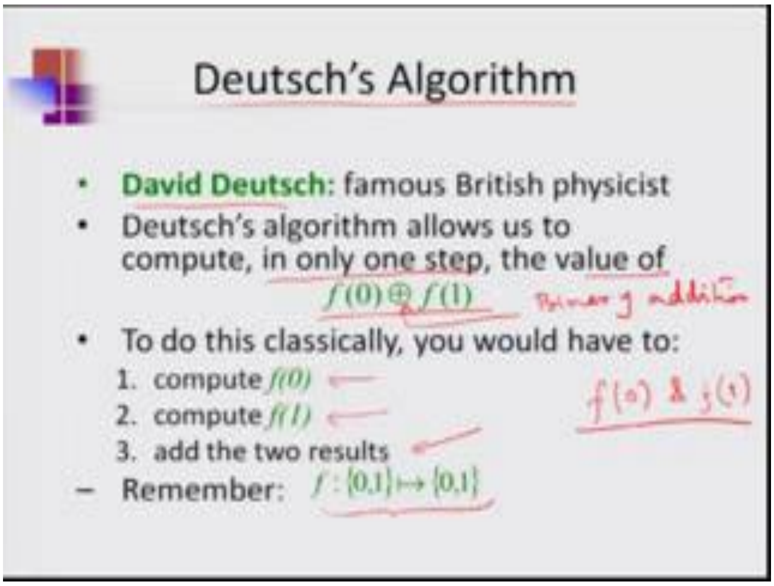


So, what is the Deutsch's Algorithm? This is by for one of the first practical applications of Quantum Information Processing or in other words Quantum Computing; in some sense which gives some ratification to this idea that Quantum Computing has some reality in it. So, David Deutsch; he is a British physicist who was the first one to crack this kind of a problem. So, Deutsch's Algorithm allows us to compute in only one step the values of the Binary Addition between two values of the function f of 0 and f of 1. In one step it achieves the Binary Addition of two functions f of 0 and f of 1 simultaneously; to do this classically you would have to compute f of 0 you would have to compute f of 1 and then add two results that is the basic idea. So, essentially you need three steps.

But in this particular case; that is the principle here the function actually will go through in this form. So, in a part of this we have done before have where we showed; then it was possible to compute f of 0 and f of 1 simultaneously as we just did by using Quantum Parallelism. Now what we have to do is we have to come out with the way so that in one step we can do this entire process; how do we do this?

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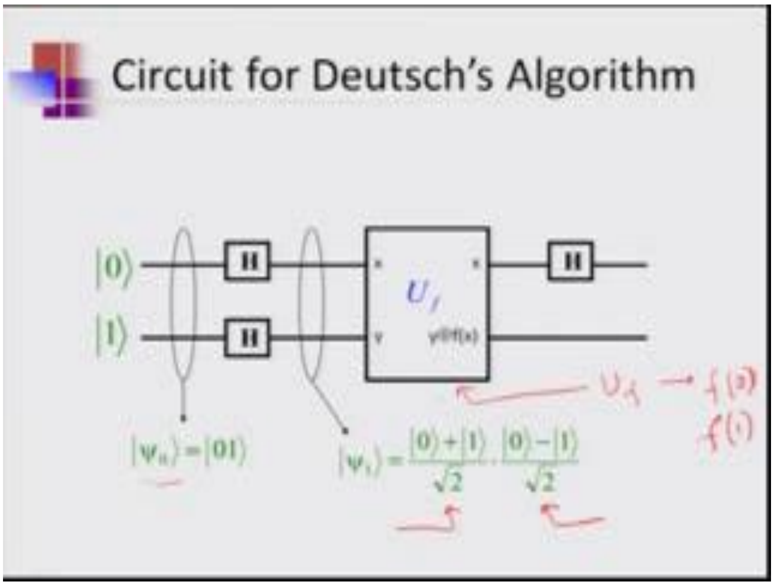


Here is the circuit for a Deutsch's Algorithm. We have the inputs 0 and 1 for example, which goes through the Hadamard Transform. If we look at right at the input level we are basically having our wave function which is the combination of 0 and 1 states. We are actually putting a Hadamard on both of those conditions. So, basically we are producing Superposition states on both of them. It is a product Superposition state of the two input cases and then we go through the same circuit or the same unitary transform which enabled us to find the two functions simultaneously f of 0 and f of 1 earlier and see what happens.

So, these are the steps. We initially set it up with sending in the any individual qubit is, putting them through two different Hadamard Transforms to produce a product state of the Superposition of the two conditions and then we go through the same unitary transform circuit which had achieved earlier by using Quantum Parallelism; the two values of the function essentially f of 0 and f of 1 and then after that we will do Hadamard one of them to get our Binary Addition.

