Implementation Aspects of Quantum Computing
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Lecture – 18
Optical Implementation 'Linear Approach'

Last week we were discussing all about lasers as a part of our Approach towards optical Aspects of Quantum Computing. We indicated that we will try to actually use that principle, but we could not finish enough in the last week to get into any of the ideas of quantum computing. We just managed to do all the details about lasers.

So, in this week let us go back to our concepts on quantum computing, a little bit with this background on optical aspects that we looked at in terms of lasers, and see from there what we can do.

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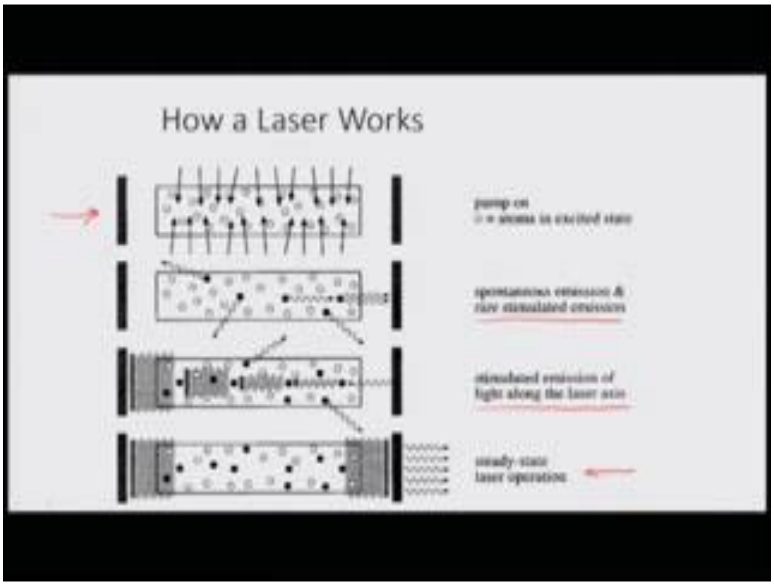

So in summary, in the last week we essentially talked about lasers where we had overall idea of pumping the system, because lasers generally would not work otherwise. So, here is a summary slide of what we were doing last week. A laser basically works on the principle of pumping the gain medium within an optical resonator to a situation that we result in population inversion. When that happen the natural process of spontaneous emission sets up the initial photons that are necessary stimulated emission process which is otherwise rare, but once the spontaneous emission photons are available they start stimulating the process inside the cavity for amplification, and that is how we get the laser.

Where the directionality of the stimulated emission of the light along the laser axis results in the amplification, that we generally need to finally get to the steady state laser operation, or as we have discussed in our previous class we can also operate in a pulse mode. So, with this background let us now go back to see some of the optical approaches to quantum gates and circuits and things like that.

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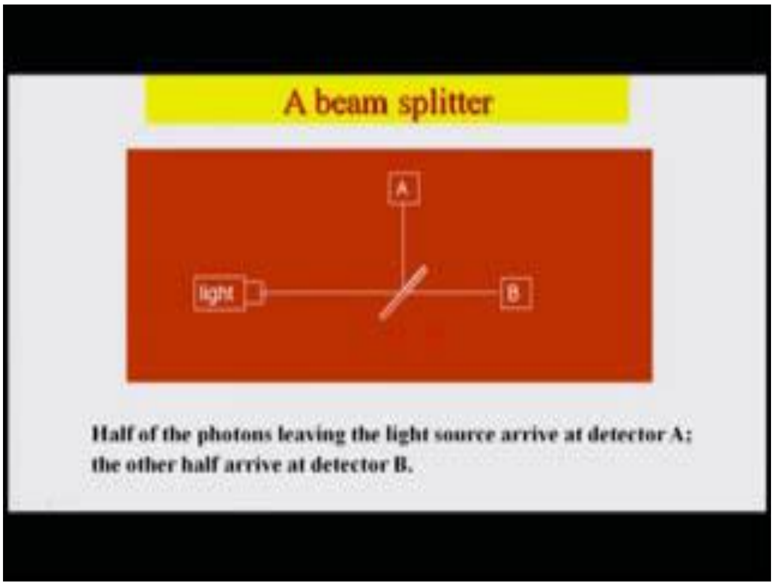

So, first let us look at Quantum Logic Circuits, using light and properties of light. So, the first one in this principle is the beam splitter, which splits the beam input into two parts; half of the photons living the light source arrive at detector A, the other half arrive at detector B. If we are using a beam splitter which is more often known as the 50, 50 beam splitter, basically 50 percent should go in one direction and 50 percent on the other side.

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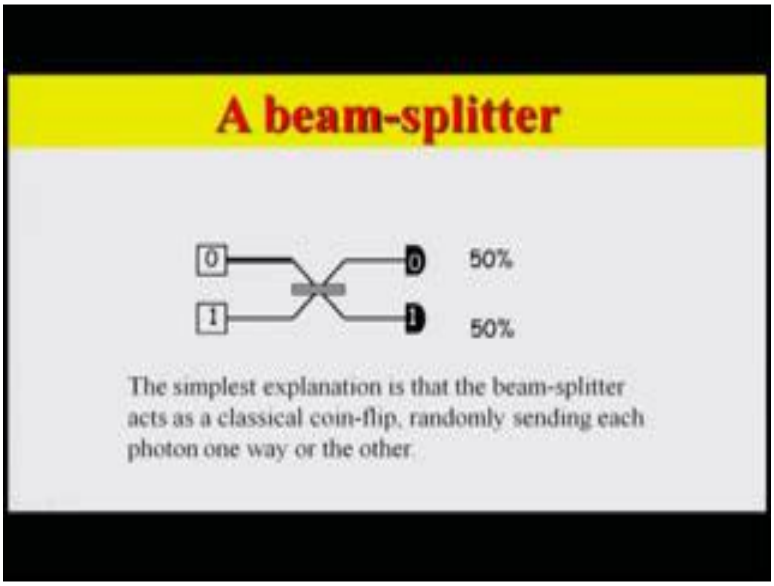

So, the simplest explanation is that the beam splitter acts as a classical coin flip randomly sending one photon one way or the other that is one way of looking at it. So, whenever we have the light coming in the option of that particular photon going to a particular detector is 50 percent always.

