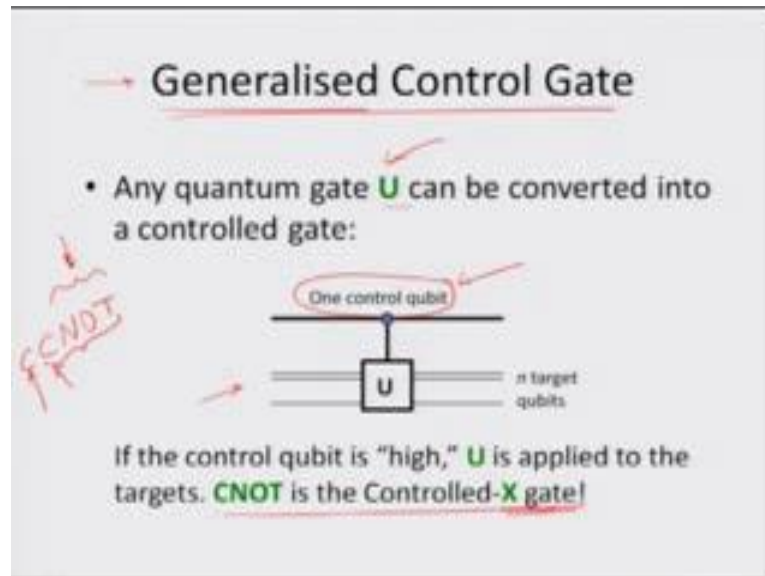


**Implementation Aspects of Quantum Computing**  
**Prof. Debabrata Goswami**  
**Department of Chemistry**  
**Indian Institute of Technology, Kanpur**

**Lecture – 05**  
**Quantum Measurement and Teleportation**

(Refer Slide Time: 00:15)

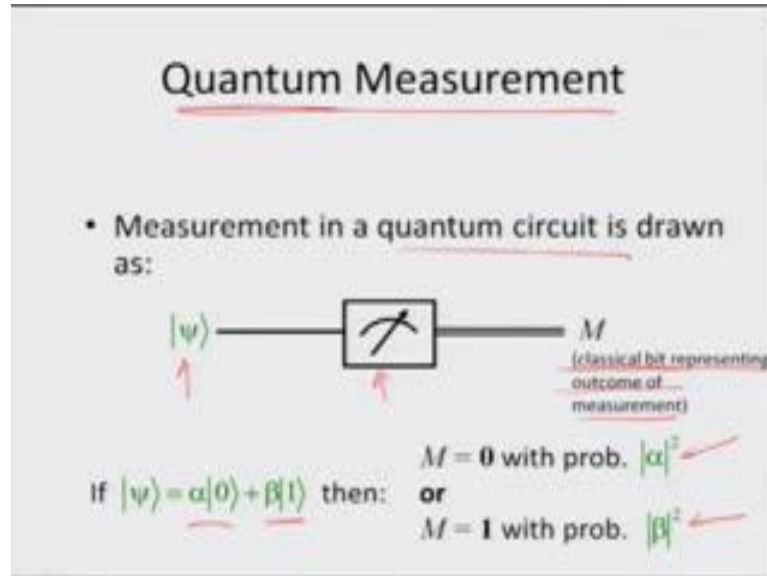


We are going to look at the Generalised Control Gate for example, here. So, any quantum gate  $U$ ,  $U$  means unitary can be converted into a control gate how do you control that, what do you need is a control qubit associated in addition to the other gate that you have then you can produce the generalized control gate, and that is the reason why CNOT is a very important gate, because taking any kind of control unit you can actually produce another gate. So, CNOT is the controlled-X gate, which is a XOR gate. You also have we also have CCNOT, which were we use another control to sort of have condition on the CNOT gate and as you know this is our starting gate.

So, you can add on controls to any particular gate that you have. So, for example, I have the XOR gate or the NOT gate the starting one, I had a control I get a CNOT, I put in another control then it becomes a CNOT gate all these are possible. So, we can also generalize saying that whenever, we take one control bit or in this case qubit we will produce a generalized control gate. So, that is the idea. Now, this is actually useful because in quantum computing having or adding a control bit or a control qubit on the

existing gates can become very use it and that is the very important parameter as you will see.

(Refer Slide Time: 02:06)



Now, the next very important thing is the measurement part. Timing again before also we have discussed this, that generally speaking we do not want to make any measurement in quantum conditions, because once you make the measurement typically that is like the end of the line you cannot any more talk in terms of what else was possible, because that is a definitive answer. So, whenever you make the measurement you have essentially decided to terminate or come to the result that you are interested. But at some point of time anyway you have make measurement and it is best to be able to make quantum measurements, so that you are able to get the solution that you are looking for.

Now, in terms of the circuitry we have been drawing the measurement in a quantum circuit is drawn in this fashion, that I have the state that we are talking about this is our measurement process, and once you have done the measurement you end up producing the classical bit representing the outcome of the measurement. Please note that now I have specifically mentioned that that is the classical bit. Now that is the very important part of this entire exercise, making it clear that the movement you have come here you have decided to look at your results and whenever you look at a result in a quantum

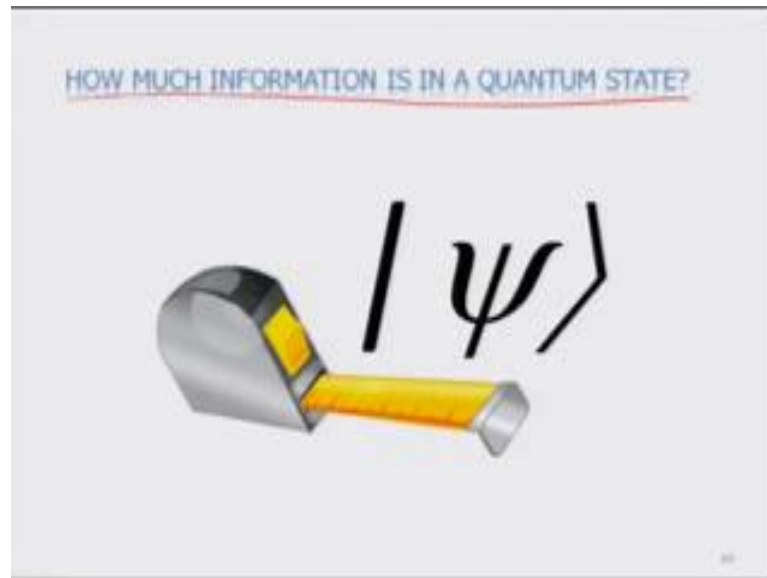
system you are essentially coming to the classical world looking is always in the classical world that is how you understand.

So, for example, for any two states, any two possibilities as long as you do not make the measurement as we have seen even in this case for example, rotation - the spinning top can be anywhere, until I measure right. Its Schrodinger's cat all the possibilities everybody who has been talking about are all based on this principle that until I make the measurement, the outcome is not clear. Just reminded Schrodinger's cat is that experiment taught experiment, which essentially talks about the case where if inside a box a cat and poison is put together what is the possibility that once you open the box you will find the cat alive or dead depends on the condition whether the cat eats the poison or does not ok.