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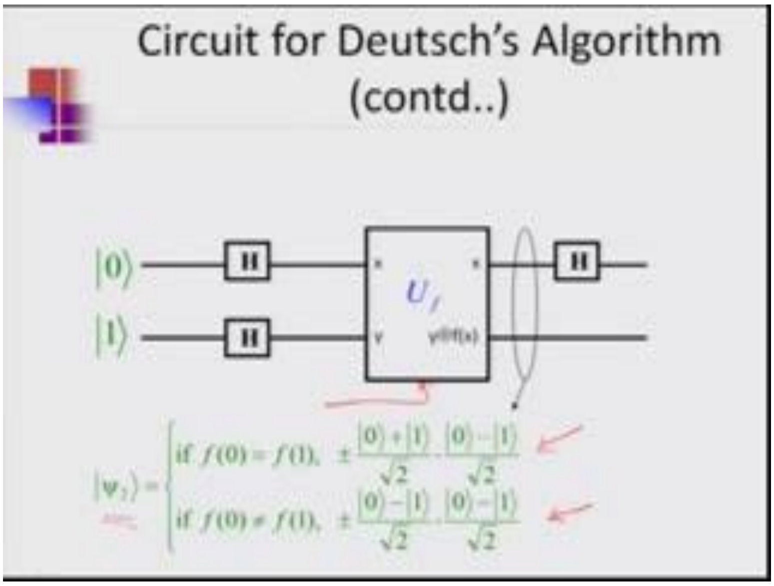


So, here once we have gone through this step through the unitary function; unitary operation; what we find is; in this circuit we find that our wave function 2 as a result of this operation will have two different conditions. If f of 0 is equal to f of 1 then we get this value where as if f of 0 and f of 1 are not equal then will end up producing this other condition.

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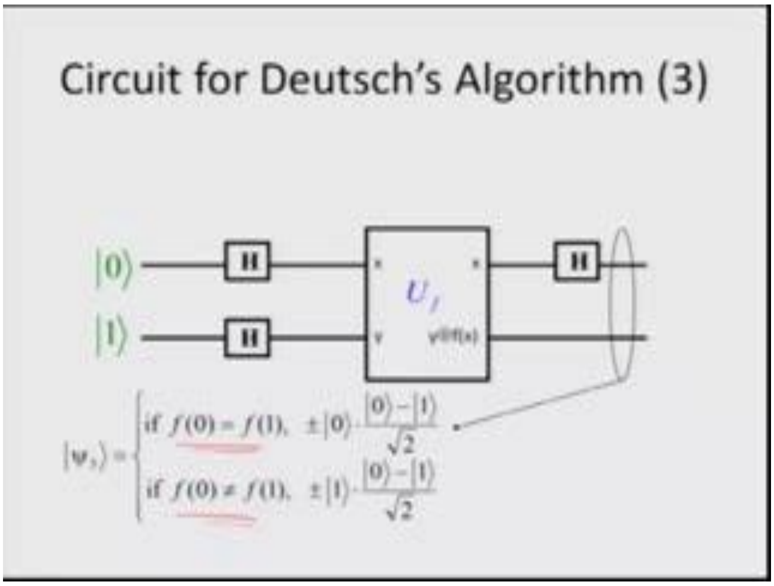


And finally, after we have put them through the Hadamard Transform on only one of the states; one of this circuit is; then what we find is that we will get the solution which will look as the wave function which has the 0 coming in first with the plus minus condition there are two conditions here and one of them is when f of 0 is equal to f of 1 and the other one is when it is not equal to it. So, correspondingly we get two results.

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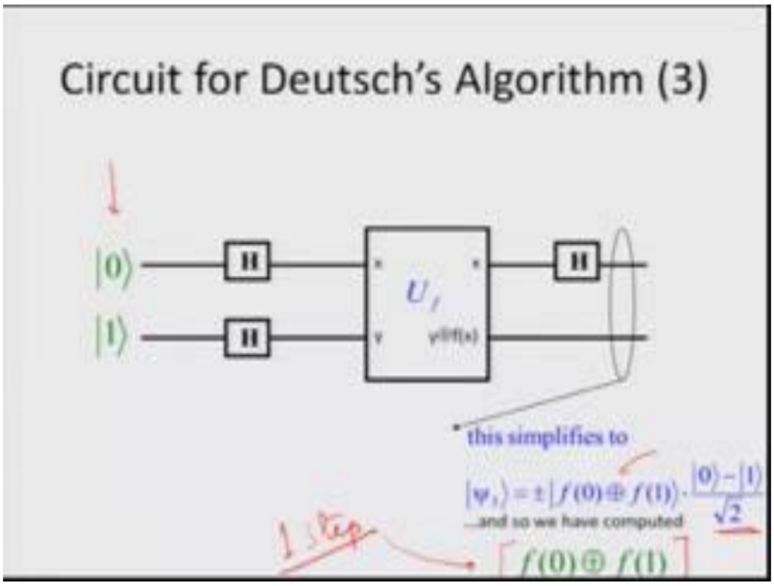


This will simplify to this condition where the entire solution can be written as f of 0 plus binary add into f of 1. This wave function; the final wave function is simplify to plus minus f of 0 binary add and then f of 1 times the Superposition state.

