

**Implementation Aspects of Quantum Computing**  
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**Lecture - 41**  
**Academic Developments in Quantum Computing and**  
**Implementation – II**

We have been looking at various different aspects of implementation of quantum computing as the final series of lectures of this course. And we have looked at many different implementations. One of the things which we have not looked at in detail which I thought I will cover in this lecture is the optical aspect of the implementation. We have covered the NMR aspects, we have covered the ion trap aspects, we have covered the commercial aspects, but the aspect involving more of the optical scheme is the one which we will be doing here.

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**Optical Approaches to Quantum Computing**

Photons could be ideal for making qubits

IN PARTICULAR:

- The fact that the photons don't interact with one another under normal circumstances
  - ⇒ A superposition state of, say, a photon's spin could be immune to decoherence by stray electromagnetic fields.
  - ✓ That suggests a need for far less error correction than for a computer based on matter qubits.

BUT

- the photons' lack of mutual interaction is also a big problem for creating two-qubit logic gates
  - This like their classical equivalents, are non-linear devices requiring qubit carriers to interact with one another.

Such interactions require nonlinear interactions that are hard with photons

So some of it we have already seen earlier and here we are going to focus on the optical approaches to quantum computing. In some sense photons could be ideal for making qubits, so there are two essential points to remember; one is the spin and the other one is the photon. Spin is very important because those are the ones which can be changed by different principles which does not affect many other interactions, as well as photons are

also very important because they can also be put together in such a way so that they do not interact with each other; also they can be transmitted.

So, here we say in particular the fact that photons do not interact with one another under normal circumstances gives an advantage. A superposition state of say a photon spin could be immune to decoherence by stray electromagnetic fields, that suggest a need for fireless error correction than for a quantum computer based on matter qubits. But the photons lack of mutual interaction is also a big problem for creating multiple qubit logic gates, this like the classical equivalents are non-linear devices requiring qubit careers to interact with one another. Such interactions required non-linear interactions that are hard with photon. So, these are the pluses and minuses with photons been used for quantum computing.

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A SUCCESSFUL APPROACH: LINEAR OPTICAL APPROACH

We discussed the approach where linear optical devices could in effect be used to carry out nonlinear operations, thanks to the fact that photons are bosons. When two photons enter a 50% reflecting beam splitter from opposite sides at the same time they will always leave the device along the same path, and this sticking together constitutes a kind of interaction.

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**Grover's search algorithm: an optical approach**

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In another linear approach discussed, we studied the case of using Laser Cavity as implementation of Grover's Search Algorithm, where the Readout was accomplished by measuring a mode with a photo-detector, which destructively determines whether one or more photons are present in a mode

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**Implementation of Quantum Search Algorithm using Classical Fourier Optics**

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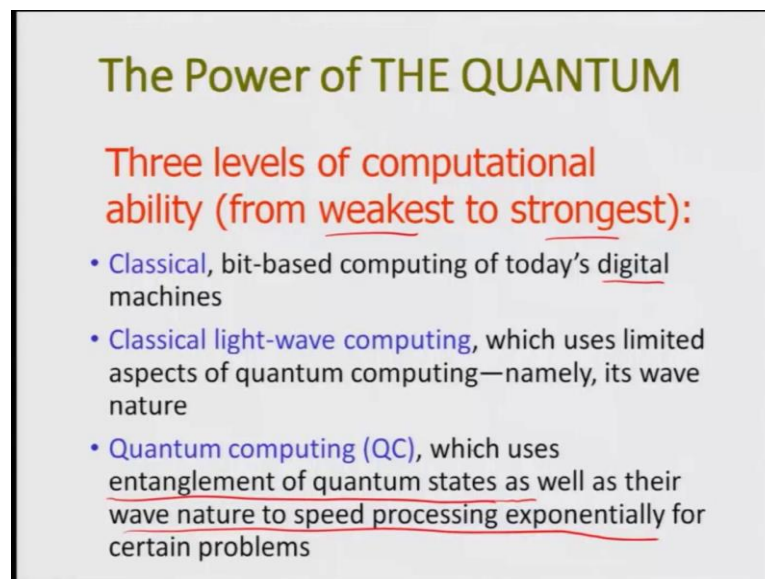
Several other ways exist to get around "lack of interaction in photons":  
E.g., Use matter as an intermediary between two sets of photons  
Today we will look at such "other" approaches of optical quantum computing...

They have been successful approaches and here is one of the ones which we discussed in the class, which is the linear optical approach wherein linear devices in optical framework can be used in effect to carry out non-linear operations and this can be possible because photons are bosons, when two photons enter 50-50 reflecting beams splitter from opposite sides at the same time, they will always leave the device along the same path and this sticking together constitutes kind of interaction.

So, this was one of the advantages which was utilized and this work was first shown in the year 2000 by Kwait and others in terms of showing that it can be applicable to Grover search algorithm. In another linear approach discussed the laser cavity was used as an implementation of the Grover's search algorithm where the read out was accomplished by measuring a mode with a photo director which destructively determined whether one or more photons represent in a given mode and so this was shown in 2002 by using classical Fourier optics.

So, there are several ways exist to get around the lack of interaction photons for example, using one of the ways is over and above these particular approaches is to use matter as an intermediary between two sets of photons which work. So in a few of the slides which follow, we would be looking at such other approaches of optical quantum computing.

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**The Power of THE QUANTUM**

**Three levels of computational ability (from weakest to strongest):**

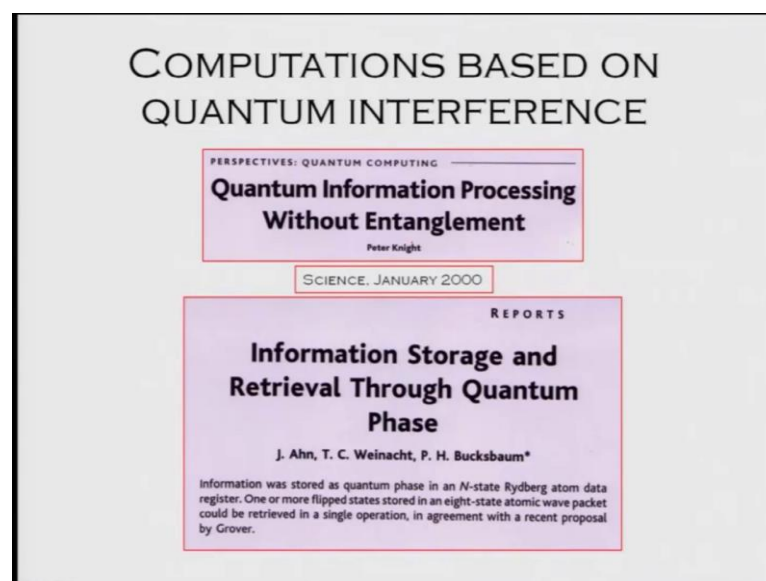
- **Classical**, bit-based computing of today's digital machines
- **Classical light-wave computing**, which uses limited aspects of quantum computing—namely, its wave nature
- **Quantum computing (QC)**, which uses entanglement of quantum states as well as their wave nature to speed processing exponentially for certain problems

The power of the quantum in this sense can be looked at as three levels of computational ability starting from its weakest to strongest. One is the simple classical bit based computing which is the today's digital machines which also uses optical technology quite a lot in terms of storage transfer and in some cases even computing. The other one is the classical light wave computing which uses limited aspects of quantum computing for example, its wave nature in terms of super position and that was often used in terms of

using say the laser cavity for demonstrating Grover's algorithm or linear optical devices to implement superposition principles in terms of classical light wave computing.

The most intricate one involves the quantum computing that take advantage of the entanglement of quantum states as well as their wave nature to speed up the processing exponentially for certain problems. Even otherwise, if it is not exponentially speeding it up, the resources used for whenever the entangle process is available in terms of quantum states, the resource handling becomes much simpler and much less requirement is there on the resources. Same problem done in a classical light wave computing versus in the quantum computing sense would require much less resources in the quantum computing sense.