So, that is always the possibility, because a cat might not be hungry might be sleeping would not interested in that food whatever, be the case it is you can find it alive. But the other possibility is equally true that it might be very hungry immediately gets the food eats it and dies and when you open you see its dead. But as long as you do not make the measurement, which is opening the box and seeing the condition of the cat you have no clue. Similarly for all quantum systems as long as you do not make the measurement which is finally, the classical bit you are not getting the result and until then you only talk about probability.

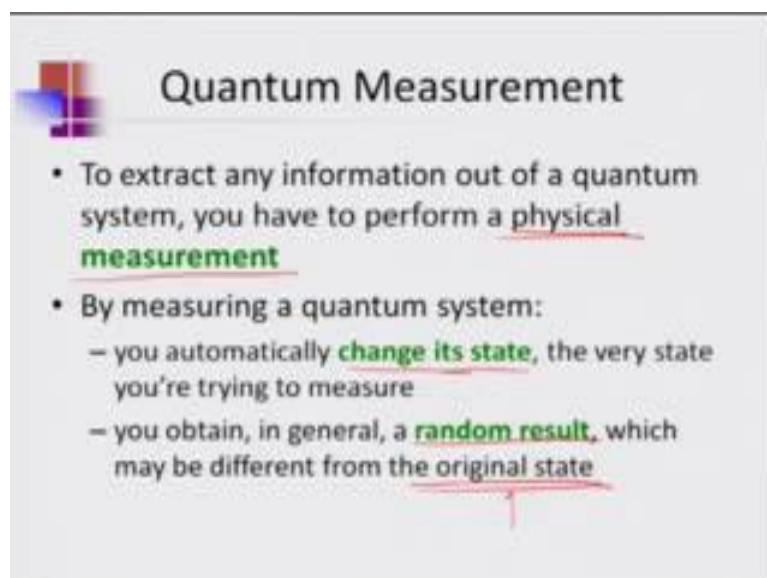
So, whenever you have two states and they can be represented either in terms of alpha or beta, which are my corresponding amplitudes then the probability will be the square of them which is associated with getting either one of the answers. So, that is the idea.

(Refer Slide Time: 05:44)



So, this is generally an important question to ask, how much information is there in a quantum state. Because all these logic seems like, if you do not measure the information quantum of quantum state can be infinite, is that a correct statement we should explore that. So, that is why this particular situation has to what do you we mean when we say how much information is there in a quantum state.

(Refer Slide Time: 06:19)

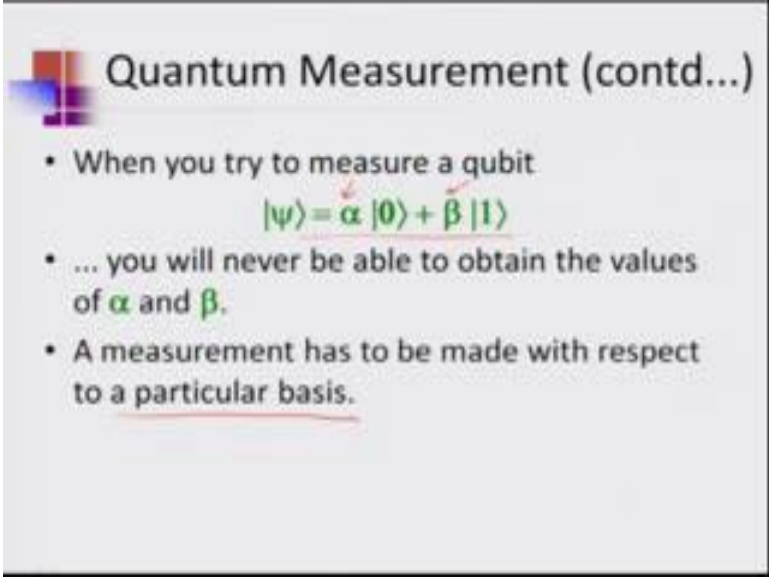


So, in order to make or extract information out of a quantum system you have to perform a physical measurement. So, that is a whole important point then this is what we have

been trying to tell. By making measurement of a quantum system you automatically change its state that we have already said. So, once you make the measurement you have change its state, you can no longer get the original condition. You obtain in general a random result which may be different from the original state now that is also a very important point. So, just making a measurement does not make sure that you are actually seeing what was there before, once again going back to the schrodingers cat it might be that in spite of a having the poison, the can miracles had no effects that is also possibility right. The poison was meant for let say cockroach, it was not big enough for potent enough for the cat nothing happen to it right it was still fine.

So, the original state cannot be talked about, it will be the other way round no matter what happens every time you open the box you find the cat is dead may be it just died out of suffocation that are nothing to do with the poison you do not give the poison even then it is dead, that could also be the case. So, the original case cannot be reflected by making this measurement. So, that is the reason why it is important to notice the difference between the fact that the measurement has not or may not always give raise to the original statement, or the information about the original statement.

(Refer Slide Time: 07:58)



**Quantum Measurement (contd...)**

- When you try to measure a qubit
$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$
- ... you will never be able to obtain the values of  $\alpha$  and  $\beta$ .
- A measurement has to be made with respect to a particular basis.

So, we have to actually known as a Quantum Measurement, to make sure that we can get repeatable answers and that is the more important thing, because given all these uncertainty if he is now want to claim that we have no knowledge about what is going to

do when we make a measurement then we cannot have a competition. So, in terms of competition we have to define it in such a way that the quantum measurement is a specific kind of measurement, which will give rise to a repeatable answer that, is what we are after.

So, when we try to make a measurement say away the given state, which is super position up to states let say you will never be able to. So, this is one of the very important reason why this is true, you will be never be able to make the independent measures of only the alpha or the beta you will be making a measure of their squares right. I mean since alpha and beta can be complex numbers knowing theirs squares does not again necessarily give the values of their original alphas and betas that is the other way of looking at the corollary.

So, it is also important therefore, to know the basic idea of the basis set, that is why when you make a measurement in the frame that you are sitting in typically or roughly known as the laboratory frame. The basis set is chosen by you in terms of the laboratory frame it might have solve the entire problem in some other frame and. So, when you make a final measurement and that is coming to you as a result of a particular basis that is due to the frames of the measurement also. So, that is another very important point to may understand. So, in this context when you have many systems together then, we talk about an easy another approach, which is known as the Density Operator.