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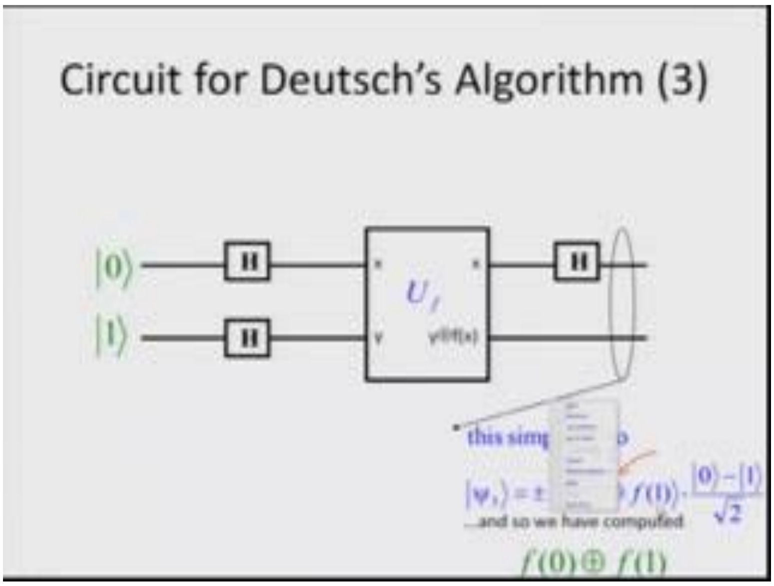

And as a result we have computed f of 0 and f of 1 binary add in a one single step because we never make a measurement after we have started our process. When we have finally, reached it if we measure then we are going to make our resultant here.

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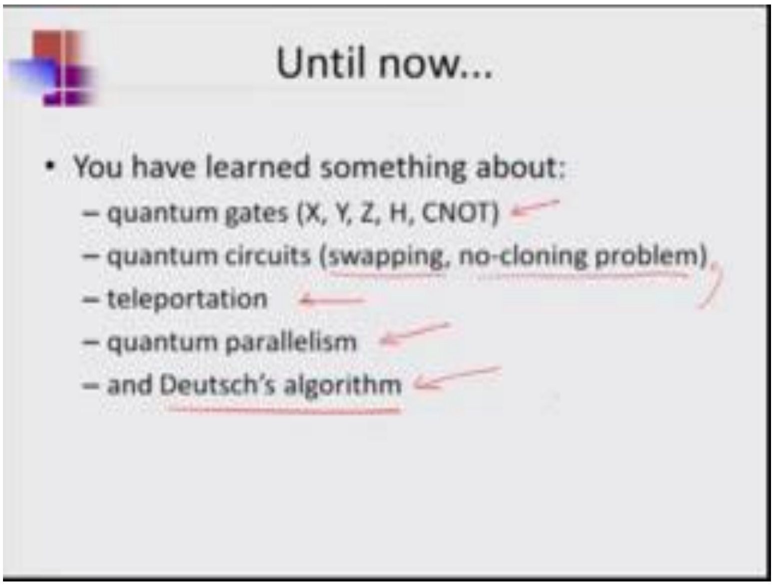


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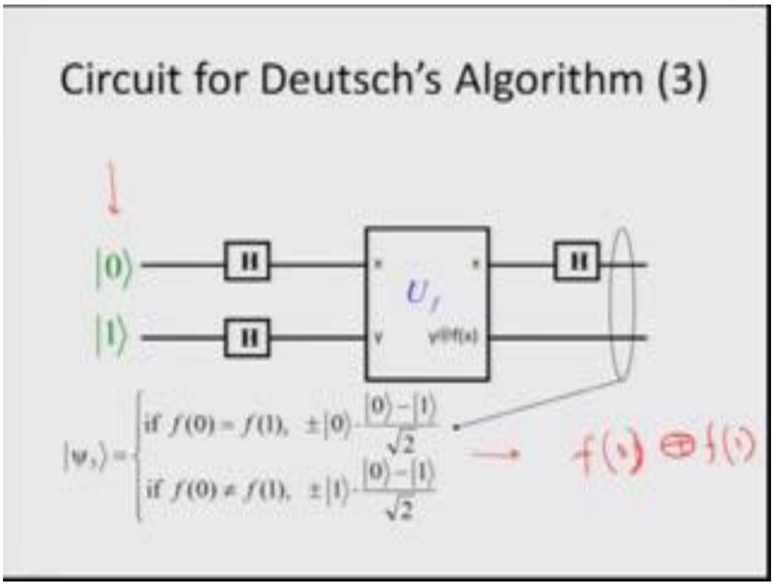


So, it is important to mention here that this particular algorithm where we are able to measure in one step f of 0 and binary add to f of 1 really utilizes the effect of Quantum Mechanics in full parallelization mode because we have essentially we started the process.

