Implementation Aspects of Quantum Computing

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Lecture – 03
Introduction: Basic Tools

Until now we have been doing mostly studies with one qubit. So, all the properties that we learnt out, which we were learning was dealing with one qubit.

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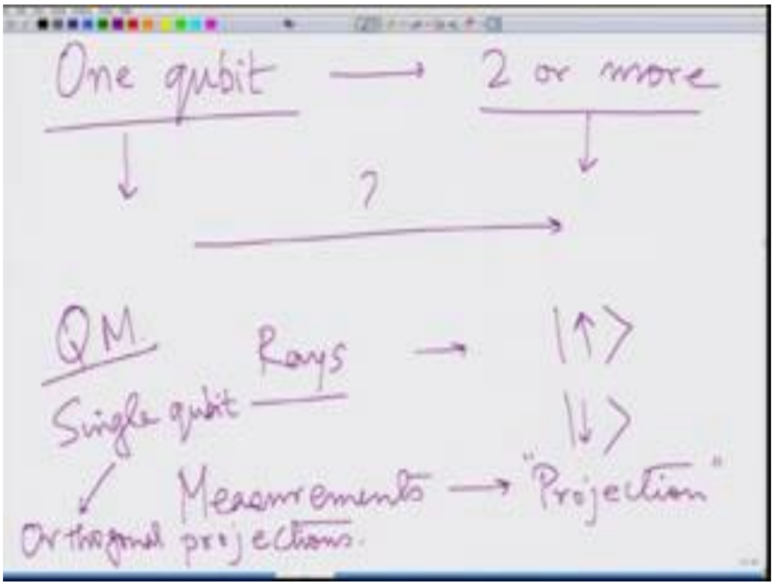

So, in terms of one qubit we rephrased our understanding of quantum mechanics, and we were able to apprehend that quite well. Now when this picture changes from one qubit to say two or more thus all the properties or the features that we did for one qubit does it remain when you go to multiple qubits; is the question that we will ask.

So, all the properties that we have learnt are they actually going to remain the same or not. In quantum mechanics the important things which we learnt about one qubit was that we were able to represent the qubit by rays and that is what we were calling them as vectors one way or the other. That was all that we learnt about it in terms of one qubit.

Then we looked at the properties associated with them and we found out that they were going as we have learnt in terms of quantum mechanics. And we were able to make statements which were resulting as a result of measurements. And in terms of our measurements what we defined was this is measurement was defined as a projection, so we had define measurements as projections. And most of the time when we use the single qubit we always landed up in projections which were orthogonal projections. These were the some of the important things what we did. We also looked at other issue which was time evolution.

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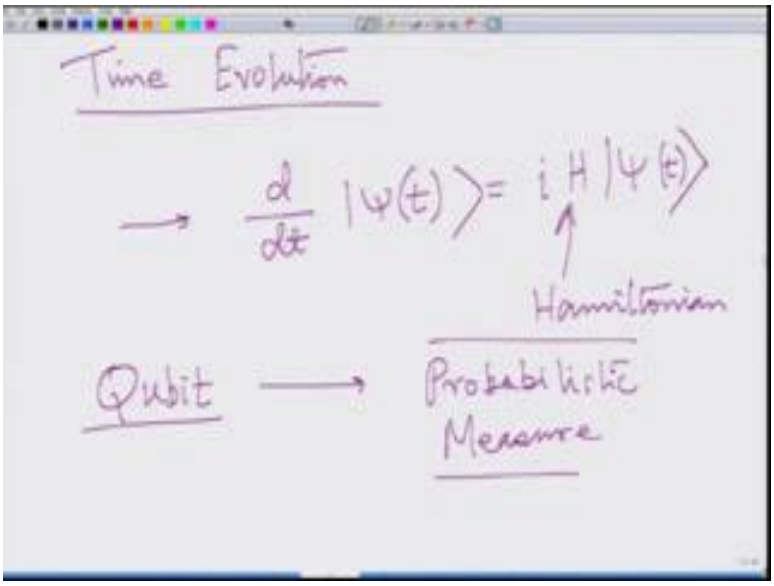

And in terms of time evolution we use the simplest possible Schrodinger equation in this form. Just to remind you I will tell you again, this was given by d of dt of say the wave function psi say time dependent one, so wave function psi t which will give rise to your minus H. So, we looked at the time evolution for a single qubit time dependent equation is linear equation. We looked at how the wave function or a single qubit changes with time, evolves with time that was the dynamics that we looked at. And, that was a linear equation.

Now the question is, when we go from here to multiple qubits what is going to happen. So, we will be looking at all of these issues, where H is the Hamiltonian. So, you know all of this before because we have done this before. Now one of the most important things we did about a qubit is we were saying that anything that we do with it will give rise to probabilistic measure, whereas this linear equation has nothing which is probabilistic in it. This is a linear expression so this is suppose to give you rise to an expression which will evolve one way versus the qubit whenever you measure it which is going to be a projective measurement or the way you look at it is going to be giving rise to probabilistic measurements depending on how much of the contribution you get from the either state. So, that was the part which was the probabilistic measure.

