Implementation Aspects of Quantum Computing

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Lecture – 33
Implementation with Solid-State Superconducting Qubits

We have been looking at quantum computing implementation aspects with regard to the quantum mechanics per say. So, carrying on in that direction I start off with the idea of quantum computation in the process of how these steps are to be labeled.

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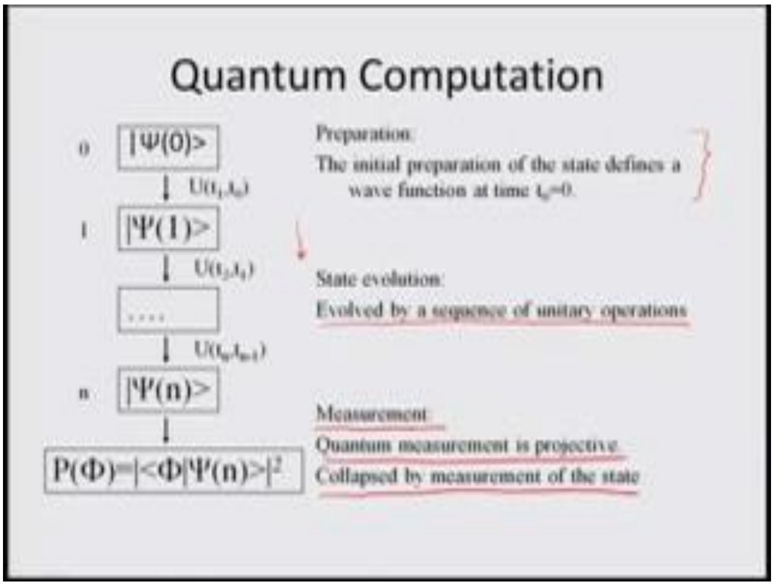

So, the 0th step or the first step is essentially the preparation phase by the initial preparation of the state defines a wave function at time t starting point t not equal to 0, so that the first step. And the first unitary operations should take it to the state which could be our state of interest which is applied to let say the unitary operations or logic gates which go ahead after that. So, these particular eventual operation procedures are sort of like state evolution operations which are evolved by a specific sequence of unitary operations.

And finally, is a measurement process where the quantum measurement is going to give rise to the projective component. And the final step is the measurements, where the projection will be made as quantum measurement is projective in their character. And essentially it is collapsed by measurement of the state. So, that is basic principle which we have been following.

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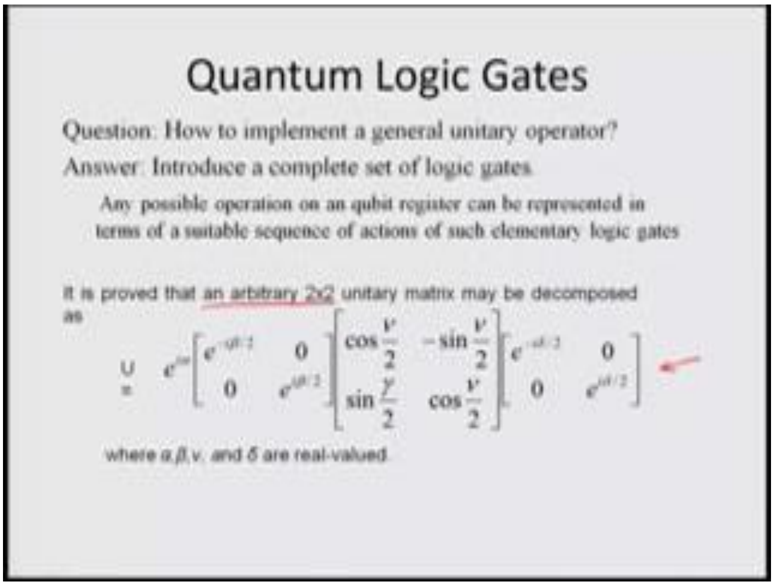

In terms of the logic gates we have been discussing the various kinds of the logic gates. The definition can be simply put as any possible operation on a qubit register that can be represented in terms of suitable sequence of actions of elementary logic gates are the ones which are in use. Introducing a complete set of logic gates are hard because, unlike classical computers the number of logic gates needed as the number of qubits increase can be very large. It is true that for a single qubit operation they are finite number of logic gates which can be defined, which could do the entire computation that are necessary. But, when it go to higher number of qubit this is not quite possible to do it.

Here is an example of a unit reoperation which sort of under goes let say the phase shift versus rotation versus once again another kind of a phase shift. Any in general it can be proved that any 2 by 2 unitary matrix can be decomposed into various kinds of rotations or amplitude changes or phase changes. And thus, the unitary operations have always possible to be labeled in terms of these kinds of several single qubit operations that can be looked at. This is true as I mentioned earlier for a single qubit concept.

So, that is the basic points which are being stated here. For any arbitrary 2 by 2 unitary matrix may be decomposed into 1 2 3 4 components, because this is how these can be represented as.

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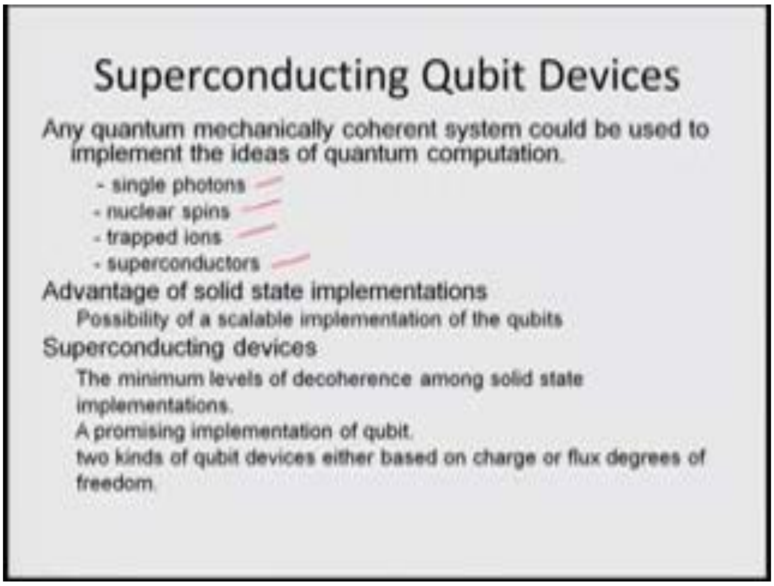

We have been discussing a lot about the superconducting qubit devices. So, let us look into this a little bit more in conjunction to these gates and qubits that we are talking. In general any quantum mechanically coherent system could be used to implement the ideas of quantum computing as we have talked about. And we discussed single photons earlier; nuclear spins NMR, trapped ions, atoms that we just discussed in last lecture. And superconductors are the other kind it will be looking at now.

Advantages of solid state implementations obviously rely on the facts that are a possibility of this scalable implementation is quite high in these cases. And superconducting devices provide the minimum levels of decoherence among solid state, partly because the energy loss for these cases are very minimal. And so this is one of the biggest advantages. So, it is a very promising implementation of the qubit.

There are two kinds of qubit devices either based on the charge or the flux degrees of freedom in these particular cases.

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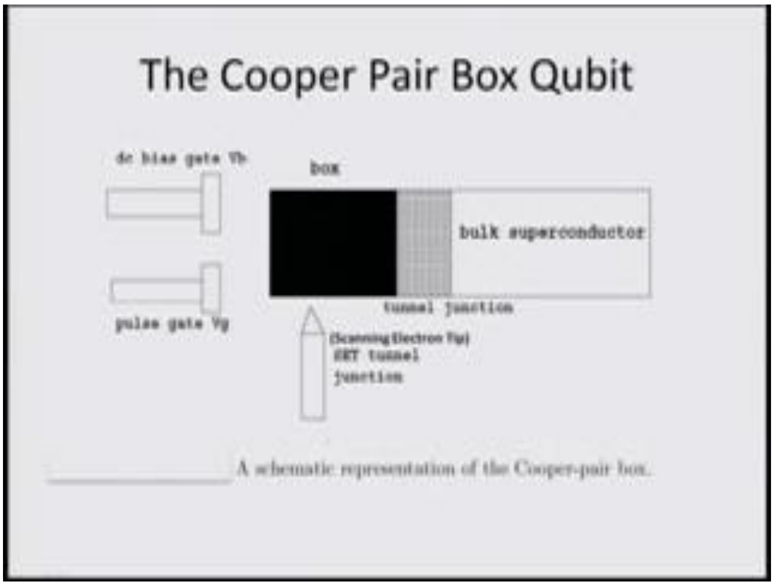

And based on these superconducting qubit devices can be looked at. The cooper pair box qubit is based on the idea that it has the it has the cooper pair look at in two parts one is the bulk superconductor and there is the part which is the main part which is connected to the box and has the d c biased gate, v b or the pulse gate v g is (Refer Time: 05:34) electron tip measures the ACT tunnel junction values, and this is the volts and this is the value which is being measured in terms of the tunnel junction signal that is been seen. This is how the schematic representation of a cooper pair box looks like.

