

Implementation Aspects of Quantum Computing
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Lecture - 16
Continuous Wave Lasers

We have been looking into lasers; one of the most important aspects which have been used for quantum computing for long time. Lasers are one of the waves of our interactions with quantum system and laser in itself can act as a quantum system. So, we are looking into various aspects of the laser in itself, and as we go along will also look into its implication in quantum computing.

So, we establish in the last lecture that population inversion is one of the key aspects of lasing. So, in order to have a laser work we need population inversion.

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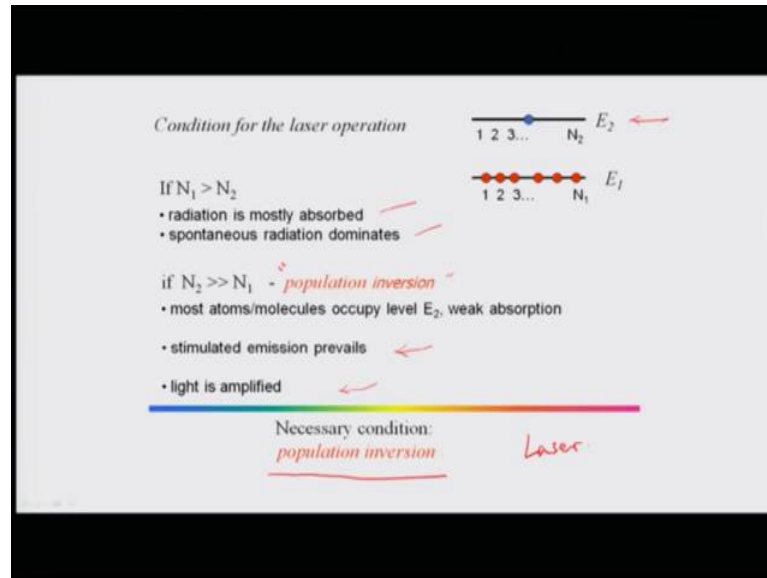
Population Inversion

- A state in which a substance has been energized, or excited to specific energy levels.]
- More atoms or molecules are in a higher excited state. ←
- The process of producing a population inversion is called pumping.]
- Examples:
 - by lamps of appropriate intensity ←
 - by electrical discharge ←

In this lecture let us look at where we left out. We stated in the last lecture we needed to have population inversion in which a state it which is a substance has been energized or excited to a specific energy levels. More atoms or molecules are in the higher excited state in the ground state. The process of producing a population inversion is called pumping.

All these process we have gone through in the last lecture. And the examples which we have been looking for population inversion are by use of lamps of appropriate intensity or by electric discharge and many other processes.

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In this regard, let us now look back as to how we define the population inversion. We looked at a simple system of a lower state and excited state. Or in other words the ground is excited with energies E_1 and E_2 , if we have N_1 occupancy in the ground or the lower energy and N_2 occupancy in the excited higher energy state. Then if it is N_1 is greater than N_2 , which is typically the case for most systems than radiation is absorbed and spontaneous radiation dominants, that is what we found.

On the other hand, if N_2 is much much greater than N_1 then is the condition which we define as the population inversion, because the other case is the regular case of the population which we generally define. So, this inverted population condition if we generate most atoms molecules occupy them the excited state. There is very weak absorption available, there is very few are available in the lower energy state. Stimulated emission prevails and the light gets amplified.

So, this is the necessary condition of population inversion which can give rise to the laser.

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Boltzmann's equation

$$\frac{N_2}{N_1} = \exp\left(\frac{-(E_2 - E_1)}{kT}\right)$$

- N_1 - the number of electrons of energy E_1
- N_2 - the number of electrons of energy E_2

example: $T=3000\text{ K}$ $E_2-E_1=2.0\text{ eV}$

$$\frac{N_2}{N_1} = 4.4 \times 10^{-4}$$

So, we generally looked at it by using the idea of equilibration and the Boltzmann equation of how the occupancy of the states are looked at, is given by the ratio of the two occupancy N_2 over N_1 , is given as exponential of the energy difference over Boltzmann constant and temperature. So, that is how the normal system is for Boltzmann condition, normal conditions.

And these are the number of electrons or occupancy of the lower energy versus higher energy and for any system even at let us say 3000 Kelvin, pretty high temperatures we still see that the relative occupancy of the excited state is very very low compare to what it is that the ground state. So, normal conditions almost always a picture which looks like this, most occupancy is in the ground state and lesser in the excited state.

