Implementation Aspects of Quantum Computing
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Lecture – 20
Laser Experimental Implementation for Grover's Algorithm

As discussed in the last lecture, we will be now looking at the Implementation of Grover's Algorithm by using optical techniques. In particular we will be looking at using laser cavity as a Grover's search algorithm implementation.

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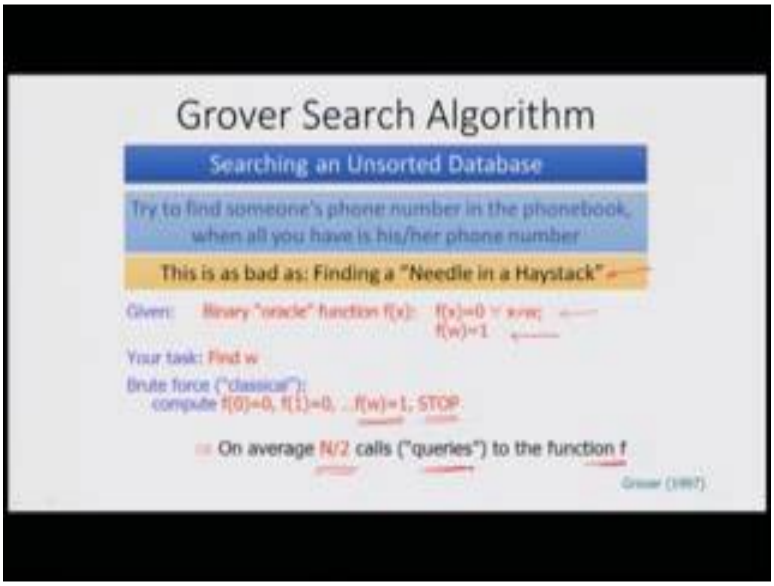

Now, this particular work is based on what Grover proposed in 1997 in his PRL, where he formulated the idea of Grover's search algorithm which gets the advantage when you are looking at searching an unsorted database. It is like trying to find someone's phone number in the phonebook when all we have is that person's phone number. Now this is as bad as finding a needle in a haystack as Grover defines it, and it can be mathematically formulated as a problem where given a binary oracle function f of x, the function would essentially be getting a value of 1 when the right function, is there otherwise when the right answer is not there if x is not equal to w then it will always give 0.

So, the task in this case is find the value of w. Now if somebody has to do in the root force or the classical approach, then every particular value has to be computed at least half time, unless the f of w which gives rise to 1 is found. So, it goes on an average you know about two call or queries are required to the function to find the value of w. So, that is the classical approach.

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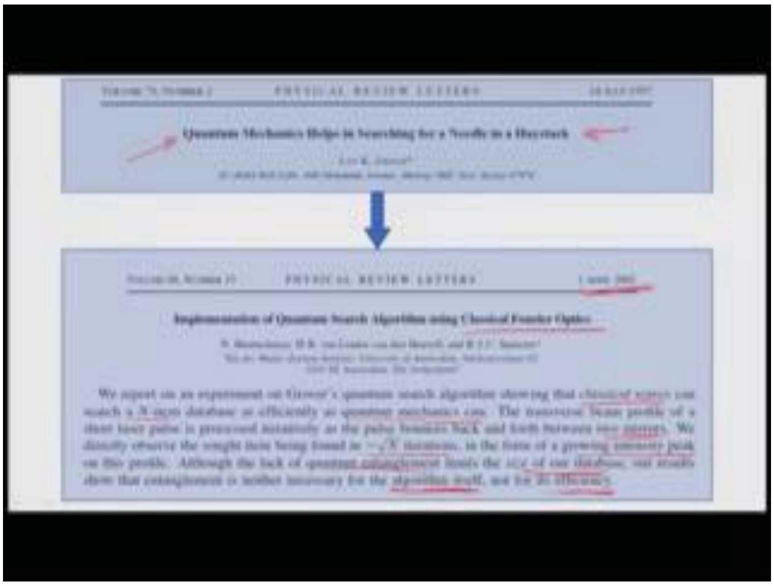

And what he was able to show was that, if the quantum oracle is used then it can be done in root n approaches and that is something we have discussed earlier in terms of plane Grover’s algorithm.

Now, in terms of implementation by this group in which came in 2002 by using classical Fourier optics in terms of using a laser, they were able to an experiment of the Grover’s quantum search algorithm showing that classical waves can search an a n item database as efficiently as quantum mechanics. And the transfers beam profile of the short laser pulse is processed iteratively as the pulse bounces back and forth between the mirrors of the laser. So, they directly observe the sought item and were able to find that in roughly root and iterations in the form of growing intensity peak of this profile that was precisely the way how Grover had predicted in his original paper in 1997.

However in this case as expected, due to the lack of quantum entanglement the size of the database gets limited, the results were showing that the entanglement is not necessary for the algorithm itself nor for it is efficiency, except that the size of the database is something that gets limited as a result of this quantum aspect being measured. But this was very important demonstration of Grover’s algorithm by using classical Fourier optics. And let us look at it in some detail.

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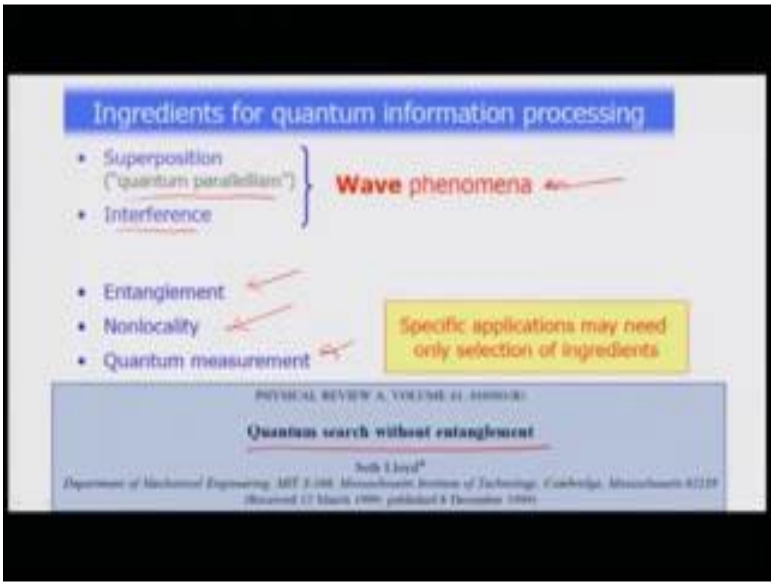

The principle which was available in this particular case which is the superposition it is a wave phenomena and that is a quantum parallelism which exists in both the cases, and it also has the interference phenomena which are wave phenomena. However, entanglement non locality and quantum measurements these specific applications need them which were not present in the wave phenomena.