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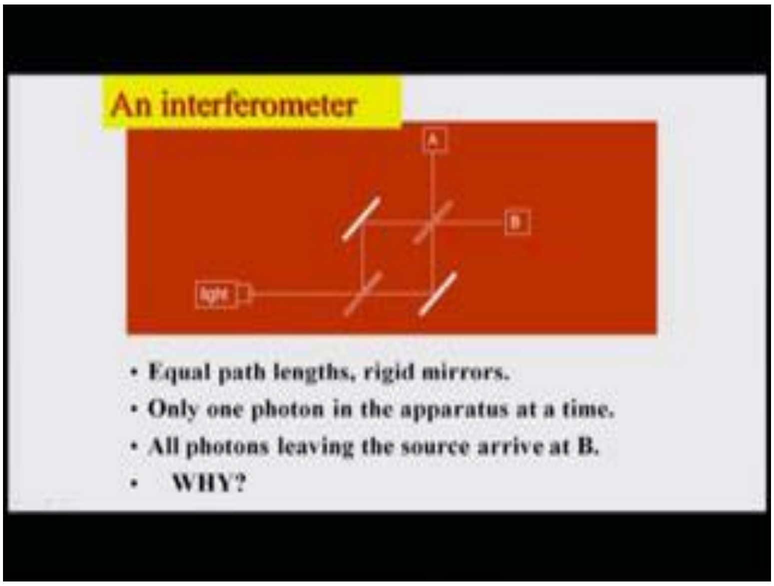

So, if we set up an interferometer as we have seen many times in our study last week that we use many different kinds of interferometers to study light and use certain aspects of light. We can have this by in a setup where the path lengths are equal with rigid mirrors and these are the beam splitters so that we can split and combine the beams. And consider a situation where we are only allowing let us say one photon to come out of the source into this setup.

So, if we do this kind of an experiment what we find is all the photons say for example, leaving the source would end up in B. This is the way how things are then whatever we just mentioned earlier in terms of the classical approach of looking at a beam splitter action in terms of coin flipping does not work. So, natural question to ask in this case if we are allowing only one photon to go through such an interferometer and detecting only photons to be in one of the detectors rather than the other is to look into this a little bit more carefully.

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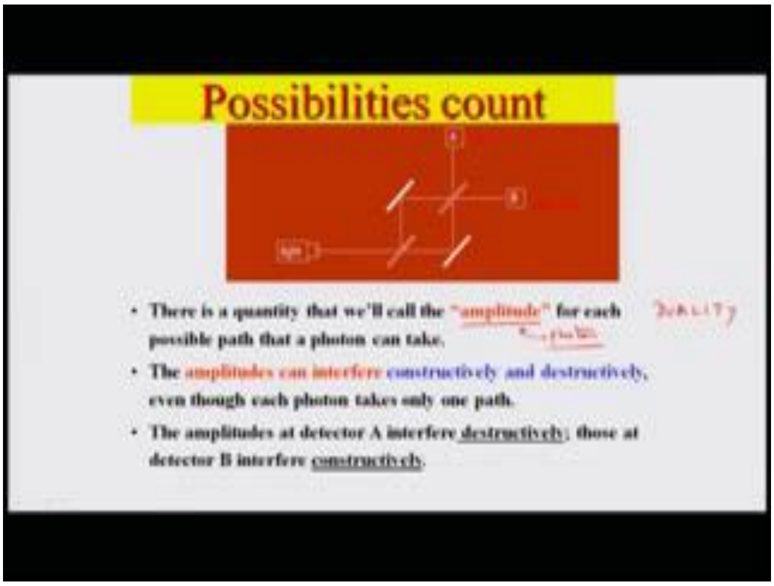

So, then we have to look into this in a slightly different way. So, we have this concept of amplitude for each possible path that a photon can take. So, this is where we are bringing in the duality picture, where we are saying that the particle photon is associated with amplitude. Now if we do that then the amplitude can constructively or destructively interfere at the beam splitter.

Even though each photon can take only one path amplitudes at detector B would be considered to be interfering constructively. So, a constructive versus destructive way of interference can be looked at once we associate the amplitude to each photon. So, that is one way of looking at it. Let us see what happens, as a result of that.

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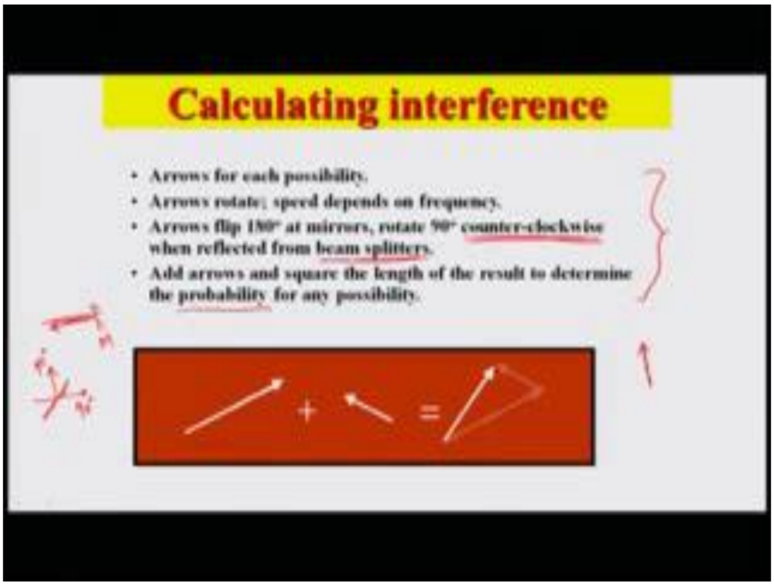

This is one way of the possibility of looking at it. So, in order to calculate the interference what we have to do is we can consider the arrows or the projection that we have for each possibility. So, the arrows rotate and their speed will depend on the frequency, the arrows would flip 180 degrees at the mirrors. So, rotate 90 degrees counter clockwise, when reflected from beam splitters and the arrows would add and square the length of the result to determine the probability of any possibility. So, this is the way if we set up our interference pattern principle then we can calculate how this interference will work.

So, the recipe for this is that we now represent, the photons by the amplitudes which are designated by the projection or the amount of the arrow that we have in any particular direction. So, as they undergo different changes or they come together they will be behaving differently and will be producing different projections, and for that we will undergo certain changes which we will define here that is what is defined here. So, 180 degree change means you are coming and going back completely on itself. So, that is the way how a mirror works which is working in this process. So, that is why the arrows will flip 180 degrees when they hit a mirror.

On the other hand, when it is hitting a beam splitter half a bit should go in and half a bit is going the other way. So, this is sort of like a 90 degree change and that is why, it will be a 90 degree counter clockwise flip, when it is being reflected by the beam splitter. So, these are one way of looking at it and let us see how we go ahead with this. And the arrows and the square of the length would result in the determination of the probability, because that is the final thing that we will be measuring as a result of all of this.