Now, what we are going to now do is to see how things change when we go from a single qubit to let us say even the simplest two qubit case.

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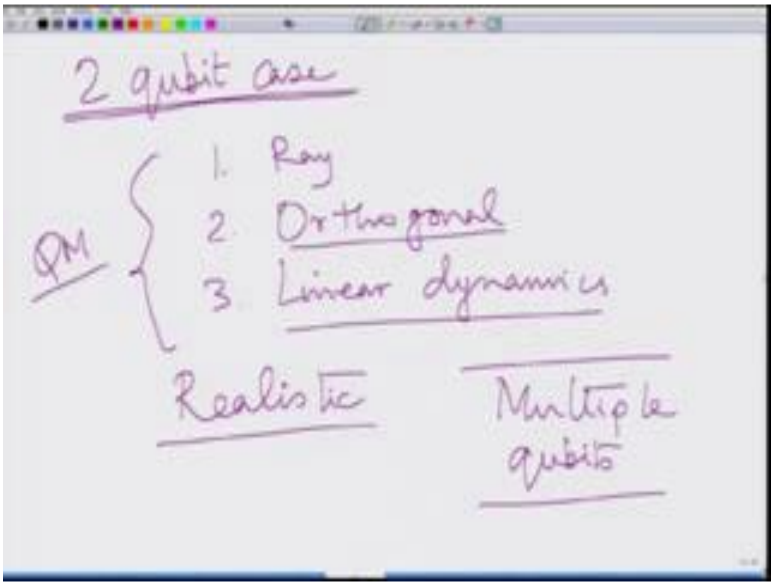

What we will notice is all the principles that we introduce in the first part that it can be represented by a single ray that it actually, I think we mentioned all of them their resultant is the projection and they are orthogonal. And the fact that this is a linear dynamics they need not be true anymore. So, that is the first thing that you will notice that whenever you go to the first condition beyond the first qubit none of these results that you are going to see from quantum mechanics they need to stay. And that is actually more close to what you see in reality.

In reality there is no reason for anything to behave or show linear dynamics all the time. And the principle that there will be an orthogonal resultant for any measurement is also not going to be something which is required. So in some sense whenever you go to multiple qubits you are getting more and more realistic answers. And what we are going to have in terms of quantum mechanics is the case where we are dealing with multiple qubits.

So, we are going to look at this in this fashion and see what is going on with it. So, let us do a little bit of a case study here.

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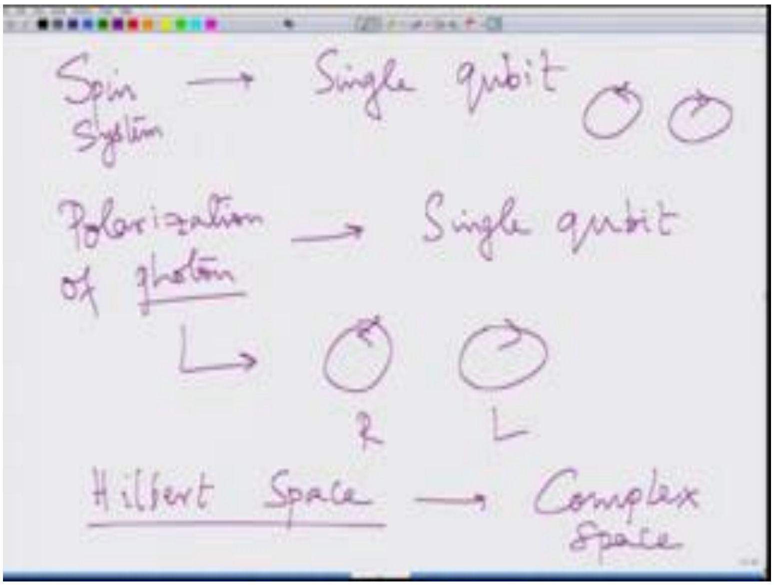

So, the first case study we did was a spin system which was a single qubit and analogous one is a polarization of light, of photon essentially light is not considered quantum, but photon is quantum and again this is single qubit. So, what are the polarization states of photon which are considered for a qubit; circular, one way versus circular the other way, right or left circular polarization? That gives rise to the polarization of the photon. Similarly here the spin system we know that it can go one way versus the other way. Now this is spin of say electron or any quantum object. Similarly, the photon being a quantum object we can look at it also as a quantum single qubit.