(Refer Slide Time: 09:53)

**Density Operator** ←

$$D = p_1 |w_1\rangle\langle w_1| + p_2 |w_2\rangle\langle w_2| + \dots + p_n |w_n\rangle\langle w_n|$$

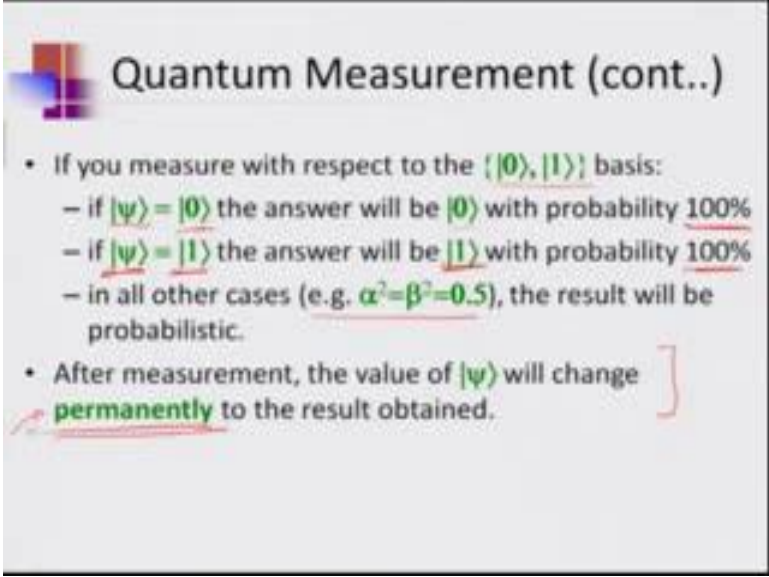
The projection of  $|v\rangle$  onto  $|w_i\rangle$

The length of the projection is the scalar product  $\langle w_i | v \rangle$ , using the fact that all  $w_i$ 's are normalized

So, how do we define density operator, the density operator is essentially a projection of one frame into the other. So, if there is a state says  $v$  cat state, and I have the  $w$  cat state is the other one they are at some angles with respect to each other the projection of  $v$  cat on to the  $w$  cat is sort of like the density, given for the particular condition that we are looking at. So, this particular projection the length of the projection is a scalar product using the fact that all the probabilities all the  $w$   $i$ 's are kind of normalized.

So, this is the case, where we are taking advantage of the fact that we are in a frame of reference, where that frame of reference is only composed of normalized states. So, anything else which is happening in any other frame of reference when we look at them we only see their projections. Those of you who have been dealing with engineering drawings and many other forms of doing things whenever, you take a solid picture and you draw its 2D projection on a piece of paper you are doing the exact same thing, your representing a 3D picture on a 2D plain. So, here also we are not really able to get the actual state in its form the particular frame that we are using we are projecting it on that frame and it is looking at. So, that is why you need an operation, which will do the job for you and this is typically known as the density operator which gives you the length of the projection in terms of the scalar product, clear, so, actually an operator right.

(Refer Slide Time: 12:20)



**Quantum Measurement (cont.)**

- If you measure with respect to the  $\{|0\rangle, |1\rangle\}$  basis:
  - if  $|\psi\rangle = |0\rangle$  the answer will be  $|0\rangle$  with probability 100%
  - if  $|\psi\rangle = |1\rangle$  the answer will be  $|1\rangle$  with probability 100%
  - in all other cases (e.g.  $\alpha^2 = \beta^2 = 0.5$ ), the result will be probabilistic.
- After measurement, the value of  $|\psi\rangle$  will change permanently to the result obtained.

So, this is all we are doing in terms of measurement, because at the end of it when wherever we are, whatever we are, we have to get the measurement. So, if we measure

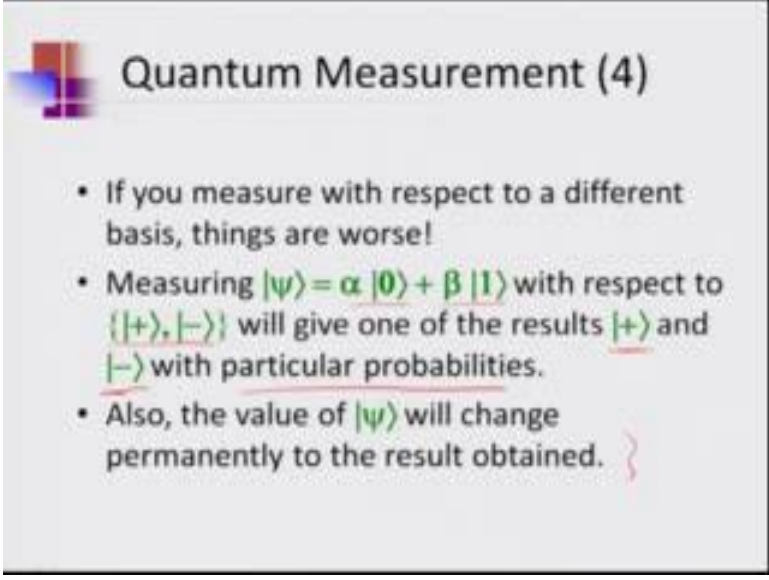
with the respect to let us say 0 1 basis, and  $\psi$  by wave function is essentially just 0 vector, then the answer will be 0 with the probability of 100 percent. Similarly if it is going to be only state 1 then the answer will be with probability 100 percent the value of state. Anytime I measure wave function I either get 1, because its only 1 or I will get 0 because its only 0, but in all other cases the result will be probabilistic it can be exactly in the middle, which means that I have a equal mixture.

So, in the large number game the typical case is always that either one of them is equally likely. So, like the coin toss problem without for a bias coin, unbiased coin the probability of getting a head or a tail is almost always equal. So, we give the probability as half right. So, no matter what you do you will always get the probability of half and this is exactly like saying that that it will get a  $\alpha^2$  and  $\beta^2$  will always be equal to 0.5. So, that is the simple thing.

The point however, to note here which is interesting, which you do not really think about when do classical measurement even say coin toss thinking. In terms of measurement after the point of making the measurement the value permanently changes to the result obtained. It is like saying that if  $\psi$ , at some point of time change from 0 to 1, but at the point of time that you make the measurement it was found to be 1 then after measurement this state will always remain in 1, because it has been made into a classical state now right. The measurement essentially takes it there, so once you have made the measurement original it could be have been an 0 or 1 with some probability, but just by chance that you measured it at a time when you just converted into the 0 case if that is the way it is then the measurement will always show forever it is going to be 1. That is the point which is very important to note that it is not exactly like the classical case there is a certain difference there.



(Refer Slide Time: 15:11)



**Quantum Measurement (4)**

- If you measure with respect to a different basis, things are worse!
- Measuring  $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$  with respect to  $\{|+\rangle, |-\rangle\}$  will give one of the results  $|+\rangle$  and  $|-\rangle$  with particular probabilities.
- Also, the value of  $|\psi\rangle$  will change permanently to the result obtained. }

If you measure with respect to a different basis things can become very complicated. So, for example, here is the case when you are measuring this is something, which I think we have there are practice, which has been done on one of the practice problems will do in class it will become clear also this is something which we will doing here. Same measurement of a function  $\psi$ , which is having the mixtures of alpha times 0 and beta times 1, now we are going to measure it with respect to the plus minus basis, which is some other basis. This will give you one of the results plus and the other one minus with some other particular probabilities.

Now, this is different from our earlier measurement, because there we were measuring with respect to 0 and 1, now we are suddenly decided to measure with the respective alpha sorry, plus and minus, so, the measurements found from this result will not be the same that we are gotten for the other basis. So, if you make a basis transform and you get answers for a quantum system they need not be necessarily the same. If by chance they are same then you can be lucky, but there is no guaranty that they will be the same most likely they will not be, and this is always true whenever you make a measurement it changes.