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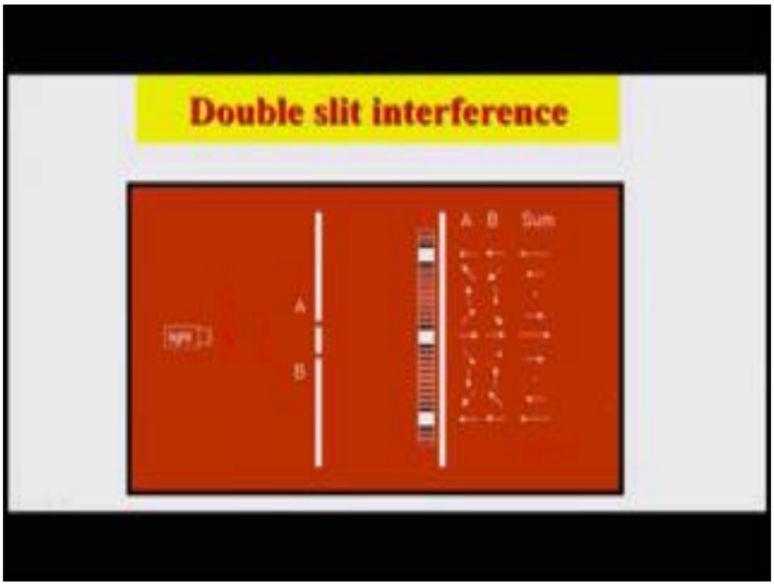

So, here is the double slit experiment as a result of this picture. So, all the arrows now can come in any direction as we have mentioned. And so the input arrows from the laser or the light source are coming through. The two slits here and they are going to have different patterns as they come through individual, but they are basically random when they come through one slit. However, when both of these slits are coming through then there some can give us a pattern which is what we see in the interference pattern.

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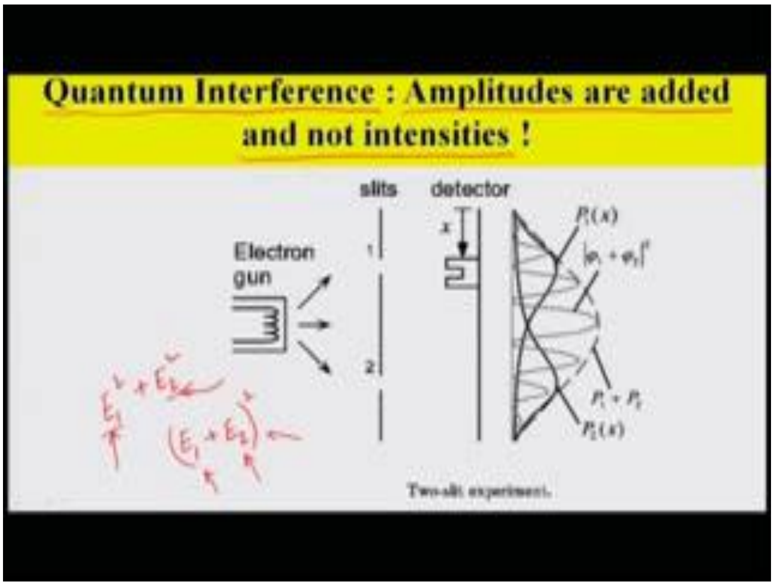

As it is shown here; so the quantum interference essentially means that we are having the amplitudes that are getting added and not the intensities that is the key part here. So, we are actually having E 1 plus E 2 which is getting formed versus E 1 squared plus E 2 square. So, whenever we measure a single slit, then we will be getting the result due to one particular source which is going to come through each of them. Whereas, when we use both the slits then they simultaneously add up together, and so the final result which is our measurement which is the square of it would be a square of the sum of the amplitudes, rather than the individual amplitudes that are squared. And that is the result of what is being shown here as a result of the two slit experiment.

So, this is the principle behind our interference which is in some sense can be considered to be the quantum interference pattern, because if we go through a classical pattern alone then it will be difficult to discuss.

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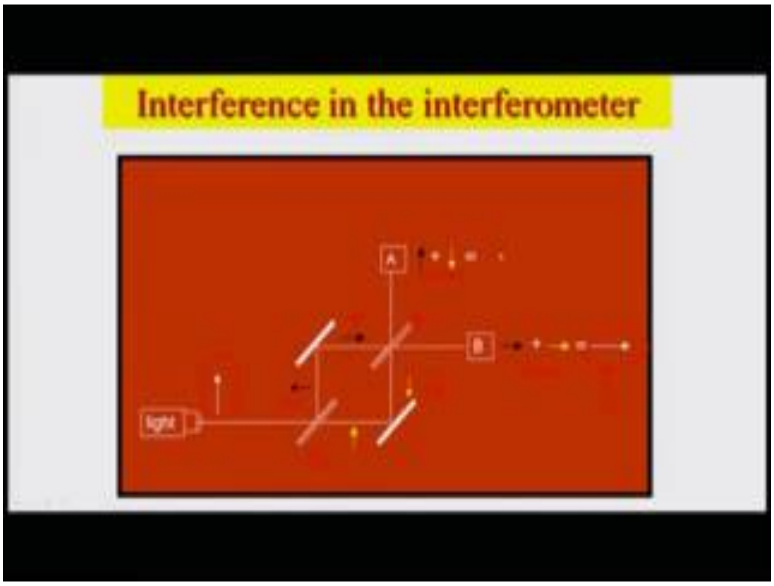

So, in the interferometer therefore what we see is that, we have a scenario where we have the light coming through, and it can undergo two different splits. When it is reflected through the beam splitter then it is 90 degree flip as we mentioned, and when it goes through it remains as it is. When it goes through the mirror it undergoes a 180 degree flip and that is why it is like that, whereas here it is 90 degree. And again when it comes to a mirror it case goes through a 180 degree.

And so when they combine here once again the one which is going through will maintain it, as it is through the beam splitter, whereas the one which is going to go in this direction will undergo a 90 degree flip. That way this one is going to flip 90 degrees. whereas this one is going to go as it is and therefore, they will destructively interfere whereas, these two will interfere constructively. So, all the photons therefore will now be found in this state where it was there to start with. And that kind of explains as to what we were just saying before.

So, if we go by this kind of a formalism that we just developed then we are able to explain how we are able to get only results in one of the detectors and not the other one even if we are getting a 50 percent split through the beam splitters at every point that it encounters it.

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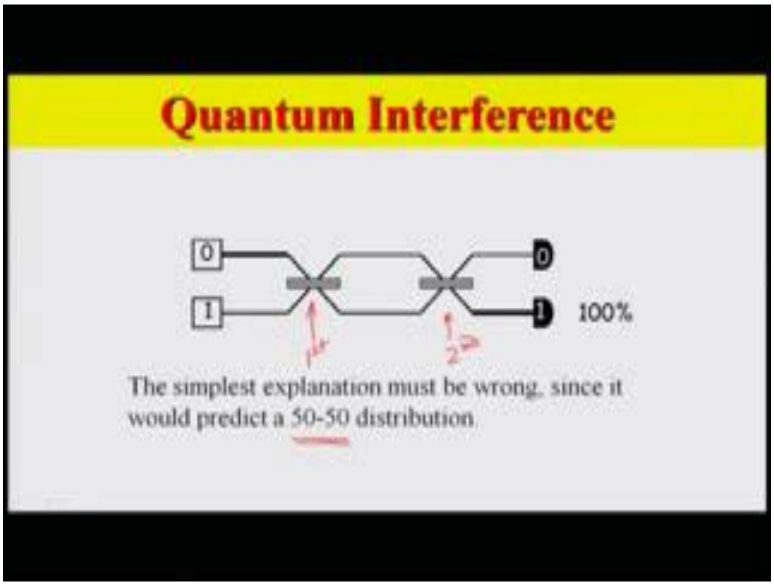

So, this quantum interference would then be having the simplest explanation which we initially thought was wrong, because it would predict an always a 50 50 distribution. So, the classical 50 50 distribution of a beam splitter, although it works when you are looking at just at one beam splitter the moment you add another second beam splitter it would not work in the classical sense unless quantum interference features are invoked, and that kind of gives rise to the idea that we are looking at more than just a classical interference, so it is quantum interference as we just said.