So, these are the examples which we have looked at in terms of single quantum. Now when you have more than one of these systems present at a given point of time then what is it going to do is the study when you get into higher number of qubits. In that respect will have to now introduce to you something which we had already discussed in not very detail earlier which is the Hilbert Space. Once again Hilbert space is the space covered by any quantum system to completion and so it is a complex plane, complex space. Instead of a real space is a complex space in which all possibilities of that quantum object which is ray in our particular case for a single quantum can exists. So, that is which we had already discussed

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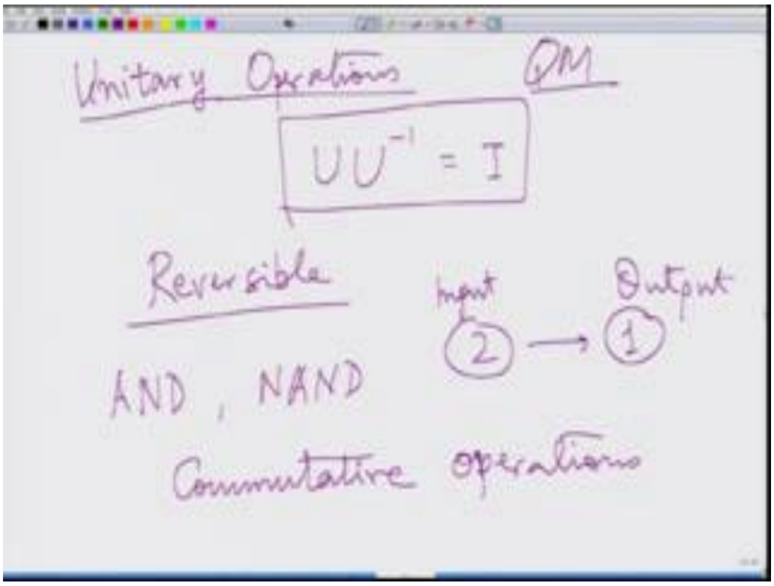

All the operations until now that we have talked about also has another very important thing which was important for quantum mechanics which were unitary operations; which means, if you have any operation going in one direction it will always have its conjugate which will give rise to an identity matrix. So that was the essential idea about unitary operations. Any operation that takes 1 state of the system to 1 would have an inverse or another unitary operation which will take it back. So, it was a perfectly reversible condition. These unitary operations were always reversible.

And in this sense we had said that the quantum operations are different from classical once in terms of comp computing definitely. Simplest example; AND, NAND which are our computing gates, basic gates; NAND is not AND. They all end up producing two objects giving rise to a one solution. Input is 2, output is 1. Whereas in case of reversibility this is not what happens, in case of reversibility with any unitary operations which will have reversible where you can put the operators back and forth again you should always have preserved the input and the output. So, that is the place where we again had a difference from the classical case which was that the fact that we had unitary operations in quantum mechanics.

Now what we are going to do is, we are going to look at what happens when you are go into more than one qubits. And we had also mentioned one other important thing which is commutative operations.

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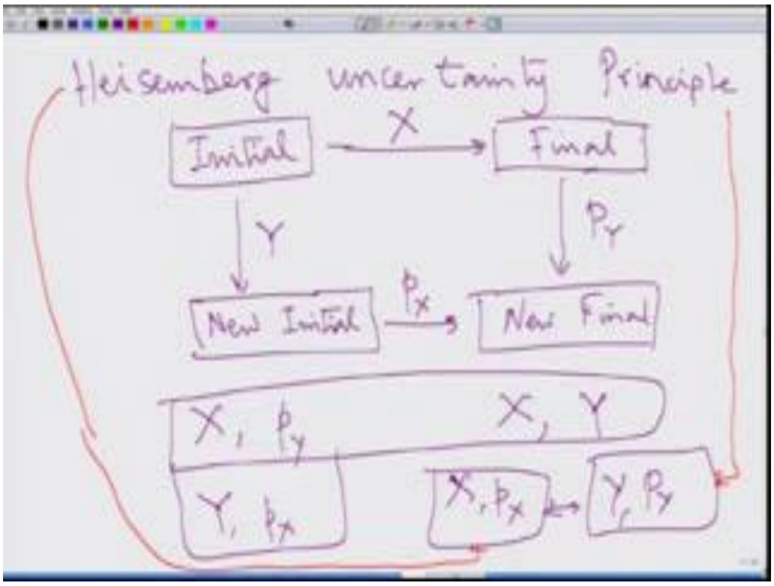

In particular commutative operations were important and we all know about the Heisenberg uncertainty, which is a one of the classic cases this commutative operators where we had say an initial condition going through a different state say which we call it a final state through one operations say operation number x. Whereas, we could have another kind of a way orthogonal operation which is say y which will take you to a new initial or a different initial state. And we can similarly have a new final state which can be connected by some other kind of say P x and say P y.

In this case what happens if they are orthogonal then what will happen is irrespective of how you do things will not be able to do see the other effect. And since they are commutative one of them will always give rise to the other you cannot get everything the same way. So, that is the orthoginality of the problem; so x and y for example or p n.

So, in this particular case when they are orthogonal then arbitrary precision is fine, but if they are not orthogonal then you have a problem. For instance x and P y has no problem x and y has no problem y, P x has no problem, but the moment we end up in pairs like x P x, y P y we cannot have the level of precision which are independent of each other. So, this and this cannot be independent whereas, all of these others are independent.