(Refer Slide Time: 16:48)

**Quantum Measurement, Formally**

- Formally, when you measure  $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$  with respect to  $\{|0\rangle, |1\rangle\}$  you will get:
  - result  $|0\rangle$  with probability  $|\alpha|^2$
  - result  $|1\rangle$  with probability  $|\beta|^2$
- If you use a different measurement basis, the result will be one of the basis states, with different probabilities

The diagram shows a vertical red bracket on the right side of the slide, grouping the first two bullet points together.

Anyways, this is all way have been telling formally all the time, that any time you make a measurement you will get their probability and the movement you change the basis of measurement the results will be one of the basis sets with different probabilities.

(Refer Slide Time: 17:07)

**Measuring many qubits**

- We want to know the possible outcomes of measuring the **two qubit** state:
$$|\psi\rangle = (\alpha |0\rangle + \beta |1\rangle) \otimes (\gamma |0\rangle + \delta |1\rangle) = \alpha\gamma |00\rangle + \alpha\delta |01\rangle + \beta\gamma |10\rangle + \beta\delta |11\rangle$$

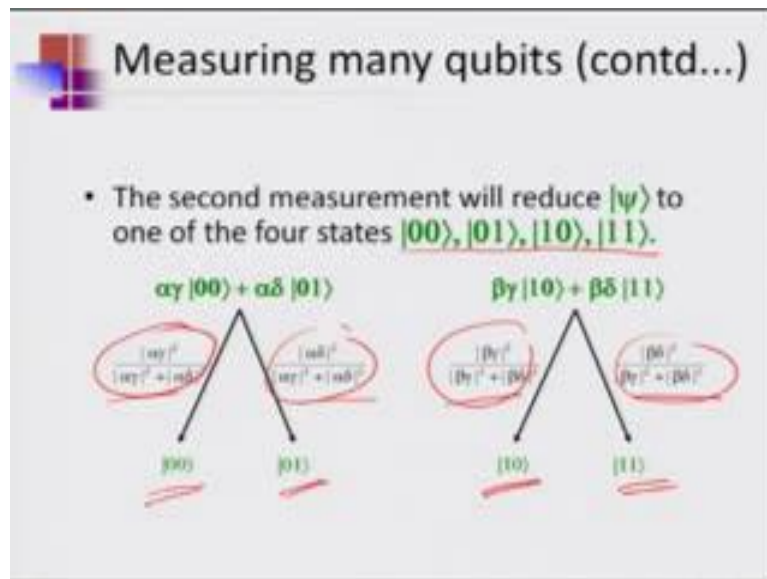
The diagram shows two horizontal brackets below the equation. The left bracket groups the terms  $\alpha\gamma |00\rangle + \alpha\delta |01\rangle$  and is labeled "prob.  $|\alpha\gamma|^2 + |\alpha\delta|^2$ ". The right bracket groups the terms  $\beta\gamma |10\rangle + \beta\delta |11\rangle$  and is labeled "prob.  $|\beta\gamma|^2 + |\beta\delta|^2$ ". Below these brackets, it says "the first measurement will reduce  $|\psi\rangle$  to one of these smaller states".

Now, when you have many qubits; as of now we had been talking about this one or the other state. So, they were essentially single qubits one state or the other, when you go for many qubits then you are now talking about different probabilities of their each combinations right. So, alpha 0, since their amplitudes are difference now they becomes

different states, and when you are looking at the wave function they are basically as we have discussed before they are tensor products, the probabilities are the final function is a larger basis set, and if you look out for what they look like then you will find that they are composites of say the 0 0 state, 0 1 state, 1 0 state, and 1 1 state they are all different.

So, we started off with 0 and on 1 all right, but we ended up with having all these different combinations because they are tensor products. So, the probabilities will then have given rise to different products. So, the probability of the first measurements will reduce psi to one of these smaller states are given by these probabilities right you can actually do these very simply and you can prove it yourself that this is how it works. Once you have made the measurements, once if you make the second measurement then you will reduce this to one of the 4 states which is these 4 states right.

(Refer Slide Time: 18:43)



Now, once you do that measurement then you get another set of probabilities, which will be of this kind because every time you are making one set of measurement we are basically converting it to go to that particular basis that you are measure.

(Refer Slide Time: 19:24)

**Measuring many qubits (contd...)**

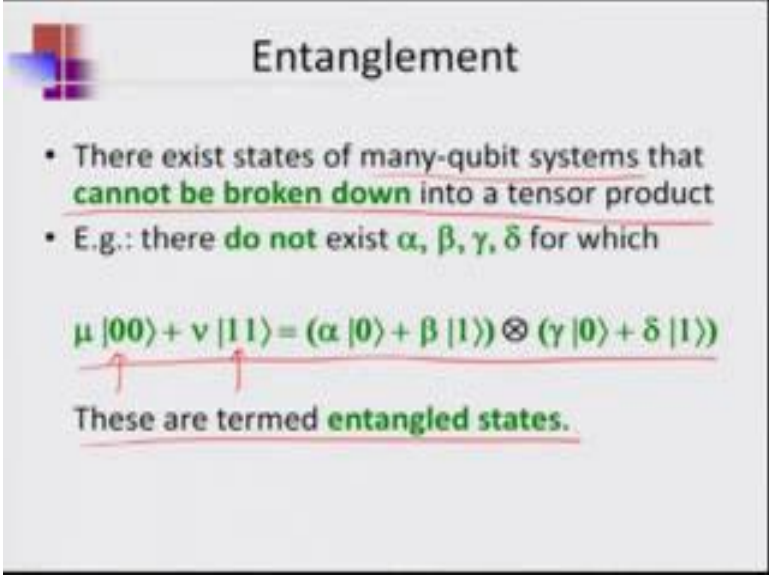
- By multiplying the branches in the overall tree, we can obtain the probability of each result. So for the state  
 $|\psi\rangle = \alpha\gamma |00\rangle + \alpha\delta |01\rangle + \beta\gamma |10\rangle + \beta\delta |11\rangle$
- two consecutive measurements will give
  - result  $|00\rangle$  with probability  $|\alpha\gamma|^2$
  - result  $|01\rangle$  with probability  $|\alpha\delta|^2$
  - result  $|10\rangle$  with probability  $|\beta\gamma|^2$
  - result  $|11\rangle$  with probability  $|\beta\delta|^2$

*Handwritten notes:  $|0\rangle$  and  $|1\rangle$  in red ink.*

So, each of them will have different probabilities as has been shown here, you can go ahead and do these maths. So, by multiplying the branches of the overall tree the way they are breaking up every time you make the measurement, we can obtain the probability of each result. So, for a state which is given by this combination, which means that it started off with just 2 qubits coming together with all these different probabilities 2 consecutive measurements will give rise to a result, which is 00 with a probabilities of this result, which is 01 with the probabilities of this result, which is 10 with a probability of this and the result of 11 with the probability of beta delta square mode of beta delta square.

So, now there is a specific thing that I wanted to mention here, that in this particular case when we talked about we were able to measure each of them individually that was important right, we measured 00, 10, 01, 11 with certain probabilities. Although we started off with states, which were 0 and 1, just remember that.