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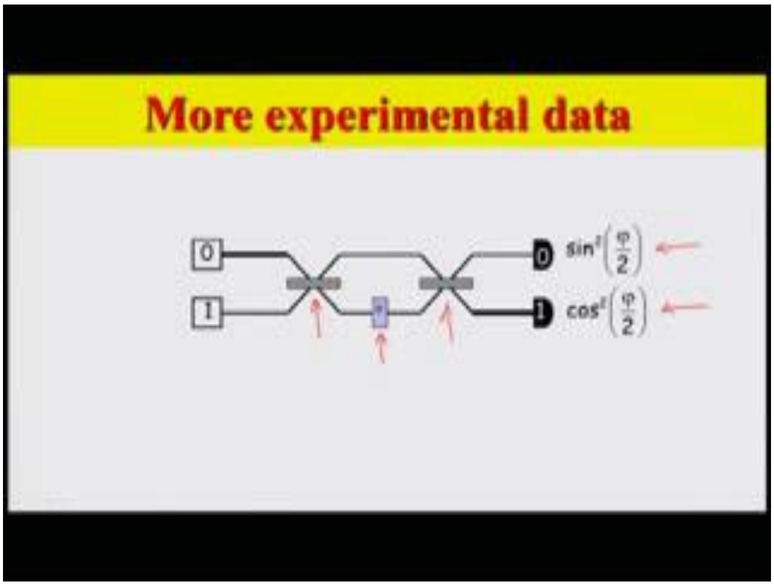

So, if we look at more experiments which have been done in this direction, what is seen is that if two states are going through these kind of two different beam splitters, and there is a way to add or put some phase in one of the arms of the interferometer then the signal detected as a result of this particular process is going to depend on the square of the sine and cosine of the amplitudes so that they can add up to 1 as before.

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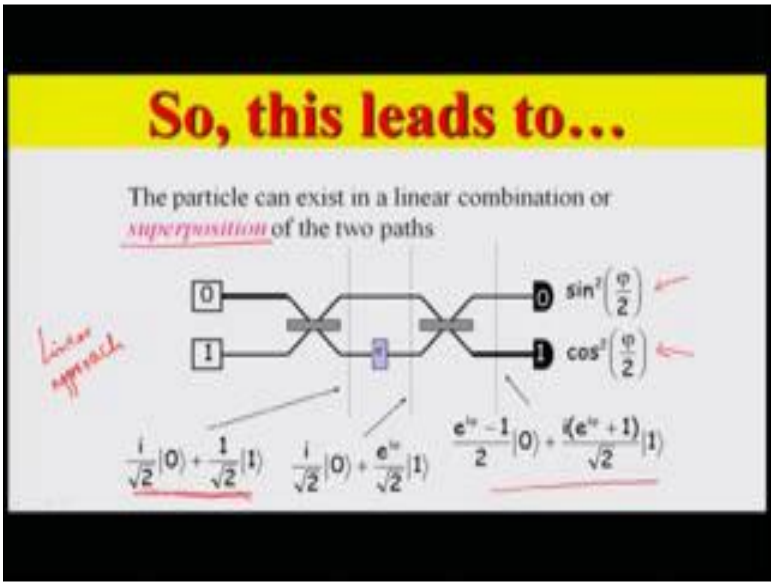

This leads to this particular story which says that the particle can exist in a linear combination or superposition of the two paths. And that is the most interesting path here and that is why we can use optical aspects for demonstrating quantum computing and quantum processes that we are discussing here. So, our state as a result of the first beam splitter essentially is generating a condition where we have an equal superposition of the two states that we start with which is then if we provide a additional phase then we will get the additional phase encoded on one of the two qubits that are superimposed. And this additional phase delay on one of those two components would then undergo additional combinations once we go through the second beam splitter.

So, this idea that the particle can exist in a linear combination or superposition of the two parts, becomes clear from the fact that we are able to have the original superposition of states right after the first beam splitter which after having the additional phase shift enables us to use the phase shift in conjunction to the superposition in the second part to give rise to the second superposition state, which can then be looked at the two detectors with probabilities which add up to 1.

So, this is a one way of proving that we are able to use the photons in superposition states so that we can use them in quantum ways of progressing with the optical approaches. So, given this background let us now see how very simple linear approach. So, this is all linear what we have just now done, linear approach of combining interferometers which gives rise to a quantum process as we are looking into.

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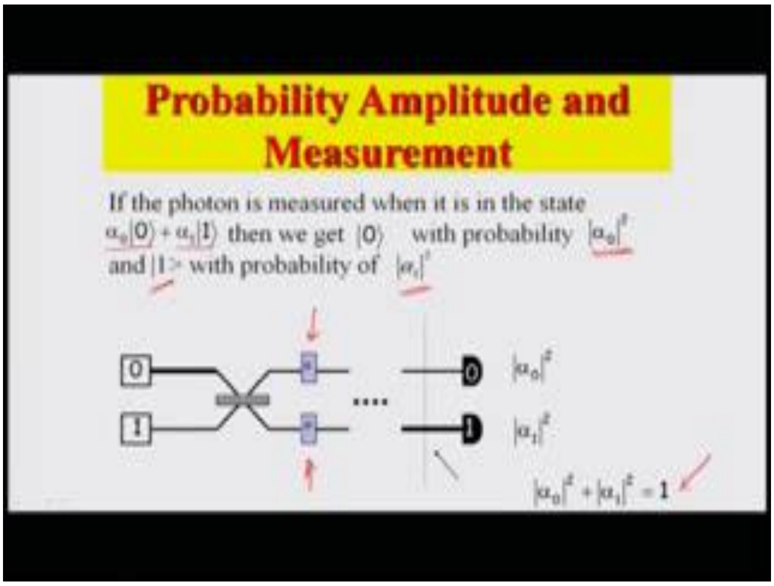

So, the probability amplitude and measurement in this particular process can lead to the same kind of probability amplitudes that we are used to. If the photon is measured when it is in state, say alpha 0 times the first qubit and alpha 1 times the second qubit, then we can measure 0 or the first qubit with probability of alpha naught mod squared and the second qubit 1 with probability of alpha 1 mod square which is basically their coefficients.