So, this is an example where independent property is possible, but whenever you have these kinds of cases then we are going to have the principle which will not let you get arbitrary levels of precision. So, these were the things that we had confronted and we looked at it. And we had also come up with some matrixes corresponding to them. Let me actually give you a few of these matrixes, whenever you were making any kind of operations.

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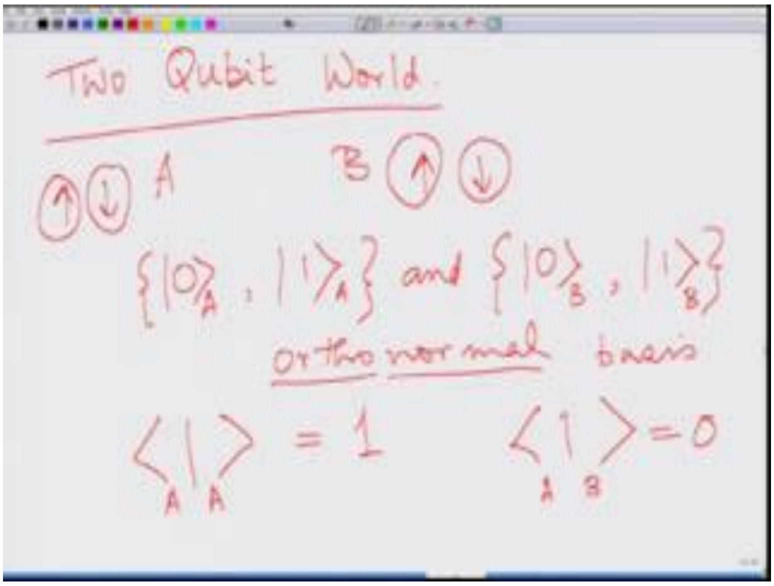

Let us start with two qubit and then we can understand what we are trying to say. In this two qubit world let us consider we have two qubits A and B, both of them are going to have two different states. So, we can use the idea that, we can represent them say by 0 A, 1 A, and then we can construct an orthonormal set which will be of this kind. So, now, these two are orthonormal. Now orthonormal basis means if I take a product of the two in this form they will give rise to a unit value; that is what we are looking for. That is normality and as long as any time we take a same kind then I will get normal one. If I take anything of other kind say A B and it will always equal to 0 that is the idea of orthonormality.

So, this is what we have chosen to be orthonormal basis so that they can have a combination of this kind.

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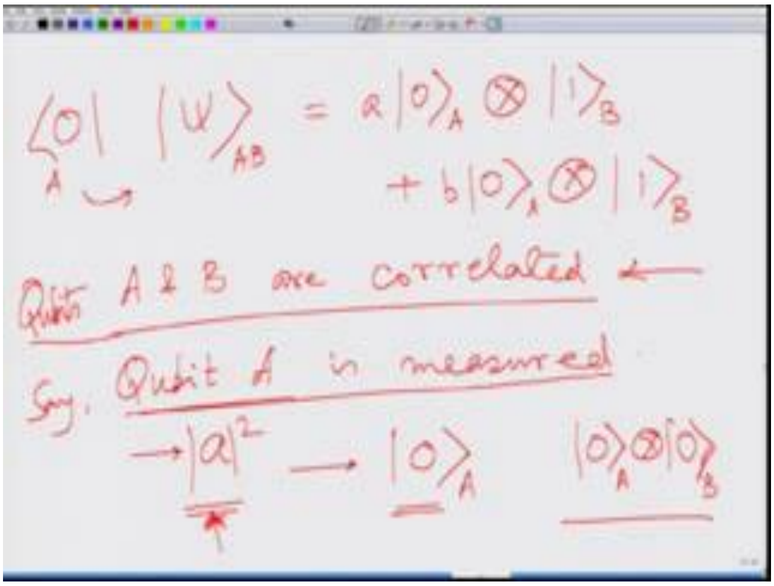

So, the wave function let us say, as a result of this will be given in terms of psi A B which is going to have some contribution of 1, and some contribution of the other. So, let us say I can use this as a A, then I will have end up producing tensor of product of the other part. So now, A and B are correlated see this is the part which is very different from the point when we were doing one single qubit. So, this is what you have to understand which is different from when you were considering single qubits.

This was not true when we were looking at single qubits because there was no issue about the distinction between one not the other. This was always one particular qubit which was under certain particular environment. And whatever happens to it was always a resultant because of that. Of all possibilities we were always getting one or the other. Now there are two of them, the vary point when they are two of them any changes to one can be essentially thought of that there is a correlated change in the other, because knowing one can result in having as knowledge about the other that is because they are correlated.

Student: Sir, greater great to accuracy retain 0 (Refer Time: 20:50) what about 0 represent 1 unit.

You can choose it the other way also, but they will again show a correlation. The idea is that matter how you chose it; whether you chose it in this form or you chose the other possibility they will all have a correlation possible. That is what we are trying to say that whenever we are choosing two qubits instead of one then we are bringing in the concept of correlation. So, that is the first thing to notice. Now, if you measure say you given this case where the A B's qubits are correlated say qubit A is measured. So, when you want to measure qubit A what will happen? You are going to create a projection along the A direction which means that you are going to now have the other side where I am going to use the part to come in.

