Implementation Aspects of Quantum Computing
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Lecture - 30
Back to Basics – II

This week we have been looking at the basics of quantum once more, as we have been looking into the implementation angles from the last week. We realized that they were certain aspects of the basic part where a little bit more understanding or relook at them would help in many ways, to better understand and go forward with implementation ideas. So, with this idea we have started relooking into the concepts of the very initial aspect of the qubits, their interactions and in this regard we just address the idea of coming together of two qubits.

We are going ahead with those and looking into their interactions and gates and will be again going back to the implementations very soon as soon as we mix and bring these ideas back on board so that they can be looked at in terms of the implementations that we are talking about.

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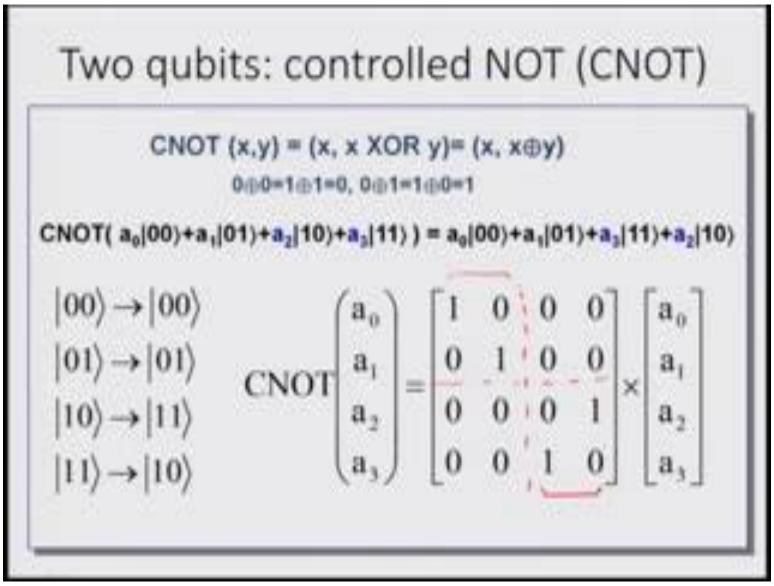

So, in terms of two qubits the controlled NOT gate is the gate which are which is going to be discussed here. We have looked at these kinds of things before also, it is just a revision in some sense of making sure that we are going back to this idea and I just mentioned earlier in the lecture that these or gate is one of the classical gates which is x or this is one of the reversible classical gates and therefore, it can be implemented quantum mechanically. As in quantum mechanics we would like to only have reversible gates, so it is also known as the addition modulo 2 in terms of the or gate, so once a c not is applied on two qubits, will be able to flip the second bit based on the property of the first qubit.

So the control axis on the first qubit which dictates what is going to happen to the second qubit. So, here is the logic here, so as long as the first qubit is 0 nothing happens to the second qubit. However, if the first qubit is 1, the second qubit is flipped, so that is the controlled NOT that we have seen. So, with 1 as the first qubit the control bit; our second qubit flips and so that is why you get to see the logic operational here and that gives raise to the matrix where the first part, where the control bit is not operational it is not going to change, is an identity whereas in the other case where the control bit is present, is going to flip, so it is going to have it in the NOT form. So, this part; the first part of this looks like an identity whereas the second part looks like the NOT gate, so that is the control NOT.

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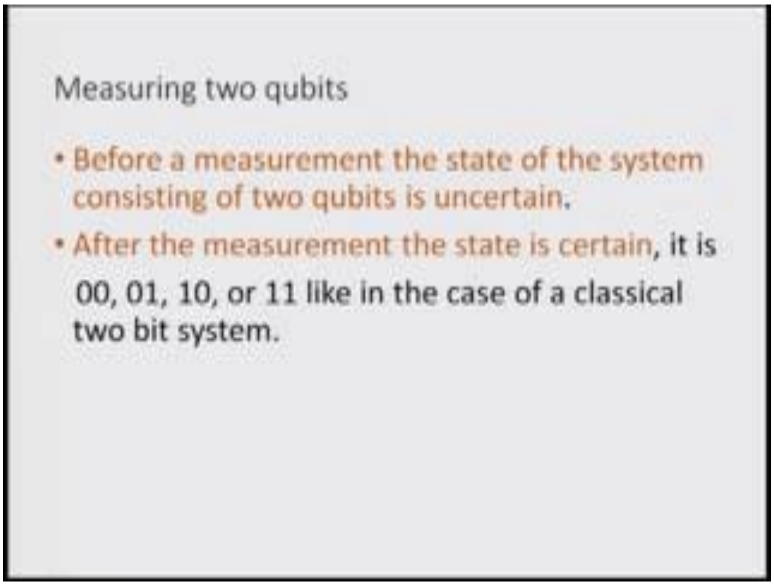

Measuring of the qubits in this case as before for all quantum system would be uncertain before the measurement is done; however, after the measurement this state is certain it is either one of these 0 0, 0 1, 1 0 or 1 1 like in the case of classical 2 bit system. We have already discussed this idea that whenever we have measurement made then it follows to the classical condition. So, before the measurement, the states of the system consisting of 2 qubits are uncertain; it is a super position state, which is being subjected to the gate applied. However, after the measurement we can collapse into one of the four possibilities.

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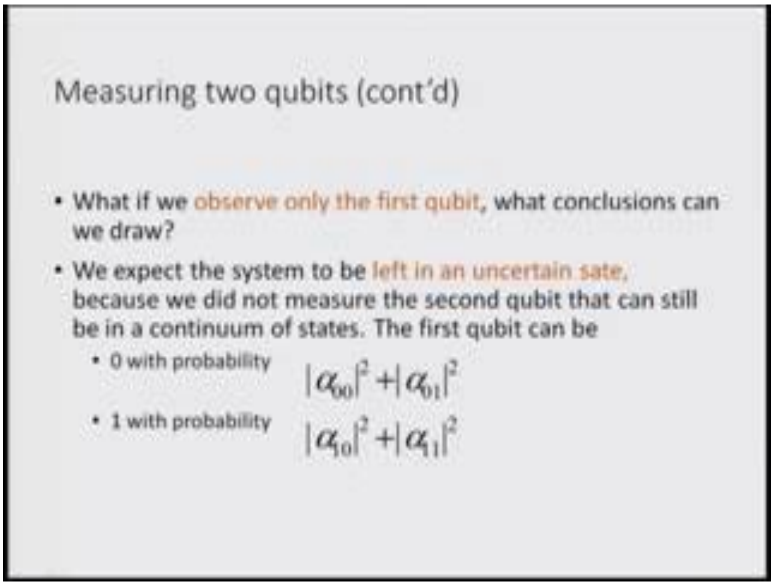

So, if we want to observe only the first qubit, what are the conclusions that can be drawn? We expect that the system to be left in an uncertain state because we did not measure the second qubit that can still be in a continuum of states. The first qubit can be 0 with probability of alpha 0 0 squared plus alpha 0 1 squared.