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Einstein's relation

Probability of stimulated absorption $R_{1,2}$

$$R_{1,2} = \rho(\nu) N_1 B_{1,2}$$

where spectral density is $\rho(\nu)$ & Einstein coeff. of absorption is $B_{1,2}$

Probability of stimulated and spontaneous emission :

$$R_{2,1} = \rho(\nu) N_2 B_{2,1} + N_2 A_{2,1}$$

Einstein coeff. of stimulated and spontaneous emission are $B_{2,1}$ & $A_{2,1}$

Assumption : For a system in thermal equilibrium, the upward and downward transition rate must be equal :

$$R_{1,2} = R_{2,1}$$

$$N_1 \rho(\nu) B_{1,2} = N_2 (\rho(\nu) B_{2,1} + A_{2,1})$$

$$\rho(\nu) = \frac{A_{2,1} / B_{2,1}}{\frac{N_1}{N_2} B_{1,2} - 1}$$

We also looked at the Einstein relation which gives rise to the conditions that now we are talking about. The coefficients that we are at defined at the Einstein A and B coefficients, and they can be little bit qualified in terms of saying that we can label them as B 1 to 2 versus 2 to 1 depending upon whether we are looking at absorption or we are looking at stimulated emission. For example, in the probability of stimulated absorption that the case when we are having not really stimulated absorption is when we have the number of lower energy states with the coefficient which is B 1 2 of the Einstein coefficients is going to be equivalent to rate of the or probability of this absorption which can be related by the spectral density and that is roughly the expression.

So, the probability of stimulated and spontaneous emission is more like this particular case where the stimulated emission is the A coefficient which is again coming from two state to; one state which is spontaneous where as the B is the stimulated case. So, the spontaneous one is the one which both of them are connected to the exercise, one of them does not need the photon but other one does need the photon. And therefore, we can write it in this form. This is another way of writing what we had written in the earlier kinetic model, and both of them are essentially giving rise to the probability of the absorption and the stimulated emission. And the Einstein coefficients at the once which are used here stimulated and spontaneous emission as we show here.

For a system in thermal equilibrium, the upward and downward transition must be equal that is what we used last time. And we can write the spectral density as a form which is of this kind, which is the ratio of the different absorption coefficients of absorptions as we had seen before along with the ratio of the occupancy number of the two states and we can come up with a ratio of that. In the last lecture we had essentially considered the case where spontaneous emission was negligible, so our upward terms essentially become equal to one; however when we consider that that also appears in the expression.

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According to Boltzmann statistics: $\frac{N_1}{N_2} = \exp(E_2 - E_1) / kT = \exp(h\nu / kT)$

$\rho(\nu) = \frac{A_{2-1}/B_{2-1}}{\frac{B_{1-2}}{B_{2-1}} \exp\left(\frac{h\nu}{kT}\right) - 1} = \frac{8\pi h \nu^3 / c^3}{\exp(h\nu / kT) - 1}$

Spectral density

Planck's law

$B_{1-2}/B_{2-1} = 1$

$\frac{A_{2-1}}{B_{2-1}} = \frac{8\pi h \nu^3}{c^3}$

According to Boltzmann statistics we have an expression, we have already mentioned which is of this kind and that can be related to the energy gap is $h\nu$ here. And this density that we just define the spectral density is essentially equivalent to this form from there to the 1 which is given as per the Planck's law because we just used $h\nu$ part come in from the Planck's law part. And, so we can use in this form the Planck's law form of the same expression in terms of spectral density.

So, by equating these we can come to situation where we have a ratio between the spontaneous emission as well as the stimulated emission. This is our spontaneous versus the stimulated, the ratio of the two can be found out by using the Planck's law and this is the expression between two. So, under the condition that both are equally provable the absorption and the stimulated emission which is what is generally the condition for any available photon; we can arrive at this important relation between spontaneous and

stimulated emission, and that is as with the help of Planck's law and we can get this ratio. So, this is another way of looking at it.

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The probability of spontaneous emission $A_{2,1}$ the probability of stimulated emission $B_{2,1}\rho(\nu)$:

$$\frac{A_{2,1}}{B_{2,1}\rho(\nu)} = \exp(h\nu/kT) - 1$$

1. Visible photons, energy: $1.6\text{eV} - 3.1\text{eV}$.
2. kT at $300\text{K} = 0.025\text{eV}$.
3. stimulated emission dominates solely when $h\nu \ll kT$!
(for microwaves: $h\nu = 0.0015\text{eV}$)

The frequency of emission acts to the absorption:

$$x = \frac{N_2 A_{2,1} + N_2 B_{2,1} \rho(\nu)}{N_1 B_{1,2} \rho(\nu)} = \left[1 + \frac{A_{2,1}}{B_{2,1} \rho(\nu)} \right] \frac{N_2}{N_1} \approx \frac{N_2}{N_1}$$

if $h\nu \ll kT$:

$$x \approx N_2/N_1$$

The probability of spontaneous emission to the probability of stimulated emission therefore is a once again given by expression. So, what we can find is that no matter what we have discussed before stimulated emission dominates only when $h\nu$ over kT is much much smaller than 1. So this is something which is difficult for visible, because for the visible photons or anything in the region the photon energies quite high and the kT is also in this range, so it is very difficult to get this condition.