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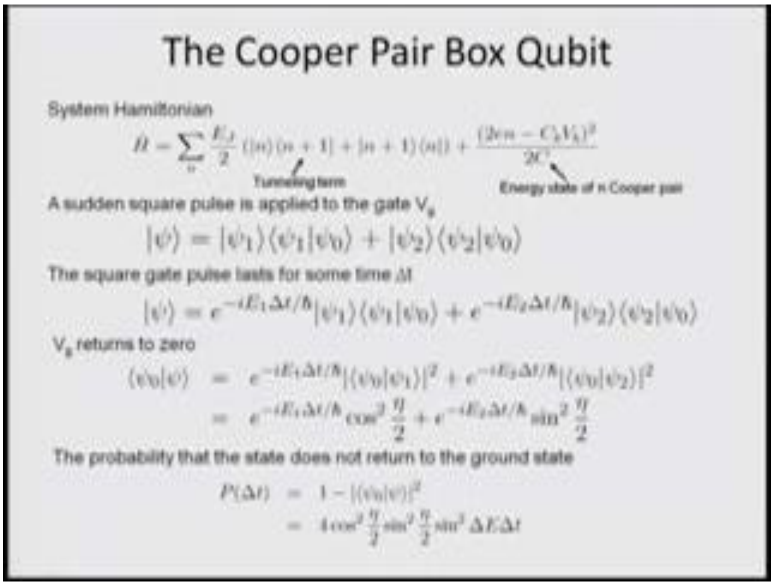

And mathematically we have this Hamiltonian, the cooper pair box qubit has a system Hamiltonian which is one part is the tunneling part, the other part is the energy state of the nth cooper pair. The two components can be controlled by using the d c voltage or the pulsed voltage. When the certain square pulse is applied to the gate by using the pulse scale v g then the coupling occurs between the states. So, we have a coupling between the 0 and 1 state as well as the 2 and 0 state.

The square gate pulse lasts for sometime delta t which ensures that there is a temporal evolution of the system and that is given in terms of these Heisenberg integrated form. And then this square gate pulse lasts for sometime delta t, which means that the system propagates in terms of the Schrodinger equation and as a result there is this exponential term which appears. When the v g returns to 0 then the resultant is a projection of the state with respect to the currently warm system.

The probability that the state does not return to the ground state therefore, can be seen as 1 minus the probability of the return and that gives rise to values which is shown here. And it depends on the energy gap and the amount of time which is been applied.

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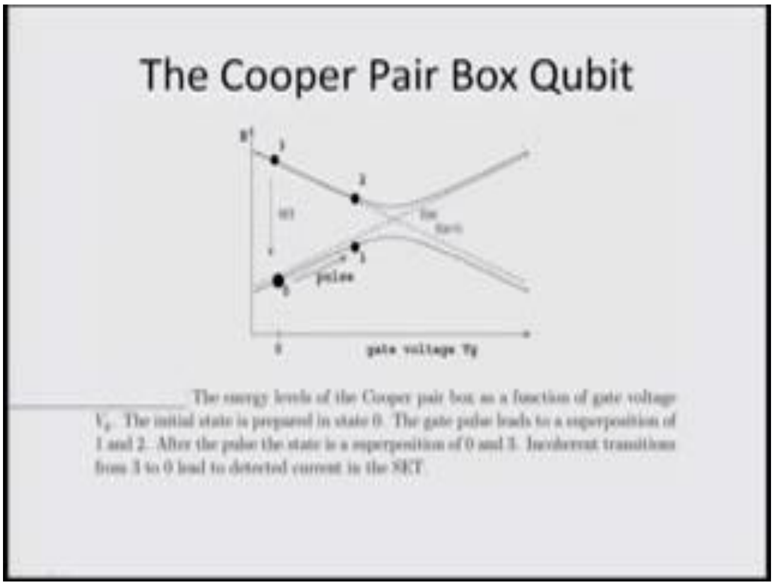

The cooper pair box qubit therefore, would evolve in terms of the applied gate voltage as shown here. The scanning electron tip voltage will be registered in terms of the value which is seen. The initial state is prepared in state 0. The gate pulse leads to a superposition of 1 and 2. After the pulse the state is super rings super position of 0 and 3; incoherent transition from 3 to 0 lead to the deducted current in the scanning electron tip.

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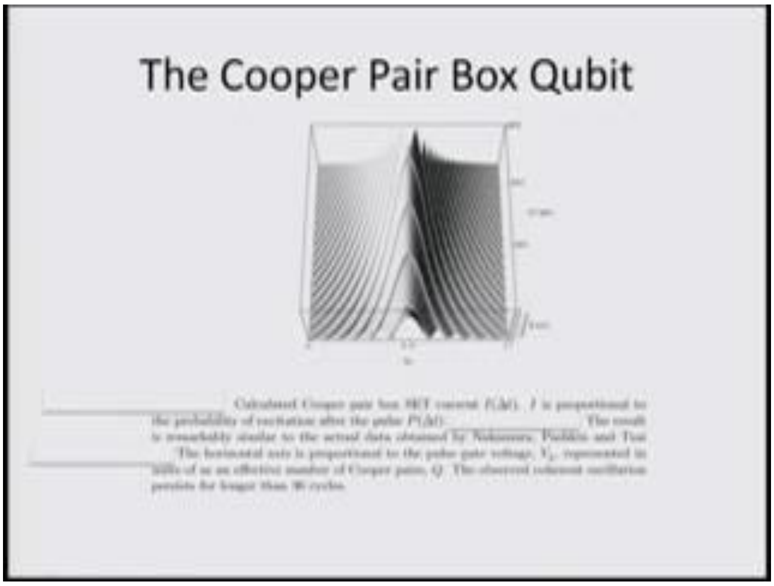

The cooper pair box qubit therefore has a pattern which essentially shows that as the current changes in terms of the applied voltage it is proportional to the excitation after the pulse. The result is remarkable similar to the actual data which have been seen by these (Refer Time: 08:48). And the horizontal axis is proportional to the pulse gate voltage represented by n units as effective number of cooper pair’s qubit.

We observe coherent oscillation process for longer than 36 cycles, essentially showing the quantum nature of this qubit.

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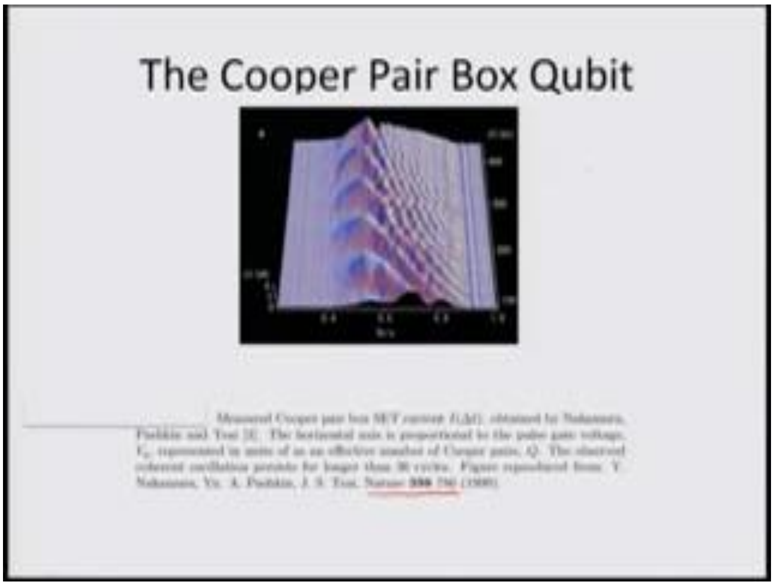

And, these have been measured by several other groups as are discussed here. This is the article which was the one where this data has been taken from. The observed coherent oscillation process for longer than 36 cycles; essentially showing that this can form an excellent coherent system which can last as long as the oscillation exists. That means, that the coherence is being maintained.