So, for quantum information processing as were shown by Seth Lloyd in 1999, it was understood that there are certain problems which can be done without using entanglement and he had already shown that it was possible to show quantum search without entanglement. So in that sense this particular approach of classical waves or pre optics to use in terms of quantum search of Grover is an interesting example.

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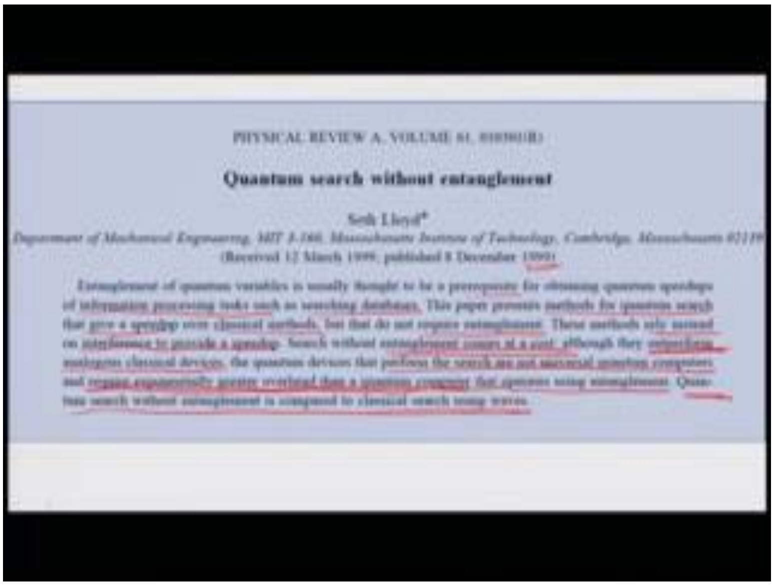

So, Seth Lloyd had presented it in 99 that the entangle of quantum variables which is usually thought to be the prerequisite for obtaining the quantum speedup’s of information processing task such as searching databases, and he was able to show that there were methods for quantum search that gives speed up over classical methods, but does not require entanglement. And these methods rely instead on interference to provide speedup. Search without entanglement comes at a cost that was the critical point that he mentioned at that time. Although they were out performing the analogous classical devices, the quantum devices that perform the search and not universal quantum computers as such they require exponentially greater overhead than a quantum computer that operates using entanglement.

So, in that sense quantum search without entanglement is compared to classical search using waves and that was what he had suggested in 1999 theoretically and had mentioned that it would require exponentially greater overhead than a quantum computer. And that is the part which in this particular implementation using waves inside the laser had the issues to do with the search in terms of the size of the problems that could be looked at. However, the principle mentioned here does work and that was every interesting.

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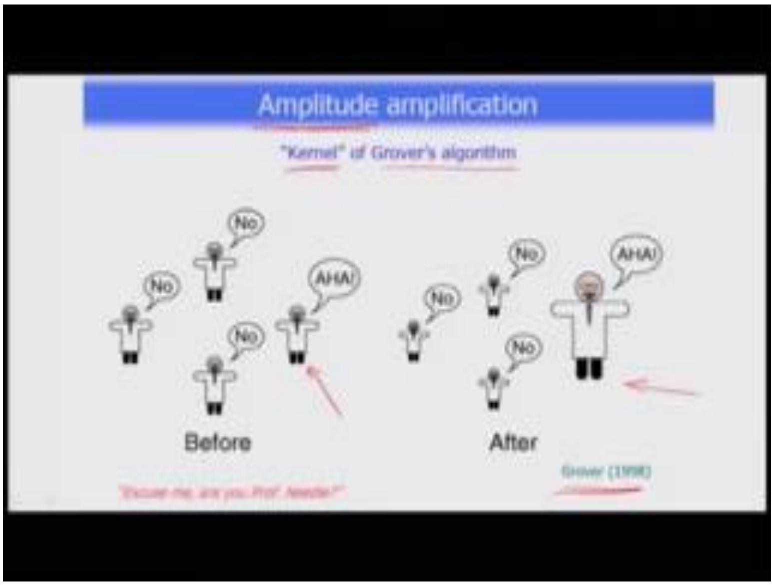

So, the idea in this entire process has had been the principle of amplification of the signal that has been sort after. So, if there is a set where the answer essentially exists, but what Grover was able to point out was that by using the oracular search approach it was possible to amplify the signal and as such be able to look at it efficiently and point to it as a result of this process.

And so this repeated questioning in a way would work when the amplification is happening every time the question essentially is being answered. So, this amplitude amplification is a critical step in the Grover's algorithm and in fact in some sense it is the kernel of the Grover's such algorithm.

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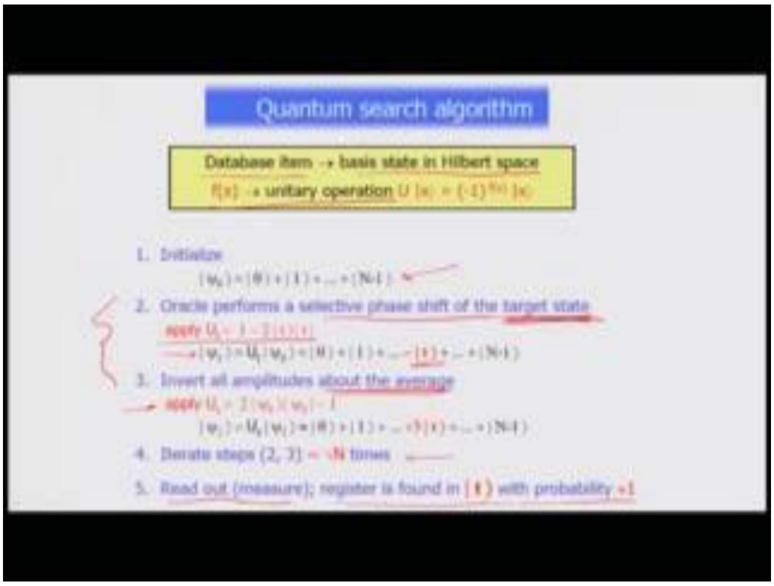

The important part of the quantum search algorithm therefore, relies on the fact that the database item is being transformed into the basis set in the Hilbert space, wherein unitary operation is being performed which sort of sets up the phase of this problem in terms of phase flipping. And so this is the unitary operator which that does that.