(Refer Slide Time: 20:46)



**Entanglement**

- There exist states of many-qubit systems that cannot be broken down into a tensor product
- E.g.: there **do not** exist  $\alpha, \beta, \gamma, \delta$  for which

$$\mu |00\rangle + \nu |11\rangle = (\alpha |0\rangle + \beta |1\rangle) \otimes (\gamma |0\rangle + \delta |1\rangle)$$

These are termed entangled states.

Now, that was possible which mean that we were going and looking at how their probabilities where when we looked at them individually in different basis. Now there can be also states in this many qubit situations, which cannot be broken down into a tensor product of a kind where I can associate individual probability to each of these kinds of states that you that we kind of discuss.

So, let us consider for example, is a condition were it looks like this, now when this happens see what I am trying to say is that these 00 and 11 are the states, where you were able to break them into these individual cases on these tensor cases, but in some cases it is such that you cannot break them and then they are known as the entangled states, I cannot really go back to my original states no matter what they are that is because, of the rule of the tensor product every time you multiply you matrix ends up multiplying every element of the other matrix that is the basic idea behind the tensor product and a vector product right.

So, the point what I am trying to do is that it is, when you do simple matrix multiplications you can always breaks them up easily, but when you are doing matrix multiplications when they are involving tensors, what we are doing is every element of the matrix is getting multiplied by the other all the elements of the matrix right and so that is different as compare to. So, these cases where we are not able to get back to the original states, where they came from they are known as entangled states, now this is a

mathematical issue which makes quantum systems behave very different from classical system, because this does not happen in classical systems.

(Refer Slide Time: 23:01)

The slide is titled "The Bell states" and features a small logo in the top left corner. It contains a bulleted list, handwritten notes, and four mathematical equations. The handwritten notes include "Quantum Teleportation" in red, "John Bell" and "EPR" in green, and "Bell states" in red. The equations are written in green and define the four Bell states:  $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ ,  $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$ ,  $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ , and  $|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ .

**The Bell states**

Quantum Teleportation

- For a two-qubit system, the four possible entangled states are named **Bell states**:

John Bell

EPR

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

So, in this period the most important condition arises, which was given by bell, John Bell is the one who looked at the EPR problem, which basically asked the question about actually we will do this in detail right now let me just do the mathematical part of it. For a 2 qubit system the 4 possible entangled states are named as bell states, because these states are the once which are possible to be transmitted and measured in a very specific manner, and then this is very important because this one helps in quantum teleportation. What is Quantum Teleportation? Quantum Teleportation is the case where you are transmitting qubits across a quantum path way and unless and until you know how to look back at it or you have the exact code to understand the particular set that has been transported you will not be able to know what has come to you. So, that is why it is extremely important in quantum teleportation and cryptography, where it is an absolutely important to able to transfer qubits securely.

(Refer Slide Time: 24:59)

**Greenberger Horne Zeilinger (GHZ) states**

- Entangled quantum state which involves at least three subsystems .
- Many measures define GHZ to be maximally entangled .
- For two dimensional subsystems :  
$$|\text{GHZ}\rangle = \frac{|0\rangle^{\otimes M} + |1\rangle^{\otimes M}}{\sqrt{2}}$$
- Simplest 3-qubit GHZ state :  
$$|\text{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

Reference: Daniel A. Greenberger, Michael A. Horne, and Anton Zeilinger, Joint Remote Bell's Theorem

So, this is the place where quantum teleportation case, where bell states are very important. If you go with more than two states, so this was with 2 qubit situations, if you go with say 3 qubit situations, or generalized m qubit situation, then the similar states which are the entangled states were given by Zeilinger and Greenberger. So, these are the three. So, here we had bell states, which were associated with EPR conditions and here we have many, many states, which were eventually generalized by Greenberger Horne and Zeilinger, where they had used m states to create the same kind of entanglement conditions, which can be transmitted across securely. So, entangled quantum states, which involve at least three subsystems, so these have to be at least three or more; anything up to two is the bells condition, satisfies the bells states, three or more are the once which I given by GHZ, GHZ or the Greenberger horn berg Zeiliger conditions many measures define GHZ to be maximally entangled.

Now, this term maximally entangled is something once again we will come back to it this is theoretical principle, but the idea of maximally entangled means no matter what happens you will not be able to get to a scenario where they can be separated out. So, let me actually tell you this the right a way here, that entanglement has its own degree also there has some cases where under certain basis set, under certain conditions you might be able to separate the states to some level then they not really maximally entangled, but they are sort of entangle to level as long as you are not doing the states transforms to a certain levels.

In a given condition you can get them to be entangle, but when they are maximally entangle no matter, what happens they are always going to remain in the entangles state you cannot actually treat them as individual units. For two dimensional subsystems GHZ can be as simple as 0 raise to the power the tensor product to m, and it is a super position with the other state over phi 2, and the simplest 3 qubit state is exactly like this. Actually, the reason why it is at least three more subsystems is because, if you go to the 2 qubit state it essentially reduces down to bells state, because it is essentially the same all the signs and everything put together the fact that this as to this tensor of the matrix on top also correct for the science that you wanted in the middle. So, that is how it works out. So, if you compare them similar terms exist this is always with 2 states where as in this case you have at least 3 or more, but they have the same characters.

So, and they cannot be sort of never taken out together. So, maximally entangled all of them all the bell states as well as, the or maximal entangle if you are doing multiple sub states or sub systems you can have other kind of situation where they are not maximally entangle conditions there we do not talk about GHZ, but if you want to get bell kind of situations taken on to larger number of qubits then this is the only kind of conditions that will be able to use and they have to be maximally entangled.

(Refer Slide Time: 29:00)

**Measuring Entangled States**

*Spooky*

- After measuring an entangled pair for the first time, the outcome of the second measurement is known 100%

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Diagram illustrating the state  $|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$  and its measurement outcomes:

- Measurement of the first qubit results in  $|0\rangle$  or  $|1\rangle$ .
- If the first qubit is measured as  $|0\rangle$ , the second qubit is  $|0\rangle$ .
- If the first qubit is measured as  $|1\rangle$ , the second qubit is  $|1\rangle$ .

Now, what is the amazing thing about entangles states, what is it which makes entangles states so important in quantum mechanics and quantum computing. Is a term associated



with this spooky, or in common language ghostly, see up to super position which has a classical analogue waves you can always justify whatever you are seeing with some sort of a physical analogue when you come to entanglement, since you cannot ever fall back to the individual states there is almost no particular condition, where you can say that I have a classical analogue is just really not there.