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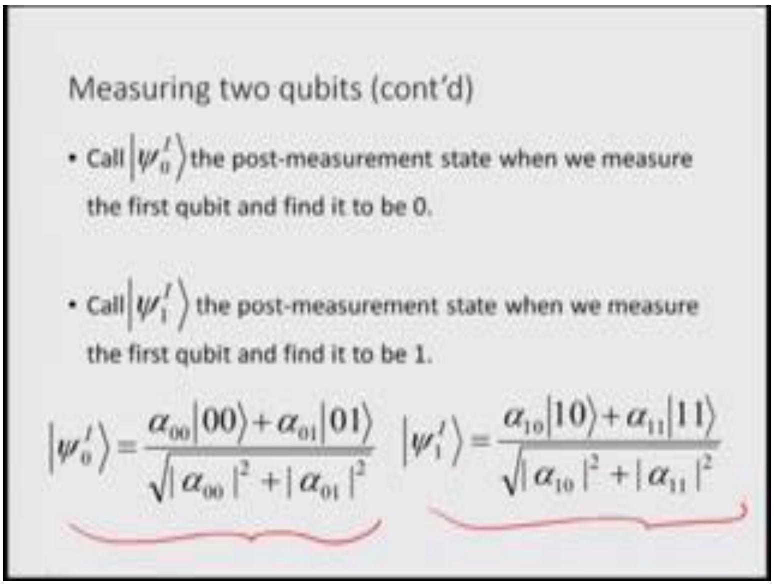

And 1 with probability of alpha 1 0 squared and alpha 1 1 squared, it is we call psi 0; superscript i as the post measurement state, when we measure the first qubit and find it to be 0 and we call the other one where we are going to find at 1 as psi i 1 then will be finding that these follow, these formulas where each of them will give raise to their probabilities.

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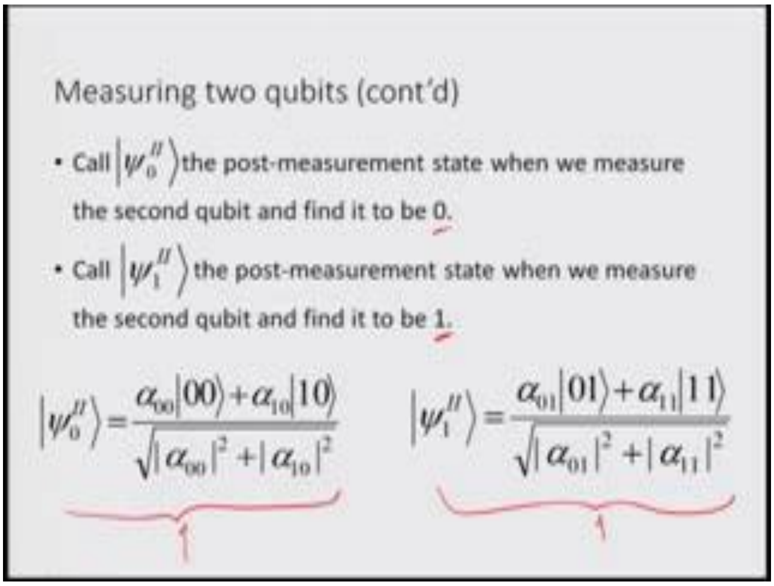

And similarly if we label the second qubit measurement in a similar fashion, we will be finding that they will follow this particular format of probability; that means, the probability of measuring 0 would be for the second qubit with be of this kind, is the probability of measuring 1 for the second qubit would be of this kind.

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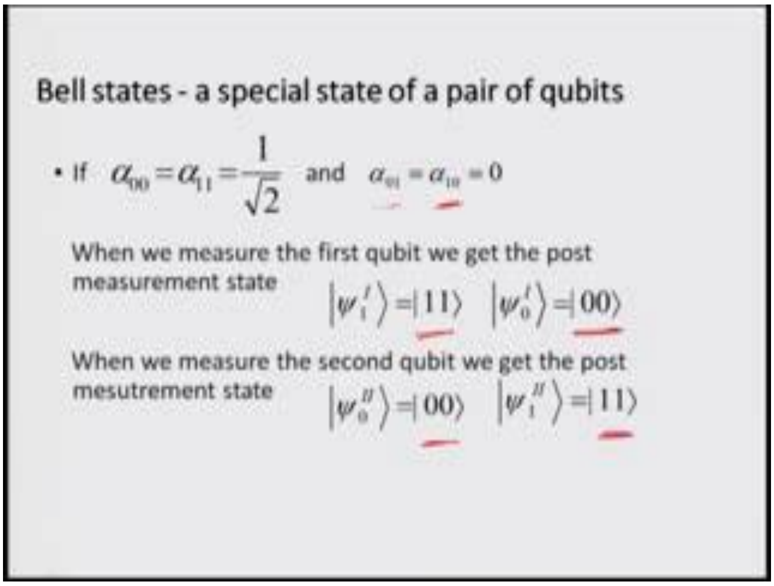

For bell states which is a very special state of pair of qubits which cannot be broken into the individual qubits that they come from the, both these alpha 0 0s and 1 1 or 1 by root 2 and 0 1 and 1 0 are going to be 0. When we measure the first qubit, we get the post measurement state as 1 1 and 0 0, but as we measure the second qubit we get the post measurement state as 0 0, 1 1.

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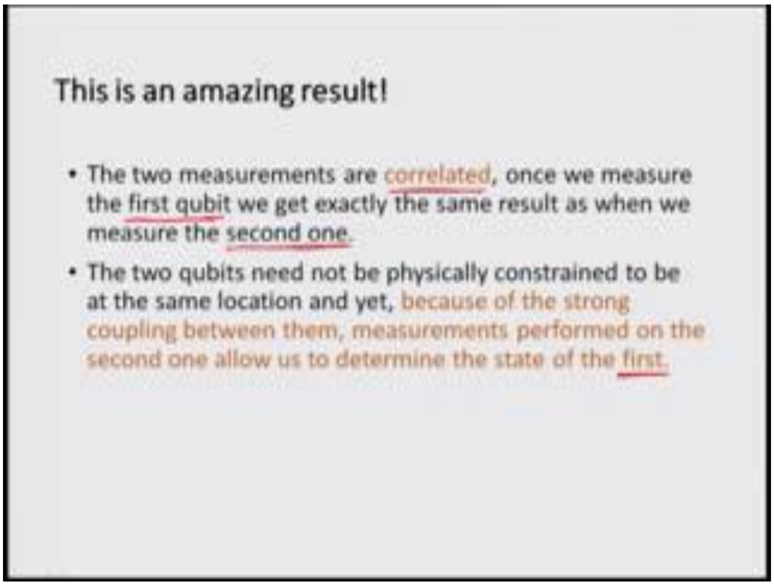

This is an amazing result because the two measurements are correlated, once we measure the first qubit we know exactly the same result as when we measure the second one. So, that is a very important result that the two measurements are correlated. So, measuring the first qubit gives us the measurement that we have going to get from the second one measure. So, the two qubits need not be physically constrained to be in the same location and yet because of the strong coupling between them measurements performed on the second one allow us to determine the state of the first one and so this is a very interesting principle which enables the measurement of one correlated to the other one.

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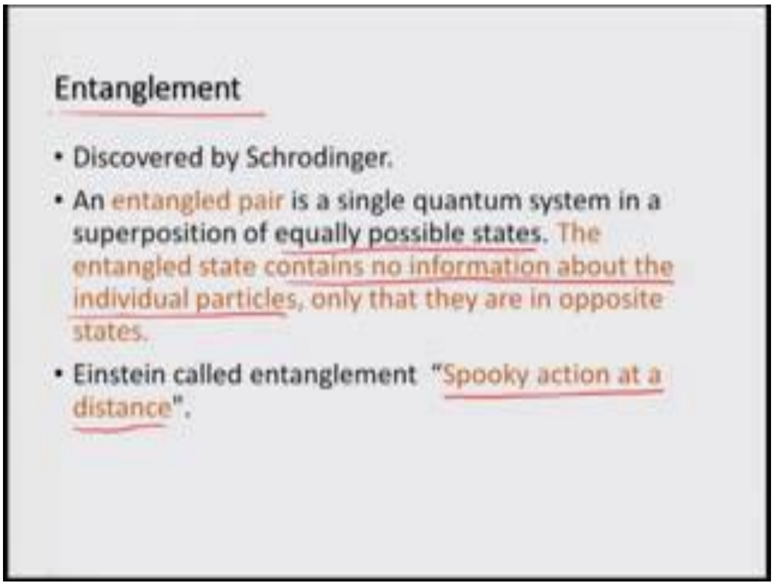

So, this arrives at the idea of entanglement which was discovered by Schrodinger, it can be formally defined as follows an entangled pair is a single quantum system in a superposition of equally possible states. The entangled state contains no information about the individual particles only that they are opposite states, so this an important point; that it does not contain any information about the individual states or in other words an entangled pair can never be decomposed back to the individual states which give raise to the entangled pair.