In the case of microwaves the $h\nu$ is less than small number. And therefore it is possible as you can immediately see that generating microwave situation is much easier. Therefore, may be you can understand as to why maser were discovered first as compare to the lasers. Any way given that that we have arrived this frequency of emission acts to the absorption can be related in terms of the ratio which is given as the N_2 over N_1 if $h\nu$ this is much much less than 1. And the that is the reason why as I just mentioned masers with the microwave are much easier concept in principle with respect to the lasers were these ratios are very favorable in that case.

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Two-level Laser System

- Unimaginable as absorption and stimulated processes neutralize one another.

MASER (NH₃)

The slide features a title 'Two-level Laser System' in purple. Below it is a bullet point: '• Unimaginable as absorption and stimulated processes neutralize one another.' The words 'neutralize' and 'one another' are underlined in red. To the right of this text is a red curly brace. Below the text, the word 'MASER' is underlined in red, followed by '(NH₃)' which is also underlined in red.

And therefore, a 2 level laser system in fact was unimaginable as absorption and stimulated processes neutralize one another. And, so this was the basic in the beginning and (Refer Time: 11:15) went ahead to show that a measure exists by using an ammonia molecule. And they were able to show that they can construct ammonia for which maser this molecule. And since the principles that we just discussed favors that it was the first demonstration of its kind of particular idea of amplification by stimulated emission and it was possible to do this.

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Laser Construction

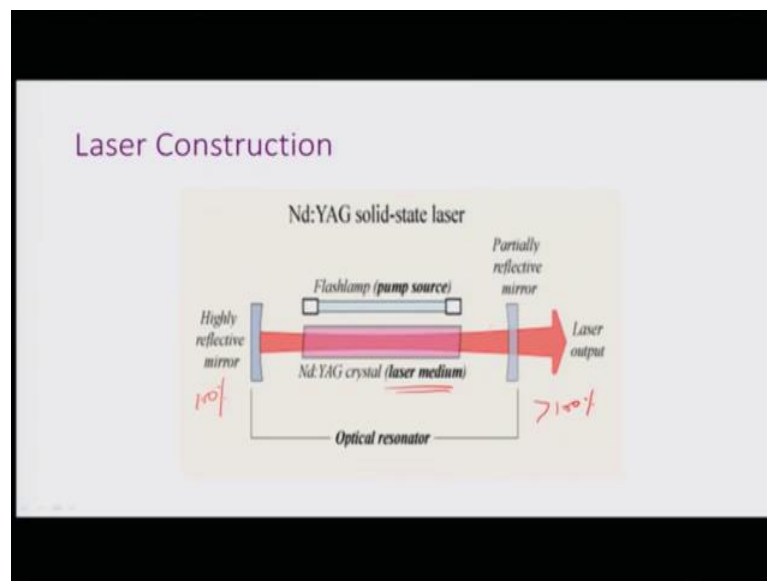
- A pump source
- A gain medium or laser medium.
- Mirrors forming an optical resonator.

The slide features a title 'Laser Construction' in purple. Below it are three bullet points: '• A pump source', '• A gain medium or laser medium.', and '• Mirrors forming an optical resonator.'. Each bullet point has a red checkmark next to it.

Now, for optical regions this sounded like a very difficult system or difficult process to do it. When the idea behind this was looked at several important aspects of laser constructions have been looked into; a pump source, a gain medium or laser medium, and mirrors forming an optical resonator where the essential of the key aspects as we will see now in terms of the laser construction. Which is as you can understand is going to be a little bit more involved than just idea demonstration principle which you just started off with.

Because idea demonstration principle works quite well and has been shown for the masers the beginning, but as it can be seen that naturally at least for a 2 level system as we discussed one of the things which are extremely difficult to achieve in the wave length regions and the temperatures that we are talking about.