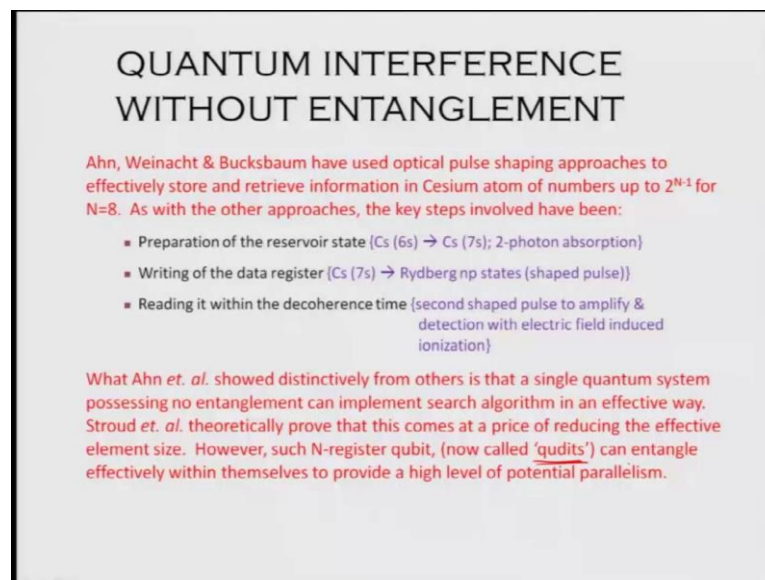
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So, computations based on quantum interference is the part which we discussed at the beginning, where the information processing was done without entanglement and the information storage and retrieval was possible through quantum phase, but it did not involve any entanglement. So, in this particular case although matter was used to have an interaction with the light, the interaction only resulted in creating supervision of states rather than entanglement and it only ended up producing similar aspects as the wave property have had.

So in this case although quantum phase was involved, it only had states which could be superimposed. So, this particular work did not use entanglement and was possible to show Grover's algorithm, but was resource heavy.

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**QUANTUM INTERFERENCE WITHOUT ENTANGLEMENT**

Ahn, Weinacht & Bucksbaum have used optical pulse shaping approaches to effectively store and retrieve information in Cesium atom of numbers up to  $2^{N-1}$  for  $N=8$ . As with the other approaches, the key steps involved have been:

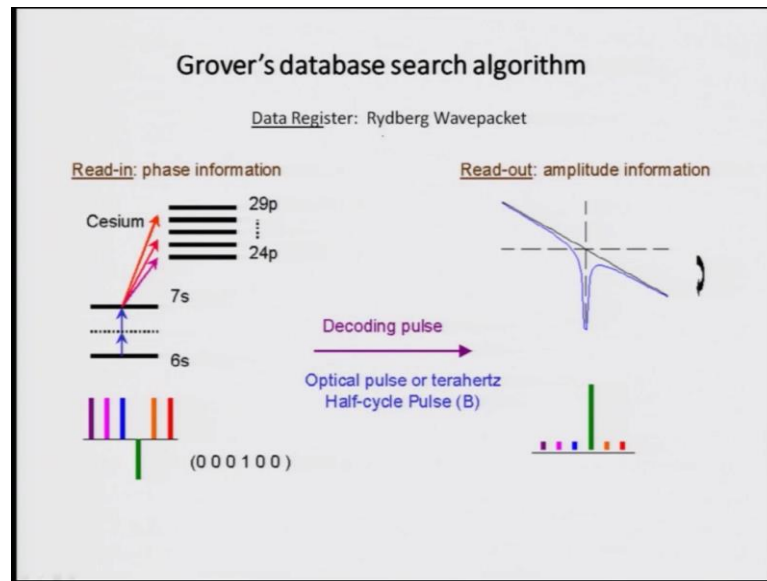
- Preparation of the reservoir state (Cs (6s)  $\rightarrow$  Cs (7s); 2-photon absorption)
- Writing of the data register (Cs (7s)  $\rightarrow$  Rydberg np states (shaped pulse))
- Reading it within the decoherence time (second shaped pulse to amplify & detection with electric field induced ionization)

What Ahn *et. al.* showed distinctively from others is that a single quantum system possessing no entanglement can implement search algorithm in an effective way. Stroud *et. al.* theoretically prove that this comes at a price of reducing the effective element size. However, such N-register qubit, (now called 'qudits') can entangle effectively within themselves to provide a high level of potential parallelism.

In particular they use the cesium atoms Rydberg state to write the data register and then read it out within decoherence time scale by using a shape laser pulse to amplify and detect the electric field induced ionization. So, there advantage was that they were able to distinctively show at single quantum systems possessing no entanglement can implement search algorithms in the same spirit as the optical waves without any entanglement could show Grover's algorithm also.

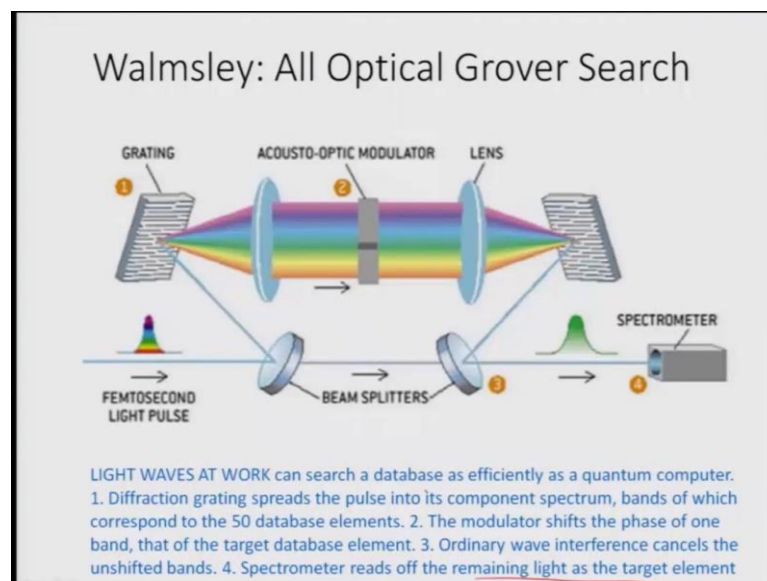
This work was later on extended by Strout theoretically to prove that this comes at a price of reducing the effective element size. However, if such n register qubits which are known as qubit size as of now, can entangle effectively within themselves to provide a high level of potential parallelism then the difficulty of this problem can again be reduced, however it is a much more difficult problem experimentally to be addressed and that is yet to be done.

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In general the data registers in these cases were loaded through optical techniques of exciting the system to a Rydberg state through a two photon excitation process by using a shape pulse two essentially write down the information in terms of phase encoding of the excise state and then using a decoding pulse which could be an optical pulsar terahertz half cycle pulses to essentially ionize the correct states to get the information out.

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So, it was essentially reading the Rydberg wave packet by the data register was stored. Instead of using a matter to interact with shape pulses (Refer Time: 09:26) Walmsley showed that the principle of pulse shaping can itself be utilized as in the previous cases of no entanglement with light waves to also do the optical Grover search principle application and in their approach, they were able to put in the data register using a pulse shaper an Acousto-Optic Modulator, wherein the input data were able to be read once again by a part of the original beam of light to find the correct answer and it was possible as the diffraction ratings spread the pulse into component spectrum bands of which corresponded to the 50 database elements. The modulator shifts the phase of one band that is the target database element, ordinary wave interference cancels the un-shifted brands and the spectrometer reads of the remaining light as the target element.

So, it is all classical and yet it was possible to do this entire processing by using the way property because in the Fourier domain this was done. So, it is a application of the optical Fourier transform principles to encode and then decode the data; however, this is all classical as I just mentioned and so in terms of using the light wave it is just acting as a wave which is using the superposition of the states; of the information content in the system.

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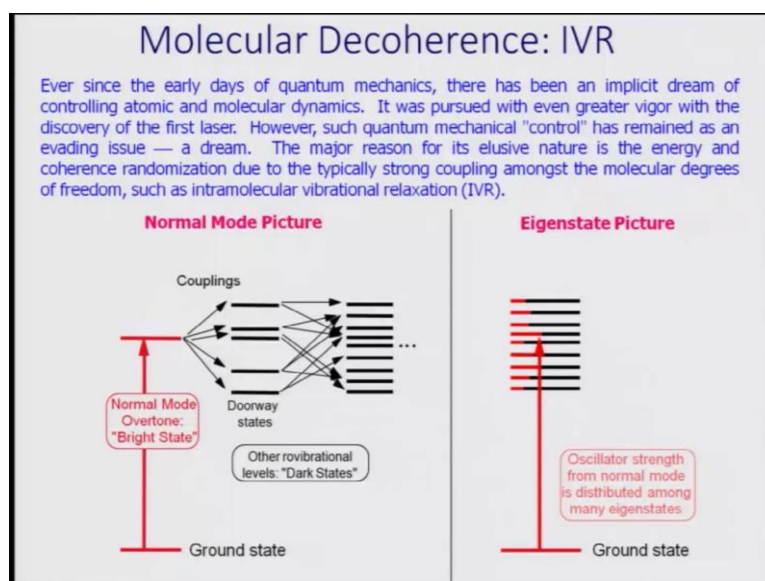
Decoherence:  
Laser Molecule Interaction

- Intermolecular—Diffusion & Mobility  
Timescales Depend on environmental conditions
- Intramolecular—Intrinsic to Molecular States  
Timescales typically vary from ns or below  
depending on whether electronic, vibrational or  
Rotational States are involved

And much more interesting problem to be looking at in terms of these interactions is the problem of decoherence which as we discussed in the earlier case of interactions for ion traps and NMR and other principles. In optical case also the laser molecular interaction has the difficulty of decoherence, which could be of different kinds; one which is intermolecular, which is due to diffusibility and mobility and their times is depend on environmental conditions; if it is a liquid, it has one time scale; if it is a gas, it has another times scale and if it is a solid.