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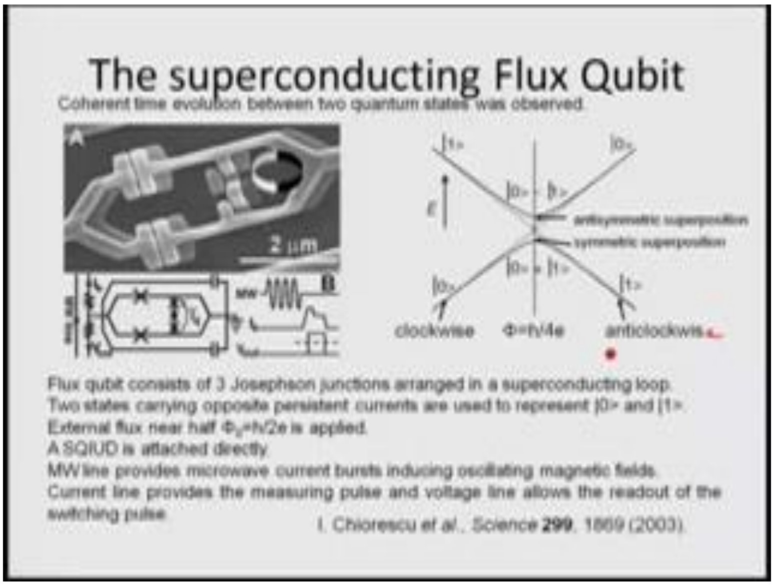

The other important aspects is the superconducting the flux qubit. So, the earlier one was the cooper pair case and this one is the two quantum states which are being looked at in terms of the flux qubit. The coherent time evolutions between the quantum states were observed and the flux qubit consists of 3 Josephson Junction arranges in a superconducting loop. Two states carrying opposite, persistent, currents were used to represents 0 and 1. The external flux near half the work function of which over 2 e is applied; this is either going counter clock wise versus clock wise.

A squid is attached directly in this particular study and so in a megawatt line provides micro wave current burs inducing oscillating magnetic fields. The current line provides a measuring pulse and the voltage line allows a read out of this switching pulse.

So, this is a technical which was discussed in this article which is shown here. And it can be shown that it is a very small device 2 microns is a sized dimension as represented. So, roughly this whole process occurs in a zone which is a few microns only.

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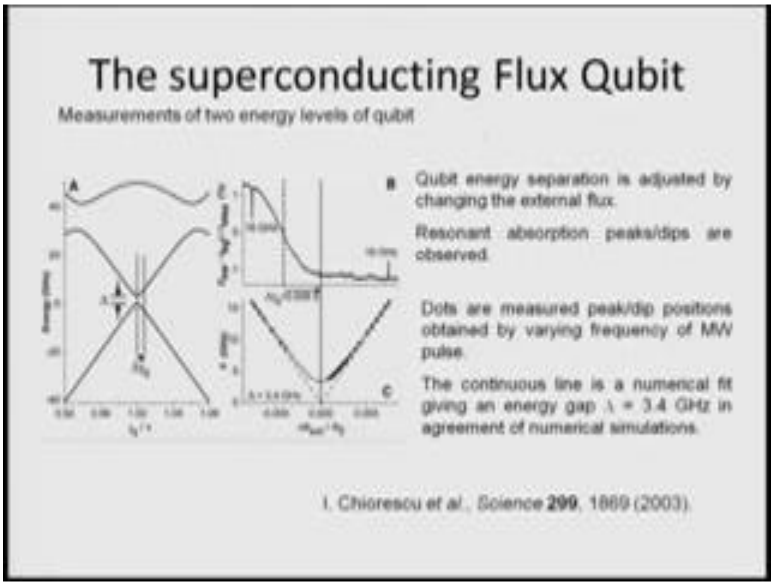

The flux qubits have been measured in terms of the two energy levels of the qubits. The qubit energy separation in adjusted by changing the external flux. The resonant absorption picks and dips are observed. The dots are measured as far as the pick dip positions obtained by varying frequency of the microwave pulse. The continuous line is a numerical fit giving an energy gap of about 3.4 gigahertz in a agreement of numerical stimulations.

Once again this is the something which has been discussed in detail in the article. We are just showing certain results, just to understand that there are different ways of using the superconducting conditions for rating the qubits.

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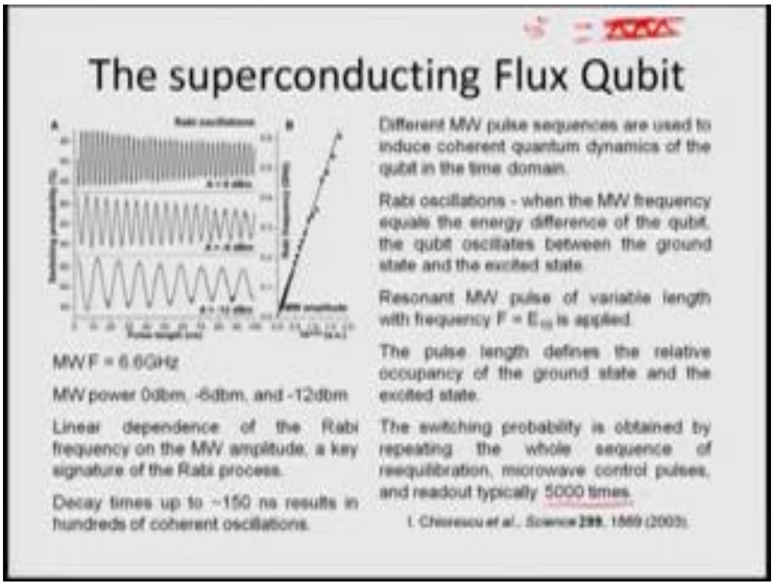

When the microwave flux is about 6.6 gigahertz, then there is a linear dependence of the Rabi frequency of the micro amplitude which is the key signature of the Rabi process. And the decay times up to 150 nano seconds in a results in 100 of coherent oscillations. So, these oscillations which have been seen here, the switching probabilities are the hallmarks of the Rabi oscillations. Rabi oscillations essentially means that the population can cycle between the ground in the excite state. In a sense that the coherence of the system is being minting between the two states as long as the field is being provided to make the oscillations work.

So, the ground and the excite state therefore have been set in such way so that there is a coherent cycling going on and that is as long as the cycling is perfect. So, it follows the science squared rule in terms of the oscillations. And different micro wave pulse sequences are used to induce coherent quantum dynamics of the qubit in time domain. Rabi oscillations occur when the microwave frequency equals the energy difference of the qubit. The qubit oscillates between the ground in excite state as mentioned. The resonant microwave pulse of the variable length with frequency that is equivalent to the gap is applied. The pulse length defines the relative occupancy of the ground state and the excite state on the switching probability is obtained by repeating the whole sequence of re-equilibrium microwave control pluses, and read out typically five thousand times.

So, this was the essential proof of the use of superconducting flux qubits.

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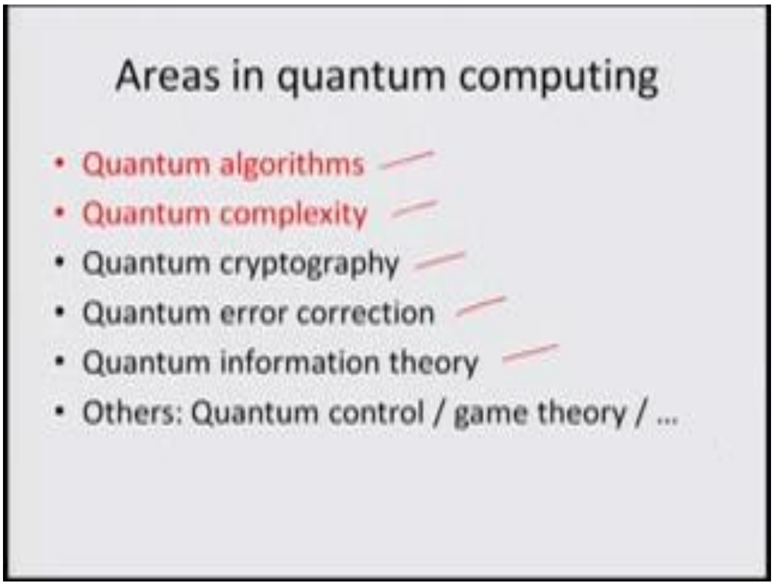

Now, let us look into a little bit of the different areas of quantum computing that we have talked about in many different ways. Quantum algorithms we have looked at, quantum complexity, quantum cryptography, quantum error correction, quantum information theory, as well as there are many others like quantum control game theory and so on and so on forth.

Let us take a stock of the developments as we do a bird's eye view of all these different applications.