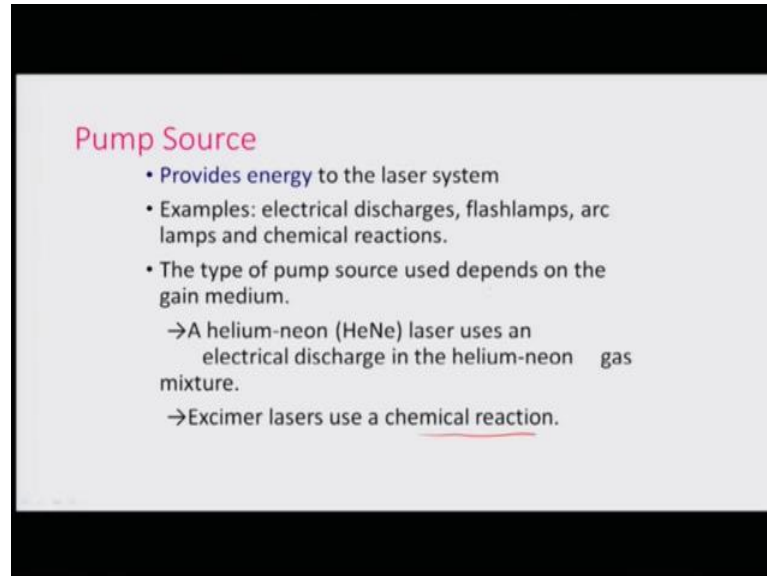
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So, here is just a picture how typical laser; this is a laser which in operation today for example, (Refer Time: 13:04) solid state laser. And in this case flash lamp pump source is pumping them system is amplifying medium which is the laser medium, which is then put inside an optical resonator. And glimpse of this kind of a laser already presented in the earlier lecture. Initially showing that we have a most 100 percent reflecting mirror on one side and say some where very high, but not exactly 100 percent just slightly less than 100 percent reflecting mirror on other side, so there little leakage comes outside output.

And most of the gain and all this process continuous within the resonating cavity, so this is the basic idea.

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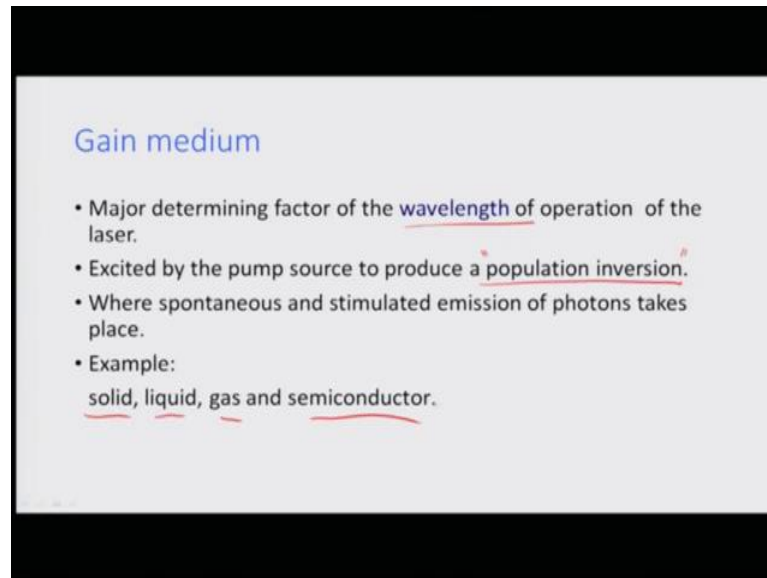
Pump Source

- Provides energy to the laser system
- Examples: electrical discharges, flashlamps, arc lamps and chemical reactions.
- The type of pump source used depends on the gain medium.
 - A helium-neon (HeNe) laser uses an electrical discharge in the helium-neon gas mixture.
 - Excimer lasers use a chemical reaction.

The pump source provides energy to the laser system. Examples include electrical discharges, flash lamps, arc lamps and chemical reaction as we mentioned. The type of the pump source depends on the gain medium. For example, helium-neon laser uses an electric discharge in the helium-neon gas mixture; excimer lasers use a chemical reaction and so on and so forth.

There are different many kinds, these are both examples in the gas phase and then there are different ways of looking it, we just looked at (Refer Time: 14:20) which is in the solid phase. And so there are different kinds we can discuss them on many different ways.

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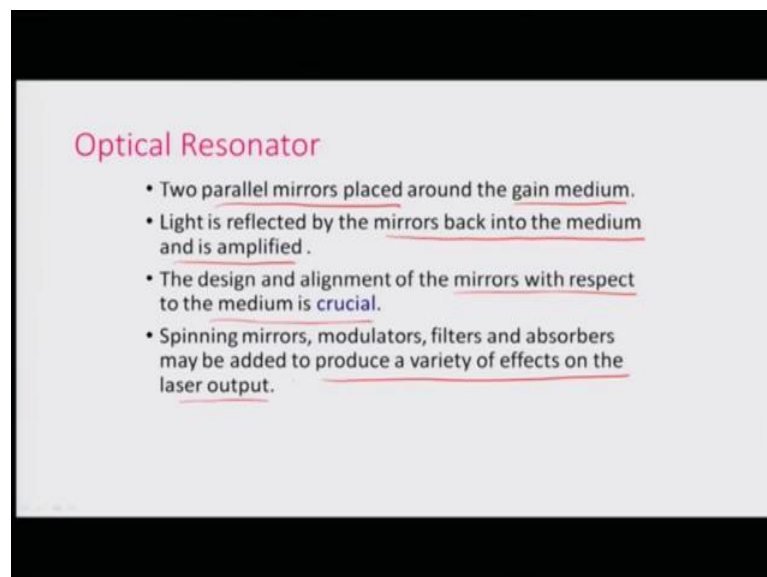


Gain medium

- Major determining factor of the wavelength of operation of the laser.
- Excited by the pump source to produce a population inversion.
- Where spontaneous and stimulated emission of photons takes place.
- Example:
solid, liquid, gas and semiconductor.