So, depending on the states of matter this can have different time scales proximity and the motion of the particles everything put together. The other intrinsic decoherence comes from the intermolecular aspect which is intrinsic to molecular states, these time skills typically vary from nanoseconds or below depending on whether electronic vibration or rotational states are involved in the process of the laws of the coherence.

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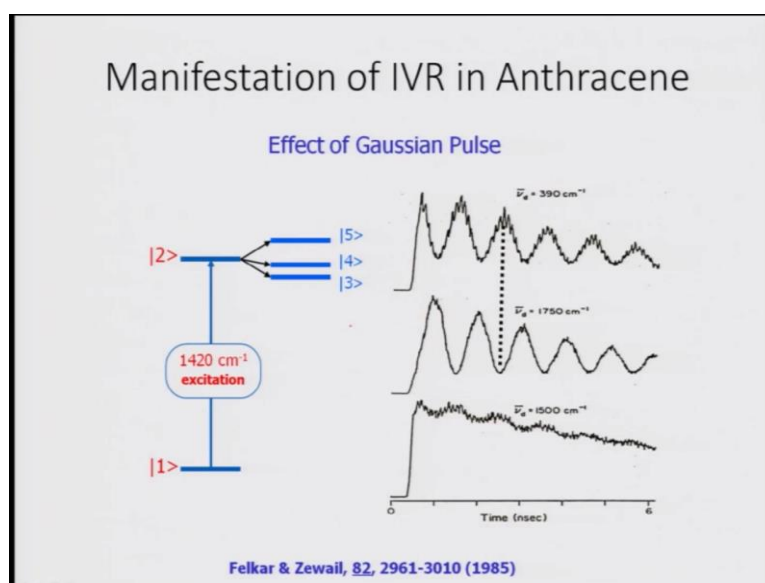
It can be looked at in this particular format where the states of the molecule are getting coupled to the other states or the nearby states which could be affected due to the environment as well to take away the coherence of this system from the quantum system.



So, this happens to be a major aspect of problem for chemistry also because ever since the early days of quantum mechanics has been an implicit dream of controlling atomic or molecular dynamics which lead to chemical changes. It was pursued with even greater figure with the discovery of the laser; however, such quantum mechanical control has remained as elusive and evading and is often considered as a dream.

The major reason of its elusive nature is the energy and coherence randomization due to the typically strong coupling amongst the molecular degrees of freedom; such as intermolecular vibration or relaxation or IVR. The equivalence of the two pictures which are provided here essentially mean that the oscillator strength from a single excitation is distributed amongst many eigenstates either way howsoever this picture is being looked at the effect of decoherence is the same that the energy localization cannot be achieved.

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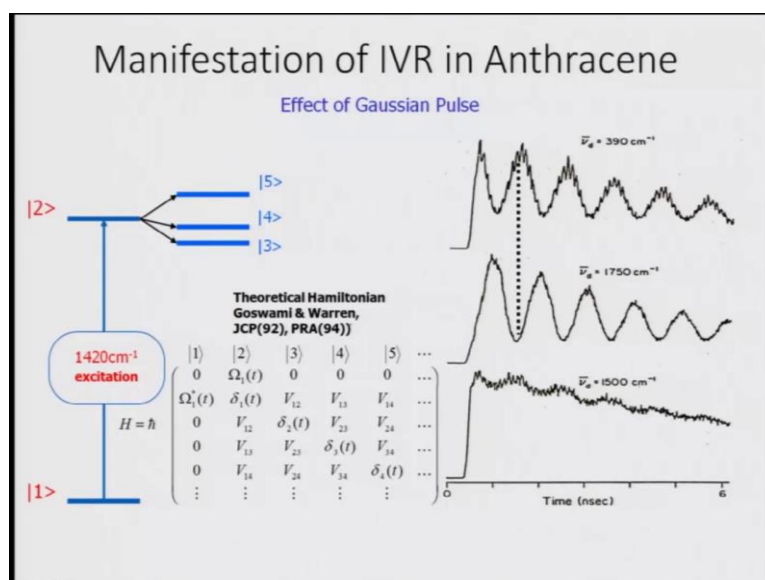


So, this has been seen in many different cases; in particular for example the work which led to the first observation of how molecular dynamics work in reality, in a chemical reaction which led to the Nobel prize of Zewail; also has features of how the intermolecular vibration or relaxation process keeps on having this feature and this is similar to the  $t^2$  oscillating feature that we discussed in the earlier case where the system essentially the quantum state got entangled with the environment and here also it

undergoes oscillations because there are few states into which the energy can flow in and then they can have a few iterations of coming back and forth and as a result you see the oscillatory behavior.

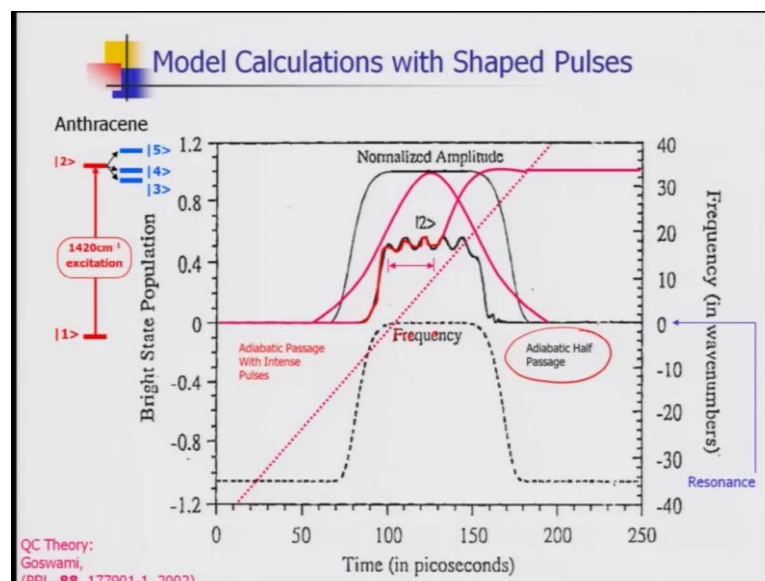
If the coupling is strong so that it essentially goes away into these other states and does not have a chance of coming back then there is a persistent decay as can be seen in these cases. The lifetime of the excited state which is otherwise labeled as the  $t_1$  state almost always have a exponential decay because that is related to the time scale on which the state can exist.

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So, based on this work because possible later to extract the Hamiltonian associated with such kind of transitions; at least for molecules where the states were recognized it was possible to do this. So, in this particular case of Anthracene study which was done initially by Zewall and Peter Felker was later on possible to have the Hamiltonian deciphered for this system and so this became a good model system for understanding and using the decoherence principle better.

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So, it was therefore possible in later years to show that the principle of decoherence which exists for these kinds of states can then be also controlled by using the pulses which can be shaped in certain way or the other. We had eluded to this in an earlier lecture where we mentioned that the different ways of controlling the coherence, one of the approaches is to actually change or modulate the pulses and here is an example; this is typically also done in the case of NMR where the decoherence or states properties are also modulated by use of pulse shaping in terms of radio frequency pulses which are used for modulating the spins in NMR.