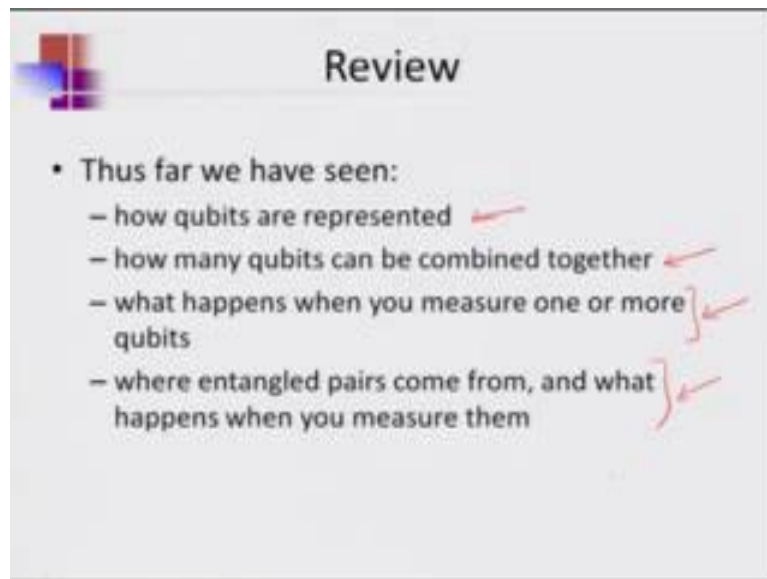
And therefore, everything in this area looks very spooky, but for the movement if you do not worry about the spooky pair there is some amazing fun associated with entangled states. So, the first thing is after measuring an entangled pair for the first time the outcome of the second measurement is known 100 percent, it is like this and that is why this question arises that information can move faster than speed of light, because according to this theory what did I just say? I said that I have an entangled pair, which I have just created right, and now I let the other. So, I have two of them let us see the simplest case two quantum systems, I have made in an entangled pair. So, I get two, one I keep with myself and the other one, I put it on say anything I mean let us say the rocket which is going to moon I put it on there it goes away to moon it is there.

Now, I measure the one that I have it with me, the theory says the movement you measure you that you have the information of other one, because that is how it is mathematically done, mathematics says that. If that is the way it is then you have just violate it what do otherwise things is speed of light, you got information faster than the speed of light, because the movement you measure the one of the pairs here, you have the information of other one, which is now sit residing in say moon and you can do thought experiments where you can say that the it has been send to the other galaxy let us say, and yet the movement you measure it here you have the fully information of the other.

Now, this is one of the fallacy which bell was able to prove there is something known as bells in quality, which proves that the EPR paradox is not leading to this kind of a situation, which says that you can have faster than light transformer information that is not possible. The reason being the concept of this entire information quantum that we just talked about is decided a priory at the time of the creation of this information, whatever you do after that has no meaning. So, it is a little difficult to come to it that is why it came all these names spooky and everything associated with it, but the basic act of the idea, which is kind of fun to at least think about it is the fact that if you have state

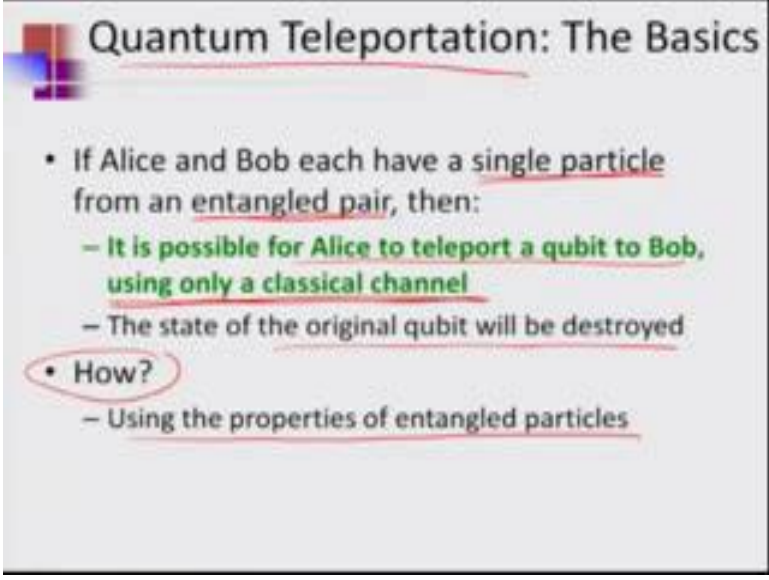
and it is a bells state for example, like this one then the movement you make any measurement of this then you have the other measurement perfectly known to you. So, that is the idea then the outcome of the second measurement is known with 100 percent precision, and this one definitely is 100 percent there is no probability associated with this at all this is deterministic, right that is the basic idea.

(Refer Slide Time: 33:12)



So, we are going to review, what we have done until now because there is kind of heavy stuff that we did though it might not look heavy, because I avoided much of the math here and hence, whatever I tried to write somewhere it did not come out properly, but I think will (Refer Time: 33:30) some of the math little bit more. But what we tried to do as we showed you in these last two lectures that today at the last one, how qubits are represented, how many qubits can be combined together, which essentially means that can be any, but for practical purposes there is some limits that also we should know, what happens when you measure one or the more, one or more qubits where entangled pair come from what happens when you measure them now these are also very important right. So, this is the review until now.

(Refer Slide Time: 34:04)



**Quantum Teleportation: The Basics**

- If Alice and Bob each have a single particle from an entangled pair, then:
  - It is possible for Alice to teleport a qubit to Bob, using only a classical channel
  - The state of the original qubit will be destroyed
- How?
  - Using the properties of entangled particles

Now, based on these that we just did we should be able to take one example, which I have already alluded to which is teleportation. Now teleportation you must have known science fiction then everything else cartoons, chambers, movies.

Student: Sir if quantum is like if entangled principle (Refer Time: 34:31) how can we prepare (Refer Time: 34:33).

Actually we are not really preparing them, what happens is whenever you have. So, the question is, how do you prepare entangled state? So, the idea is whenever you can interact states together in a certain manner then you will be always creating the entangled state or super position state, now it depends on how you are making the interaction. So, that is the reason why I through the gate before that because, we wanted to make you appreciate that how these operations are actually giving rise to the different results that we are getting. So, one part is measurement, but point of measurement is the once you are making the measurement is the final answer which is like a classical answer, but going before that are all the different steps which were gates. So, first was definition of a qubit next was the operations which were our gates.

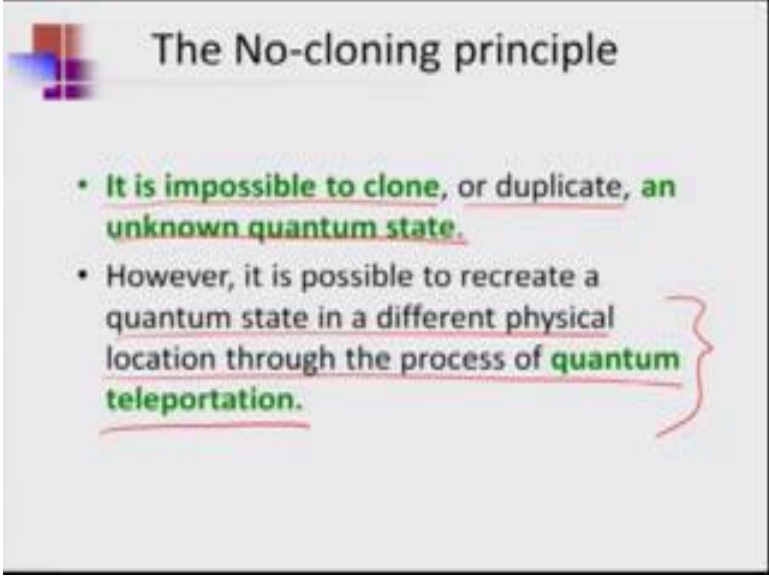
Now, these operations sort of had different results as a result of interactions some of them can end up producing only super position or only amplitude enhancement or something like that nothing very amazing, and they can be completely ratified as for all our knowledge in terms of classical mechanic or classical understanding. But there are

some cases where the movement you do that you are not able to get back to the individual behaviors or characteristics that is where we defined entanglement we said that although we had two super positions of states. So, generally it starts off like this, when you have one qubit which is basically two states that is the simplest case. Once I take one qubit mix with another qubit I get the next level of problem, which can be as simple a super position right it is only some of the two conditions all possible sums.