When I do that what will happen? The probability will be based on what a squared will be. So, from here if I am only going to be interested in what happens to my probability related to a square then I will be getting anything to do with the measurement of qubit A, because that is all I have in terms of o A that is the part which is giving rise to the measurement of A. And this measurement will actually prepare me in this state.

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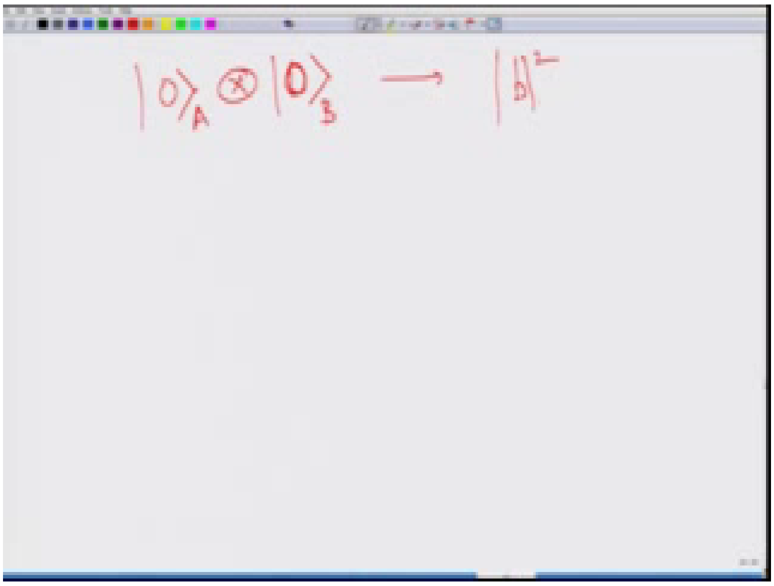

So, what I am going to do is, I going to create this measurement will actually create a state which is of this kind. And this will have a probability of b squared. Now to be correct absolutely we should have each of them put as the mod square values. So, we will go back again. Say I prepare qubits, I have qubits A and B and I prepared them qubit A B in such a way that I have this mixture, and this mixture is when I say that my qubits A and B are correlated. Under this condition if I measure qubit A then I will be looking for a probability of A mod square which is connected to this state 0 A state, which means that whatever will be left is the state which will be A B. Because, in order to measure this I have to multiply by this; in order to get my value of qubit A I will have to multiply by this from this side which means that I will actually create this qubit which will be. And measurement that we will get as a result of that is my probability of the qubit A, is that clear?

Now tensor product is ok, so that is going back further whenever we have more than one qubit we are going to have a tensor product between the qubits, that is the definition. Because we said that all possibilities have to be covered and only way the all possibilities are covered when we a tensor product not a vector product; every possibility is covered which means I am going to expand rather than the look at the sub group. When we do vector product we only have a sub set from the overall possibilities. So, the all possibilities are covered only through tensor product.

All possibilities would mean this and once we are considering the case where they are correlated then we have one of these cases. And from one of these cases what we are saying is we are going to measure say one which is qubit A, then we are going to come from this side with this projection, that is what we want to do. We are only going to take that plain which is due to o A plain which is what we apply. When we apply this we end up producing this particular state which is left over. And whatever measurement we make because of A is the probability associated with a; small a, that is my contribution from the o A state.

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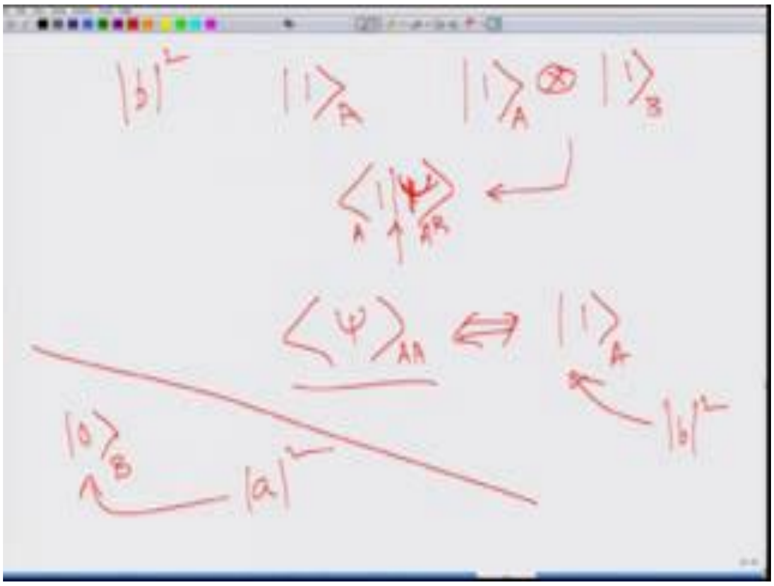

Similarly, if instead I measure the probability corresponding to b mod square so then what I am doing is, I am measuring the probability which is corresponding to this and then we are producing the state which corresponds to A. Now maybe you remember from your earlier classes whenever we write this, this was a short hand notation. Even in the last case when I said I coming from here I am essentially multiplying producing these kinds of states which are nothing but the way we had applied it.