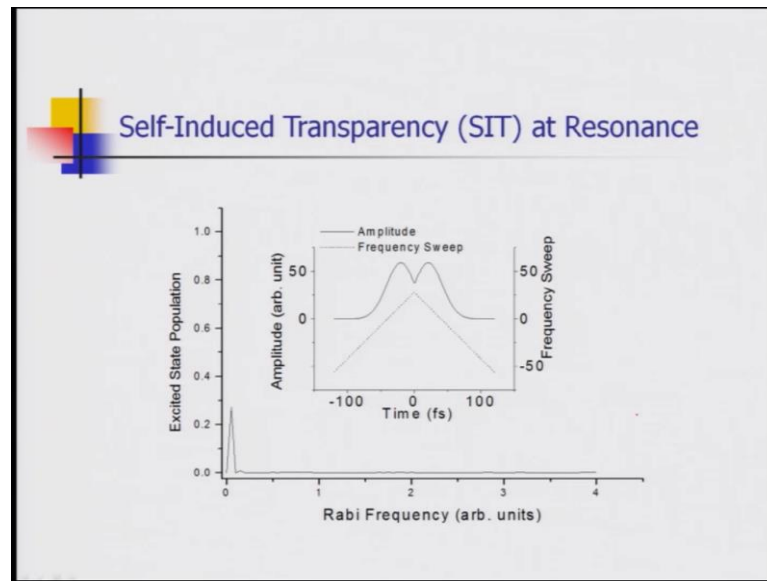
In case of optics it gets much more complicated because the time scales involved are much faster, it has the benefit of the faster time scales which means that the gates can be operated and worked at a much faster rate. However, it has to be carefully applied because for example here, there are two different shapes of pulse which is being discussed here and this is based on the principle of adiabatic interactions of the system with the applied field and if the field applied has its frequency changing linearly and if the change in linear frequency sweep is slow enough then there will be a period over which as is indicated by the horizontal arrow here and there will be a condition where the two states will remain in coherence and the 1 and 2 states and it will not be possible for the other states 3, 4, 5 to be able to extract energy from these two states.

So, under that condition when the 2 states are not going to talk to the other states then it is possible to have the energy localized only between the ground excited state or other words they can be incoherence and the decoherence arising from the decay into the states 3, 4 and 5 does not occur; however, when the frequency sweep goes more further away from resonance then the system again follows the applied field and it goes into the excited state.

So, this is the principle of adiabatic passage wherein this can be seen at certain frequencies are at certain sweeps of the frequency. A better approach would be to essentially bring the field in such a way so that the frequency is swept to resonance and is kept there and that is typically known as adiabatic half passage and this is a typical principle which is applied in NMR to bring systems into resonance and keep it there and in doing so adiabatically, it is possible to ensure that the system essentially comes to coherence and stays there.

Unfortunately in optics such pulse shapes are very difficult to do because such a pulse shape would require change of frequencies occurring over very rapid time scales and so these are quite challenging unless certain property of the system itself enables search frequency components to be generated. So, these are the reasons why this technology is often difficult in optics; however, if possible to be achieved; it enables complete coherence between the ground in excise states in such a way so that the other decoherent states have no role to play in such interactions. So, that is one of the important aspects of decoherence which can be looked at and the theory on this was possible to be developed only based on the fact that the Hamiltonian existed from earlier experimental results.

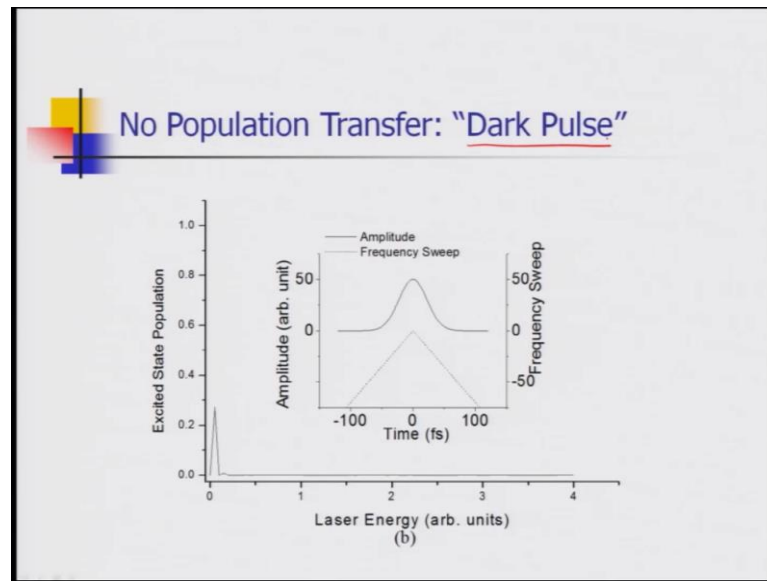
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Going forward on these; the idea of pulse shaping can still be further put to use by having the pulses go through furthermore shaped conditions either ramping up and then going down and in this particular case this could be two different pulses which are coming with linear sweep frequencies and getting overlapped, as such that the interaction in essence brings down the system back to its ground state.

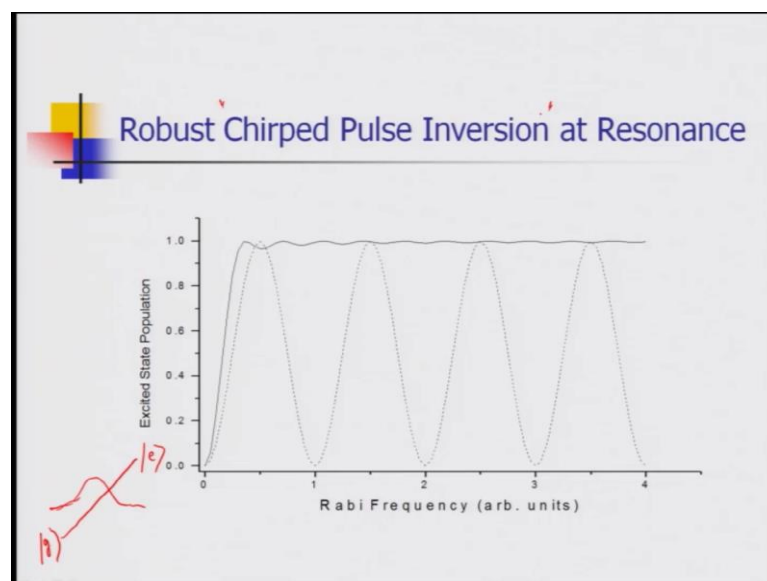
So it is not that the system does not interact with the applied field; it does however, the interaction balances out in terms of bring the system back down to its ground state in terms of cycling it through and this particular process of keeping the system, although it is being interacting with the applied field is self induced transparency at resonance and that is because at that point; the light will be able to pass through the system without having a net interaction happening to the system because it goes cycles through back to the ground state.

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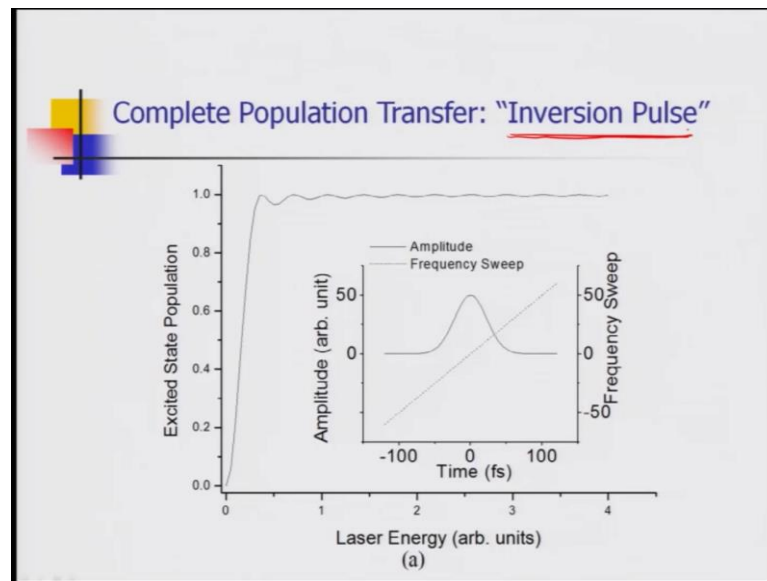
So, this particular idea of producing an interaction where no population transfer may occur has sometimes been termed in literature as dark pulses because the pulse interacts with the system and yet there is no net change in the system as a result of this interaction as if the pulse was non existence, so in some sense it is a dark pulse.