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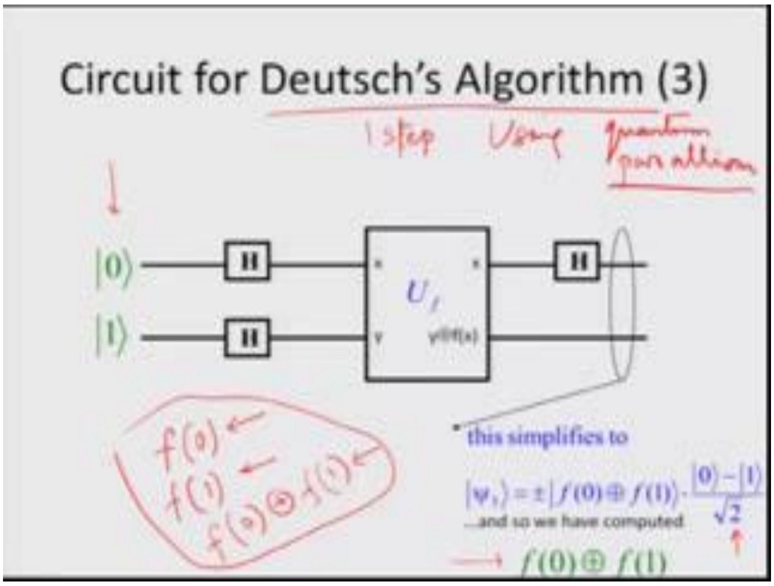


And we finally, make a measure of the final state psi 3 which is equivalent to this particular process where I have the Superposition state of the original states and the measure of f of 0 binary add into f of 1 which gives me the solution that we looking for.

So, that is the reason why Deutsch's Algorithm is very important because that is able to achieve these result in one step using Quantum Parallelism. No classical computer would have been able to achieve this because as I mentioned before you needed to find out f of 0, f of 1, one at a time, evaluate them and then you needed to do the binary add on top of that. So, it would have taken at least three steps to get there whereas, in this case we were able to do this whole thing in one step. So, that is about Deutsch's Algorithm which is a very important one and they have been cases where the Deutsch's Algorithm has actually been implemented this is one of the first algorithms to be implemented because it actually shows the power of the quantum computer through Parallelism.

So, what have we achieved until now? We have learnt about Quantum Gates as we went along in this entire period of time. We have looked at Quantum Circuit is the simple once for example, Swapping and we have shown No-Cloning Problem, we have achieved Teleportation and we have shown Quantum Parallelism which has been utilized to get to the Deutsch's Algorithm. So, that was one of our first algorithms which we were able to show as a result of the way we have been progressing.

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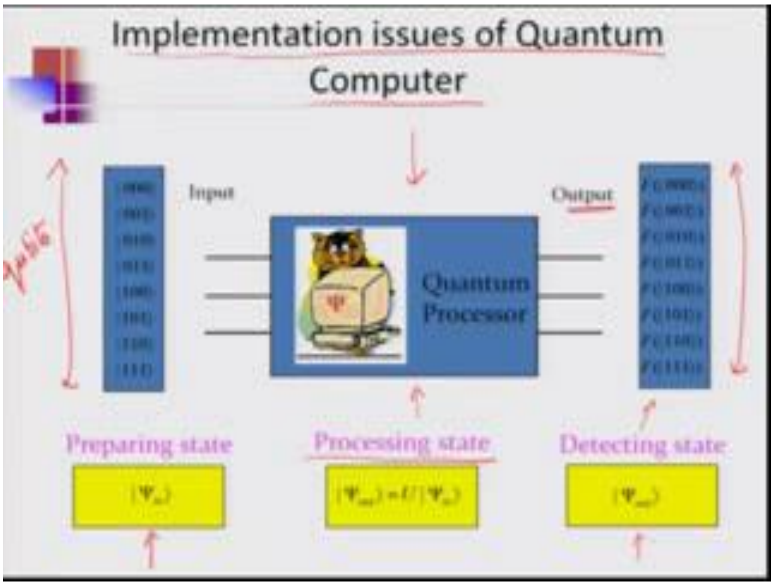


In order to look in to this entire process; the way we have done; the main issue has been Implementation and the Implementation Issues of Quantum Computing is one of the most difficult ones.

Before going into more complicated Quantum Algorithms let us look at the Implementation Issues of Quantum Computing here. What we are essentially looking at is the idea of having a series of inputs which are our qubit is; this is how we have progressed that is our input point and then there is this entire principle which is the Quantum Processer. While the Quantum Processer is working we are generally not looking at it, there is no measurement done. As a result most of the time we are talking about the steps in a different way and finally, we have this output where we are getting our qubit is and the resultant in a way which is what we are looking for which is our output where we make the measurement.

In other words we have a few task (Refer Time: 19:24) one of the most important one is the preparation step which are where we are making our quantum states. So, this is the Preparing of States are important; the input point. Next of paramount importance is definitely the Processing State; we have to have our Quantum Processor which is all the principles of the gates; that is our Processing State and then the final point is our Detecting State which is what is coming out of this entire process.