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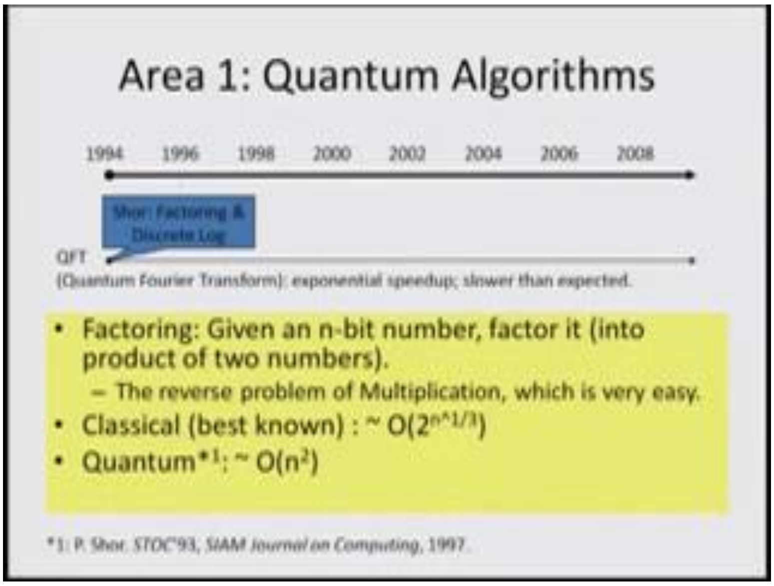

So, in terms of different areas let us see how the field also has developed in terms of chronology. The area of quantum algorithms have shown a major development form 1994 in conjunction with the quantum Fourier transform concepts which gives rise to exponential speed up. And Shor was able to show factoring and discreet log applications. It says that given an n bit number it can be factored into products of two numbers; the reverse problem of multiplication is very easy.

The classical best known system is of the order which can be presented in terms of exponential, whereas the quantum is of the order of n square. So, definitely there is a exponential benefit in going to the Shor’s case. This was first reported in the 1997 paper by Peter Shor

So, the basic discovery came in terms of the quantum Fourier transform implementation as well as error analysis which he was able to show successfully in 1994.

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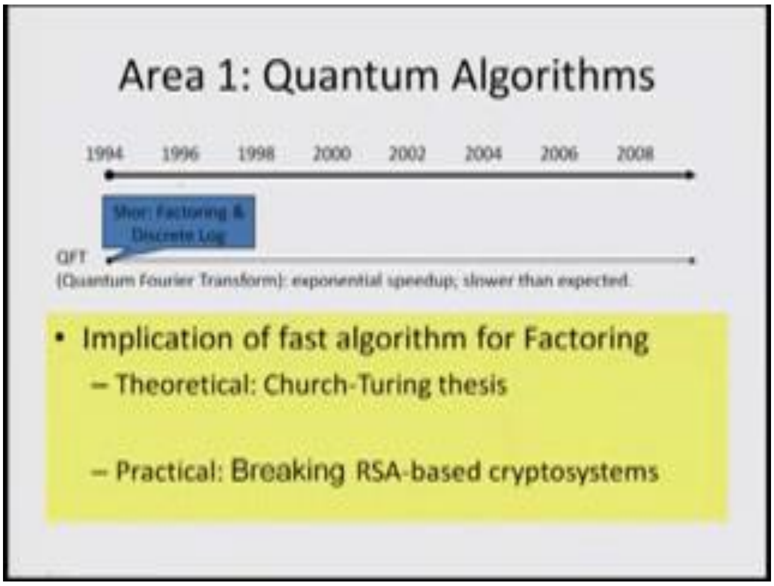

And implication of the fast algorithms for factoring has impact on the Church-Turing thesis in terms of theoretical aspects. As well as it also has the most important practically implementations in terms of the breaking of cryptosystems codes, because the RSA- the popular crypto graphic analysis of the system is based on the idea of having a large prime member that cannot be a factorized that easily. So, that is the point where the practicality of this became a very important point.

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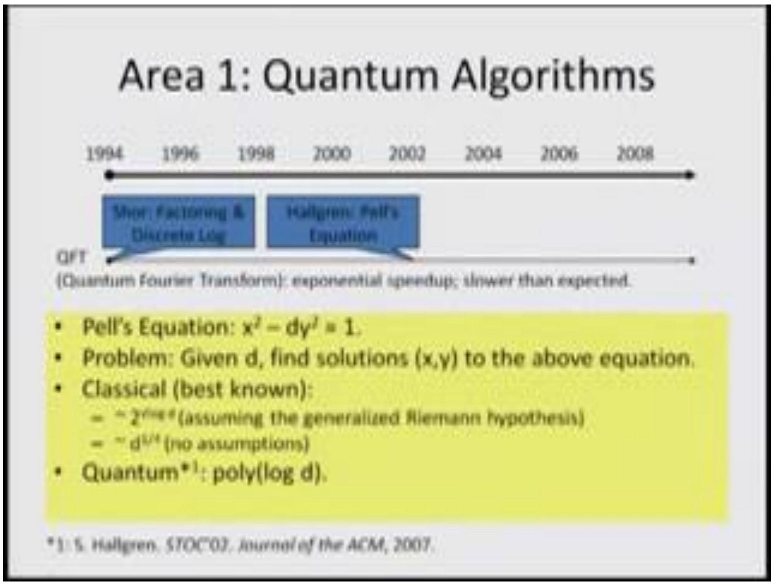

Next came in that region came some more developments which were shown in recent times where its say Hallgren Pell's equations. So, mathematical problem which finds that given a term d find the solution to x y to the above x square minus d y square equal to 1 solution.

The classical best known in this case is 2 to the power root log n assuming the generalizing Riemann hypothesis, whereas in without any assumptions its j to the power one fourth. In quantum, its polynomial in terms of log d, so this also a very important development in recent years which has come through.

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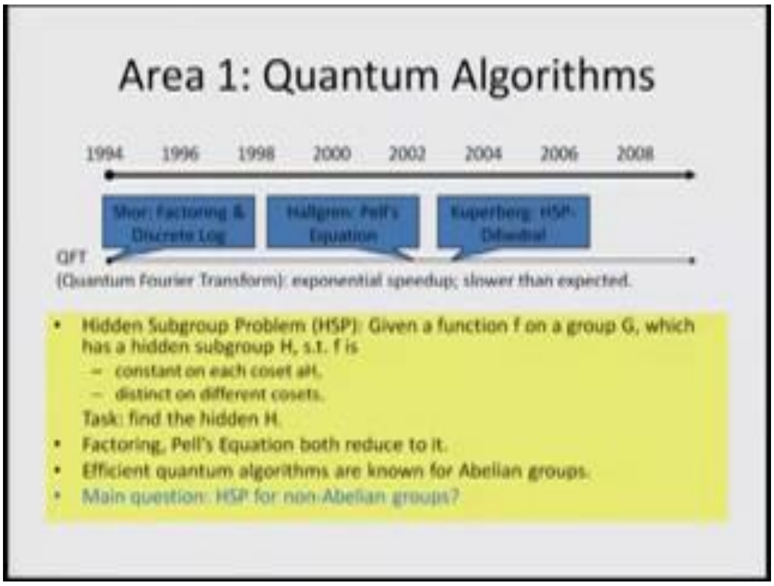

There also been some more development in terms of the hidden subgroup problem, and given a function f on group g which has a hidden subgroup H. The state function f is a constant on each co-set H, distant on different co-sets. The task is to find the hidden H function.

The factoring of these Pell's equations both reduces efficient quantum algorithms are known for these kinds of cases which is this abelian groups. The question is whether the hidden subgroup problem for non abelian groups exists or not.

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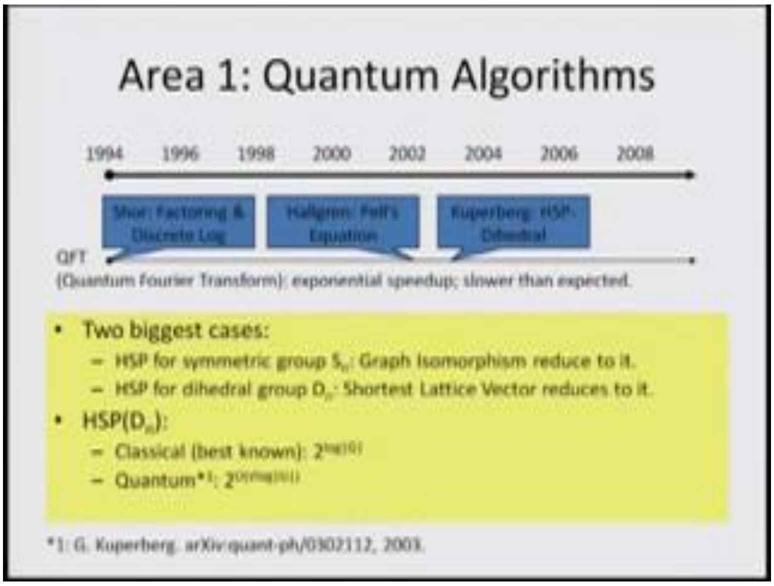

So, there are two biggest cases one is through the graph isomorphonism to reduce it which is possible for the symmetric group cases of s n, whereas the other of the dihedral cases shortest lattice vector reduces into it.