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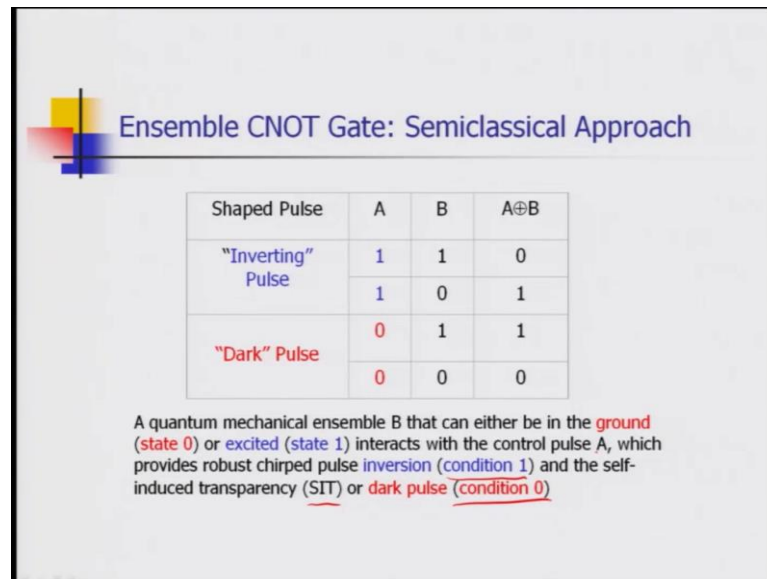
On the other hand, when a system is interacting with the pulse such that because of adiabatic sweeping of the applied field, the system follows the applied field to go from ground to the excise state by the end of the pulse; we end up generating Robust Chirped Pulse Inversion at Resonance and this is known as Chirped Pulse inversion process due to adiabatic inversion and this is another way of achieving 100 percent inversion.

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So, complete population transfer or inversion pulse are therefore, possible in case of pulses which have this character where the frequency sweep is able to make sure that the ground and the excise state population are neatly transferred from the ground to the excise state. So, by using these kinds of pulses either which is something which is a dark pulse which does not create any change in population in spite of its interaction with the system or an inversion pulse which changes the population from the ground to excited state on interaction with the laser.

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The slide features a logo on the left consisting of overlapping yellow, red, and blue squares. The title "Ensemble CNOT Gate: Semiclassical Approach" is centered at the top. Below the title is a truth table with four columns: "Shaped Pulse", "A", "B", and "A⊕B". The table is divided into two sections: "Inverting Pulse" and "Dark Pulse". The "Inverting Pulse" section has two rows where A=1, and the "Dark Pulse" section has two rows where A=0. Below the table is a paragraph of text explaining the quantum mechanical ensemble B and its interaction with control pulse A.

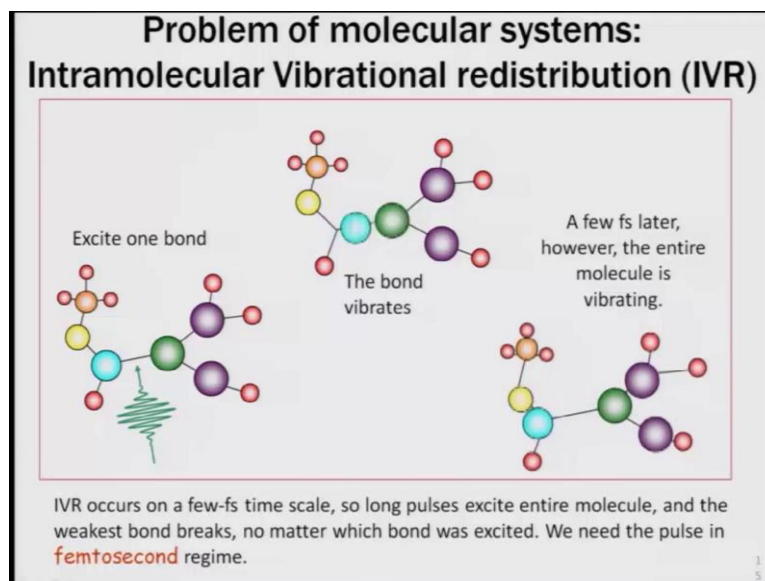
Shaped Pulse	A	B	A⊕B
"Inverting" Pulse	1	1	0
	1	0	1
"Dark" Pulse	0	1	1
	0	0	0

A quantum mechanical ensemble B that can either be in the **ground (state 0)** or **excited (state 1)** interacts with the control pulse A, which provides robust chirped pulse **inversion (condition 1)** and the self-induced transparency (SIT) or **dark pulse (condition 0)**

It is possible to come up with an ensemble control not gate in a semi classical sense as the inverting and the dark pulse interaction creates a system to undergo changes in this particular manner, a quantum mechanical ensemble be that can either be in the ground state 0 or excited state 1, interacts with the control pulse A which provides robust chirp pulse inversion condition 1 and self induced transparency set your dark pulse condition 0 and based on these conditions it can undergo this. So, such interactions and such gates have been shown to operate modeled based on these principles.

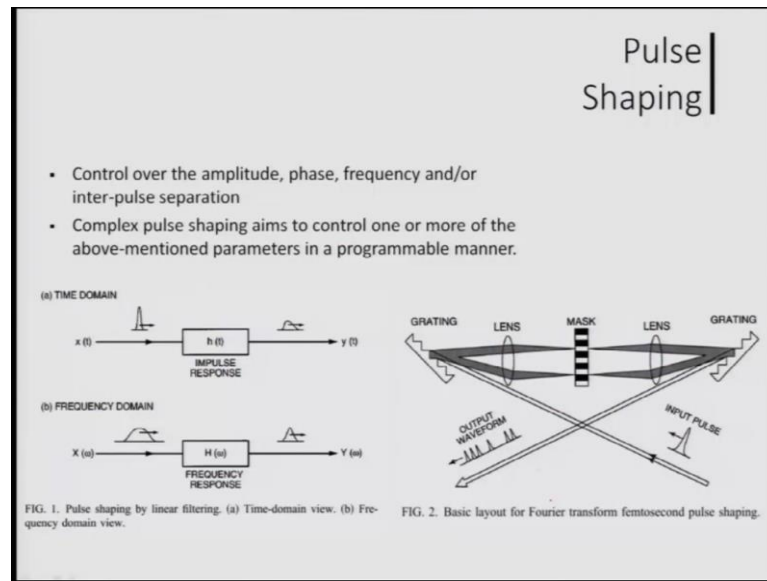


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So, the overall problem that we have been discussing is the idea of molecular interactions due to intermolecular redistributions and whenever molecules are going to be used for quantum computing, these are important steps to be understood because no matter how hard we try to excite a single bond and even if a single bond vibrates within a few femtosecond. However, the entire molecule starts vibrating and that is because of this principle that we just showed that the energy is not possible to be localized and they leak out and start vibrating as soon as time elapses.

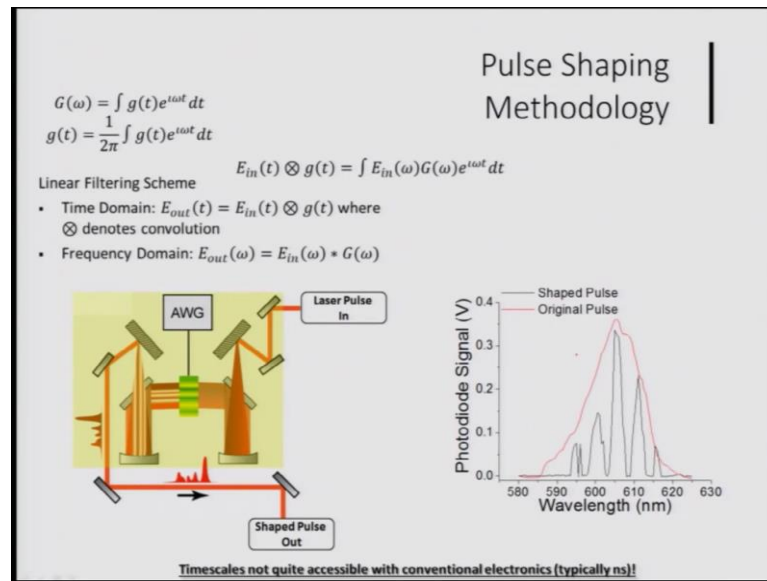
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So based on this; what we had discussed about was the use of shape pulses and the principle of shape pulses is something where we showed already that it has been utilized in Fourier domain by either using modulators or masks or programmable components to be able to create the output wave front which is going to be based on the pulse shaping principles that are necessary.

So, this is the overall idea of pulse shaping and typically indirect approaches using Fourier transforms are necessary because the interaction time scales for shorter and shorter pulses become very difficult to be of much use when the pulses become very short and therefore, it is much easier to have this process happen when it is converted into the Fourier domain and the interaction works without much of a problem.

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So, this methodology is essentially is linear filtering scheme either in the time domain or in the frequency domain. In time domain it is convolution whereas, in frequency domain it is a multiplication and vice versa depending on how it is being utilized. So, this has been a quite a useful technique which has been applied for many cases in terms of optics approaches and some of the technologies that we discussed in terms of how modulations or gates another thing can be built are based on this methodology ideas.