So, that was a tensor product itself, but it is done or represented in this form. There is a short hand for this as you know where we even do not present this part, sometimes we just say what this corresponds to say making the measure for A, because you are only taking; sorry this was this was A B part where we came from this it became a measure only for this. And that is nothing but it is equivalent to saying that I have measured A. And this corresponds to the probability which will be given by in this particular case by b mod square. And on the other hand whenever you measured the B case we had a probability associated with a, that was the other case, clear.

See this is important, if necessary we can go back once more this is not a problem. What we can do is we can also do problem set in this direction in this one which will give you some exercise on how to do this which will help you in understanding this better. Maybe that is what we will do the problem set that I design will have an exercise which will test your ability on setting these up.

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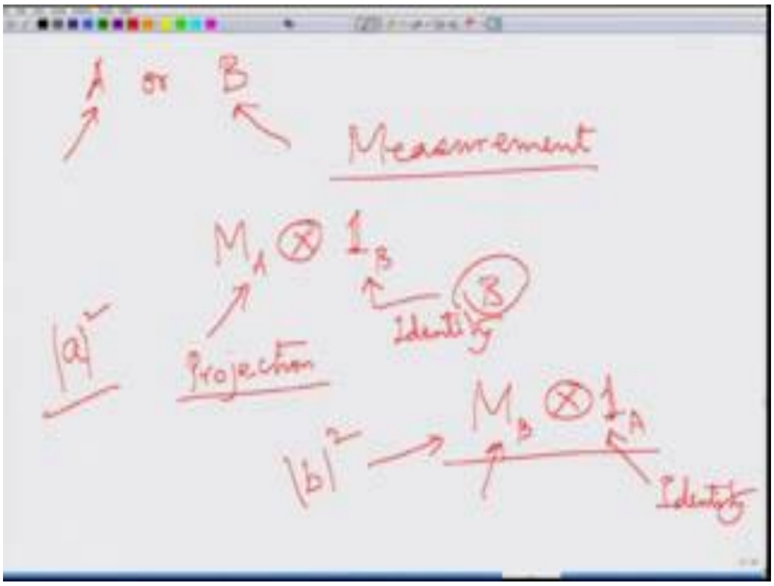

So the basic idea behind all of this is, I making a measurement now which is picking A or B that was a basic idea what we did. We were picking up on A or B through my concept of measurement.

But it is not really the way we have talked about in terms of projection. Is no longer simple idea of projection, because now you have more than one state and you are not looking at only one or the other; that is property of one or the other, it is something else you are actually trying to pick one versus the other? So, that is the reason why the way of looking at this is slightly different compared to the way we were looking at earlier. So, they have differences.

In this case if you are going to define a measurement operator then you have to make a measurement operator which looks like this, say for the state which is corresponding to this or more importantly I think it is better represent in terms of the identity matrix. The identity matrix on works on B which means that the B is remaining unchanged A is the one whose amount you are going to make measure. That means, A projection comes from A.

So, this is the first measure that we talked about where we were making measurements. If instead I used this one then we are doing the other measurement which is my b measurements, and because I am taking a projection in this direction and this 1 again is identity. Similarly here it is identity which means it does not change B.

So, we will actually do a couple of practical cases now. So let us see; what is the actual effect of this.

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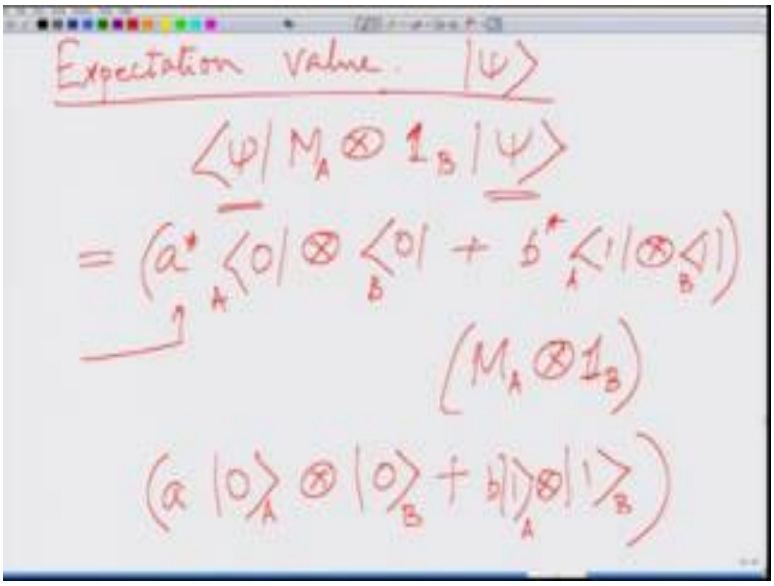

So, ultimately we are looking at expectation value, so the expectation value of the observable state which is our state is psi depends on what kind of measurements I am going to make. If I make this measurement that I just mentioned say M A identity B psi this is what I am applying. I have already defined these psi's for you. So, I am going to put that back, so that is going to be a star. So, whenever I write my bra vector then everything associated with that is complex. Whenever I write my ket vector everything associated with that is real. I mean one is complex conjugate and the other one regular complex number. So, this is my complex conjugate and the other one which will come is going to be a complex number.