So, the classically best known for this particular case is a 2 to the power log g k case, whereas in as shown in the recent work it is of the order root log mod g.

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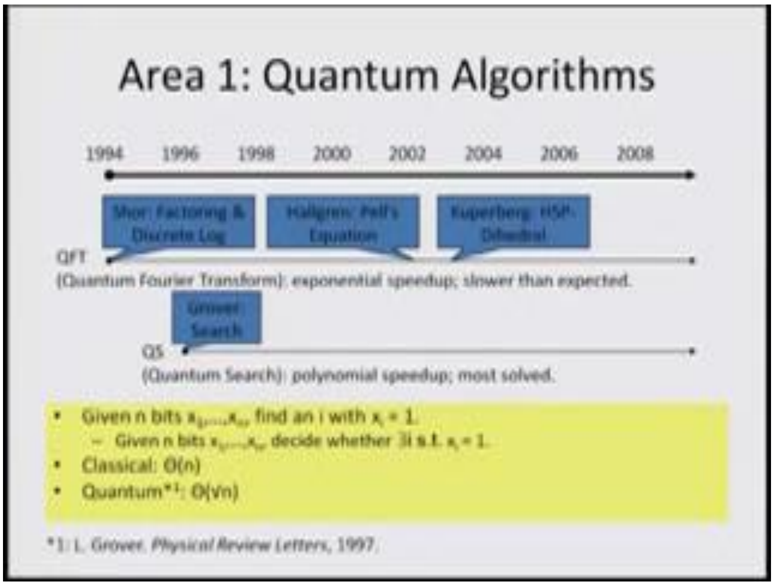

The other important development in terms of quantum algorithms has been the quantum search which shows the polynomial speed up and it is mostly important to and that is the grower search. It states that given n bits find an i with x i equal to 1 which is the solution. And given the bits we have to decide whether it stays within x i equal to 1 classical. Terms for this is the in terms of order n where is in quantum case its route n, and so that is a polynomial speedup in this particular cases. And this was first show in 1997 by Lov Grover, and since then it has became a very useful one.

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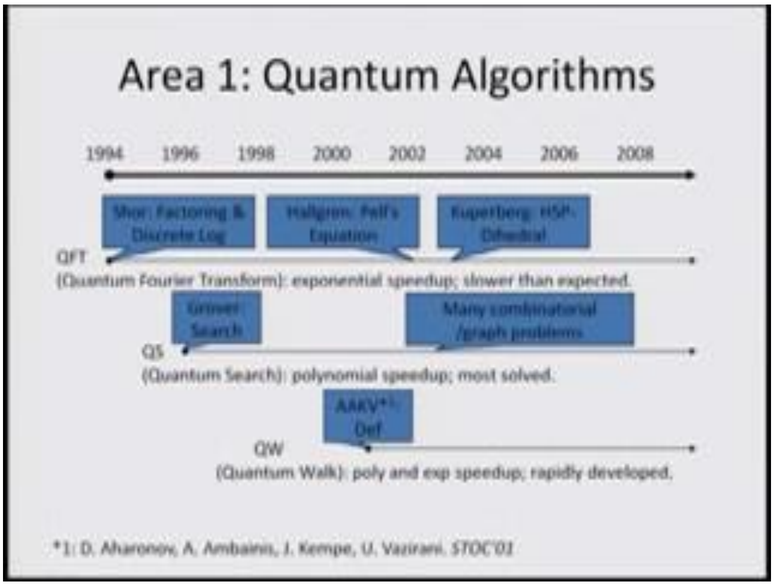

In recent years there have been many (Refer Time: 19:35) and graph problems which have also been utilizing this principle. This other area of quantum walk which helps in terms of poly and explanation speedup, and it is been rapidly developed. As a results of some of the recent work by (Refer Time: 19:53) and others.

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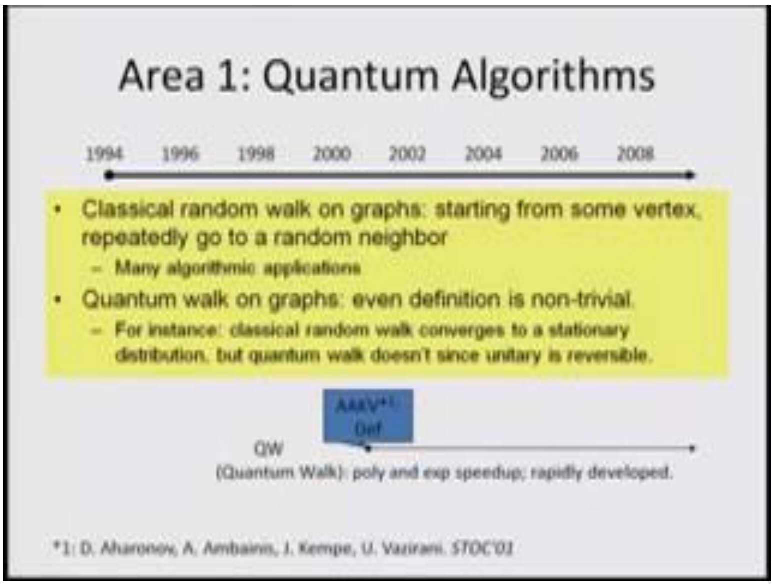

And say, classical random work on graphs starting from some vertex a repeatedly go to a random neighbor and many algorithmic applications exists on this. Is a quantum work on graphs even the definition is nontrivial. For instance, classical random walk converges to a stationary distribution, but quantum walk does not sense because it is a unitary is reversible. And therefore, suppose to have a huge development possible.

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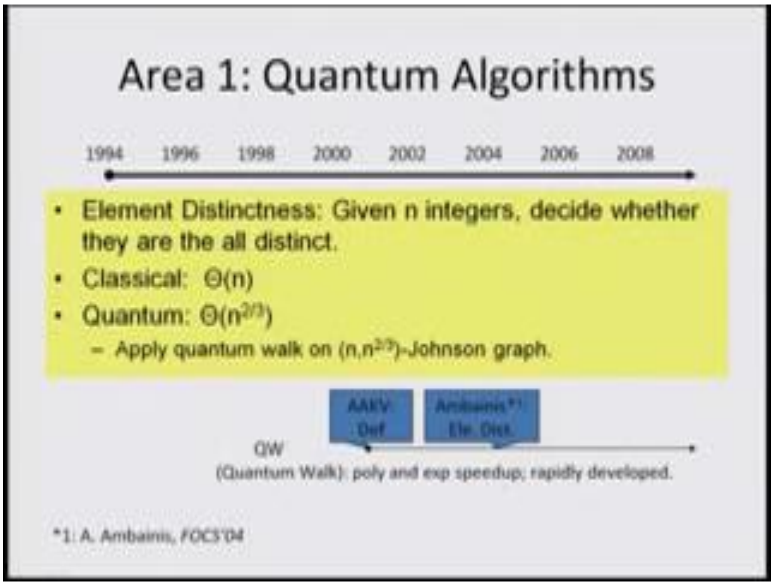

And elementary discreetness which involving n integers decide whether they are the all distinct or the same; so this is related to the quantum walk problem. In case of classically it takes order n where is n case of quantum it takes at n to the power two third and it is important for applying quantum walk on Johnson graphs and some applications in this area has been looked at. These are mostly very theoretical based problems which have been developed and some of these have been just shown here for completeness.

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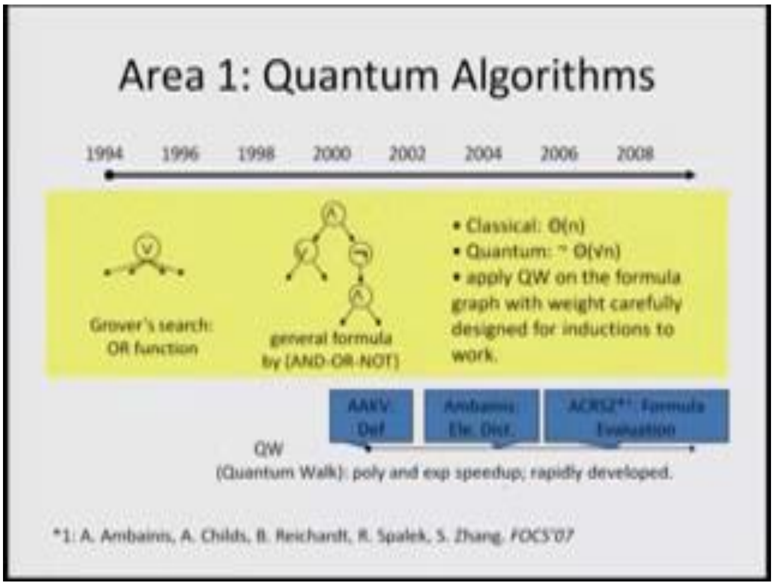

Another area in recent times has been the development of formula evolution point in the quantum walk area which utilizes the idea of the global search of function. And, in this is case this is a generalized formula and where again the classical would be of the order n has in terms of the search problem, whereas the quantum would go an in order of root n. We can apply the quantum walk principles of the formula graph by very carefully designed for induction to the walk.