So, the gain medium is the major determining factor in the wave length of operation of the laser and it is excited by a pump source to produce a population inversion. So, this is the critical part which we have to think about how we achieve that, whereas spontaneous and stimulated emission of photon takes place, for example. And this can be of different kind's solids, liquid, gases and semiconductor, some material part.

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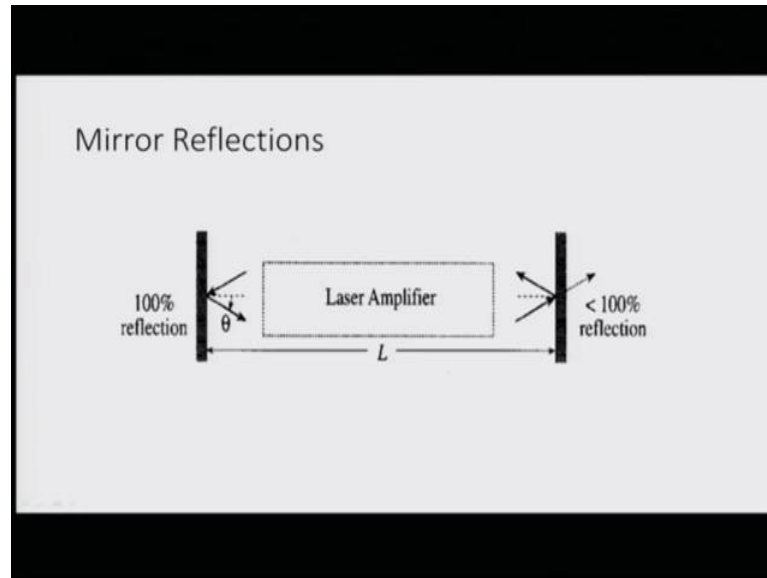
Optical Resonator

- Two parallel mirrors placed around the gain medium.
- Light is reflected by the mirrors back into the medium and is amplified.
- The design and alignment of the mirrors with respect to the medium is crucial.
- Spinning mirrors, modulators, filters and absorbers may be added to produce a variety of effects on the laser output.

The resonator is the two parallel mirrors placed around the gain medium. The light is reflected by the mirrors back and forth into the medium and is amplified. The design and

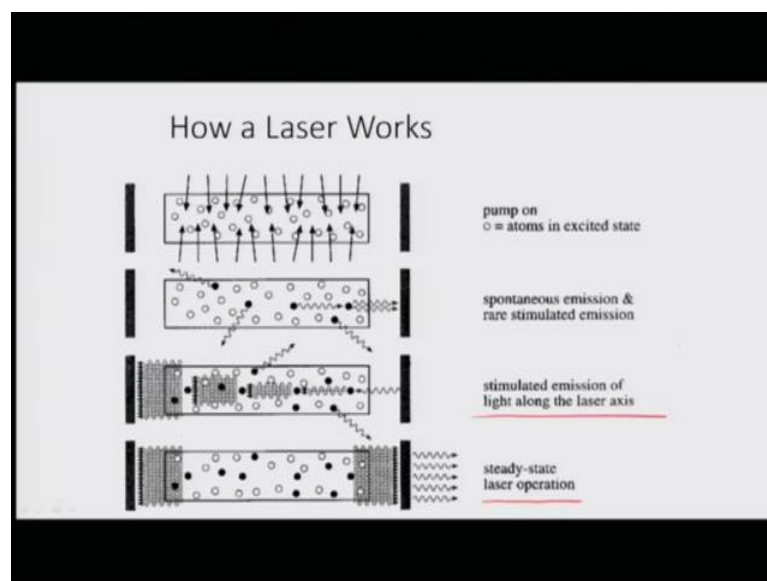
alignment of the mirrors with respect to the medium is crucial. Spinning mirrors, modulators filters and absorbers may be added to produce a variety of effects on the laser output. So, this are issues which can be later on looked into more detail.

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But roughly speaking this is the concept that we are talking about, that there is a 100 percent reflector one end and just about less than 100 percent on the other end its been maintained within that, and there is a length within the whole system is been put.