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### Coherence in Two Level Systems

- Linearly polarized laser pulse  $E(t) = \varepsilon(t)e^{i[\omega t + \phi(t)]}$  to  $|1\rangle \rightarrow |2\rangle$
- $\phi(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + \dots$
- $\dot{\phi}(t) = b_1 + 2b_2 t + 3b_3 t^2 + 4b_4 t^3 + \dots$
- $\dot{\phi}(t) = \sum_{n=1}^{\infty} n b_n t^{n-1}$
- Louville equation  $\frac{d\rho(t)}{dt} = \frac{i}{\hbar} [\rho(t), H^{FM}(t)]$  (numeric integration)
- $H^{FM} = \hbar \begin{pmatrix} \Delta + N\dot{\phi}(t) & \frac{\Omega_1}{2} \\ \frac{\Omega_1^*}{2} & 0 \end{pmatrix}$  &  $\dot{\phi}(t)$  is an additional resonance offset
- $\Delta = \omega_R - N\omega$
- $\Omega_1(t) = \frac{k(\mu_{eff}\varepsilon(t))N}{\eta}$      $\Omega_1 = \frac{k(\mu_{eff}\varepsilon^*)N}{\eta}$  (Multiphoton Rabi frequencies)
- For  $|1\rangle \rightarrow |2\rangle$   $\omega_R = \omega_2 - \omega_1$  is the single photon resonance frequency
- Assume  $\mu_{eff}^N = \prod_{n=1}^N \mu_n$  (Neglect virtual level dynamics for multiphoton interactions)



**Notation**

$|1\rangle \rightarrow$  Ground eigenlevel  
 $|0\rangle \rightarrow$  Excited eigenlevel  
 $\omega \rightarrow$  Laser carrier frequency  
 $\varepsilon(t) \rightarrow$  Instantaneous Amplitude  
 $\phi(t) \rightarrow$  Instantaneous Phase  
 $\dot{\phi}(t) \rightarrow$  Rate of change of  $\phi(t)$   
 $^{FM} \rightarrow$  Rotating Frequency

Modulated Frame  
 $\rho(t) \rightarrow$  Density matrix. (Diagonal represents populations in the ground and excited states; Off diagonal represents coherent superposition of states)  
 $H \rightarrow$  Hamiltonian for an N-photon transition  
 $\Delta \rightarrow$  Time independent detuning

In terms of coherence of the two levels; linearly polarized light which is available from such laser pulses can make the two states go through the interaction where they are getting coupled in the dipole limit, which can then be written in terms of either the phase been expanded in terms of the Taylor series is or its derivative which is the frequency and the entire process of their interaction when it is being looked at theoretically is by done through simulation of the Louville equation which is a time dependent form of evolution for the ensemble rather than a single wave function and we have done a lot of density matrices and its interactions throughout this course and we have also mentioned the time dependent evolutions of these density matrices which are nothing, but the Louville equation and the result of these essentially lead to the numeric integration of how the equation or how the system evolves with time as the interaction goes on.

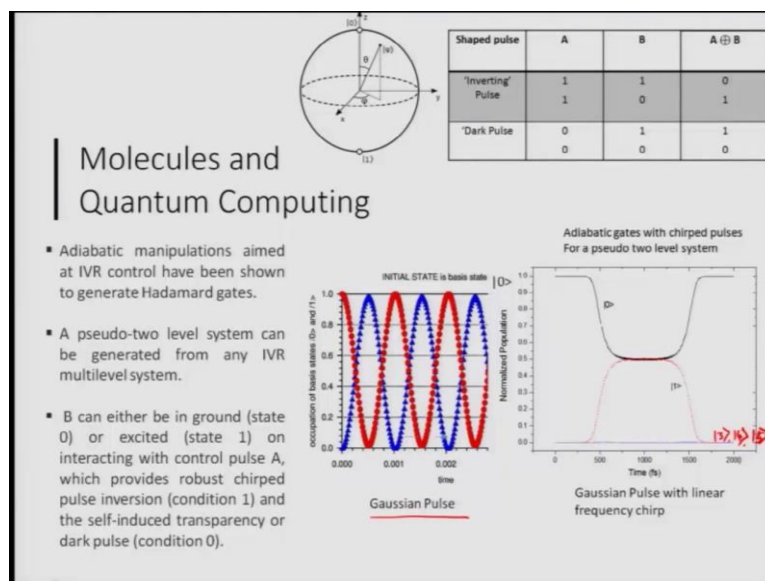
The time dependent phase change is the frequency change which occurs within the pulse and this appears as additional resonance offset for the frequency modulated Hamiltonian what is been used when these are been continuously interacting. So, these are the different parameters which are used and the Rabi is the exact resonance energy gap, the difference in energy from the applied field to the energy gap of the two states is the omega R is the resonance energy gap and the difference of this energy gap to the applied energy field is the one which gives rise to the detuning delta as it been shown here. Now

if it is not a one single photon interaction, but if it is a multiple photon interaction then it can be in terms of  $N$  photon interaction and therefore, the overall detuning can be written for the most general case as  $\omega R - N \omega$  which could then be related to the energy of the applied field and the dipole moment and all these properties which can be effective values in terms of multi photon conditions.

And they can be made to work in similar ways as the single photon problem has been developed over the years. There are some cases where these simple logics will not work, but in most cases when a single photon property does not exist and only multi photon properties are going to happen then this simplification is possible and that is often true when 1 intense laser pulse interacts with a simple system. So, here are the basic notations of the states or the system that we have used in this frame and rotational frequency is the frequency over which the system is rotating. It is similar to the principal of how we are standing on the earth and still not facing the fact that the earth is rotating all the time. So, that is the modulation frame that we are in where the effect of the overall rotation is not going to be of much effect on the system which is being looked at.

So, these are the different ways of setting up the problem so that this can be modeled to get to the results that we have been discussing in terms of the applied electric field with the system interacting. It has to be noted though similar principles work also for magnetic fields and so similar ideas of these properties can be extended to magnetic interactions also.

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So, here is the cartoon of the block sphere that I was trying to draw in the other particular case and we have also mentioned before in one of the lectures that when we have mix states instead of pure states then these points that we have they do not actually traverse only on the surface they are not coming on the way to the surface, but those are for cases when they get mixed, but in for pure states the representation is only on the surface of the states that we are looking at sphere.

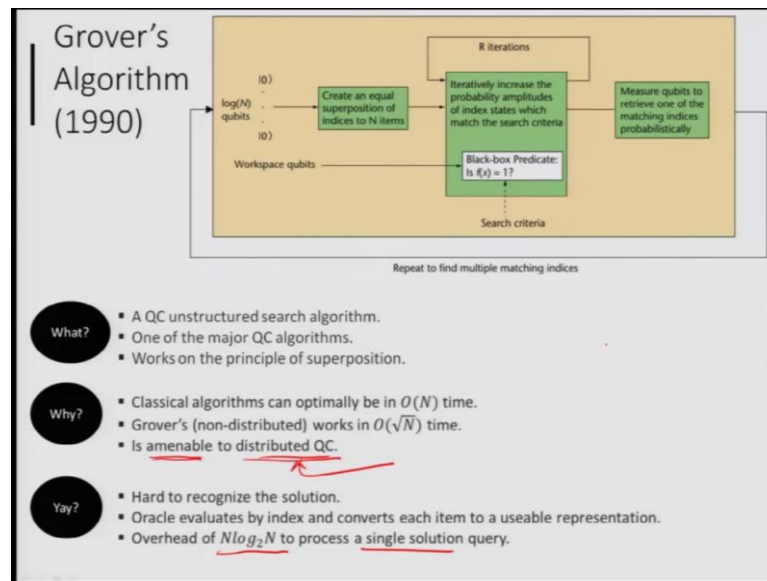
So, in terms of molecules and quantum computing adiabatic manipulations are aimed at IVR control which have been shown to generate Hadamard gates which is how it is being shown here in this case where two level system interaction is going to have them become equally probable and they can be put to work together. A pseudo two level systems can be generated from any IVR multilevel system because we are able to turn off the interactions with other states, which are all bunched here can be 3, 4, 5 for instance; depending on the number of states that have been used.

The street could be either in the ground state 0 excited state on interaction and this is what we just described in terms of the control not that we talked about, when we interacted with the state. In terms of a simple interaction with a laser pulse, the system keeps on oscillating between ground and excited state; however, when we have the

adiabatic principle applied and we apply chirp pulse it is possible to have only one interaction or the other.

So, here is for example adiabatic gates with chirp pulse pseudo two level system which are generated based on the idea that the other states are not interacting with the system.