So, the moment we have this advantage of coupling the interferometers in addition to these additional phase changes that we can provide inside these interferometers that we can combine, we can have this capability of combining the states and going ahead and working with them. With always having the unitary properties which we can set up, as well as having the total probability given always as 1 as expected.

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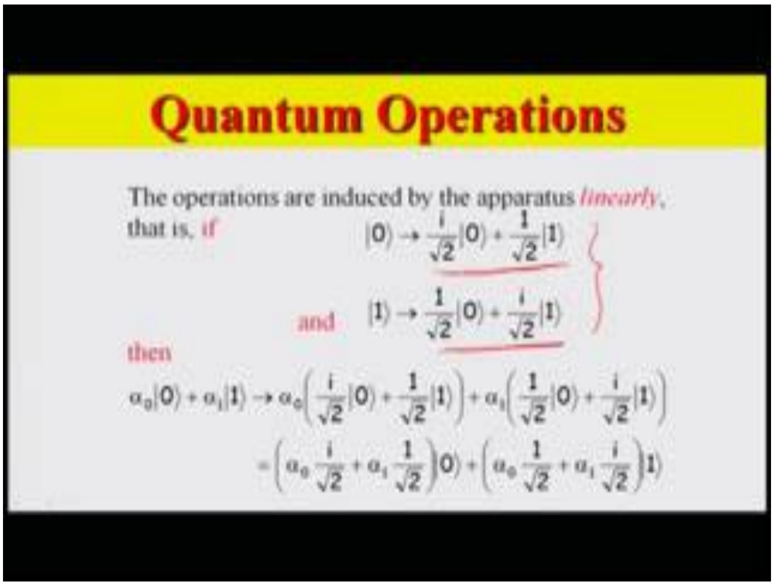

So, when we look at quantum operators in this particular linear approach, the operations are induced by the apparatus linearly; that is if one can be made to have a combination or a superposition state. So, the first qubit is superposed with the second one been put together into a superposition state, then the multiple operations that we execute on each of them ends up making several combination of these qubits one after the other. And that is the basic idea.

So, in some sense you are applying hadamard at every stage. You are combining the 2 qubits with equal probability and this is the hadamard kind of an approach when you are mixing up the qubits together, more and more. And this way the mixture of these states can be continued linearly as the number of interferometers or the numbers of beam splitters that we employ are higher.

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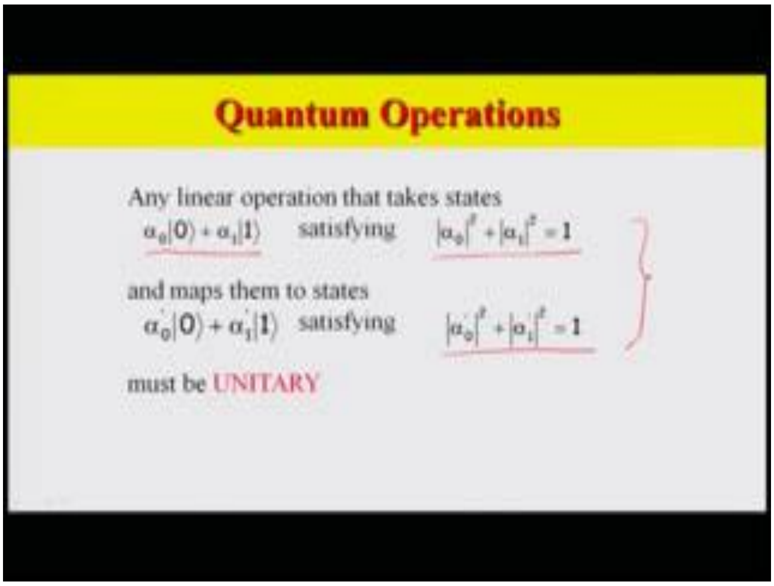

So, any linear operator that takes the states from linear combination of them satisfying there the total probability of their measurements are 1 and maps them to the states satisfying another set is going to be unitary. And this unitary transform would help us to do a basis transform. So, we can actually do a very efficient basis transform by using this technique.

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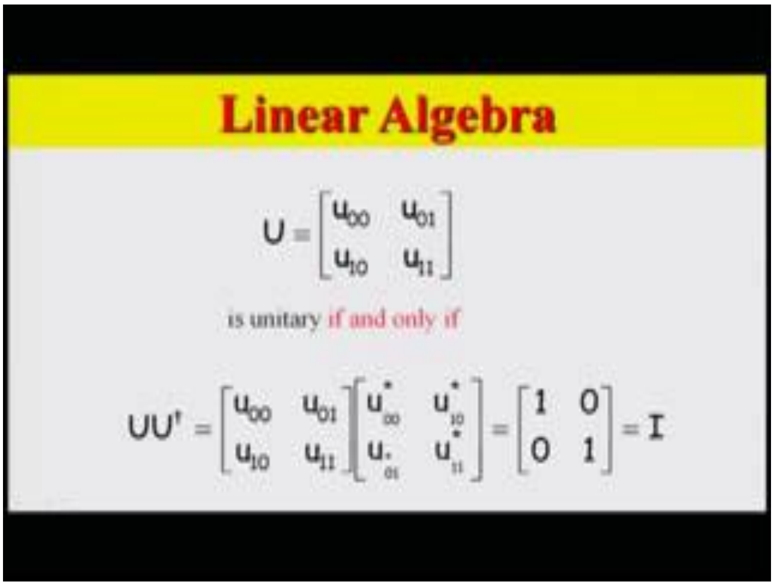

So, it is a matter of linear algebra where we can define a unitary operator, which will satisfy the principle that it will be its adjoint is going to be an identity.

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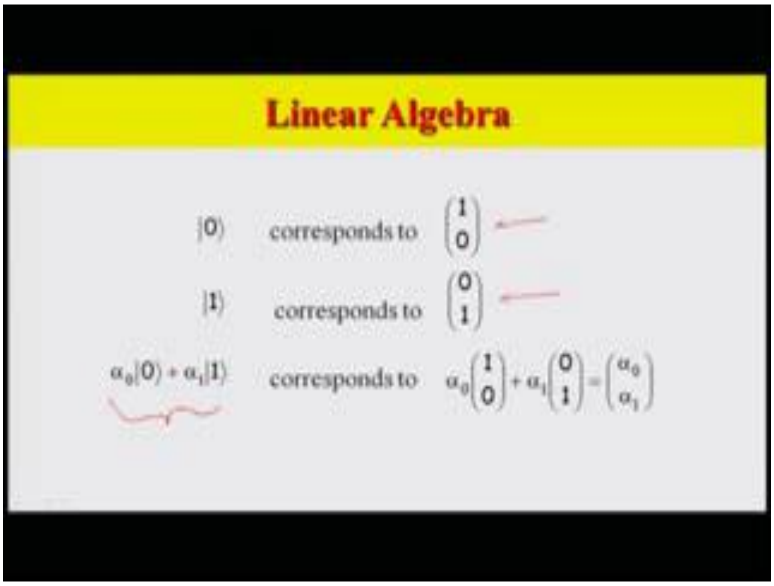

And, in our particular case what we have is that the state 0 corresponds to this and state 1 corresponds to the other version of this as before. And we can add them in terms of the coefficients in the form that we have already used before.