So, this is one of the principles which led Einstein to lose faith on quantum mechanics and he came up with this principle that it says spooky action at a distance because there is no requirement of how close these states are to be for this particular property to be maintained.

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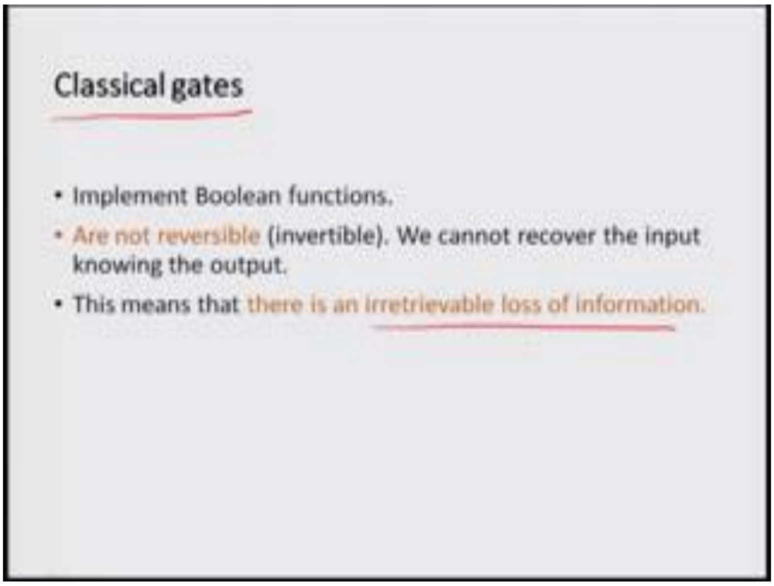

We have looked at classical gates and classical gates are there to generally implement Boolean functions, they are not reversible, we cannot recover the input knowing the output which means that there is an irreversible loss of information when we are looking at classical gate, that is one important aspect of the classical nature of computing and these are the typical classical gates which are essential for the basis of a computer to work.

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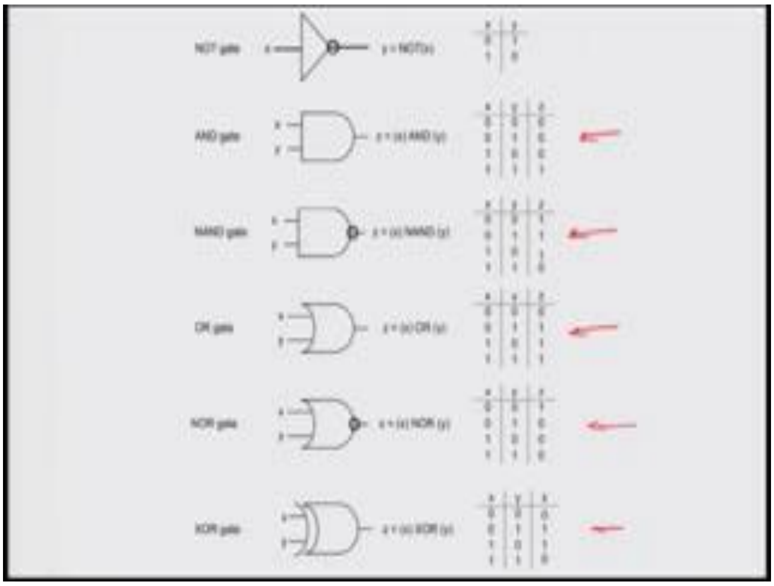

So, these are the set of basis gates on the basis of which a computation can be performed, so as the NOT gate, AND gate, the NAND gate, the OR gate, the NOR gate and the XOR gate, most of the computations can be belt on this particular few fundamental basis gates in terms of classical computation.

So, the ideas of these a very clear; NOT gate is essentially just opposite. So, y is equal to not of x and gate always involves more than two states. So, we would have two states come in to give raise to the third state, the NAND gate the not and gate also requires minimum of three states the OR gate also has two inputs to give raise to one output which is one of the 2. So, NOT reversible, NOT reversible, NOT reversible, the NOT or gate is also not reversible because the input and the output are not the same number.

However, the XOR gate is something where as we will see later on can be correlated to a condition which can be made reversible, but as it directly looks like here it is also a non-reversible condition for the classical case, the only reversible gate in that sense is the NOT gate.

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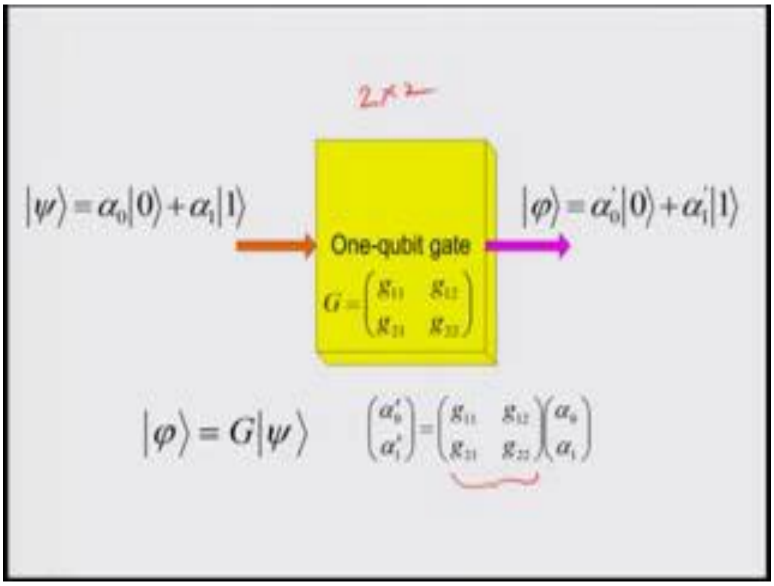

So, the idea of the gate in terms of quantum mechanics where is the operation of a square matrix on the qubits that we have. So, for a single qubit it will be a 2 by 2 matrix which will operate on the input qubit to give raise to the final result, so this is the square matrix which defines the operation.