So, the 1st step as always required for a quantum implementation process is to initialize the register, quantum register. Then the oracle is performs a selective phase shift of the target state and that is the kind of processing is which sort of is essential in terms of having an appropriate oracle which recognizes the solution, but does not know the answer. So, in that sense basically the oracles unitary operation keeps on amplifying it through a selective phase shift operations and so this unitary operation which is being applied to the quantum register which has been loaded with the data, gives rise to this marked state which can then undergo an inversion, wherein the all the amplitudes about the average is being rotated or inverted.

So, all the amplitudes about the average gets inverted in by the application of the unitary invert operator. And this entire process is being applied. So, this step 2 and 3 is repeatedly applied or iterated in such a way that it on an average requires root n’s steps to get to the answer. So, the readout or the measure the register is found to be in the state with probability almost equivalent to unity in terms of the right state that is the mark state which is the target state which was being looked for.

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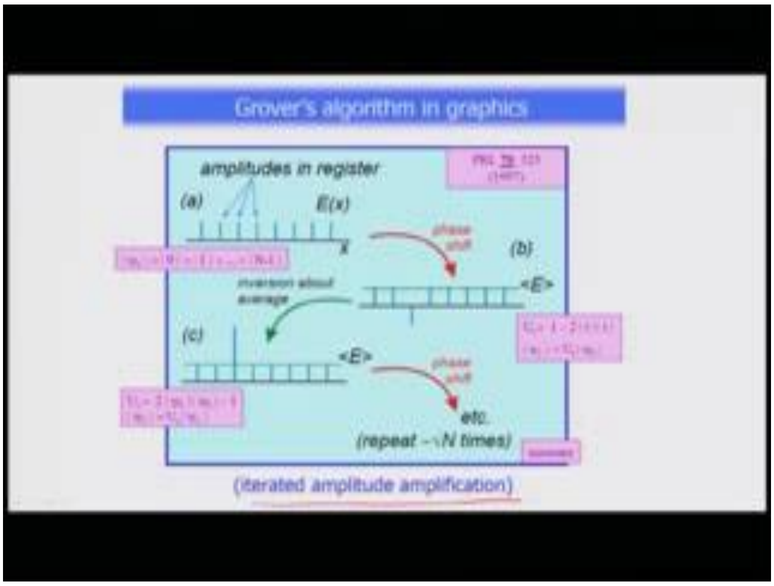

So, in graphical sense this is what Grover showed in his 1997 paper is that the amplitudes in the register are given the phase shift, in such a way that the mach state is undergoes a transformed inversion about the average then flips it back up to a point where it gets amplified and this undergoes phase shift and is being repeated root n times to until the measure is done and the iterated amplitude amplification results as a result of this process. So, this is the essence of the Grover's algorithm graphically.

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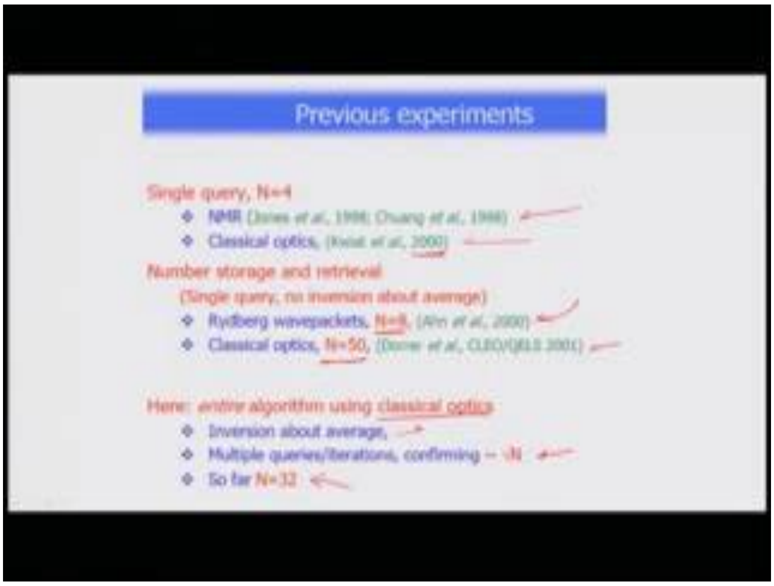

There have been many previous experiments as we have also discussed earlier which is based on the NMR process that we looked at. We have been discussing about the classical approach optics approaches, but have not really exactly discussed the Paul Kuwait's approach that was shown in the year 2000 which we will next look up in the future lecture which also involves classical optics. And once again the scalability depends on the elements that we talk about.

The number of storage and retrieval requires single query and there is no inversion about average and there have been also other ways of looking at it that is which has been done by the Phil Bucksbaum and his group and et al in 2000 where they were able to populate Rydberg wave packets with the states. And they also have the same problem of the number of states which could be addressed and there the marked or the encoded state was possible to be measured also as per Grover's terms by using the Rydberg wave packet states that were written; and then looked at in the Grover sense of root 8, but again the number of qubits that could be utilized in that process was also very limited and precisely n equal to 8.

In this particular approach of using a laser and to do this algorithm it uses classical optic sense again that the entire algorithm is through classical optics, it is again undergoes the inversion about average, it goes the multiple enquiries iterations confirming the same process. And as far as the particular principle was been demonstrated in the year 2001 it had 32 elements that could be encoded for doing this. Although in 2001 some more work by using classical optics showed that could be there can be other studies of classical optics which can take the number of encoding states to about 50.

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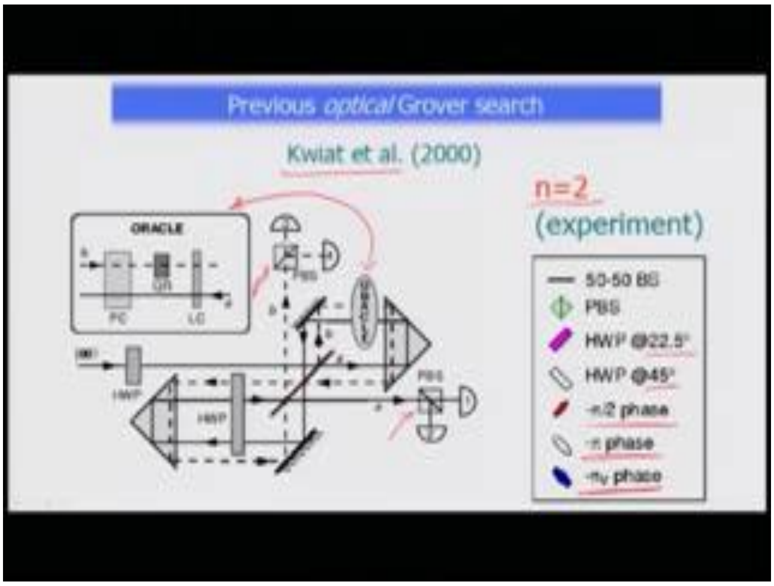

So, ongoing efforts are there in this Grover search methods and all these has been attempted in different ways. The 2000 approach of Kwait et al is actually shown here in brief I mentioned all be doing it in next lecture, but it is known it is not necessary I will actually explain it here itself. Where what was done was they used a halfway plate and beam splitter and polarisers to sort of to utilize this principle of linear optics to encode and decode and search the encoding of the states that were being looked at and look for the database. So, this p c here is the pocal cell and l c is the liquid crystal and q r quarter wave retarder and this is the oracle which works which is being used in terms of marking.