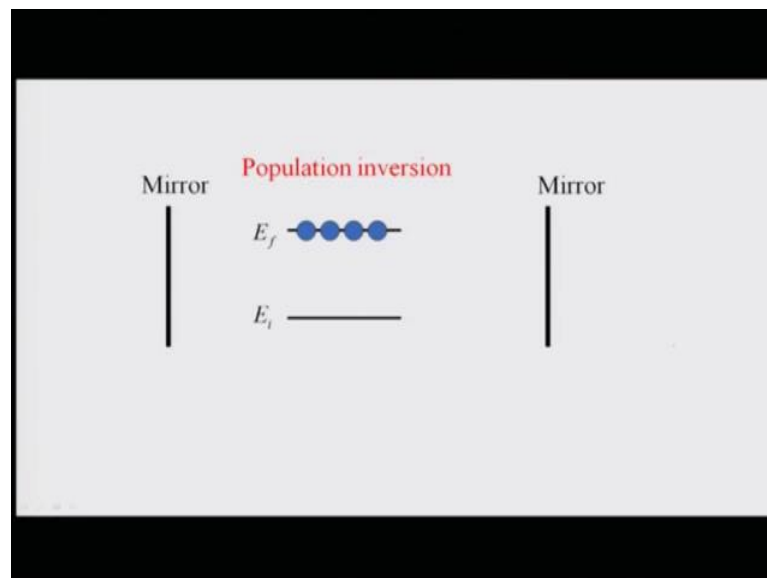
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So, crudely speaking this is how a laser works. We have a the laser beam is pumped, atoms in the excited state has been generated, spontaneous emission and rare stimulated emission is happening, and that is being put inside and because stimulated emission is something which is along one direction and is directional so what happens along direction of the cavity its gets amplified.

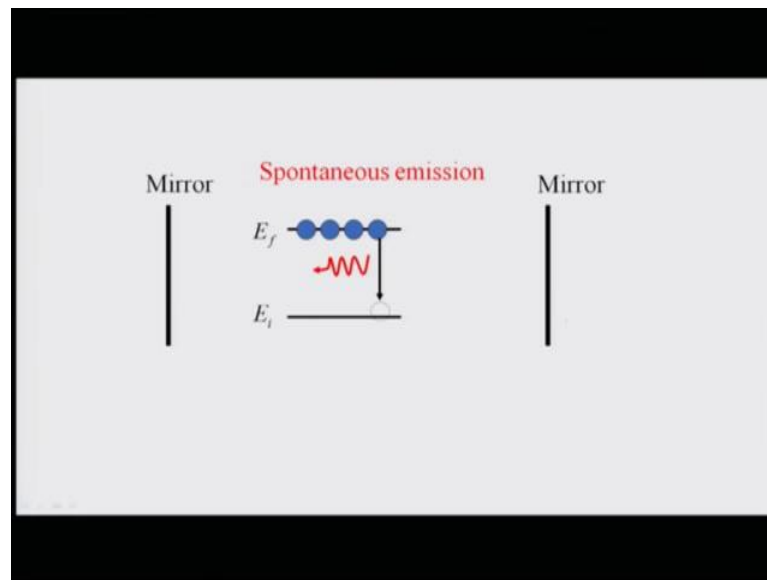
And, initially it is a rare process, but then again that one which is dominating because spontaneous emission goes in every direction and gets lost, whereas the stimulated one keeps on getting saved that is what gets amplified. And finally, along the laser axis you can start the gain happening and the study state laser operation can occur in this matter. This is the basic typical idea about how laser occurs.

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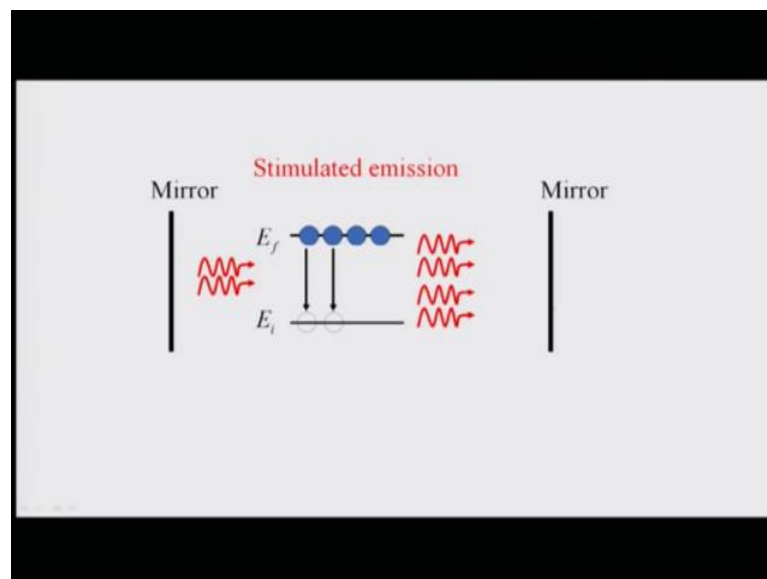
So, a little bit more in terms of the how we are looking at it.