So, these are the various different kinds of algorithm developments which have been seen in recent years. This is been provided just to ensures that there is the completeness in the development that we are discussing; because most of the implementation that we have looked at we are not really focused on the idea of the algorithm development much because that is more of theoretical and development angle.

In terms of algorithm we have been using most of the ideas of the algorithm developments in terms of the implement ability and showing that the implementation realistically is doing the job of the quantum problems that we are looking at.

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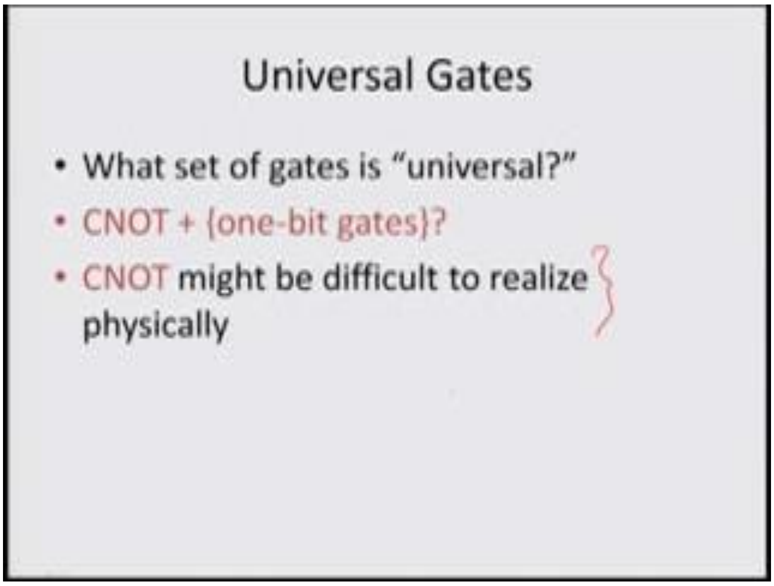

The other very important aspect of the development that we have been looking at is the understanding of the fact that we are indeed using the appropriate gates of the universal in the sense of applicability to the quantum world. As we know the CNOT which is a 2 bit gates. And this set of all possible 1 bit gates forms a complete set which we have discussed earlier for your set number of qubits. As the set size increases multiple combined qubit set gates can be redefined, but the main basis of the quaint bit gate as well as the CNOT are the important ones to initially represent. CNOT often may have difficulty to realize physically because of the implement ability aspects of quantum computing.

And so it has often found the bench mark for the idea of understanding with the implementation of the so called quantum system for quantum computing is really working in the right direction.

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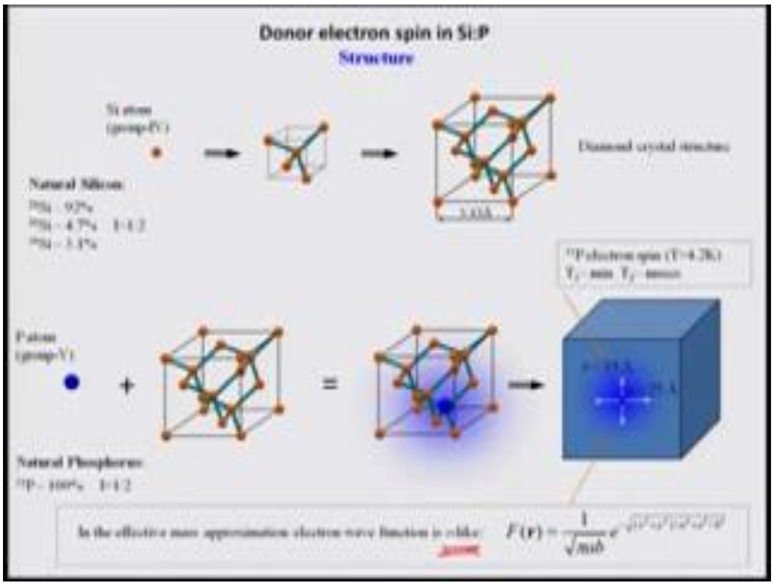

Let us in this particular case look back into one of the other implementations which we have also discussed earlier. Which is the diamond crystal structure in which we use the donors spin electron has a qubit in this case. And the idea is that the silicon atom which forms the basic diamond structure is a group four element. So, any dopant in terms of first process in this would essentially mean that its extra electrons that is coming because of the group five elements and that is the idea of having the donor electron spin as a qubit, in this particular arrangement.

So, a phosphorus 31 electron spin is the one which is being utilized as a quantum system for this computing application. Here are certain aspects of the facts that isotopes of natural silicon are the ones which are shown here; 28 silicon, 29 silicon, 30 silicon the most common abundant one is the silicon 28 which has 92 percent availability in this particular case and so it is a spin half system in terms of the nuclear part. Whereas the donor electron if we are considering from the phosphorus setup to be the spin of the system which will be looking at. And so the natural phosphorus which is P 31 also has a spin half system. Then natural phosphorus is good thing about it is that it is a 100 percent P 31.

So, in the effective mass of approximation electron way function is s like because it is single electron which is going to be around all the spin half systems of the nuclei. And so it will have a spherical cemetery for the electron way function that can be utilized.

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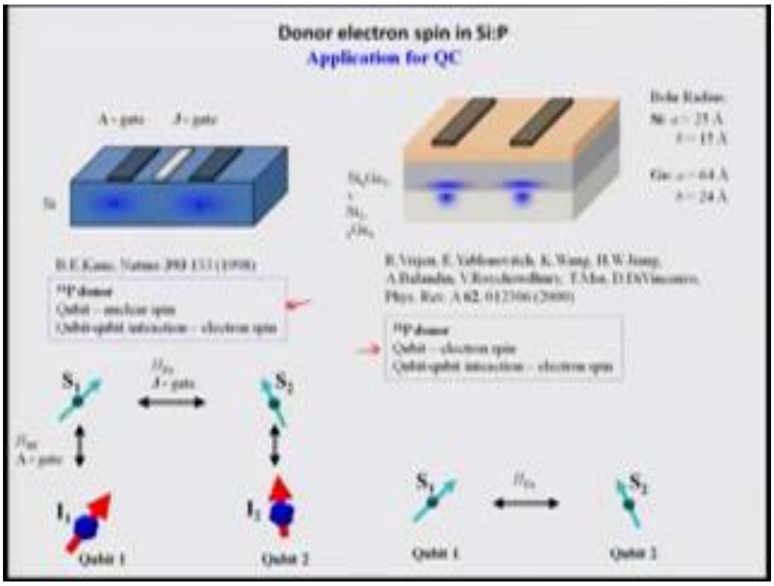

So, in terms of application for quantum computing this would be almost like a solid state atom, in some sense where there is a single electron which is being looked at. And that was the part of the work which was shown where there were two different works which basically focused on this particular area. One was the Khan’s work in nature in 1998 and followed by the work in (Refer Time: 26:55) 2000, were in both cases the donor is the phosphorus 31 and the qubit therefore is the nuclear spin and the qubit qubit interaction is due to the electrons spin.

So, this is an interesting concept where the qubit itself is the nuclear spin which is the spin half system of the silicon in one case. And silicon germanium in the other case and both cases the interaction part is provided by the donor electron where the electron spin acts in terms of interacting that to qubits. One of the most important aspects of the quantum computing qubits is the fact that not only the qubit has to be very precise and proper and be controllable in alright. However, it also needs to have a situation where the qubit may be able to couple very effectively with the other qubits.

In many cases the difficulty often has been that although a qubit has been very effective in itself as a good qubit, quote unquote in its different aspects of interactions, gates decoherence even. However, it may suffer from the fact that once the time comes for it to be able to communicate or translates is information or do something collectively the inter qubit interaction is something which often very very difficult to and take care off. So, there are situations where inter qubit interactions are going to be extremely important in defining as to how good of the quantum computing can be implemented as a implementable device.

So, in these cases their individual qubits are represented by the nuclear spins which are very conveniently being connected across the qubit qubit interaction through the electrons spin which is coming from the donor. So, that is the beauty in both these particular aspects that has been shown. And this is one example of how a spin shown here where the j gate of the particular electrons spin is being applied from the external field, whereas the internal qubits are being connected by these two cases.