So, this is going to get applied by and this is my B plus this is 1 this is my B. This is the effect of the identity matrix remains it same, then I am going to have the rest of it M A B. This is the left hand side, now the right hand side; a, make sense. So, it is basically writing out whatever we wrote earlier in this form.

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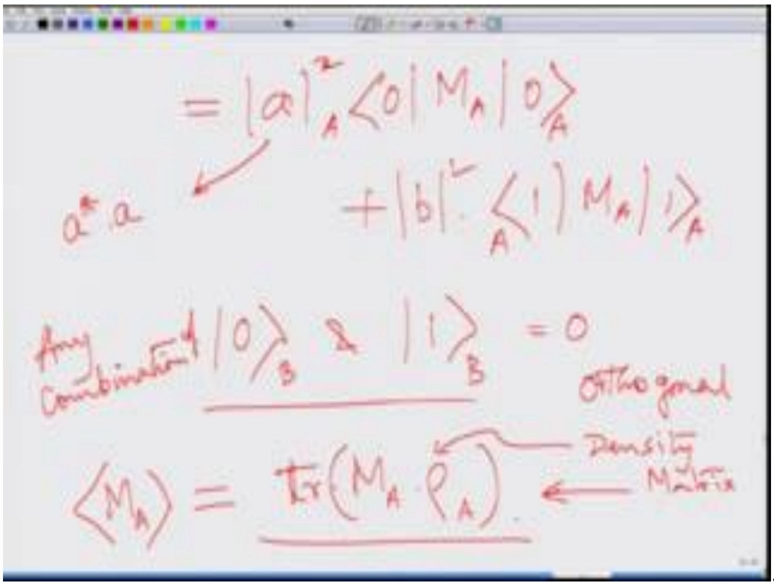

So, this one of them should be a, this should be a I think right. So, once we do that then we will get the next one which is you can go ahead and get that, so then sorry mod square. So, this by the way as we know it represents a, that is all. So, this is this and then we have is for the A part plus b mod squared 1 once again A A. So, this is after we use the originality of this and this. Since these are orthogonal the product of them with respect to each other will vanish. So, you will be only left with those where they are, so basically anything to do with this appearing is going to be 0; any combination of these two going to be 0, so this is your orthogonal.

Once we get this it essential means that we are looking at nothing but the projection of M sub A which is the trace of M A times the state that we started off. This we have seen before, this is getting to the same point which says that once you have; so in other words this particular representation of psi here is called the density matrix.

This replaces our notion of the cross products and all that and on thus tensor products and gives rise to final answer which we are looking at. This way you know you basically have an idea what is going on just to remind you trace of a matrix is just the sum of the diagonal elements.

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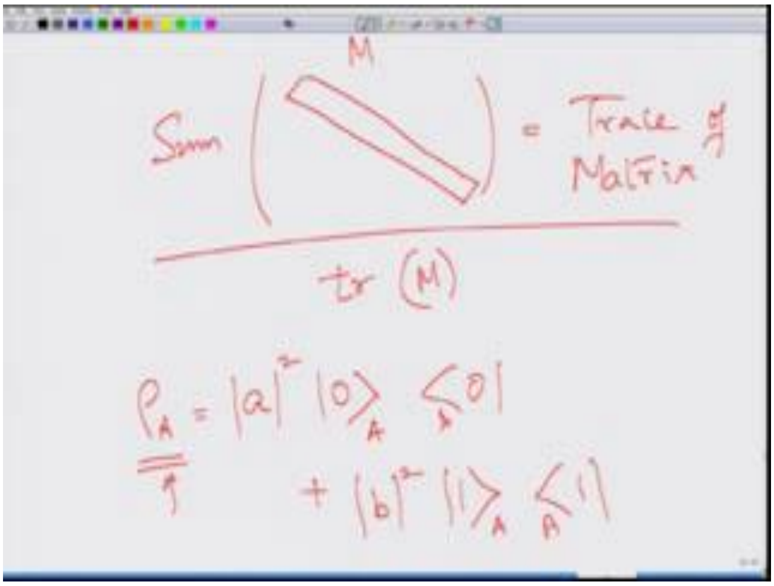

So, if you have any diagonals, any matrix then all the diagonal sum this is equal to the trace of matrix, it is just a memory revise. So, this is what we use when we said the trace out say this is M. So, this is the idea behind this how you get to your projection. So, from now on whenever you have more than one qubits this is the way you will go about finding out the solution.