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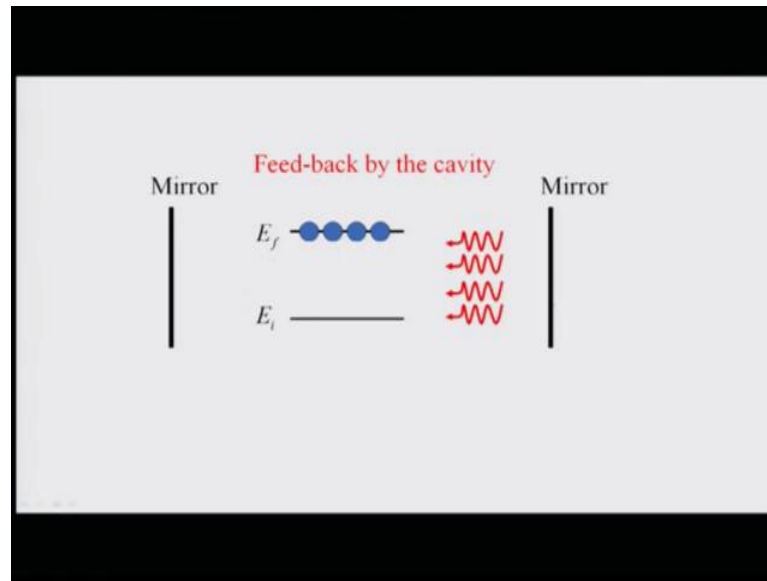


Some initial to some final state we have the cavity in which we are having this.

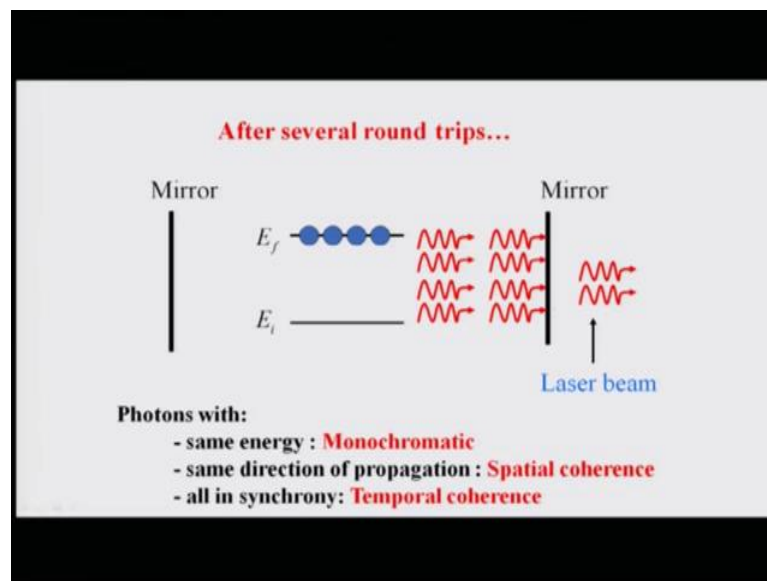
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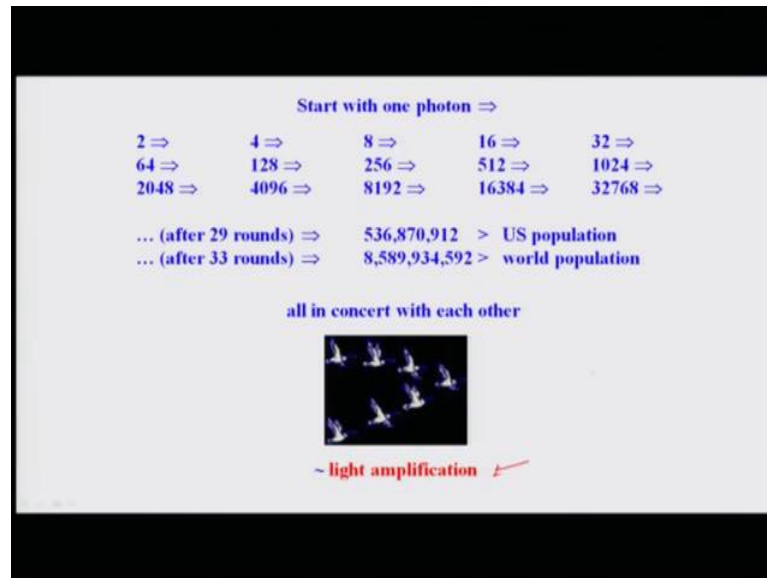


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We are having this spontaneous emission which is doing the process of taking the photons in the direction which leads to feedback in the cavity keeps on getting into the process. And after several round trips of feedbacks you get the laser beam which is monochromatic spatial, coherence, temporal coherence and all this.