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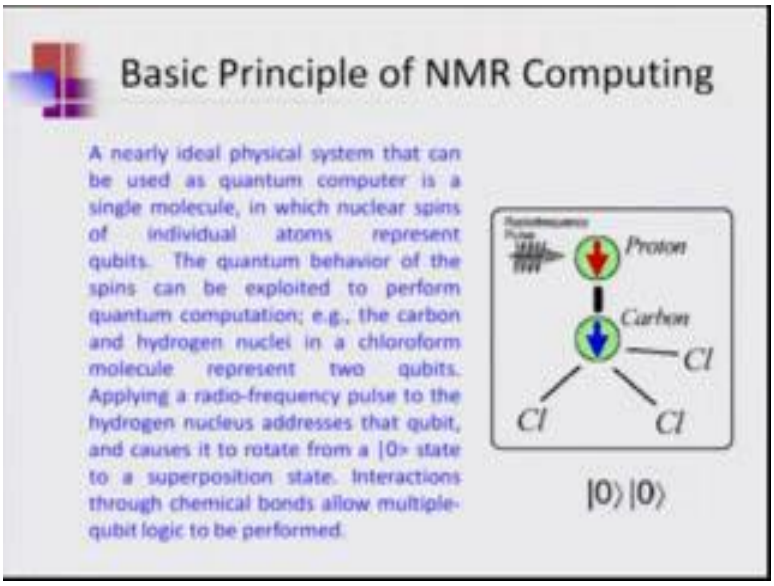


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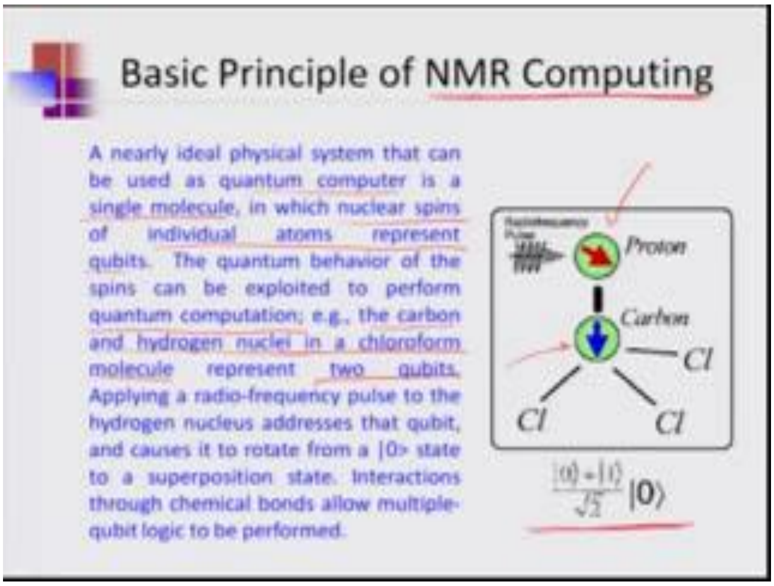


So, one of the examples where this has been used quite effectively in terms of implementation; one of the first areas where a lot of progress was made was a surprise area which is NMR because in NMR typically you have a lot of molecules and yet the principle of Quantum Computing was sort of explored quite effectively in NMR and it became to know as NMR Computing; NMR Quantum Computing. The basic principle behind this lies in the fact that it is almost an ideal physical system that can be used as a quantum computer since it actually explores a single molecule.

The NMR principle is based on single molecules in which the nucleus spins of individual atoms represents qubit is. In the NMR Computing; the quantum computer is the molecule essentially a single molecule in which nucleus spins of individual atoms represents qubit is. So, here is the cartoon what we are trying to show here is a one of the very simple molecules for example, a proton containing carbon and chlorine and all that. So, it is a chloroform molecule where we have 1 proton, 3 chlorines and 1 carbon. When the radio- frequency pulse is applied it is going to act on the spin of the proton only because the rest of them does not matter.

But the carbon spin with respect to the protons spin that will be having to different conditions and based on that this particular molecule can act like a computer quantum computer. So, the quantum behavior of the spins can be exploited to perform Quantum Computation for example, this C Cl for the carbon and hydrogen nuclei in a chloroform molecule which is this particular molecule C Cl 4 represents 2 qubits. One of the qubit is the proton coming from the hydrogen atom and the other one is the spin state of the carbon 13 molecule, carbon 13 atom inside the molecule. The radio frequency pulse to the hydrogen nucleus only addresses that qubit and it causes to rotate from a state which let us say is the 0 state to a Superposition state.

So, is the 0, 0 state changing to the Superposition state because that is rotating as a result of the applied the radio frequency pulse only. Interactions through chemical bonds allow multiple logics to be performed in this condition. So, basically this particular set whatever is generated through chemical bond is happening because you have the chemical bond in between the proton and the carbon and this interaction through that is the reason for this computation to work. So, this is one of the ways Quantum Computing has been attempted.