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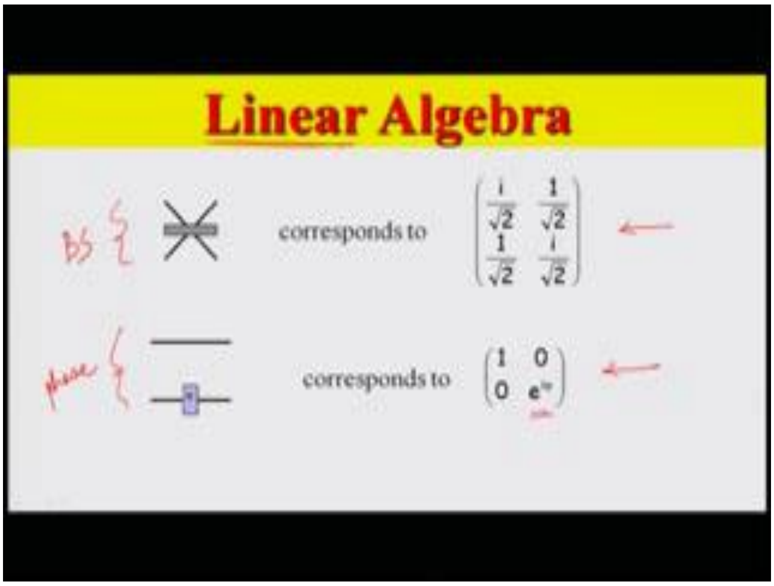

What we are now saying is our beam splitter which we use essentially corresponds to an operation of this kind. Whereas, the phase that we introduced in one of the arms of the beam splitter corresponds to a phase change in the between the two states as we did.

So, these are our unitary operators that we have just now introduced you to base on the optical elements that we have just added. So, in terms of optical quantum computing a beam splitter and this is all under the linear and domain, these are the key elements that we can think of using in terms of optical approaches to quantum computing.

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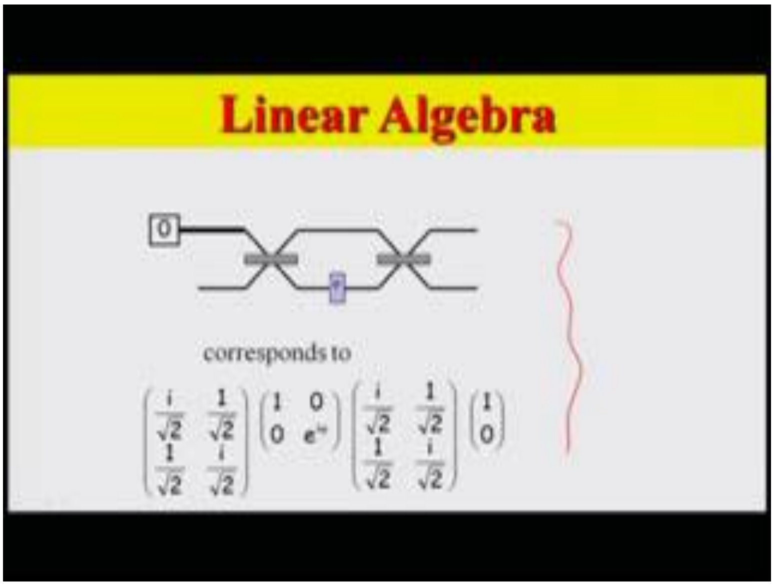

So, we can go ahead and show what we just did before in terms of the linear algebra that we have just now discussed. Is obvious that this is how the linear algebra would correspond to. The states would get superimposed as we discuss in terms of this as well as have the phase transforms. And finally would give rise to the result that we are looking for.

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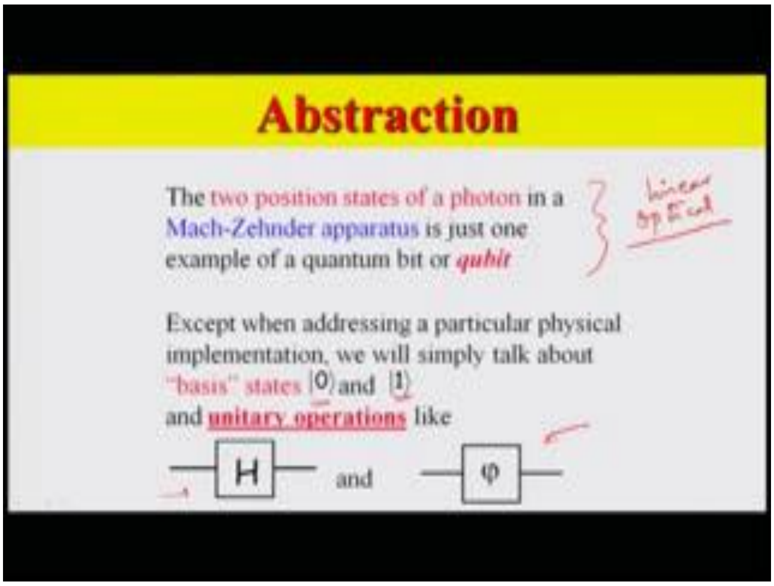

The two position states of a photon in the Mach-Zehnder apparatus is just one example of a quantum bit or qubit. That is what we have just now proved to you as a result of all this discussions that we went through, where we used our beam splitter as one of the important elements to set up the Mach-Zehnder apparatus. And the position states of the photon were used as our quantum bit.

So, except when addressing a particular physical implementation we will simply talk about the basis sets 0 and 1, and unitary operators like hadamard and phase, and so this are that we were just talking about. And use them in terms of this quantum bit or the qubit that we have just discussed in the optical (Refer Time: 21:38). So, this essential principle works under the linear optical scheme of quantum processing using photons.

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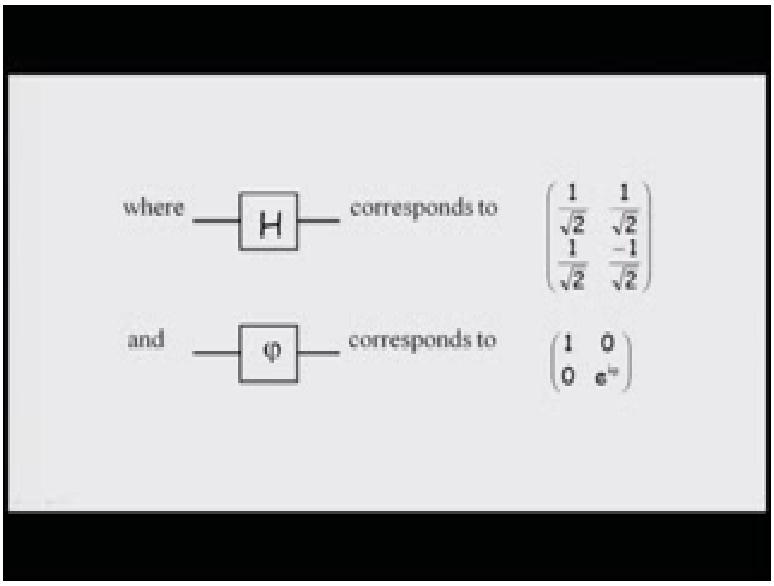

So, once again these are formally the hadamard would correspond to what we have just shown before and the phase transform would correspond to this.