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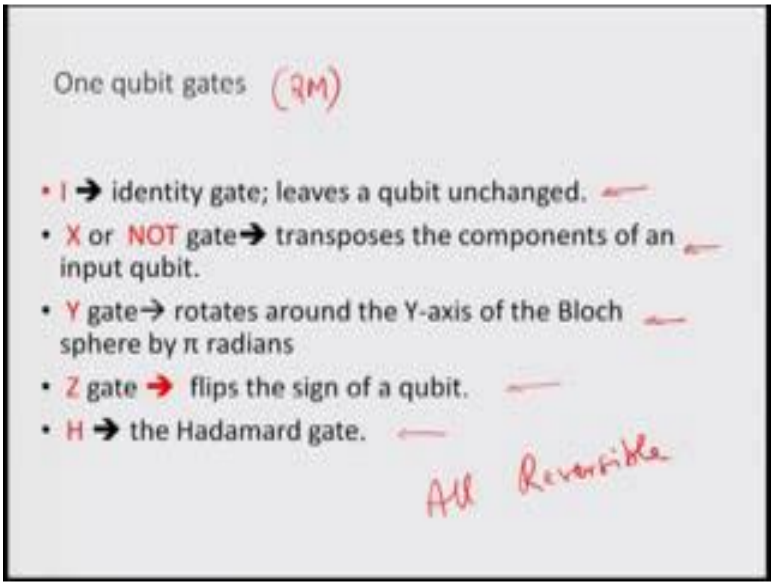

So, the basic 1 qubit gates are the identity gate, which leaves the qubit unchanged. The X or the NOT gate which transposes the components of an input qubit, the Y gate which rotates the qubit around the Y axis of the Bloch sphere by pi radians, the Z gate which flips the sign of a qubit and the Hadamard gate which makes equal superposition of the individual qubits. Now all of these qubit gates in terms of quantum mechanics are going to be reversible because that is the basic requirement of quantum mechanics.

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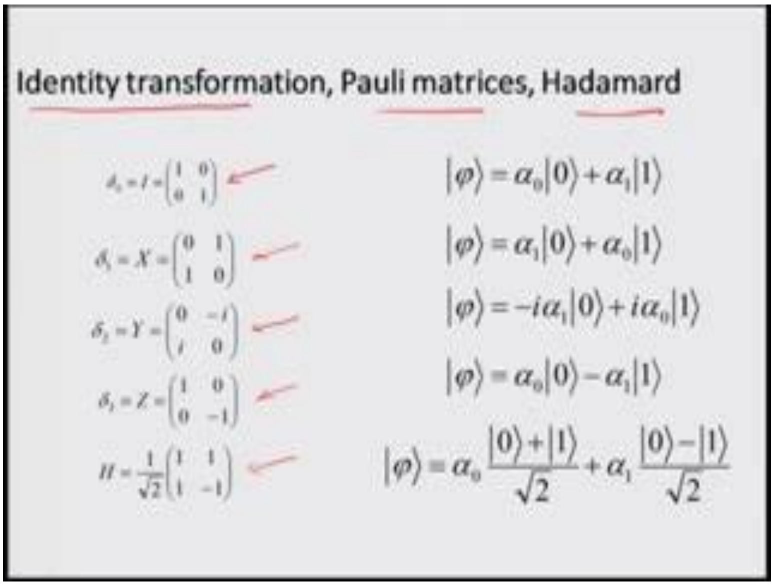

The other important aspects of these gates are for identity transformations the Pauli matrices are the once which work on the spin basically they rotate or producer it identity matrix and the Hadamard which makes an equals super position. So, these three are the most important aspects of the gates that we have looking at, the identity transformation is the one which basically just keeps the same qubit. The x gate is the NOT gate, this also a part of the Pauli matrices, the y which I described is the rotation and the z gate which was the one which flips the sign of the qubit and the Hadamard one, these are one basically next equals super position of the two qubits.

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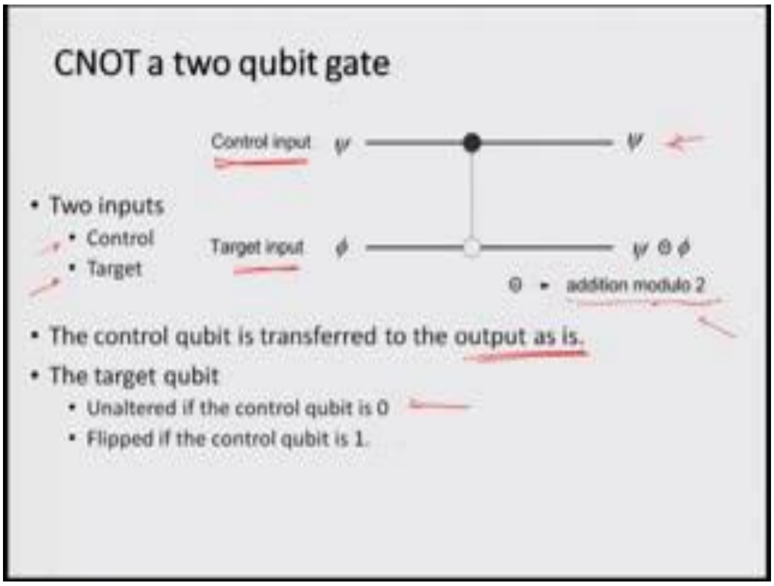

The CNOT that we had looked at earlier also is a two qubit gate, which requires an control input as a result of which the target input is going to be undergoing addition modulo two. The control bit is transferred to the output as it is and that is the quantum nature of this particular control gate going to be reversible. So, there are two inputs control and target and there are two outputs one is the control (Refer Time: 14:22) as it is and the output which is going to be addition modulo 2. The target qubit is unaltered if the control qubit is 0 and is flipped if the control qubit is 1.

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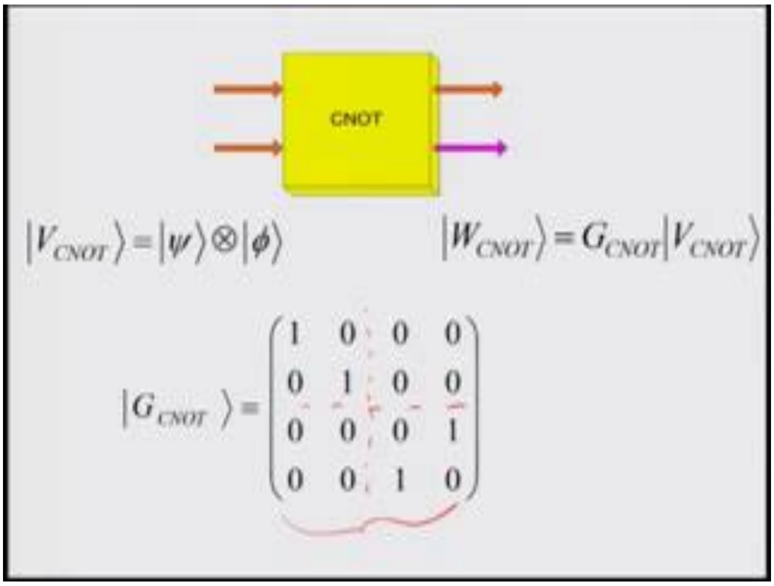

So, that is the one that we have shown here which keeps raise to this again the same matrix that we discussed before with the first upper part to be the one which essential it is identity does not do anything, keeps the same form but is the other one flips it, it is a NOT gate.

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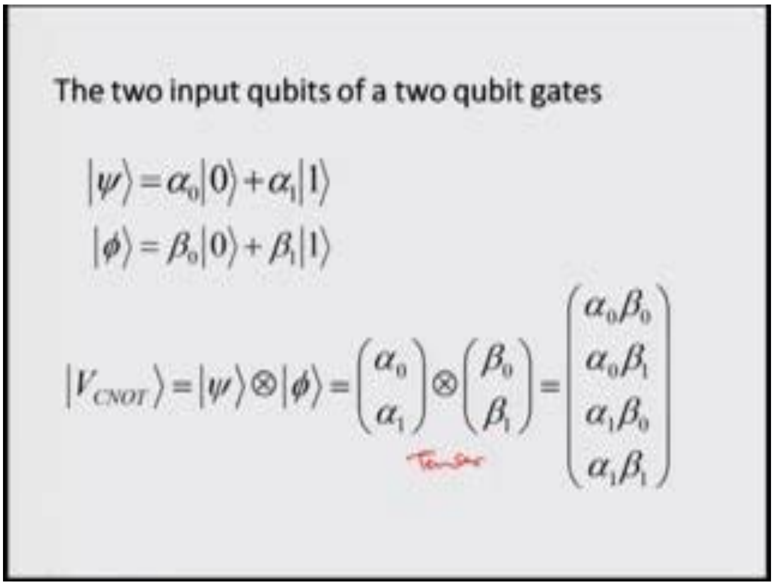

The two input qubits of a two qubit gates and the super position of the two states and then they can be put together as a matrix multiplication form, to give raise to the overall states which are going to undergo tensor multiplication to give raise to the final results.