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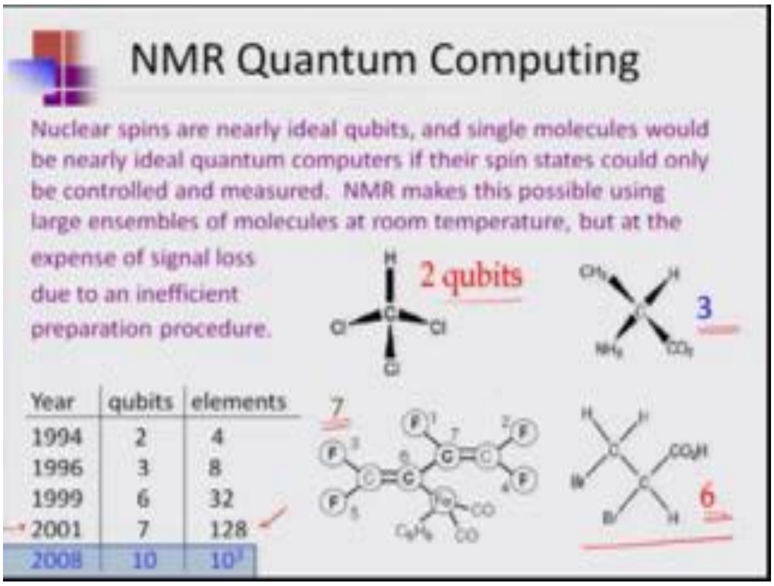


Let us see how it has been shown. So, that was the case of 2 qubit is which we just discussed; chloroform which has carbon and hydrogen and it turns out that these are almost nearly ideal qubit is and single molecules would be nearly ideal quantum computers if their spin states could only be controlled and measured. NMR makes this possible using large ensemble of molecules at room temperatures, but the expense of signal loss due to an inefficient preparation procedure.

So, basically here the problem is preparation of states what we have to do is somehow manage to get this particular state interact only with the applied field or the radio-frequency pulse under the condition when they are in a certain condition. For example, when I call this a 0, 0 case when my proton spin is in the same direction as let us say the carbon spin and when I apply the radio-frequency to only effect the proton then I am able to get this to work as a computational step making the Superposition. So, basically a Hadamard Transform.

But in order do that this preparation step of where I am going to have this particular proton only be in a particular orientation with respect to the other carbon spin is a difficult step; that is my difficult step here because there are many of these molecules only those who are in this kind of condition will be my chosen kind of the qubit. So, that is the reason why it becomes a little bit difficult you can do that and that is why people have done it, but it is an inefficient preparation procedure and so the signal loss beyond a certain point can make it difficult to achieve the goals that you would like to but any way with this they have been 2 qubit is as we mentioned then it has gone ahead with the say this other molecule where 3 qubit is have been achieved and similarly much more fancier molecules like this particular one has shown 6 qubit is. And finally, this particular molecule which has been used by Chwang to show Shor's Algorithm which we will do later on; has been shown to have 7 qubit is and here are the qubit is; the fluorine and the carbons spins are been utilized in this particular case.

So, 1, 2, 3, 4, 5; these are the fluorine and then 6 and 7 are the two carbons. So, these forms the 8 qubit is of this particular molecule. As you can see the progress have been difficult, but it has happened. In 2001 for example, this 7 qubit is was done where the elements that could be utilize as a result of 7 qubit is was; is 2 to the power 7 is 128 elements could be achieved and so on and forth.

I think in 2008 which will also again discuss later on; it was possible to show up to about 10 qubit is which will be looked at.

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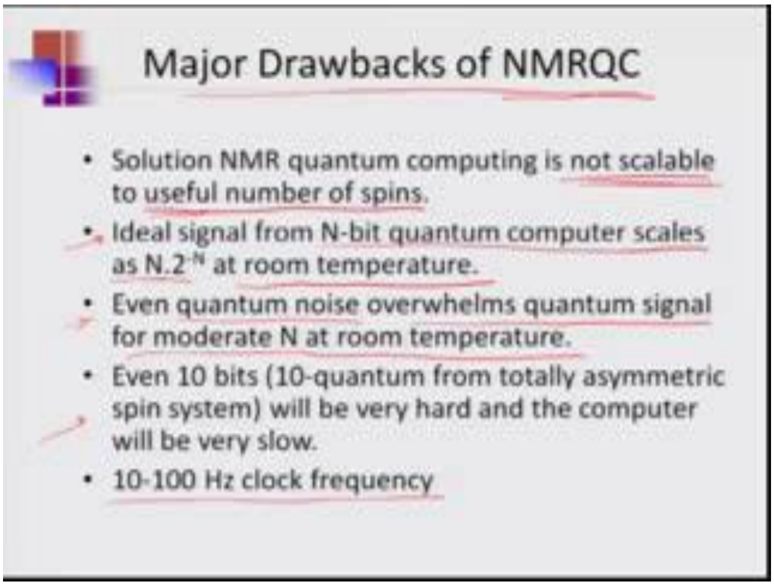


So, that is roughly the way have things have been evolved. One of the major drawback; however, of the NMR Quantum Computing lies in the fact that the as I mentioned; the Preparation State part and you are always fighting for signal to noise. A solution NMR Quantum Computing is difficult to scale; in fact, it is not quite useful in terms of that fact it is not scalable to useful number of spins because what happens is beyond a certain point of time the signal to noise conditions overwhelm; the noise becomes too much and you cannot really get the required number of spins or the useful number as we would like to say.

As initial demonstration purpose is, but generally we would like to have in the orders of quite a few number of spin; qubit is to be able to do some realistic problems; that is why this is one problem. The ideal signal from N bit quantum computers scales as N times to the power minus N at room temperature. This is the ideal case, but the problem is even quantum noise would overwhelm quantum signals for moderate N at room temperature because of the way this equation is N times to the power minus N and that is why as the number of qubit is increase the noise would finally overwhelm the quantum signal and so that is one of the biggest challenge here.