So, quarter wave retarder essentially sets the particular state which is marked through this iterative state of rotation which enables it to being thrown out if it is being the mark state. And so this is how it operates you have the right enter with 2 qubits encoded through this halfway plate which undergoes these interferometric pathways of the beam being split by beam splitter into the different pathways. This is the oracle which is being looked at where the light beam is following 2 pathways; one where it has been displaced and other where it has not been displaced.

Now, the displaced part is the 1 where it actually goes through the other part and is being trapped to get to the information that you would like to have and these are the basic points which are being shown here. So, this is the 50 50 beam splitter; the polarising beam splitters are sitting here and here in terms of this and these halfway plates which are at different angles some of them have 2 and 2.5 means it will essentially act like a quarter wave plate and the other halfway plate 45 degrees has been used.

So, in 1 case double pass of that will act like a halfway plate otherwise it will be a single phase. And so they would give rise to pi by 2 phase shift pi phase shift and pi over pi v phase shift depending on their rotation axis that they undergo. With this n equal to 2, 2 qubits that came in was possible to search for the pathway.

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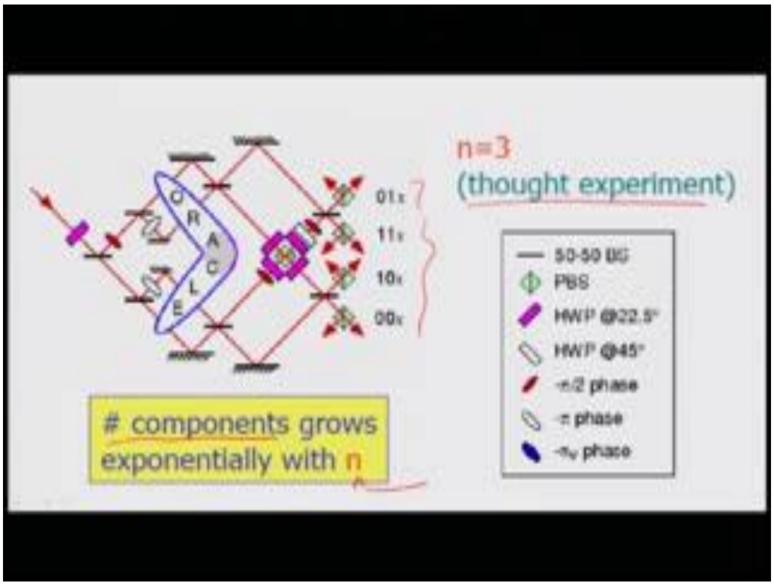

Now, in this process the biggest problem this is a thought experiment which was thought of doing here. And although in principle the number of qubits can go up higher it gets complicated because the components at every point leads to have more and more of these individual beam splitters and interferometers which are being port through, with multiple oracles doing the same bits of job in each of these to finally, give rise to the qubits that we are looking at.

So, in this process it was estimated that it could be scaled up to a certain number which theoretically can go up to a certain value, but as you can see the now number of components grows exponentially with the number of qubits that was being implemented. In this sense the classical approach of quantum computing in this process is very very limited because of the exponential growth of the resources necessary for this.

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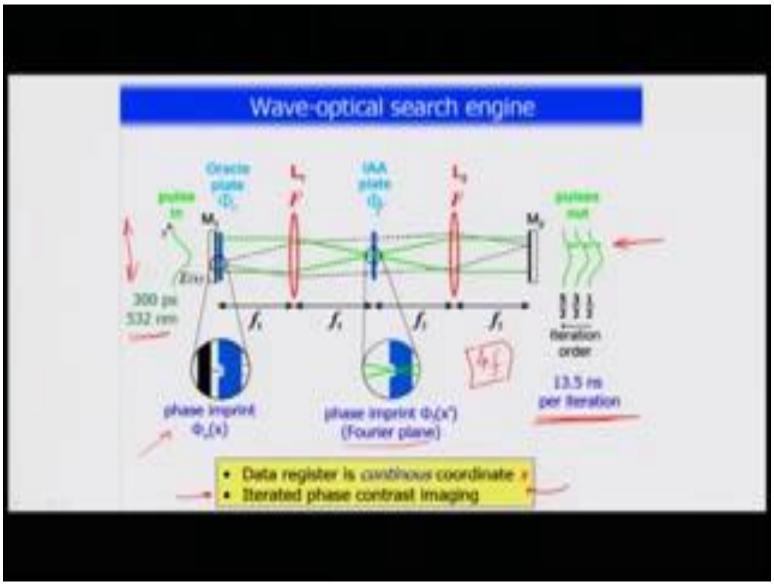

On the other hand, the process which is being implemented as discussed by this particular paper in 2001 by using a laser cavity is much more efficient because what it does is it uses a notch filter in a plate which is a phase in notch phase imprint which is been put right next to the high reflector of a laser cavity such that; so this is the high reflector. So, it is a almost hundred percent reflective. But there is this phase imprint which is being inputted right at that point, where that is the one whose Fourier domain point. So, one of them the one at the 1 which is right next to the high reflector acts like the oracle plate and one at the Fourier plane does the phase imprint in it is Fourier domain and it is being looked at in 2 different ways.

One, when it is the marked state where it actually focuses goes through a different geometry to the mirror and so out comes the pattern at one point which is being amplified as it undergoes through the number of iterations. So, it is a 4 f geometry where the amplification process occurs and this 13 and half nanosecond time repetition per iteration that is being taken care which is being doing it in this process. The data register thus is continuous in it is x coordinate and the iterated phase contrast imaging is what is being applied to use this.

So, this 4 f geometry amplification scheme in the optics approach works out very nicely because the plate which is being used is also essentially the path where the process can go through the amplification process. So, a laser which is having say 300 pecosecond pulse with at 532 nanometer say YAG laser which is being put through this cavity, where these phase plate in it is Fourier are put together at the f and 2 f points end with respect to the lens to give rise to the this phase amplification scheme which undergoes this iteration. And under the condition that there is a 13 and half 0.5 nanosecond time lag between the iteration is enough to have no interactions between them.