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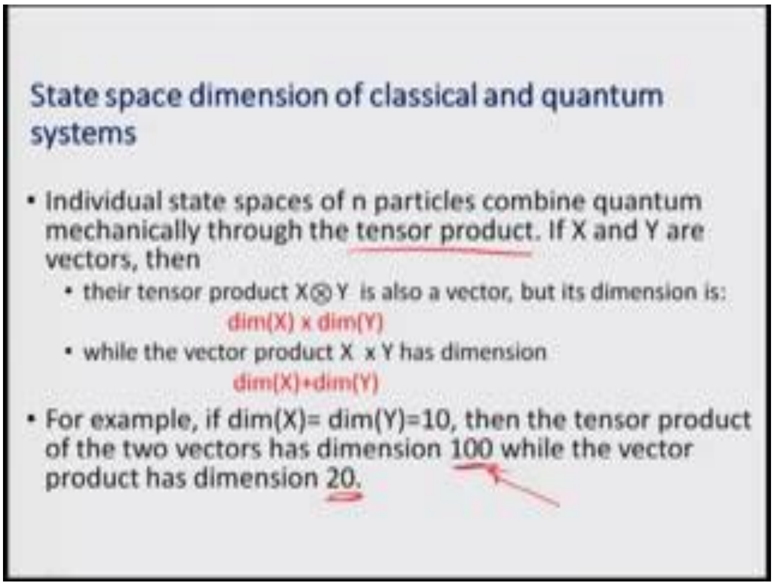

The state space dimension of the classical and quantum systems are also quite different, the individual states space of n particles combine quantum mechanically through the tensor product. So, if X and Y are vectors then that tensor product is also a vector, but its dimension is now the multiple of the 2, while is the vector (Refer Time: 15:51) product has addition of the dimensions of the 2. So, for example, if dimension of x and dimension of one, dimension of y is 10 then the tensor product of the two vectors has dimension 100 while the vector product has dimension of 20 and this is the reason for the exponential nature of the quantum computing process.

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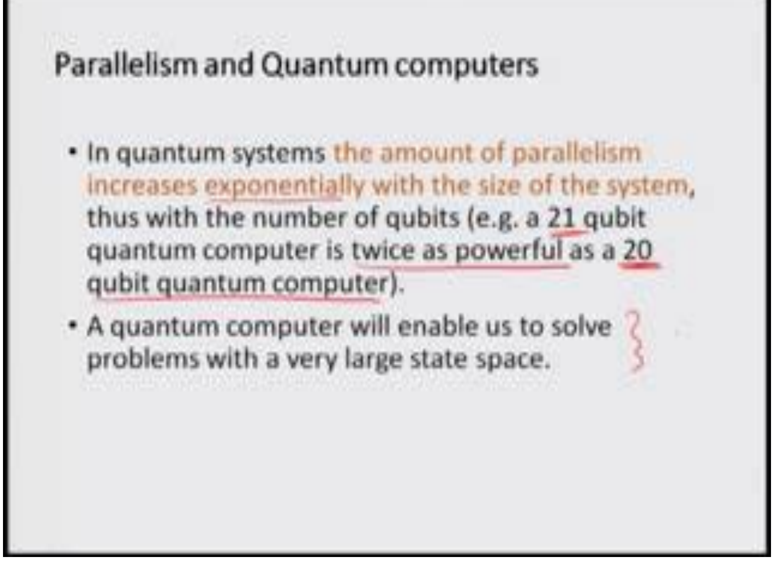

Parallelism and quantum computers in the quantum system, the amount of parallelism increases exponentially with the size of the system thus with the number of qubits in quantum systems, the amount of parallelism increases exponentially with the size of the system. Thus with the number of qubits for example, 21 qubit quantum computer is twice as powerful as a 20 qubit quantum computer. So, that is the exponential nature of the problem that is the advantage in the quantum computers, a quantum computer will enable us to solve problems the very large state space that is the biggest advantage of the quantum parallelism that we take advantage of.

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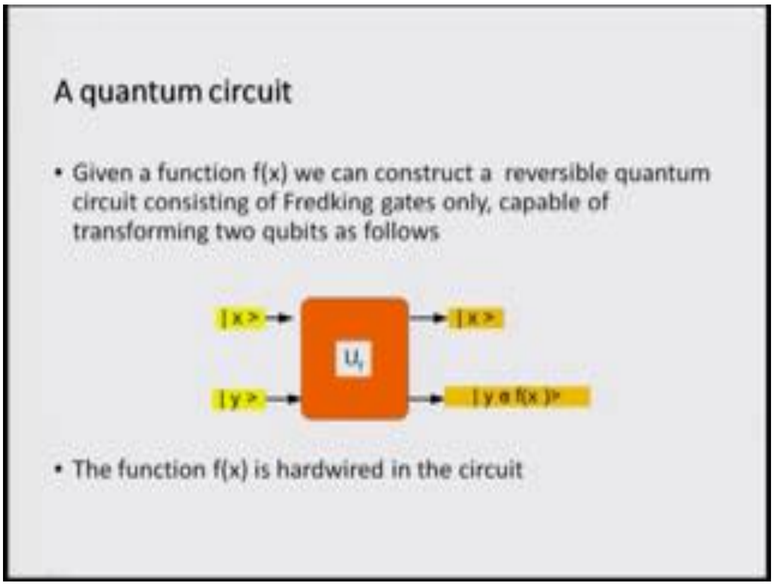

In case of the quantum circuit as we have been discussing, if we have a given function f of x, we can construct a reversible quantum circuit consisting of say the Fredking gates only capable of transforming two qubits as follows, the function f of x is hardwired into the circuit.

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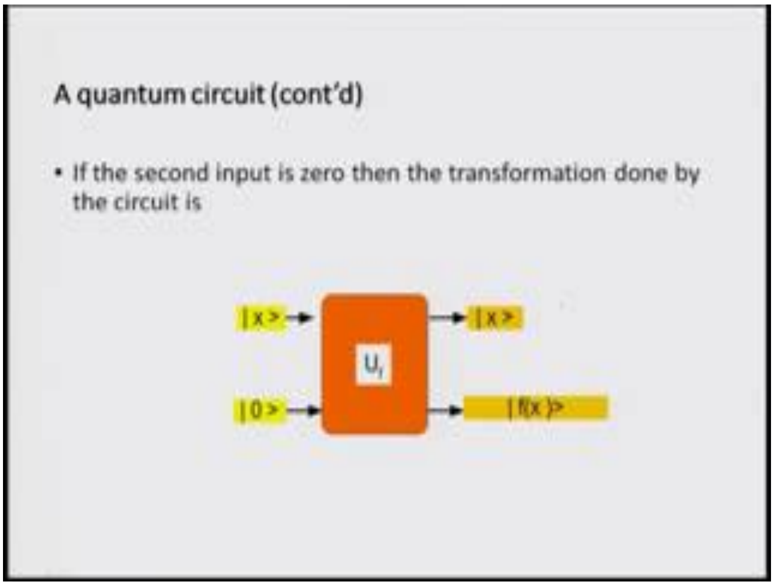

So, this is how looks like which is sort of the CNOT that we looked at, if the second input is 0 then the transformation is done by the circuit is given by as this one.