For example, even a 10 bit 10-quantum from a totally asymmetric spin system could be very hard and the computer will be extremely slow this is one of the problems which had been discussed and have been looked at and so although it is one of the first positive implementation aspects of NMR Quantum Computing happened of late the attempts in this area has slow down because it is very difficult to scale it up to something realistic. 10 to 100 Hertz clock frequency is roughly because it is overall NMR is a slow process. So, whenever you want to reset the qubit that also take a lots of time and therefore, the frequency at which you can actually do computing is quite slow and. So, solving a problem and then looking at a different one becomes quite slow in some sense of the other.

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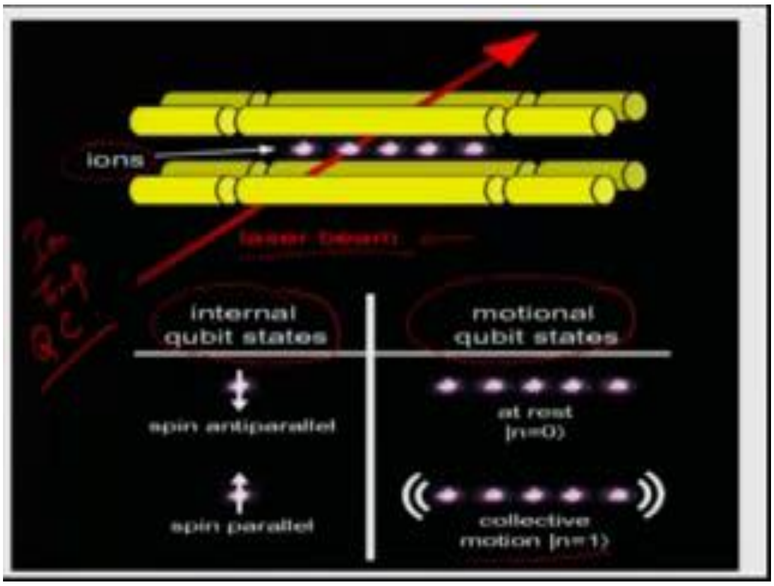


And therefore, as a demonstration purpose it may be fine, but it is difficult to be thinking as a future realistic quantum computerized as a result of that.

So, the other area where a lot of efforts have gone in are these (Refer Time: 30:25) ions where what happens is that you basically set the ions inside a cavity in such a way that they become individually addressable. When you do that then it is possible to have each of these ions act like qubits as long as they can be close enough so that there is an interaction or they can be connected one way or the other and this particular idea of having some interaction and having a result occurring because of that is done in this case by using a laser beam.

If you remember in case of NMR; we discussed it. We were saying that we use radio-frequency pulses; here you have to you generally use a laser beam because the laser beam is going to interact and do something or which will be considered as our operations on these individual ions and that is how will be getting this to happen. The Internal Qubit States are roughly say the spins. So, if the spins are anti parallel or the spins are parallel they form; they could be our qubit state. It could be also to be our Motional Qubit States which could be at rest and collectively can have a motion which could be another kind of a state. You could have different levels of qubit interactions which can be utilized as a result of this kind of activity and with this kind of Ion Traps; these are essential known as Ion Traps Quantum Computers; quite a bit of progress have also been made, but once again the region over which such ions can co exist in a way so that the interaction works together.

It is again a very difficult experiment and the a level of effort to be put in to have this kind of activity work would also again be a difficult thing, but still once again like in NMR these are one other places where implementation aspects have been (Refer Time: 32:47) and the initial efforts to try to show that Quantum Computing can be implemented has been shown in either of these two cases although it requires a heavy instrumentation aspect.

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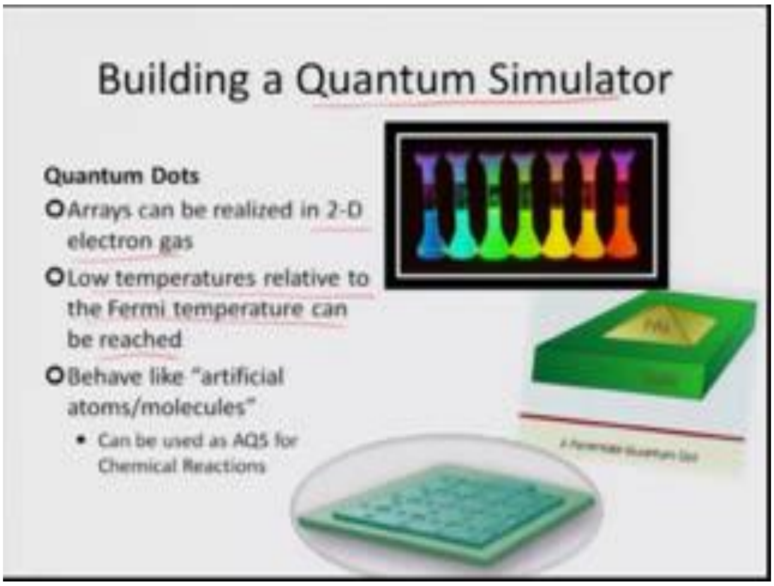


The other important development which has happened over the recent years well everything has been happening, but as we discussed this is sort of like the time skills initially Quantum Computing efforts where shown surprisingly successfully by NMR and then right along at the time little bit more progress and simultaneously Ion Traps and then sequentially Quantum Dots have also walked into this problem which are kind of interesting because it is sort of mimics the Ion Trap situation, but not in that kind of a secluded environment; it is a little bit more useful kind of a scenario and in this case for instance you could have different levels of Quantum Dots that can be realized in 2-D electron gas for example.