Now, when I take one super position state, and I have another super position state come together their interaction is going to be defining as to how they will interact do they choose to remain to be like super position like in the sense that each of them will behave as independent qubits and they will again only produce up to super position, but that is the rare, in quantum system that does not happen, they lose their individual identity at that very point they all become something else, when they come together. So, those four qubits now which I initially started off has now actually no it was just 2 plus 2 right. So, I had 2 1, which had two conditions had another one, which had two conditions we put them together they already started in interacting in a different way, right that is the bell condition where I can produce entanglement when I can put another one that is getting even harder and that is where the starting point of this other conditions starts and then as you when you make it grow is the same idea mathematically it is to be represented by tensors and before that whatever you do they are going to be represented as vectors that is the idea.

And the biggest advantage we have as a result of all of this one example is quantum teleportation, which is a reality by the way this is one thing which I should first say that this part of quantum computing or quantum information processing is essentially a reality, and there are 100 percent proves that this one exists and it is being used. So, this is the basic idea that there are two individual communicators, let say Alice or Bob that the common nomenclature in this area and if they have a single particles. So, here we start the first one a single particle, from an entangled pair. Then it is possible for Alice to teleport a qubit to Bob using only a classical channel, the state of the original qubit will be destroyed, because the movement measurement is made state is destroyed. So, the point is only using a classical channel this can be done and therefore, it is kind of interesting to note that you can do this when you have an entangle pair and this is the question of this is the using the properties of entangled particles this can be done.

(Refer Slide Time: 39:15)



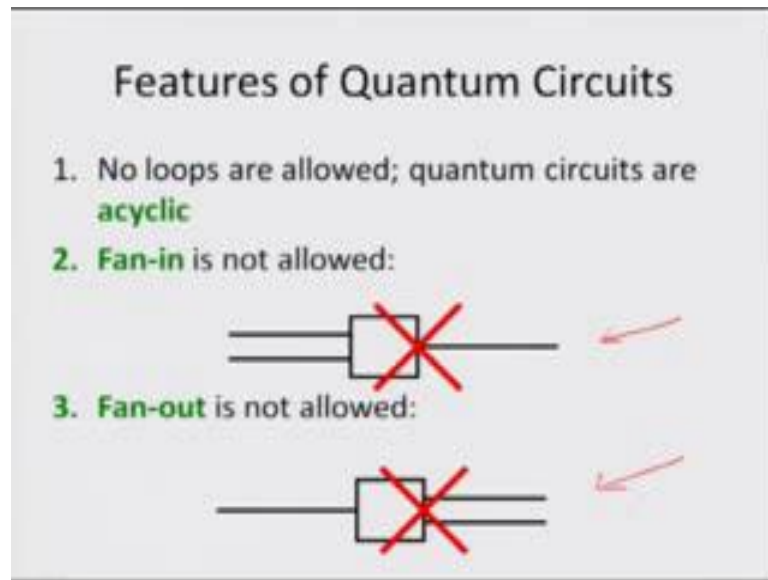
**The No-cloning principle**

- It is impossible to clone, or duplicate, an unknown quantum state.
- However, it is possible to recreate a quantum state in a different physical location through the process of quantum teleportation.

So, what is the idea behind this particular idea of a teleportation? The first thing to remember in this cases is, this is something which we have to done before in quantum mechanics we cannot create something, we cannot analysis something, nothing can be created nothing to (Refer Time: 39:28) that is known, which means that it is impossible to clone or duplicate an unknown quantum state. When you know a quantum state then it is classical anyway right, so that part is gone; however, it is possible to recreate a quantum state in a different physical location through the process of quantum teleportation.

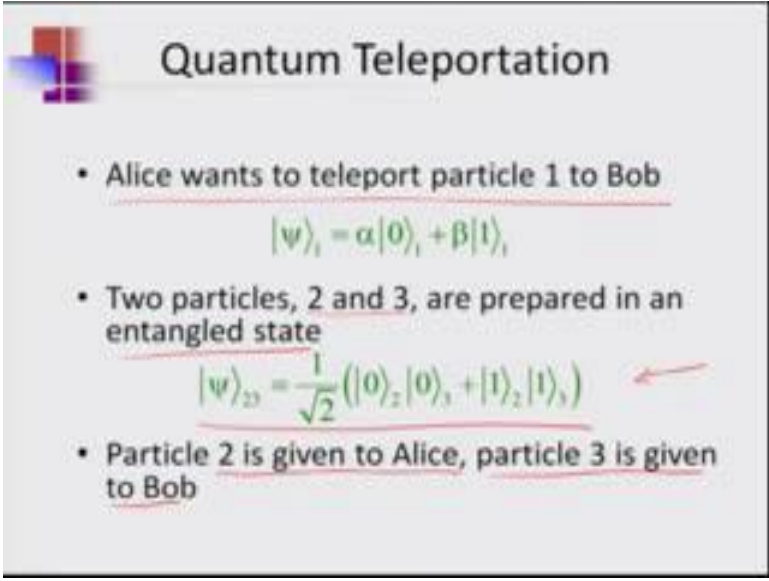
Now, this is kind of very interesting, which means that although. So, these are the things why quantum computing in some sense can become a prove reality, because once I went if we go back we saw the cases where we decline this cannot be done that cannot be in let me quickly go back. So, let see we started off here.

(Refer Slide Time: 40:19)



Now, although it looked very pale it was very important to realize that in a actual computer, in a actual computation, it would be necessary to have these situations to arise also now the very fact in quantum systems, which is supposed which has to be reversible these are not going to happen, which means that I will not be actually able to create a quantum computer in this sense of how we know about a computer. If this is strictly all how it is going to happen, but the very fact now I am telling you, when I come here is that it is true that I cannot do this. So, this is the no cloning I cannot really do that that I already just told that, that was my no cloning, but it is also true that since I have teleportation I can actually do this in a different way, at a different location see. So, I have found a way out of some of the necessary gates, which I could not otherwise I have implemented if I only go by the logics and laws of quantum mechanics this is perfectly allowed through quantum mechanical system that I can actually do at teleportation quantum teleportation.

(Refer Slide Time: 41:57)

A slide titled "Quantum Teleportation" with a small logo in the top left corner. It contains three bullet points and two mathematical equations. The first bullet point is "Alice wants to teleport particle 1 to Bob" with the equation  $|\psi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$  below it. The second bullet point is "Two particles, 2 and 3, are prepared in an entangled state" with the equation  $|\psi\rangle_{23} = \frac{1}{\sqrt{2}}(|0\rangle_2|0\rangle_3 + |1\rangle_2|1\rangle_3)$  below it. The third bullet point is "Particle 2 is given to Alice, particle 3 is given to Bob".