So, it is a continuous data register in the along the x dimension which is the dimension where the pulse is being inputted through the mirror and in the spatial domain therefore this phase input x as the amplifier point which is being marked out as it goes through the iteration order.

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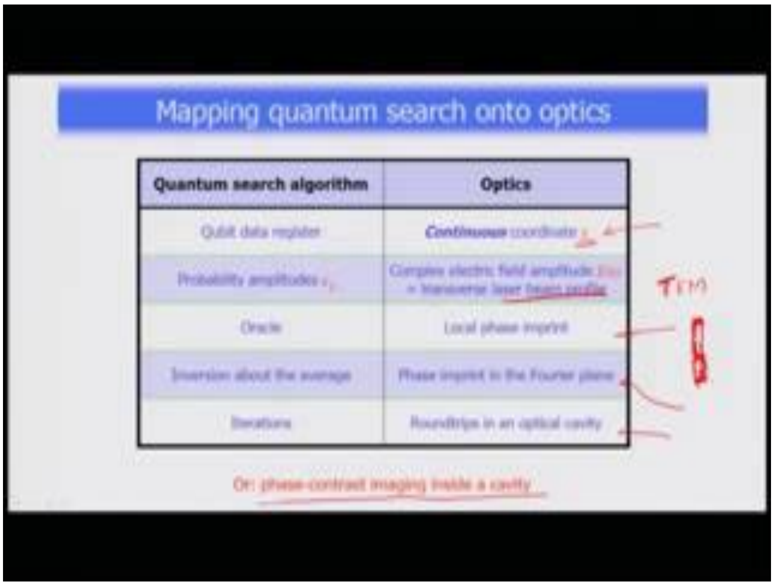

So, the quantum search in this particular approach is being mapped by using a qubit data register in the optics domain by ussing the continuous coordinate in the x dimension. The probability amplitudes are then correspondingly the complex electronic field amplitudes in the later on the transverse laser beam profile. So, this is the transverse electromagnetic mode; TEM mode of the laser beam which is being played around with the coordinate space which is undergoing a phase change depending on the notch which is being provided in the spatial domain for the phase which continuously operates by using the optical geometry which is being iteratively gone through.

So, the oracle in this case therefore acts in terms of the local phase imprint which is being inputted through the phase notch that has been shown at the input mirror or the high reflector end of the laser cavity which has been utilized for this process as shown in the figure. Here, that is the phase notch which is the oracle that has been used and the inversion about the average is being done through the phase input the analogous 1 in the Fourier plane which is able to undergo the same Fourier input because of the phase imprint which is there in the Fourier domain.

Once again, in the Fourier domain this phase plate essentially this is the same job as the inversion about the average. So, the number of iterations therefore depends is given by the roundtrips in an all optical cavity which is where this whole work is being done. So, the phase contrasting imaging inside the cavity is the way this particular approach of quantum data processing has been done to reach the Grover's limit.

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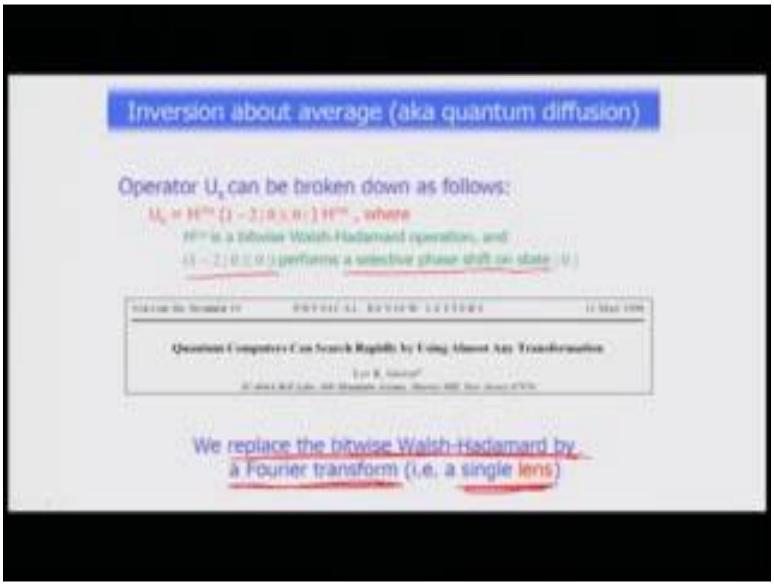

So, the operators in this case can be broken down into the bitwise Walsh Hadamard operation and the particular operation essentially performs a selective phase shift on the state input which is the 0 state. And we therefore in this case it is the replacement is happening by the bitwise Walsh Hadamard is being replaced by a Fourier transform which is essentially a single lens in this particular approach. So, the single lens here essentially acts like the Walsh Hadamard operator. So, the Walsh Hadamard operator is being replaced by the Fourier transform element which in case is essentially the single lens.

So, the analogy to the original work is hundred percent there except that this is a physical device in this optical domain which essentially limits pack to the same operation which is achieved through the Walsh Hadamard gate.

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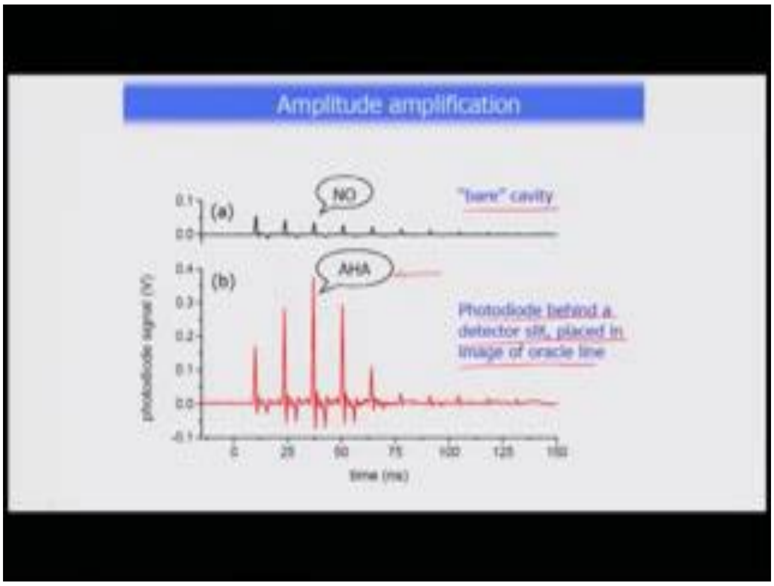

So, in terms of the bare cavity where there is a no data which has been inputted in terms of the phase mask there would be no enhancement happening. So, it will always be a no for an answer. However, when a photodiode behind a detector slit placed in the image of the oracle line, then there will be amplification corresponding to the marked or the target bit which can then be looked at. And that is how the solution appears.