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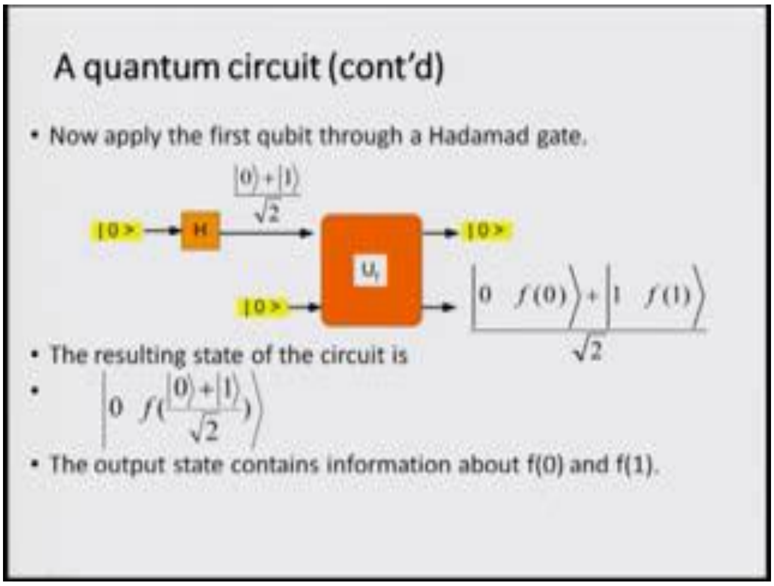

And we apply the first qubit through a Hadamard gate then produce state whereas, the resulting state of the circuit is this, the output state contains information about f of 0 and f of 1.

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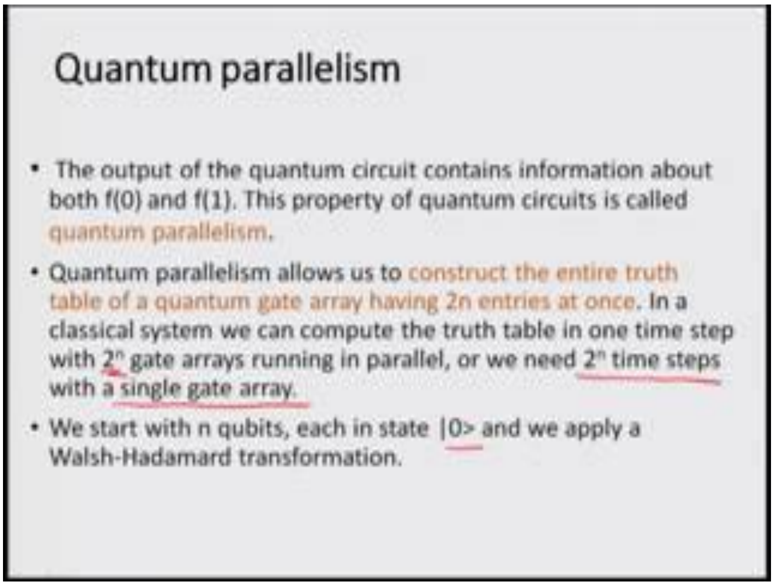

The output of the quantum circuit contains information of both f 0 and f 1, this property of quantum circuit is called quantum parallelism. The quantum parallelism allows us to construct the entire truth table of quantum gate arrays having 2 n entries at once, in a classical system we can compute the truth table in one time step with 2 to the power n gate arrays running in parallel or we need 2 n times steps with a single gate array.

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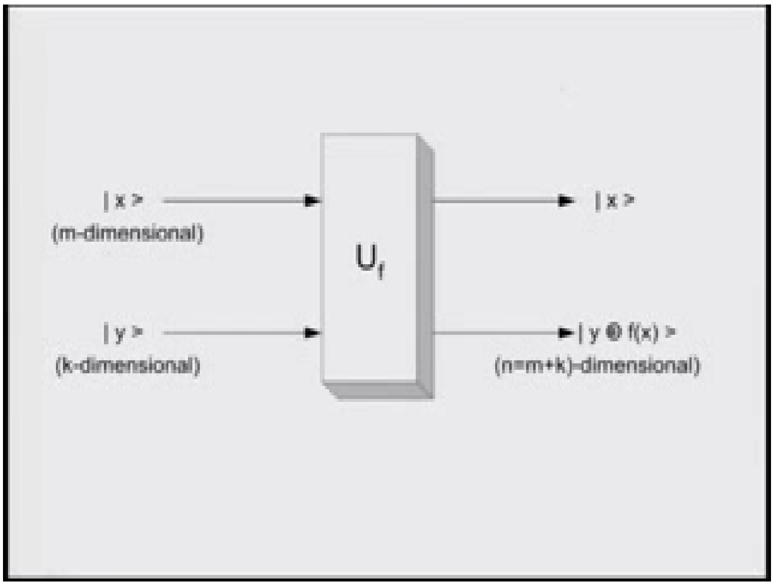

If we start with n qubits, each of the state 0 and we apply Walsh-Hadamard transformation, we are able to do this particular gate principle. So, we have seen all these gates before, so I am not going into a details of it Walsh-Hadamard is the one which we use for the Grover's algorithm and so we can go on with these number of dimensions which go higher and we have given this new principle where we have actually telling you how these dimensionality of the problem is increasing because you are doing quantum way your dimensionality is going as the product of the cases.

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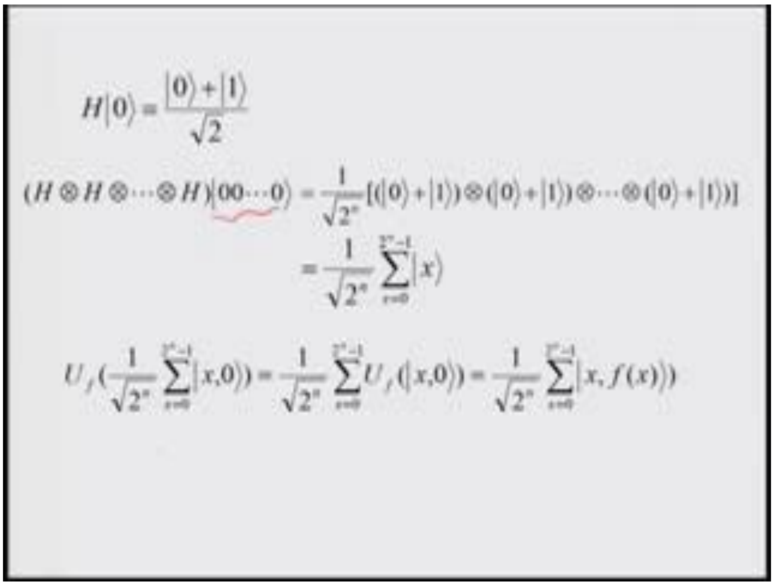

So, here we can take the Hadamard on a particular step and we can repeatedly apply the Hadamard on n qubits which are undergoing tensor products and as result we can get the superposition of states, which have undergone the Hadamard operation. So, we can apply this unitary transform on that as a result of this operation, so a Hadamard operation of this kind, so do looks like this.