So, rho A then is given by a mod squared of A is, right. This is everything to do with A. When similarly you can have the B, every time the index change will be getting the similar one, and whenever you have A B then each of them will start altering. It could be B A also then it will be the other way round. But that is actually harder to write, because that would mean that you are not going to have, so we cannot write away first. So, when we had a its nice, when we have b its nice, but when we have the two of them then what would you mean is that will be having independent parts coming from there which will be a part which is not provided by each of them.

For orthonormal conditions we automatically got them to be 0. If they are not orthonormal then I can arrive at rho A B where I have that situation, but as of now that is not the case.

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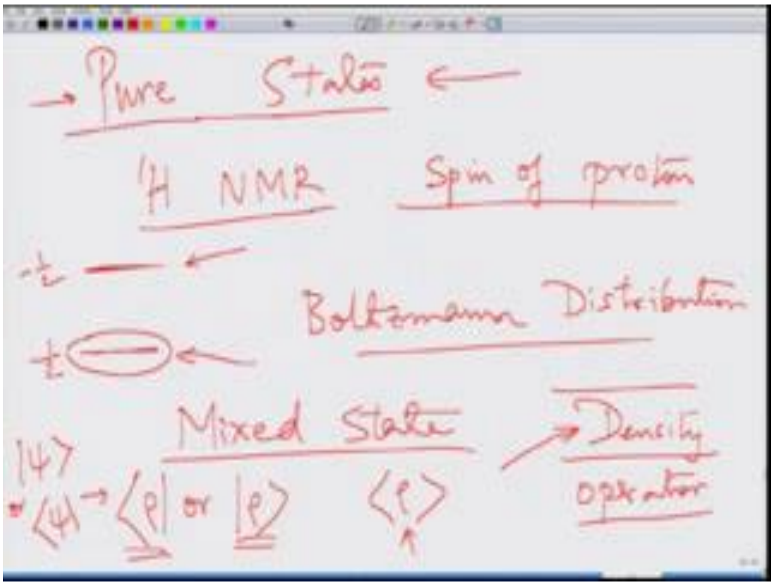

So, this is typically the case which is represented by something known as Pure States. So, this is the basic idea between pure states where the appearance of this cross term down does not exist. It is easy to look at states like this, because at the end of it you will either find one or the other. However, this does not preclude the fact that any other possibilities do not exists.

So, that is the reason why I just mentioned when we started traditional idea of projection will no longer work when you have multi qubits, because the very fact that only under the condition when you have pure states then this traditional idea exists. But that need not be true, you can always perhaps mathematically design a measurement where you will get this kind a way answer, which means that you can define or you can start off with something known as a pure state.

So I will give you an example, say you have a spin of an electron or a state or let us say in NMR we are taking the proton and we are looking at its states. So, let us say the proton NMR, in this we look at the spin of the proton. Now this is not for one proton there are many protons which are available, and so if you have a condition where the spin of all the protons can be represented by something. Then that is the condition where we have the pure state, but that is actually not true.

Even in the traditional NMR when we talk about although we happily represent the spins by plus half and minus half and we say that there is this let us say the plus half state and the minus half state, we represent them. We often know that it is not true that everything is here. There is some probability that it will have a population of states here and then there are certain other populations of states which are there. Typically we know that there is something called Boltzmann Distribution; that sort of establishes how much of the population is going to be in this state versus this state at any point of time.

If you make any measurement at any point of time just to say that I have everything at let us say the minus half condition or I have everything at the plus half condition is not possible. So, the idea of getting a pure state for any physical system is almost impossible. In the definition itself we have an issue about defining pure state, but if you do have a pure state no matter how many states you have it can be made two behave in the way that we have started our discussion with a single qubit.

That we can go ahead and step by step understand how the correlation works. But the moment you do not have this situation which is how the things are in reality, then you will have to confront the idea that this principle of projection along the particular axis or doing something along that way is not going to work out. So, that condition is known as Mixed State.

The reason why we can still get away in terms of let us say NMR and it becomes a good example case for starting or understanding quantum computing partly is that you can always define at a certain condition everything to be under the pick pure state to start with and that is why in real NMR condition do this kinds of studies. So, that was one reason why initial ideas of quantum computing were based on NMR. NMR has its limitation, but still this very principle that you want to start with mixed with pure states was possible to be defined in terms of NMR so that was one thing. Let us see what else we can tell you about this.

But most importantly this whole thing is starting to deal with something called Density. This density is an important term because everything else after this will be treated with respect to the density and this is a density operator correspondingly which will now look more like. The density matrix is something which is this, but a density operator is something which has this notation as this or this. So, the density operator is something which you apply and you give rise to a result that we looking for which can have a density matrix as a solution.

So, we will be replacing our size with these densities when we are entering this field, because whenever we deal with multiple qubits will be having that. So, that is the basic idea over here. We are going to stop today here we will pick out a pick it up from the next in this class.

Thank you.