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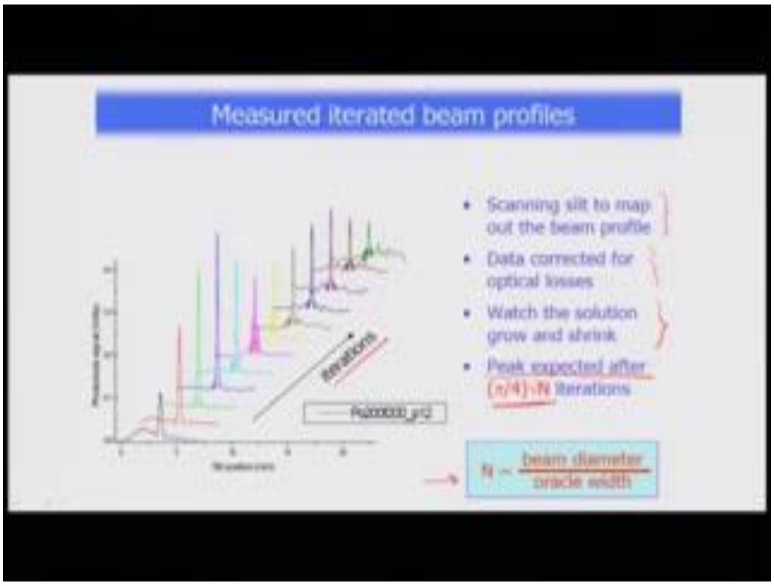

It essentially goes through the iteration bits, the scanning slit to the map out of the beam profile it shows the iterative measurement which is going on for when the number of round trips, the data can be corrected for the optical losses and the solution can be watched to grow as well as shrink depending on the number of iterations which are being performed.

The peak of the actual result is expected after roughly pi over 4 times root n iterations. And the number of iteration is generally decided by the beam diameter over the oracle width. The oracle width is essentially the phase notch filter which is being provided in the spatial domain.

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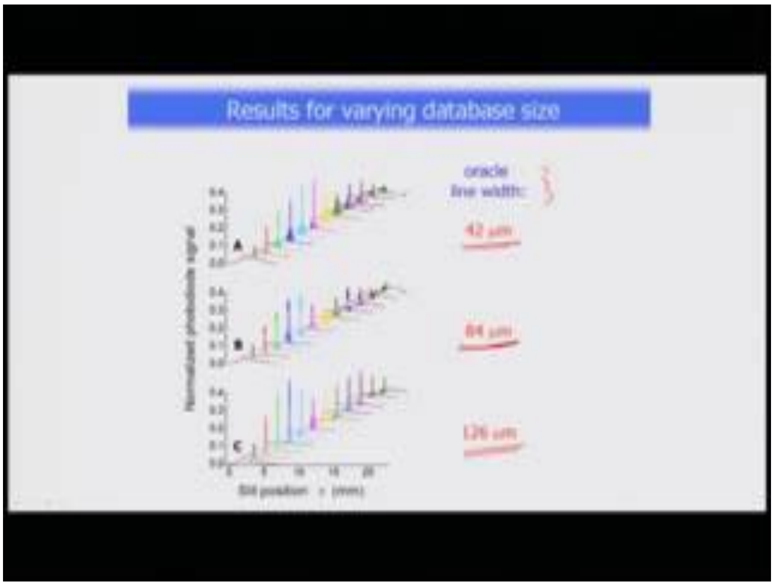

So, given the oracle line width as we just mentioned when it is as short as 42 microns 2 and it is about 126 micron the results seem to follow as similar kind of principles and it is possible to get the same kind of results depending on the number of iterations.

Given the understanding that the number is being defined as we just discussed in the last slide, which is the beam diameter over the oracle width, which is now for a fixed beam diameter the oracle width has been changed and so the number keeps on changing.

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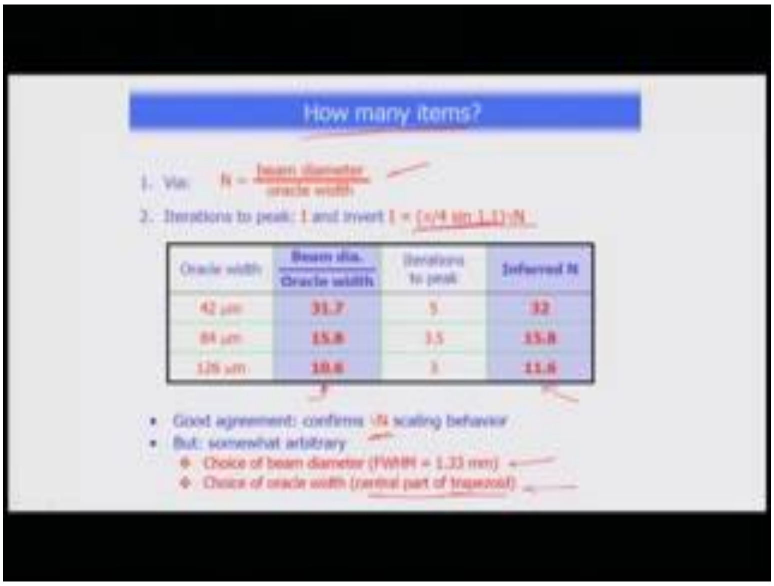

So the critical question therefore, now comes us to how many times by using the particular process that we just defined the iterations will peak at every i and will invert at that many root i's. So, given the oracle width of different kinds. So, the experimentally the beam diameter of this where also the ratio was found to be that that is roughly the n value which is what is being looked at the number of iterations to peak would be the once which are roughly the root n here.

So, based on that the inferred N which for each of these operations were pretty close to the experimental data. So, this 32 with respect to 31.7, 15.8 so roughly exactly the same 11.6, 1.6. So, this is in good agreement and confirms that the root n scaling behavior.

So, we were using this particular formula to generate all these values. And however, the some of the parts which were somewhat arbitrary has been in this particular case the choice of the beam diameter which was set to be 1.33 millimeters. The choice of the oracle width which was the central part of the trapezoid which was being matched on it was also sort of arbitrary in terms of finding these, but having said that the agreement was essentially extremely good in terms to getting these results.