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So, these are basically the points which we have looked at, it is then possible to extend such an optical scheme into a case where we can apply the Grover's algorithm. So, in general a Grover's algorithm essentially utilizes a superposition of qubits, which is being iteratively manipulated to increase the probability amplitudes of the index state which match search criteria and the search criteria is provided through in oracle and then finally, the qubits are measured to retrieve one of the matching indices probabilistically and so this is being followed multiple times for the matching indices to work. So, it is basically a quantum computing unstructure.

So, we need a unstructured search algorithm. So, the Grover's algorithm is a unstructured search algorithm in the quantum computing sense and it is one of the major quantum computing algorithms the other one as we know is the source algorithm, it works on the principle of superposition typically classical algorithms can optimally work in order time

$\sqrt{N}$  by 2 to be precise minimum and Grover's non distributed principal works in order root  $\sqrt{N}$  time and one of the things which is being now we will look at is whether it is possible to distributed quantum computing.

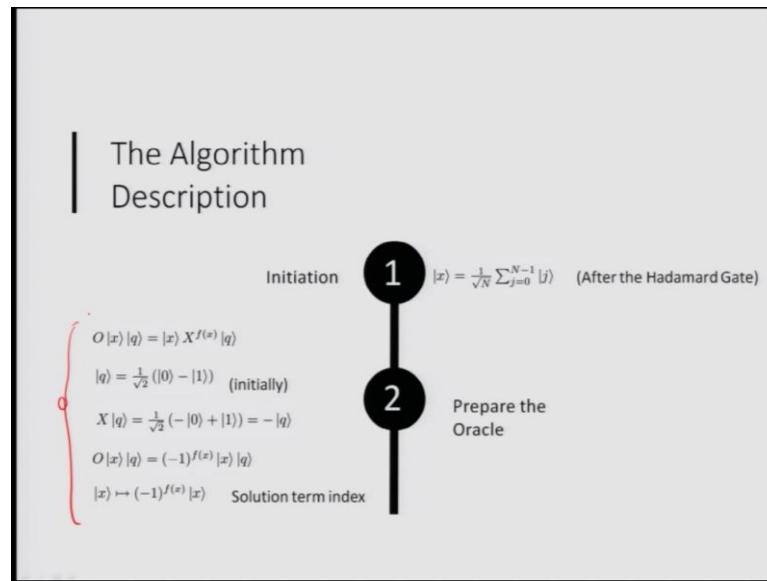
As until now what we have been discussing is the principle of only using a single quantum computer. Why would anybody think of distributed quantum computing? That is because many of the issues of implementing quantum computing that we have done throughout this course have come to the point of saying that scaling of the quantum computer is always a problem. So, one of the ways to get rid of this problem would be to be able to have a principle of distributed quantum computing so that the smaller nodes of available quantum computers can be put to use to develop a larger quantum computer.

Now the easy or the hard part of the problem is that it is hard to recognize a solution, when we are going to go ahead with the distributed quantum computing. Typically the oracle which is used in Grover's algorithm evaluates by index and converts each item to a usable representation and this can be utilized to ensure that when the system is getting into the distributed quantum computing mode, this particular oracle evaluation is put to a proper use and the overhead of this process can be put back into the principal to see if we can have an advantage and if one goes by this approach, it is found that given an overhead of  $N \log_2 N$  in addition to the single solution query approach, it is possible to have a distributed quantum computing approach to Grover's algorithm.

Now, this is a very important step because as this is developed it essentially brings back the scalability issue of quantum computer into practice and so an optical approach of doing scalable quantum computing in this sense is very effective. The reason why this would work most fruitfully with optical computing approaches is because the optical qubits are the best in terms of distributing because they are the bits, they are the qubits which can be made to be distributed across different nodes in terms of other kinds of computing environments like NMR or other principles, it may be very difficult to do that, but in case of optics this is possible. So, that way it is an attractive approach.

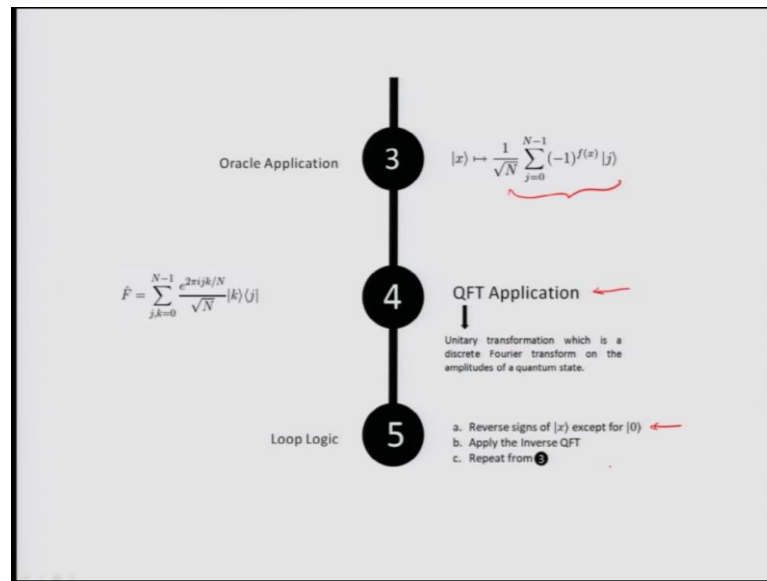


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So, the description of the algorithm is similar to what it is been always, it has the initialization step which is essentially a what is gotten after ahead amount gate in terms of all the states been put as equally state. Then there is an oracle which sets the stage for the entire problem to work and that part is a critical part in terms of generating the preparing the oracle and that we have done several times is been provided here for completeness in this second step here and the important part is to recognize that we have a solution term index which we are going to use as we go ahead.

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The oracle is an applied to the initial states that are prepared and the better way of this entire Grover's algorithm process involves the QFT application process where it is a unitary transformation which is a discrete Fourier transform the amplitudes of the quantum state which is being done in the subsequent step and finally, we can go through a loop where the oracle and the QFT keeps on applying on each other before doing that; however, would be reversing the sign of the quantum state in question and then keep on applying the inverse QFT and this particular loop approach, it is possible to get to the marked signal to be amplified and that is the basic point here in terms of doing the Grover's search.

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Distribution of Grover's Algorithm (2015) LNCS

**What?**

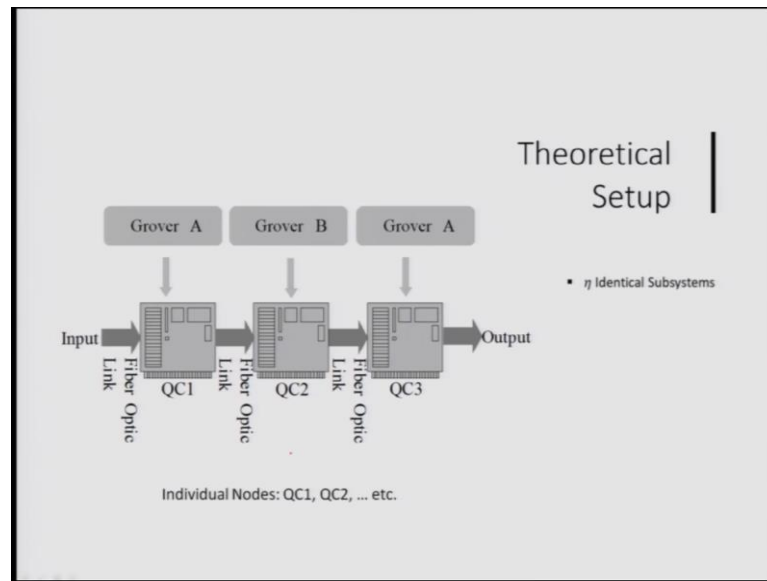
- Distributes the computing load on a set of quantum computers.

**Why?**

- Single query solution has an overhead of  $N \log_2 N$  steps to process and prepare the query.
- Allows for a virtually large pool of available qubits by networking and data sharing.