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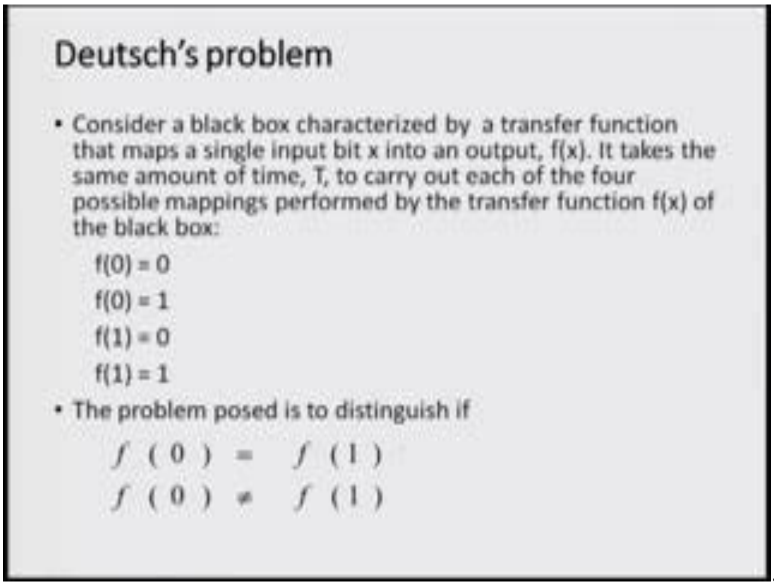

And this can be utilized for looking at the Deutsch's problem once again the Deutsch's problem was the one where, it was going to look for the balanced function, whether the function is balanced or not. So, if we consider black box characterized by a transfer function that maps a single input bit into an output, takes the same amount of time t to carry on each of the four possible mapping performed by transfer function f of x in the black box.

The problem posed is to distinguish if the function is going to be equal for both f equal to 0 and f equal to 1 or it is going to be unbalanced, this is f 0 is NOT equal to f 1.

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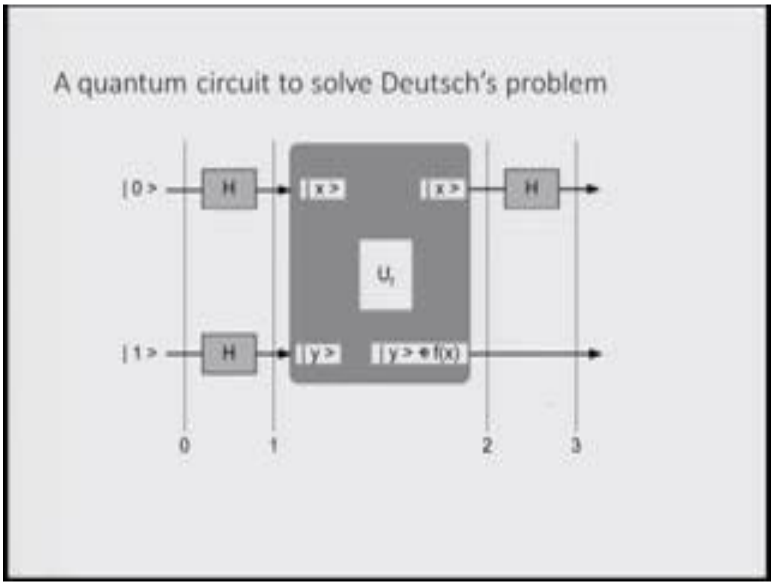

So, the times scales involved in each of these cases are shown here, the unitary operation which goes for this takes this into the operation procedure where we can do all of this in one shot. So, the quantum circuit to solve the Deutsch's problem can be given in this particular form which has been discussed earlier also, but here we are just telling you the details in some sense, we have the initial inputs 0 and 1, which goes through first Hadamard operation on both the inputs and then it is undergoing a unitary transform where the essential part of the unitary transform is to take one part of bit into the modulo function whereas, the other one remains the same and then the first part undergoes the Hadamard transform while the second one is essentially gives raise to the final answer.

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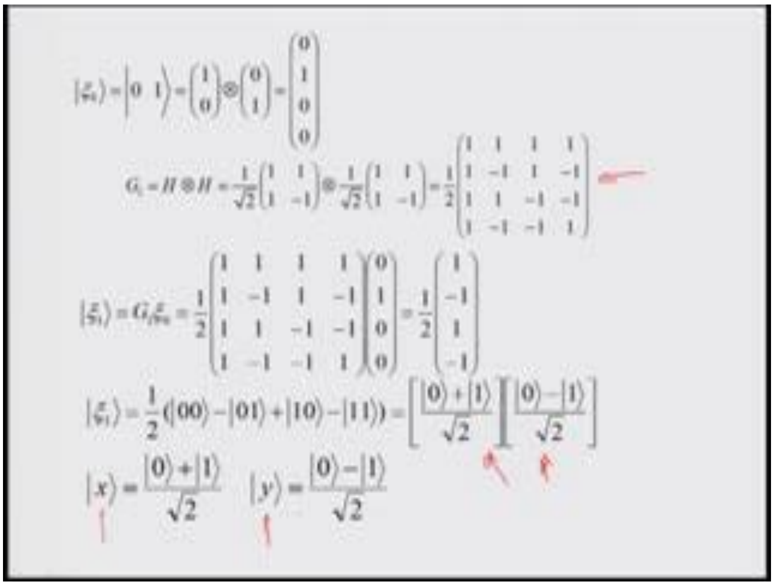

So, in terms of the matrix operations and the tensor products, these are the basic steps which undergo in terms of how this entire process goes, the overall gates that we are applying are multiple Hadamard gates which are getting applied in a tensor form which grows in terms of the size, these are then looked into by applying the overall gate. So, here is the g 1 gate which has been created and that is been applied to the original Hadamard transformed input which then gives raise to the final result in first step and then that is been looked into as a combine state which can be broken down into two different states as that then forms a basis sets of x and y.

So this is the first form of the part which goes in when we look at this point, so at 1 this is what we have just achieved, this is the process which takes us from 0 to 1, so here this entire step goes from 0 to 1 in our earlier slide here.

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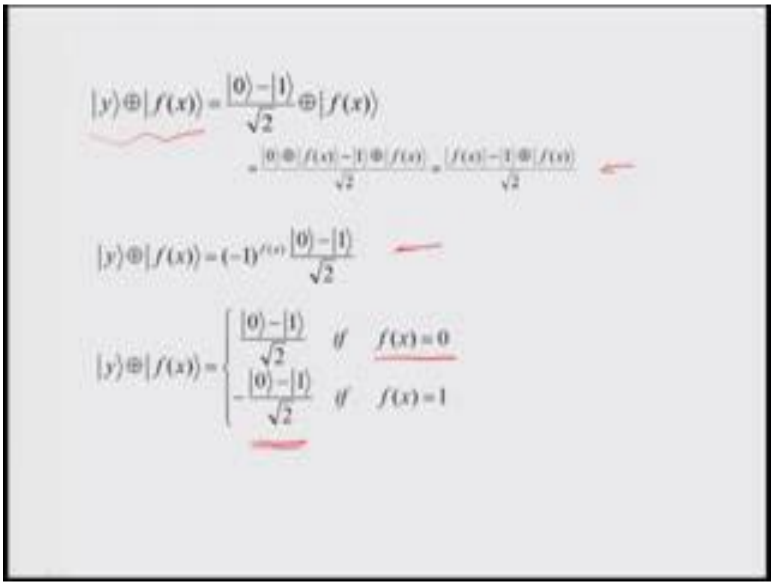

Next it undergoes an interaction with the unitary transform where it undergoes the function modulo applying on the y qubit and it gives rise to a functional form which would be of this kind. So, this is the part where we have the form of the function interacting with a y qubit and so the y qubit takes this particular format, follows the principles that if f of x is equal to 0, it will be the positive value whereas, if it is 1 it will give the negative value.