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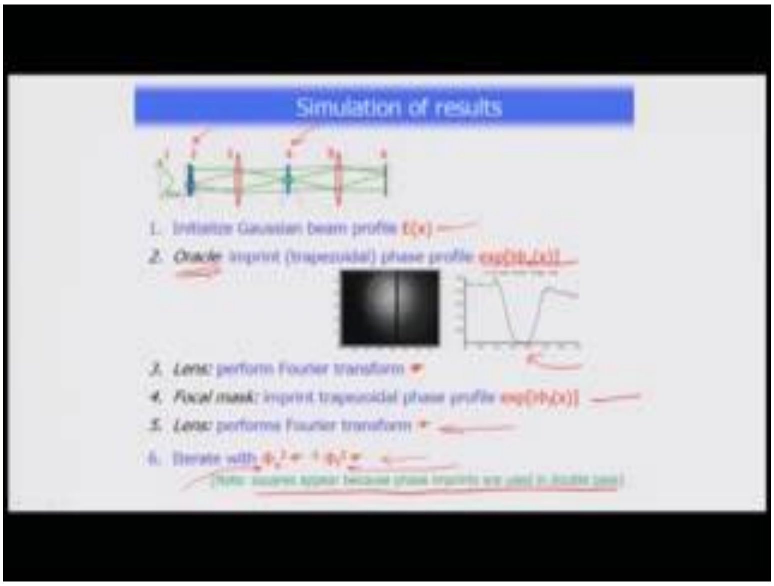

So, in some sense the simulation of the results for these particular case would be confirming what had been attempted in terms of the experimentation and in that sense it would be essential to have a model in which the input beam with it is x axis been defined is going through a set up 4 f set up in the same geometry as we have discussed. We initialize the Gaussian beam profile e of x. The oracle in this case theoretically would be the imprint or the trapezoidal phase profile of a certain exponential.

So, in this exponential I phi naught of x which gives rise to this kind of a trapezoid function. And use a lens to perform Fourier transform, it is a focal mask to imprint the trapezoidal phase profile and again use another lens to perform the 2nd Fourier transform to bring back to the normal plane and then keep iterating this with this functional form. And what it would be found is these squares would appear because the phase imprints are used in the double pass.

So, that is because the actual results would essentially have the squares because the phase imprints were used in the double pairs in both the cases of these process. So, instead of having 1 interaction there were always having double interaction. So, the operator was acted up on twice and that is why the iterations were in terms of squares.

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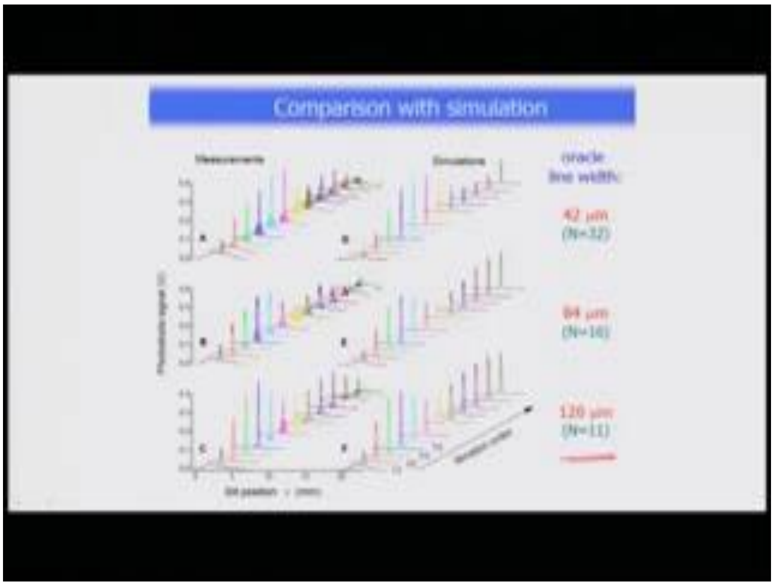

So, when this was compared with the measurements the simulations were in quite good agreement and it was found that the process as discussed through the experimental process sort of remained similar to what was expected.

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And the idea was that even if the iterations involved required finding the values which were not as clear cut as in the experimental case, like you were pushing the limits and you were trying to find it at the position which were not having enough photons, even then the preliminary conclusions were thought of as to item was even found behind (Refer Time: 32:09) the qualitative change of the oscillations were there and in some sense some more work was thought of being required.

However, it was not pretty good agreement in terms of the actual signal that was being looked at. The part which created a little bit of a problem was this region where some sort of a qualitative change in the oscillating feature came in terms of these simulated results versus the others fact being that there was some aspect of this noise and these are the things which came on. And the item was being found even beyond the full width that I have my switch remained like a little bit of a questionable parameter when it was done in theory.

However, 1 thing to note here is that since the qualitative features of the results were still maintained the theory behind this remained quite good.

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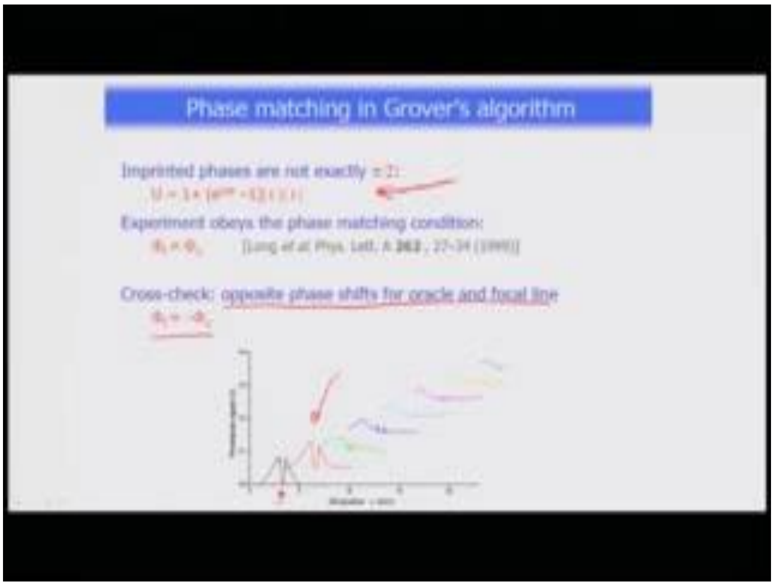

The idea understanding where much more realistic when it was thought or understood that the imprinted phases were not exactly pi by 2 in the real sense of the experiment. The experiment obeys the phase matching condition instead of having arbitrary cases where they were having different levels. The cross checking essentially meant that the opposite phase shifts for the oracle and the focal line were in the same pathway and that created better approach where it was possible to see how this signal interaction worked out.

So, theoretically it was therefore possible to even go a little bit further, but I think the most important take home message was the fact that the principle of the Grover’s algorithm was met with irrespective of how the experiment design was being looked at.