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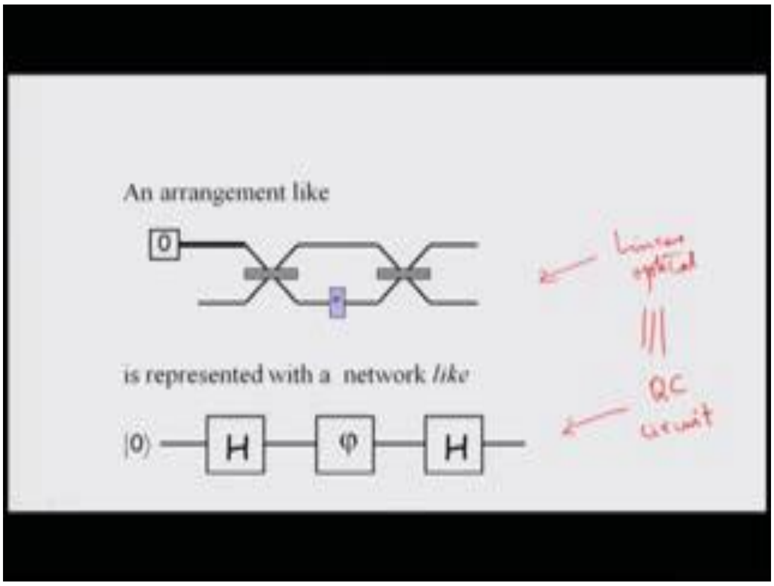

And in an arrangement as we just discussed we can just represent the correspondingness between the linear optical. So, this is our linear optical approach, which would correspond to the standard quantum computing network circuit as we know it. So, these are how they can be correspondent in similar sense that we are now used to.

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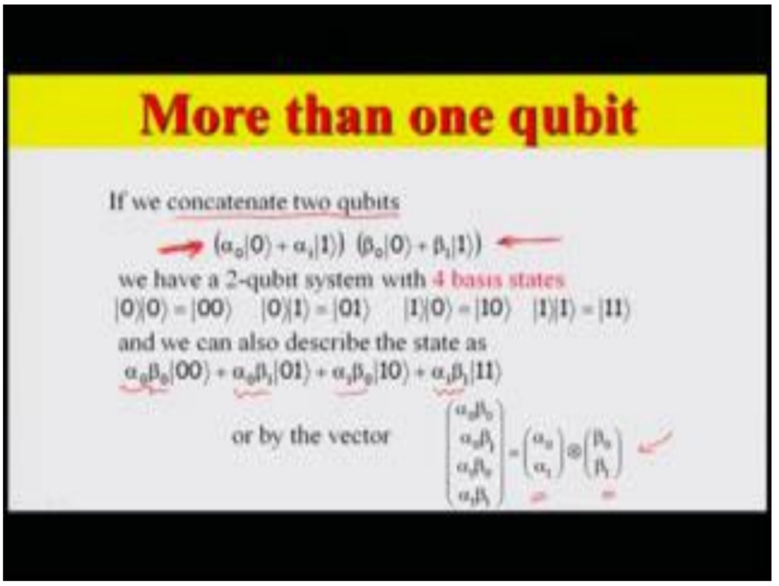

So, once we have more than 1 qubit, we can get more of these things happen. So, we can actually concatenate 2 qubits in this fashion. For example, we can have one marks and the set up a superposition state as we show here, and another one with another set of superposition and then we can have the 2 qubit system working in this way, with 4 basis states now because we basically in this fashion can concatenate 2 qubits, but both of these as we have shown can be made to arise from optical approaches, and these 4 basis sets can then be utilized as a multiple qubit system.

So, instead of a 1 qubit system where we start from we can simultaneously use more than 1 qubit. And so we can have a four basis state starting from this particular approach and write them in our standard approach as we have been writing before. The only thing we have to be clear is that our alpha beta, alpha naught beta, naught alpha 1, beta 1 are being generated appropriately when we are a creating these superposition states in the optical cells.

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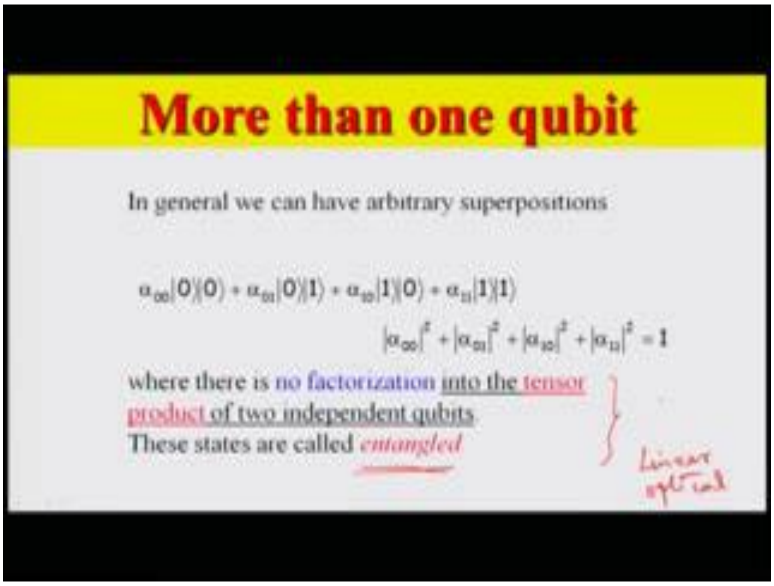

So, once we know that then we can go ahead and do the operations as is being shown here, and we can have the standard operations can be written in terms of tensors some tens of multiplications between the coefficient matrices and they can be made to generalize the arbitrary superposition in this fashion. So that there can be which cannot be factorized into tensor product of independent qubits, we can go back to the familiar concept of the entangled photons, entangle states by using multiple qubits as we have put them together here.

So, this is how we can build up the optical quantum system by simply based on linear optical approaches.

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So, this is the basic idea of building it up in this particular process. Now in this case the entanglements in this multi qubit system are not independent as they become entangled and to represent the states of this n qubits we then can use this 2 to the power n complex number amplitudes.

So, here we are always talking in terms of amplitudes as we are providing the amplitude aspect of the photon to these optical states which are being modulated and put together simultaneously through interferometers which act as our ways of combining the qubits.