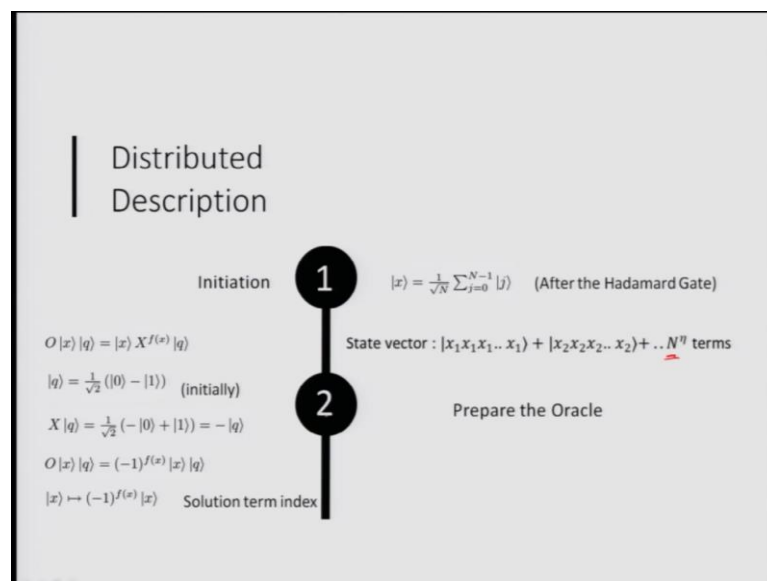
In order to distribute the Grover's algorithm which was first proposed as late as 2015 lecture notes of computer science, it was possible to show that it distributes the computing load on a set of quantum computers and the critical part on this was a single query solution has an overhead of  $N \log_2 N$  steps to process and prepare the query and this allows for virtually large pool of available qubits by networking and data sharing.

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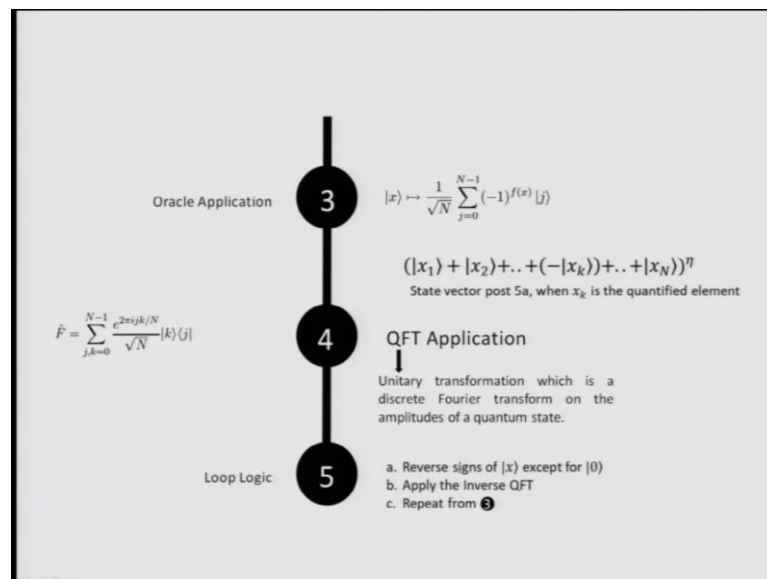
So, here is the principal where eta identical subsets have been setup and the interaction of these subsets which are quantum computing nodes are being connected through optical fiber links because the information content or the points of interaction with them can be done through optical connects and so this is how this principle have been looked at through various different aspects of the nodes being put to work simultaneously.

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And in this case also the principle is the same, it has an initiation process where the states are all in equilibrium; equally placed and each state vector is in  $N$  to the power eta terms because they have been put in each of these individual qubits states; individual quantum computers  $q \in 1 \ q \in 2$  for example, the second step involves preparation the oracle which again remains the same as before in terms of the regular Grover's algorithm.

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Application of the oracle is now going to be on most; the oracle will now be applied to each of these different quantum states and the state vectors would then be prepared as a result of the application and then the quantum Fourier transform which is the unitary transform of discrete Fourier transform; the amplitude of the quantum state will again be operating on them. Each of these states will then undergo the interactions as we have discussed before except that the reversal of the states that we now do in terms of these as we have discussed is going to work for all except the zeroth state and we apply the inverse of QFT and repeat the loop from step three by the oracle application process starts.

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The slide is titled "Results and Caveats" and contains the following content:

- Results**
  - After  $n$  cycles.  $(|x_1| + |x_2| + \dots + (2n+1) * |x_k| + \dots + |x_N|)^2$
  - General Case is  $4n^2 K \pm O(\sqrt{K})$
  - Small sizes (eg.  $n = \sqrt{\log_2 N}$ )  $\rightarrow 4 \log_2 N \pm O(1)$  when  $\eta$  is of the order  $N$
  - Upper limit of applicability is  $n \leq \frac{\sqrt{N}}{2}$  else all the subsystems are in the qualified state.
  - At  $n \leq \frac{\sqrt{N}}{2} \rightarrow O(\sqrt{N})$  overall

Annotations on the slide include:

- A red box labeled "Caveats" containing: "Classical limit for small sizes." with an arrow pointing to the small sizes result.
- A red box labeled "Note" containing: "Distribution for a small size is impractical anyway." with an arrow pointing to the upper limit of applicability.

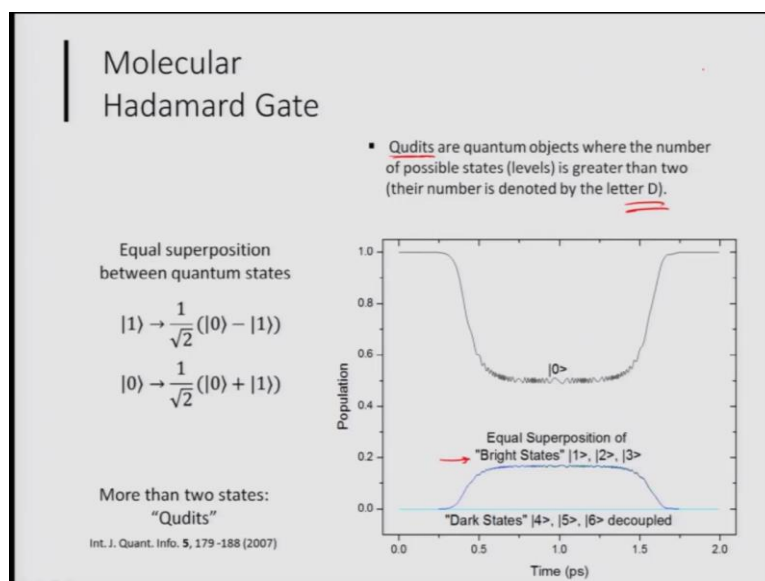
So, this loop continues and what we generate after  $N$  cycles is a case where each of these have been undergoing independently the process. The general case is that would be an order of root  $K$  and the advantage is that when there are small sizes then that is when your  $\eta$  of this order of  $N$  then it goes back to the original condition of log base  $N$  and the upper limit of applicability is when  $N$  is less than half of root  $N$  else all the subsystems are in the qualified state.

So, overall we find that we have a limit over which this particular process is possible to give rise to the same order as per the Grover's overall club order of applicability. There are a couple of (Refer Time: 44:19) to worry which is the classical limit is what the answer comes out for smaller sizes and so when the size gets really small, then there is no advantage of going into this distributed mode because it finally, goes back to the classical search condition, there is no advantage in some sense by applying this.

However a point to note is distribution for the small size is impractical anyway. So, nobody would want to do a distribution for a very small size of a search algorithm. The reason why it is going to be applicable is the case when the size of the problem is large enough that you would like to distribute the problem. So, the point to note here is that unless the problem size is large; there is no advantage or there is no point of given

distributing this principle of quantum computing or in this particular case of doing Grover's algorithm in this particular manner. However, when the data size is large and it is not possible to be done by using the traditional Grover search algorithm because of the size of the problem, it does make sense to distribute the problem as there is an advantage while going into the system.

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And last, but not the least I thought, I will mention the molecular Hadamard gate, so many times, I should also show it here and this is one of the things where in the use of these many many states which are these IVR states that we have been discussing; it is possible to in fact create a situation where all the states would have equal superposition between the quantum states and this could be possible, if we have a bunch of states which are coupled to the ground state and once this couplings work, then we call them bright state.

So, in typical approaches we have used only one bright state with respect to the ground state and the rest of them are all states which are going to take away the energy from them so that we do not want them to be coupled and so we call them dark states as we can decouple them. However if there are states which are directly coupled; dipole couple to the ground state.

So, that they all get excited when the energy is available and by applying of principle of adiabatic sweeping of the laser, we can keep the unwanted states to draw energy away from these optically couple states, we will be having unequal superposition of bride states and such multiple coherent states; if can be generated then they are nowadays called qudits which are quantum objects where the number of possible states or levels is greater than 2, the numbers are denoted by the letter D and this was first given by Stroud as I mentioned in this particular work more than 2 states called qudits.

So, that was the final set that I wanted to mention and with that I would like to end today's lecture on the optical aspects of quantum computing that in this particular end state, we have managed to show a little bit more than what we have actually discussed earlier; in the sense that there is a possibility in research at least or in principles that has been developed as of now that there is indicated that distribution of quantum notes or architecture may perhaps be possible; although this is too preliminary to make those statements. But if they are in the scalability would become important and with that I would like to thank you and see you next time.