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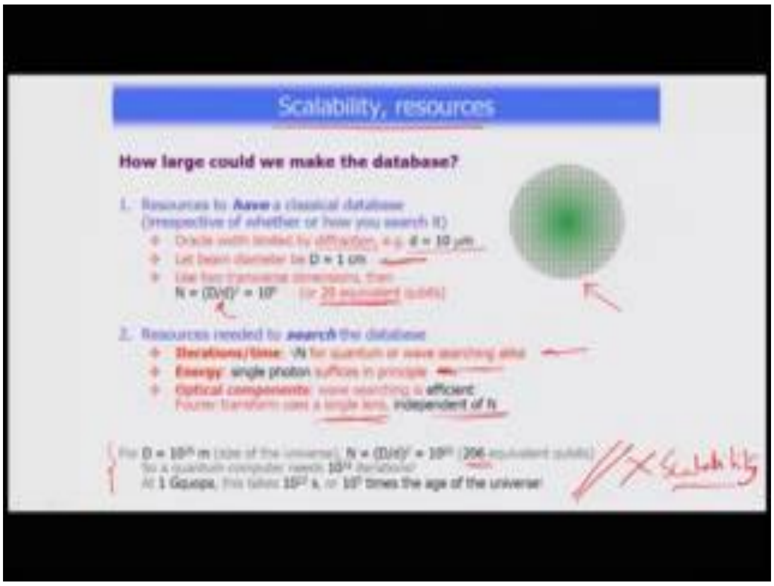

So, now the question which came immediately was the scalability at the resource used. So, if you look at a beam profile it roughly in the x y plane it looks like this where we are basically using a grid pattern to figure out, where can we put in the notch in the phase front which would essentially encode the data in it. So, in some sense resources has to be a classical database irrespective of whether or how we can search it and that in sort of imprinted on the beam profile that is being looked at.

So, the oracle width limit is going to go down to the diffraction limit and that is in terms of roughly 10 microns or so, because if it goes below that then will not be possible to distinguish between the notches which are right next to each other. So, by using a beam diameter of about say 1 centimetre although it was much smaller beam diameter that was being actually practically used and by using 2 transverse dimensions or about roughly 20 equivalence of qubits were being talked about in this particular approach. Because, the distance over which the measurement and the other processes was being done had to be respect with respect to the diameter of the beam and in that regard the beam divergence became an issue, as well as the signal intensity because as this went through further and further in this entire process of going through the system there were losses in the optical process which had to be also encountered.

So, the resources needed to search the data by required the iteration time for the quantum of the wave search was sort of a like which was very important. In typical sense single photon energy would suffice in principle. However, for measurement purposes and to get beyond the beyond the noise limits this number had to be substantial so that it can be measured. The optical components wave searching is quite efficient, the Fourier transfer using a single lens and that is why which is that is independent when that really helps this whole process. So, certain number in principle could be attempted which could go for say how large of a beam diameter which could be used; for example, the size of universe or if you were going to see at a distance which could be of a certain cases.

But, the scalability in some senses extremely poor because even if you take a size which is as large as the universe size it only results in 206 equivalent qubits whereas, in some sense getting up to 1 centimetre diameter which is also a pretty large case, but it is doable is we are talking about in terms of 20 equivalent qubits. So, it is only a scaling factor of 10 which would demand it to grow to a huge size factor. And therefore it is something which is becomes very difficult.

For instance, talking in terms of numbers this is also extremely difficult to reach which are 10 to the power 31 iterations and this is not possible to do in terms of scalability. So, in some sense the scalability of this is an issue, however it is kind of q as to how it is possible to completely picturize this entire process of Grover search by mapping an optical beam, optical profile; profilometer in some sense and writing out the information on a laser beam of light and then putting it through a Fourier device to get the result. And, up to some 10 to 20 equivalence of qubits though it might look reasonable it just scales terribly beyond that.

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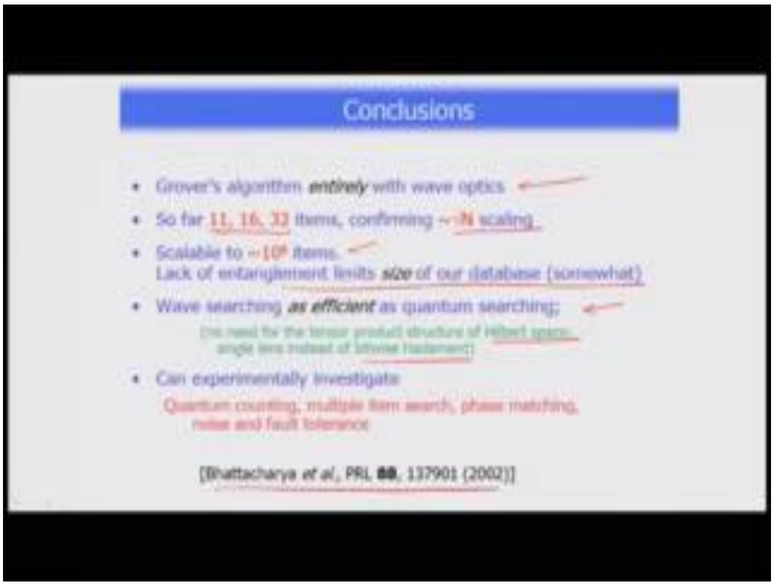

So, the idea of being able to show Grover's algorithm work entirely with wave optics is quite impressive. And in some sense the numbers in the range of 11, 16, 32 items confirming root n scalability has been shown. And in principle it is scalable up to 10 to the power 6 items the lack of entanglement limits size of the database to a large extent.

Now the wave searching is in fact as efficient as the quantum searching and there is no need for the tensor product wise structure of Hilbert space single lens instead of bitwise hadamard does the job and that is a huge advantage in terms of Walsh Hadmard gates that are necessary in other approaches, here just using lens for a Fourier transform becomes very effective. And it can experimentally investigate perhaps some aspects of quantum counting, multiple search items phase matching noise and cauldrons and this was reported in the PRL as mentioned below in the year 2001 earlier.

And with this I am going to end today's lecture, where we have basically shown you how to look at classical optical approaches either in terms of laser which is being specially encoded or which is being put through beam splitters and polarizer's to be able to do as important jobs of Grover's search. The only problem that ends up in all these approaches although the laser part was much better than the plane linear optics approach, the scalability was definitely much better than the linear optics method. But yet it still suffered from the scalability approach, but either way we have managed to show that it is possible to show Grover's Algorithm's effectiveness and efficiency even in these classical roots by using so position alone.

However, if it needs to go further in terms of source algorithm this is not is what going to happen anymore. So, there some other approaches will be needed and we will take up on these things later. As far as the next part of this week is concerned think we will be looking at the problems at solutions in the time if we have left in this week. For the questions that now have been asked over the forum for clarifications which we can perhaps do here in the little bit of time that is left over in this week. And with that I am going to end today's lecture.

Thank you.