**Quantum Teleportation**

- Alice wants to teleport particle 1 to Bob  
 $|\psi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$
- Two particles, 2 and 3, are prepared in an entangled state  
 $|\psi\rangle_{23} = \frac{1}{\sqrt{2}}(|0\rangle_2|0\rangle_3 + |1\rangle_2|1\rangle_3)$
- Particle 2 is given to Alice, particle 3 is given to Bob

So, that is the reason why quantum teleportation is important. So, although opened it by saying how we are going to do this, here is the mathematical point of how we are going to actually achieve the idea of quantum teleportation. So, Alice wants to teleport the particle 1 as I mentioned to Bob. So, in order to do that what is necessarily required is we need two other particles 2 and 3 that are prepared in an entangled state. So, this is basically the bell pair as we have done. So, this is state 0 and 0 having of 2 and 3, which is we superpose super imposed with the 2 and 3 or 1 and 1 of root 2. So, this is the bells state that is created. Now, particle 2 is given to Alice, particle 3 is given to Bob, once you upgraded this.

(Refer Slide Time: 43:01)

**Quantum Teleportation (contd..)**

- In order to teleport particle 1, **Alice** now entangles it with her particle using the **CNot** and **Hadamard** gates:  
$$\text{CNot}(|\psi\rangle_1, |\psi\rangle_2); \text{H}(|\psi\rangle_1)$$
- Thus, particle 1 is "disassembled" and combined with the entangled pair
- **Alice** measures particles 1 and 2, producing a classical outcome: **00, 01, 10 or 11.**

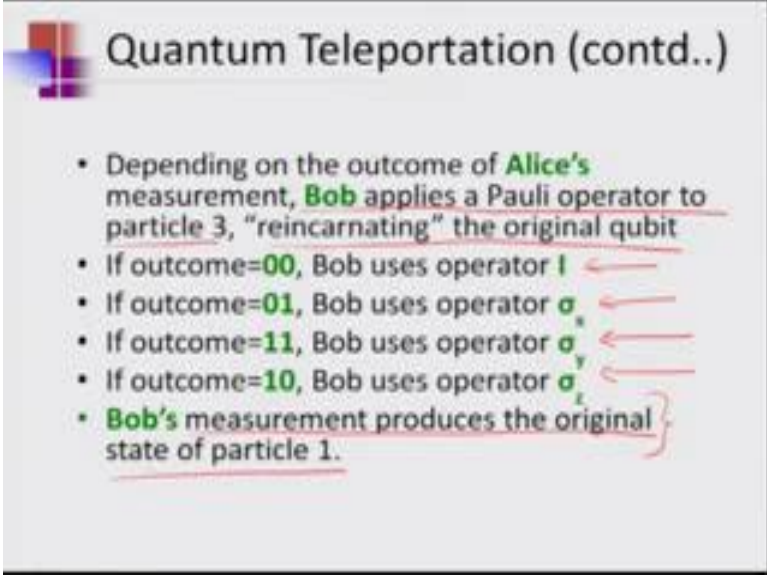
In order to teleport particle 1, Alice now entangles it with her particle using the CNOT and Hadamard gates: now you have to look back into your NOTs in the gates that you did before. So, CNOT you are going to apply on one and two states and Hadamard we are going to apply on the one states. So, now, the particle 1 is disassembled and combined with the entangled pair.

So, this was the process that I was telling you about earlier now you have to understand one thing very important in this particular course is that in a implementation quantum course this is the part, which is the extremely important how is the process happening knowing the mathematics as we have been doing. In terms of saying that this is the math this is the way it is fine, but we have to somehow get to a point where we are discussing the scenario as to how these are happening.

So, once you have done this part where you have manage to put your particle 1 with the entangled particle where sorry, where you have managed to entangle your particle 1 with the particle which is now with alice then you are in a disassembled these assembled state, this assembled state of particle 1 with entangle pair, which Alice has now Alice measures particles 1 and 2 producing a classical outcome which is all these possibilities.



(Refer Slide Time: 44:53)

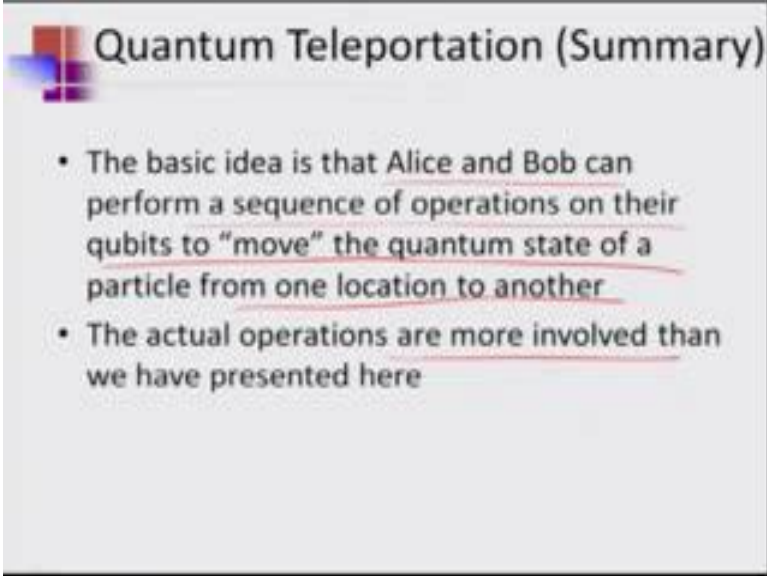


### Quantum Teleportation (contd..)

- Depending on the outcome of **Alice's** measurement, **Bob** applies a Pauli operator to particle 3, "reincarnating" the original qubit
- If outcome=**00**, Bob uses operator **I** ←
- If outcome=**01**, Bob uses operator  **$\sigma_x$**  ←
- If outcome=**11**, Bob uses operator  **$\sigma_y$**  ←
- If outcome=**10**, Bob uses operator  **$\sigma_z$**  ←
- **Bob's** measurement produces the original state of particle 1.

Now, depending on the outcome of Alices measurement, Bob applies a Pauli operator to particle 3 reincarnating the original qubit, now if the outcome is 00, Bob uses operator I if the outcome is 01, Bob uses operator sigma x, now this is the Pauli set as you know, if it is 11, then he uses sigma y, if it is 10 uses sigma z.

(Refer Slide Time: 45:39)



### Quantum Teleportation (Summary)

- The basic idea is that Alice and Bob can perform a sequence of operations on their qubits to "move" the quantum state of a particle from one location to another
- The actual operations are more involved than we have presented here

So, based on this measurement Bob is able to produce the original state of particle 1. So, the basic idea behind this, is that Alice and Bob can perform a sequence of measurements on their qubits to move the quantum state of the particle from one location

to the to the other, the actual operations are more involved than what we have presented here, and the actual of operations are something, which will actually be doing in detail here this is just summary. So, this is just to set you up as to how will be doing it. I think I will be stopping here, because we almost come into the point of here; we will be stopping here and we will be exploring more details on this areas from the next class.

Thank you.