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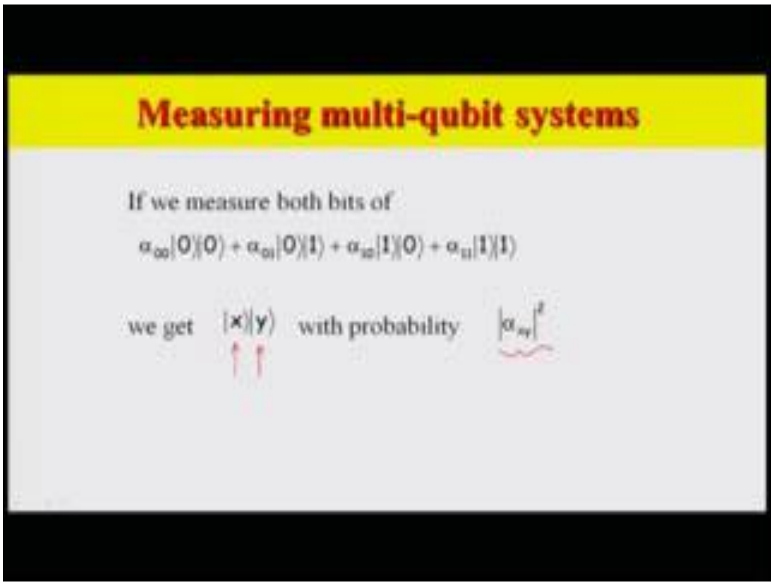

Now, once we measure the multi qubit system, we end up having the same kind of scenarios as before where we can measure the individual joint probability of each of them with probability of alpha x y mod squared as when we have the combined qubits which are put together, and we can measure both bits of the independent of these entangled qubit pair and we can have familiar measurement principles which rely on this system.

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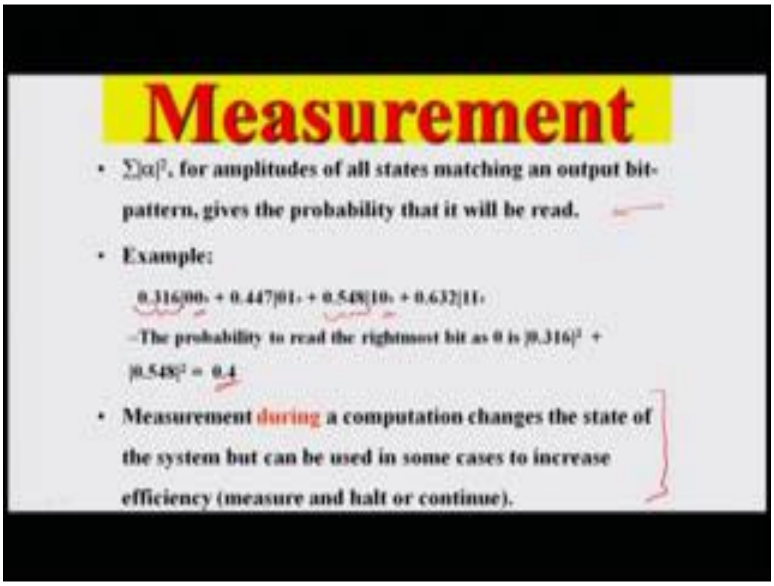

So, the summation of all these square of the amplitudes, of all the states matching an output bit pattern would give the probability that it will be read as. And as before we can see that the entangle states will have the probabilities, defined in the way that we have done before.

For example, if we have say the 2 qubits 0 and 1, put together as 0 0 0 1 1 0 and 1 1 qubits, each of which having some probability or amplitude factors as defined here in terms of alpha beta, betas then the probability to read a particular bit would be given in terms of the square of the individual probabilities and mod of the amplitude squares. And here is an example where it is looking at. The right most bit for the particular ones which are 0. So, it is in this particular case these are the two different amplitudes that concern us and the probability of the readout in that case would be the squares of individual of them being added together. And in this particular case for example, it is like 0.4.

Let me also mention here, that from now on we would also be doing a little bit more of these problems that I have been given you even during your assignments earlier. Maybe formally we will address some of the difficult questions later on in this particular week also for some of the problems which have been we gave you the answers, but maybe we did not actually go through some of the steps. So, we will be doing some of those also, but this is just an example that this is a part of bit while we are I am doing a lecture. But, maybe I will also spend some time separately on some of these cases where the problems have to be worked out a little bit more than just looking at these solutions. So, we will do that also as we go along now on.

So anyway, a measurement during a competition changes the state of the system, but can be used in some cases to increase efficiency. And these are sort of things that we have encountered before also, but this is just a way of making sure that we understand that we are going we have entered the realm of doing quantum measurements as we expected by using simple linear optical schemes.

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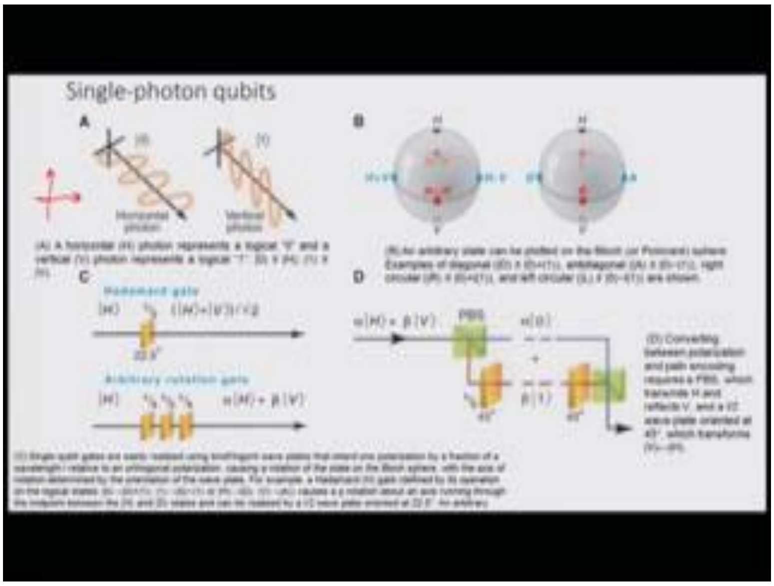

And, in the later on parts of this week we will be formalizing this in a lot more fashion. Here, are a few ways of looking at single photon qubits either by looking at them as horizontal photons or vertical photons these are basically the polarizations as we talked about. And they can then be represented as arrows in some sense which are perpendicular or horizontal based on how they are being looked at.

So, these are just examples and we will deal with this more from the next lecture onwards. I would like to end todays lecture with the idea that we just have introduced the principles that the photons that we have been discussing with, the main principle of the last weeks lecture which went on discussing about light and it is all features and the way to get generate; the most and the best possible light that we need for these activities the laser which is coherent, and has a proper polarization and other properties associated with it.

We have just introduced you here, as to how even the simplest way of using linear optical techniques can give rise to the quantum kind of approaches that you are interested in to get to quantum computing and quantum information processing as we are dealing with in this course. See you in the next class.