So, this spin 1 and the spin 2, these exchange interaction between the two are the interesting use of coupling in the two qubits which have been enabled by a result of these kind of interaction.

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However, even here these sources of decoherence exists and that could be due to a interaction with photons and could also be due to the application gate errors which have being provided. There has been a lot of work on this area in terms of the different interactions that can happen for the silicone nuclear spins and lot of work has then on been carried on. As this seem to have a very interesting development in terms of quantum computing.

One of the most important aspects as we have been discussing all the time in terms of implementation is that we would like to have quantum in systems qubits which are most relatable to the present conditions. For example, silicon is a very important device in terms of the present electronics industry. And so if qubit is built based on its property then it has a very high chance of its applicability. So, these are the issues which also come into play when these we looked at.

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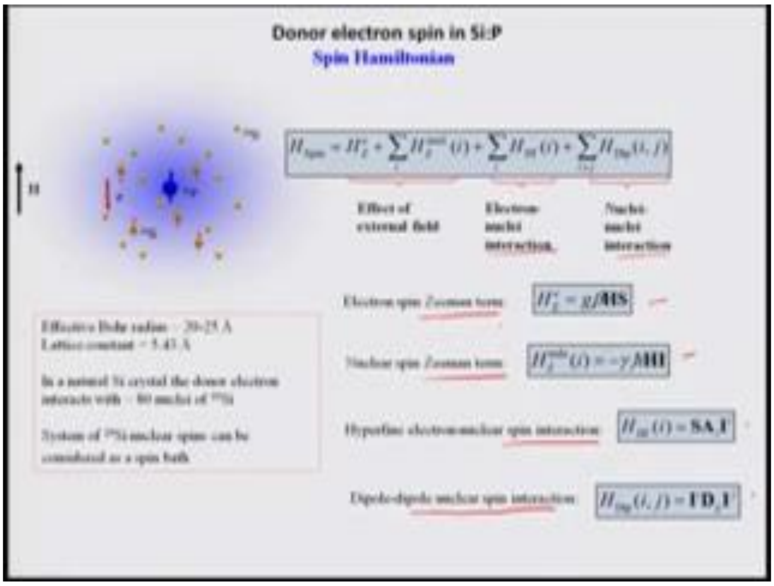

Here for example, the spin Hamiltonian can be looked into mathematical sense now. So, there is this applied field which is interacting for these conditions. As a result of the external field in the spin of the system can have different parts have which are shown here, effect of the in total Hamiltonian spin is a result of these different parts, external field electron nuclear interactions, nuclear nuclei interactions, and they are composed of different parts. The effect of external field is due to Zeeman's effect because of the applied magnetic field it can give raise to that.

The nuclear spin could also have Zeeman term due to the. There can be there would be two parts; one is due to the electrons spin, other one would be the nuclear spin. And then the hyperfine electron nuclear spin interactions are the other part which is due to the electron nucleon interaction. Then finally, is the dipole, dipole nuclear spin interaction which is due to the nuclear part by themselves.

So, there are all those different parts which give rise to these different interaction Hamiltonians which are forming the part of the total spin Hamiltonian which have been looked at.

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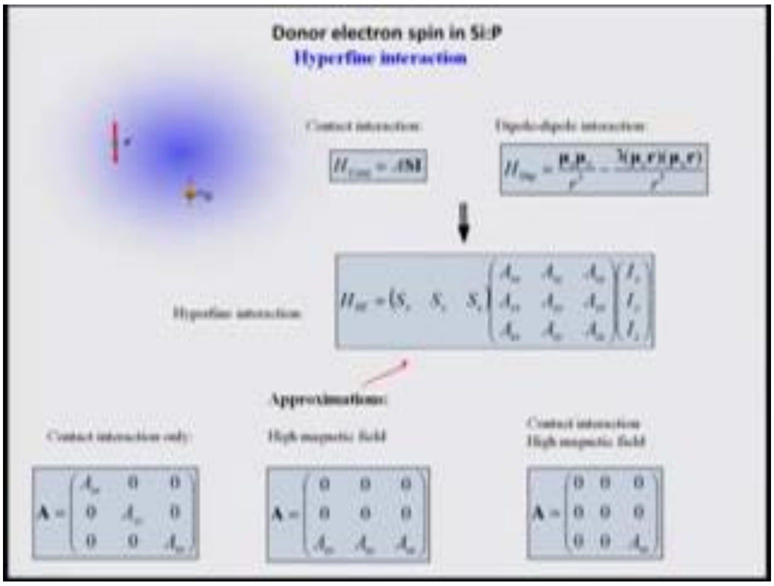

The hyperfine interaction essentially is result of two different interactions one is due to the contract interaction; other one is due to the dipole, dipole interaction giving rise to final composite hyperfine reactions which is given by a matrix as shown here. And they will essentially give rise to the spin conditions of the electrons being changed. There are certain approximations which have been utilized in this particular final statement which is being written in terms of the hyperfine interaction.

So, contact only interaction is being considered under high magnetic field the contact interacting high magnetic field interaction is only along that direction of the applied high magnetic field. And so this makes this interaction much more easy to be handled as compare to a general term.

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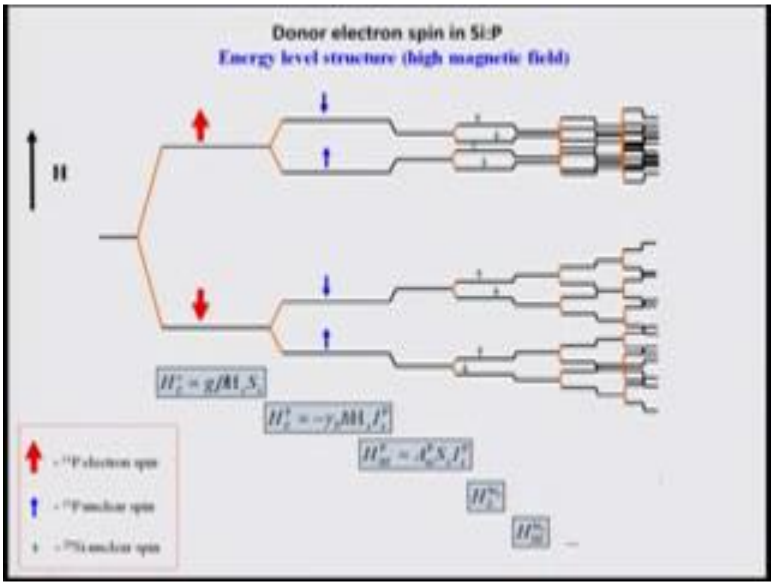

So, the energy levels structure under the high magnetic field therefore, results in this kind of is splitting into different terms where the interaction finally is left to the hyperfine part which is going to be doing most of the job here.

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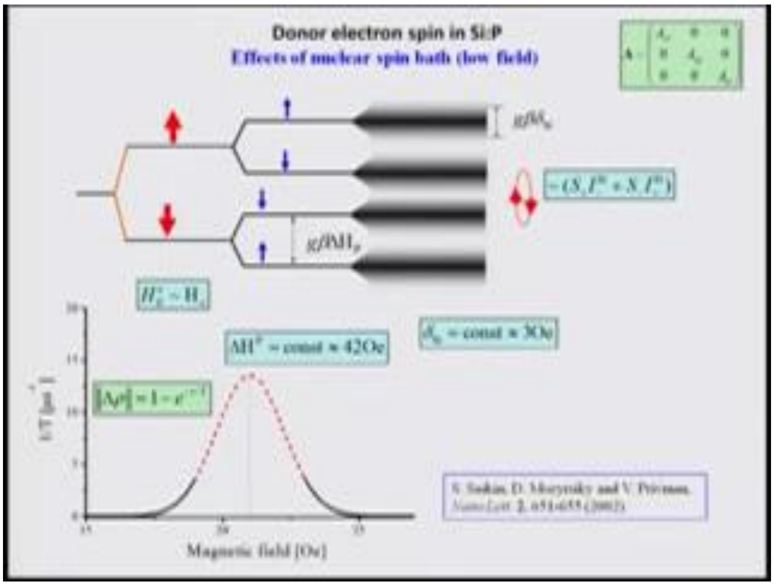

And so the donor electron part in this case is going to interact. However, if the nuclear under the low spin condition, it is another approach. So, the donor electrons spin in case of the high magnetic field makes it is possible for the different states to be easily coupled and there are so many different options which become available as a result of this interactions. And in some ways easier because it is only the z interaction which is to be considered because of the highly strong field which is being applied.