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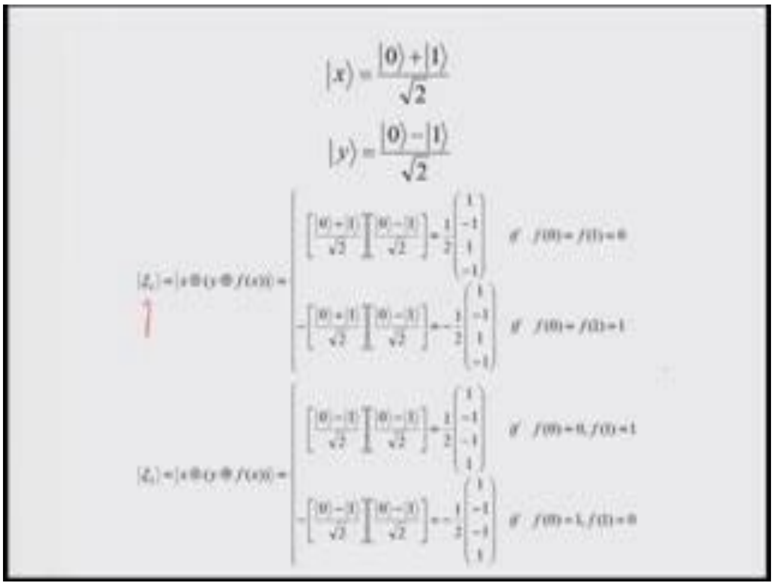

So, the x and y are the ones that we have started off with after we have gone through the Hadamard transform then we have applied the gate. So, these gates essentially take the first qubit into the unitary transform which gives rise to the solution whereas the second qubit undergoes the part where the solutions would be of this kind and based on their individual results will be able to have the different conditions that we give raise to them.

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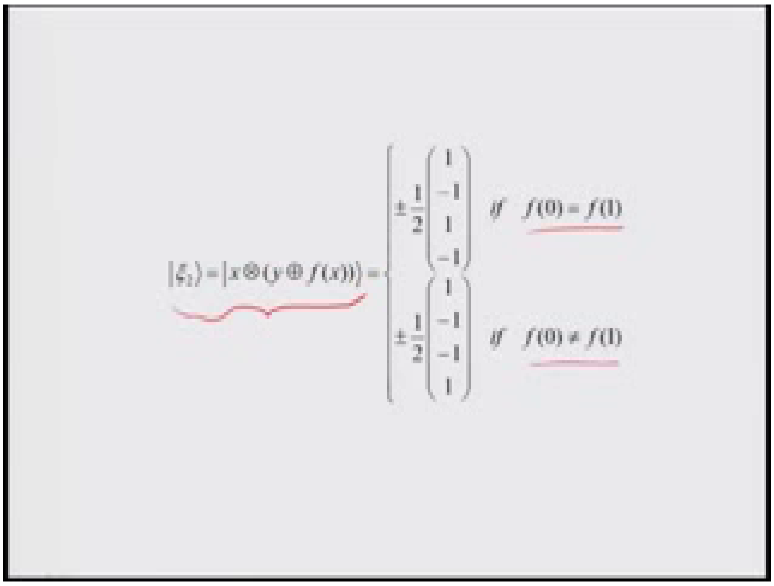

So, in the end will be having two conditions, one when our function is equal in one case and it will be different when our function is not equal and so by idea that our interaction for the part which is the second qubit would give raise to this results will be able to understand how this interaction goes on.

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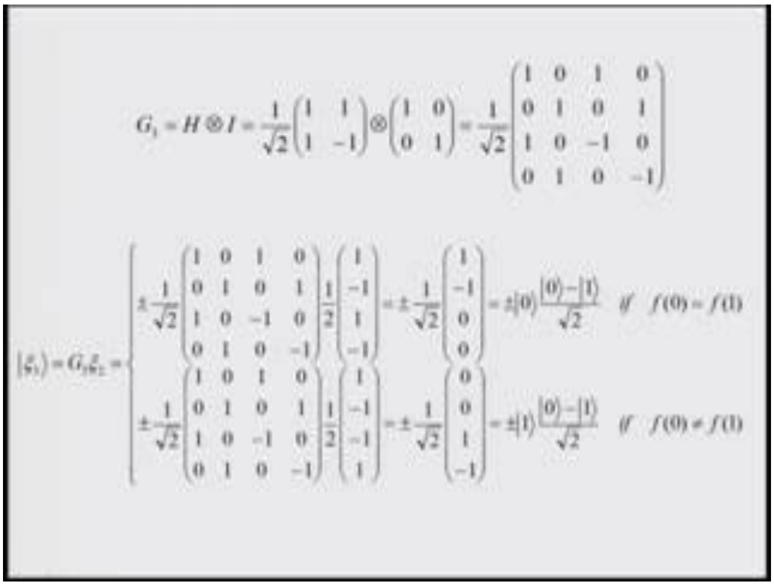

So, in the third part is again the first part goes to Hadamard transfer and so that the upper qubit goes through a Hadamard transfer rate; however, the second part of the qubit remains as is.

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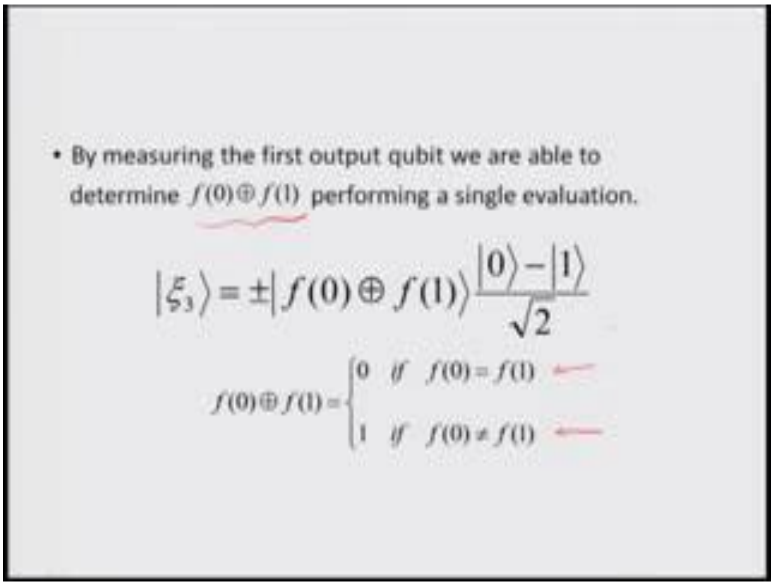

So by measuring the first output qubit, we are able to determine whether f of 0 addition modulo of; f 1 is performing a single evaluation or not. So, if we go ahead with this result will be actually getting either 0, when f of 0 is equal to for 1 and it will be 1 when f of 0 is not equal to 1.

So, that is the basic idea behind the Grover's algorithm that we had looked at before and in many cases this implementations have been utilized as we have done in our last few lecture. In this lecture we have been revisiting the aspects of quantum computing basics in relation to the implementations that we have been undergoing over the last few weeks.

In the next lecture, we will be dealing with some more of the basics and some other implementation aspects as we develop the basics in relation to the implementations and I look forward to having you in the class.

Thank you.