Since low temperatures relative to the Fermi temperatures can be reached; the noise aspects in these cases are also something which can be looked at not to a very difficult case. These can be behaved like artificial atoms or molecules and can be used as atomic quantum states for chemical reactions. This have also been attempted as our model of Quantum Computing so, it is the Quantum Simulator kind of an approach has been taken into these cases. So, these are just a broad over view to just get you started on this idea of how a quantum computer is suppose to be built and how it is going to work out and thinks like that; although there a lot more essential details which we will be now getting into delving into as we have looked at the basic initial steps.

We will now take on very important one at a time example cases and we will try to see how we can go ahead and do this; how it has been implemented, how where do we stand in it is applications and so on and so forth. But today's lecture was first more to do with some of the basics and then connecting it to how the current scenario is in some sense although a lot more has happened since these and will be dealing with them as we go along. This is just to give you a glimpse of the implementation parts which are also very important as some point.

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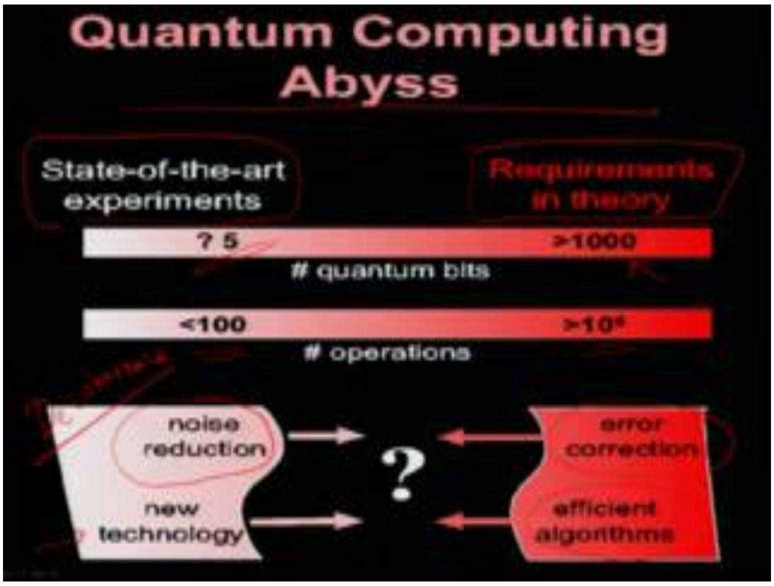


Let me actually end today's lecture in terms of giving you this picture which is sort of like a an Abyss of the Quantum Computing arena where the state of the art experiments as of now lie somewhere in the range; this number has been updated to about 7 qubits at least 7 to 8 qubits. But they are all state of the art experiment. This is one thing to be remembered that at this point of time everything we talk about in terms of experimentation, implementations are all state of the art.

Typical requirements in theory to do something realistic and break in to the arena where the current classical computers are having difficulty would require at least in theory around 1000 qubits which is quite difficult. We are at least couple of orders in magnitude away from it; at least 1 order magnitude away before we can start doing something which is very difficult for a classical computer to do.

And similarly in terms of operation we are roughly in the region of about 100 where as typically in actual case of doing a large a problem you would like to have billions of operations going on. So, that is one of the most important situations where we stand (Refer Time: 37:06) where we are stuck in terms of our developments are in Noise Reductions, in terms of Error Corrections this is one of the areas where there is lot of effort has been given, Efficient Algorithms and in getting rid of one very important point which we have not mention although it is sort of perhaps clubbed into noise reduction is a Loss of Coherence or Decoherence; so that is one very important area also which comes into this problem case.

This is an Inherent problem that we also have and obviously, there is a need to get more and more new technology to be able to look at into this areas and with that we would like to see how we can go about discussing about producing more and more kinds of Quantum Computing implementations which will be of futuristic use. I do not want to get too much into this aspect yet because we have not done some of our more important algorithms and principles which are necessary what is the; so we should actually built up from here for example, as to what are our basic requirements that we would like to have you know implementation conditions and so on and so forth.

So, in the next few classes I would actually try to bring you up to date with those kinds of things. In the middle we will also do a few of these problem solving sessions in the class now; so that you are able to understand based on some of the background material that we have been providing you to look at some simple problems how they have been solved and so on and so forth, how can they be solved and so on and so forth so that you can be up to date and up to speed with the way we are doing this approach and then we will again go back and start addressing each individual problems one at a time. So, with that I would like to end today's lecture.

Thank you.