However, under the low field a condition it is not that easy because there is a lot of coupling which still remains. And the separations are not very large and the couplings cannot be just due to only one of the interactions there are many other interactions which become possible. As a result there are a the interactions can be looked at as a result of the integrated effect of all this different conditions which have been addressed also in recent work by another group where they have shown how the magnetic field is interacting in terms of the low field conditions, and that there is a decay time associated with this process. And, within the decay time however where it is possible to still do the interaction and study them.

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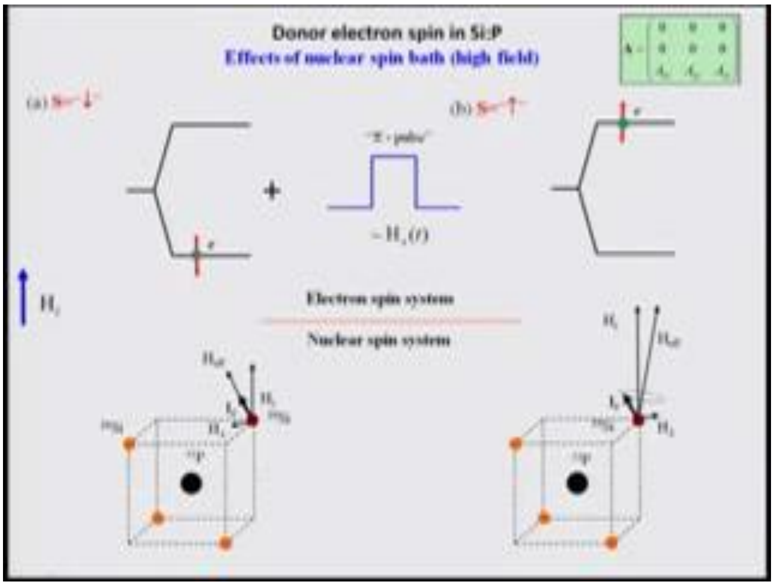

In case of the high field case is possible to apply specific pulses to look at the interaction of one kind the other. The electron spin system can be separately looked at with respect to the nuclear spin system, and their overall interactions can then be studied interesting.

For example, the electrons in case of the high field system would be going from say the ground state to the excise state, whereas in the nuclear spin system the rotation would be changing with respect to the applied field system.

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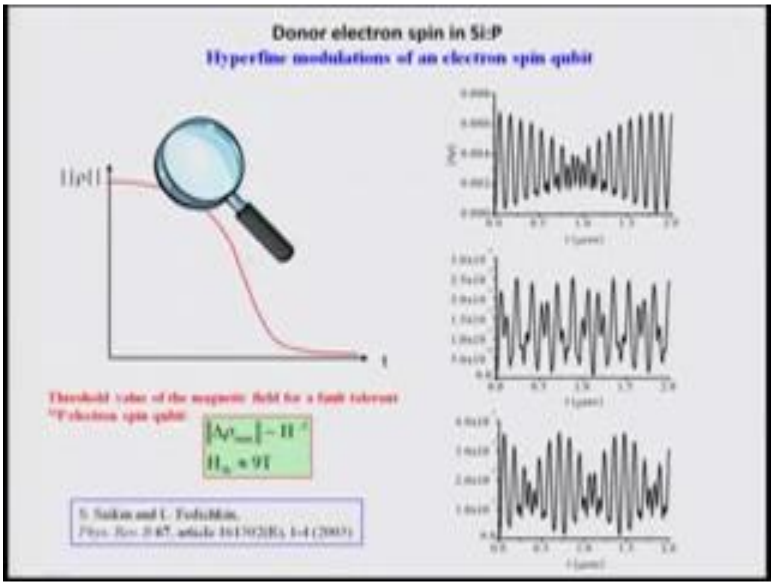

So, the hyperfine moralizations of electrons spin qubit would be possible to be seen when this is happen. A threshold value for the magnetic field for a fault tolerant phosphorus 31 electrons spin qubit has been determine to be quite low at a threshold of 9 tesla.

So, this is some of the work which have basically been applied to show how these can be the modulations of the electrons spin qubit can be looked at very carefully, and it can be essentially made to be fault tolerant another conditions of the experiment.

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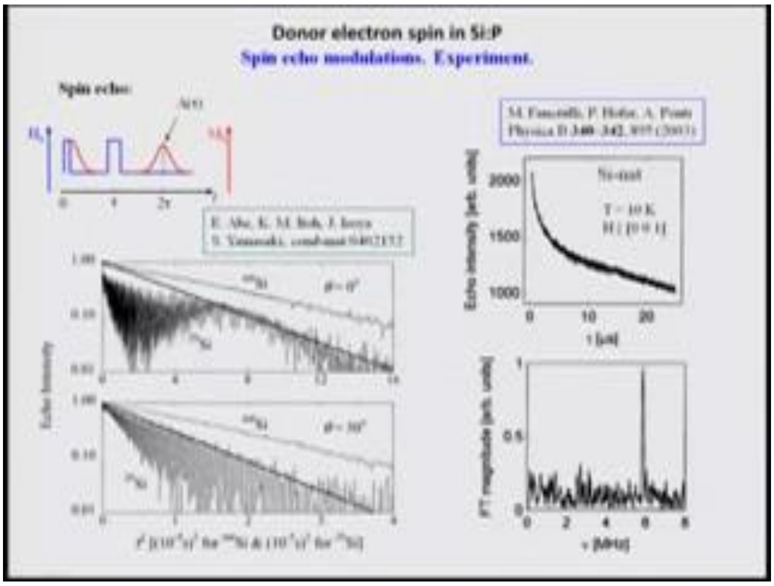

Aspects which are similar to NMR have also been attempted here, because it is essentially the nuclear spin. So, a spin eco condition of where the nuclear spin will be recovered has also been showing that the coherence of the system is very easy to be a kept and modulated and it can be transferred as a result of these interactions. So, it is a very useful approach in the high field limit it is easier to understand and work ahead with it.

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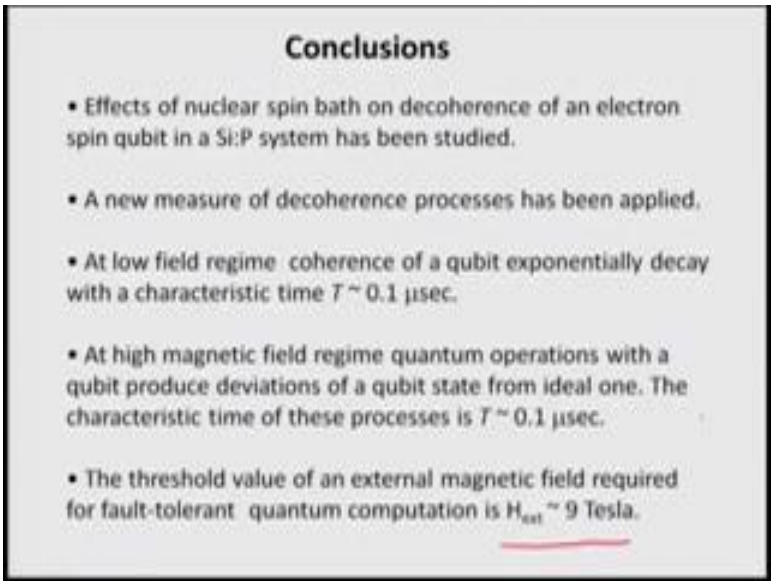

So, the conclusions of this part of the effect in terms of the donor electron interaction for nuclear spin; has been that the effect of the nuclear spin path on decoherence of the electron spin qubit in a silicon phosphorus system have been studied heavily by many groups. New measure of the decoherence process have been developed over the years by several groups in this process. And at low field regime the coherence of a qubit exponentially decreases with certain characteristic time which is roughly the claim of about a 0.1 micro second.

At high magnetic field regime the quantum of operations with qubit produces deviations of a quantum state from ideal one the characteristic time of these process are about 0.1 micro seconds. The threshold value of an external magnetic field required for a fault tolerant quantum computer for using an electron spin qubit in a silicon phosphorus dope system is of the order of an external magnetic field of 9 tesla. So, it is a large magnetic field which is necessary, but under those conditions which is possible to use it as a very effective system for quantum computing implementation.

So, we have tried to look at several other peripherical aspects of implementations of quantum computing which are close enough to the ideas that we discussed earlier. But there are many many variances as we have actually shown in many of these areas that we looked at in this particular lecture. So, in this week we have focused on trying to see how the very different approaches of quantum computing in implementations are sort of connected to the basic idea what are developments that we are doing in terms of understanding the quantum computer.

Although it is quite difficult to still have a very easy implementation over the overall quantum computer it is at a step where certain implementations are growing to a level that at least one system as we discussed earlier is commercial available and there are many more which are waiting to get to that phase. So, hope exists and will see how these are now translating to further advances in the next few lectures that are the left with us in the upcoming weeks.

Thank you.