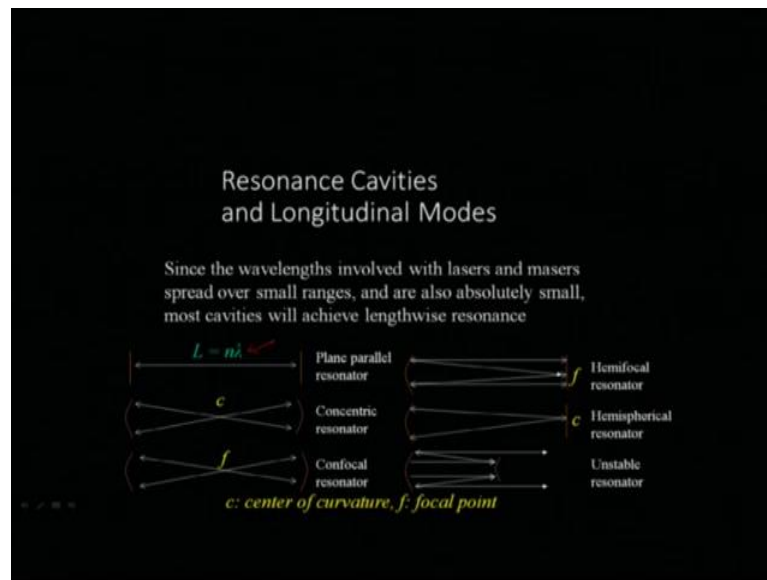
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So, if you want actually you can do some number game here. We can start with one photon goes into two just keeps on going in order so magnitude going higher and so with like every round trip, the number amplifiers and we can see that within a very few round trips to; 1 2 3 4 5 6 and so on. As you can see it is orders of magnitude the numbers keep going and so roughly in like 29 or 33 round trips some population which is in this case can be like as high as the one countries population or it could be as high as the world population after certain number of round trips.

And they are all in tandem with each other which gives rise to the very important aspect of the light amplification. So, that is just the way of looking at it.

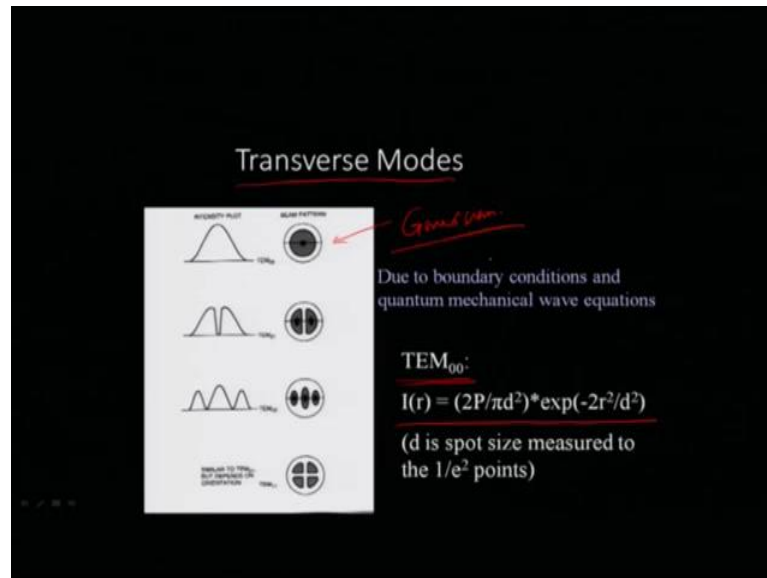
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So, the resonance cavities and longitudinal modes which are also important in this particular process need to be now looked at. I want to remind you that we have come to realize that just simple principle of exciting and looking at it, it is not going to work out. So, we need to look into other aspects which are very important in the process of laser it is a design aspect that is what we are going into it. We will also show as what is the best materials and best ideas of choosing the level structures very soon, but let me go over some of these design aspects.

So, the other aspect is the resonance cavity and longitudinal modes which occur, the wave lengths involved with lasers and masers for example, spread over small ranges and also absolutely small so most cavities will achieve length wise resonance because that is only in terms of some λ . And so you can have the cavities set up in terms of how the curvatures of the mirrors are or if one of the curvatures both is concave. So, basically whether you have a concentric resonator, confocal resonator or a whether you have hemifocal resonator where one of the mirrors is flat other one is the curved one have; hemispherical resonator unstable resonator depending on the choice of the combination of the mirror combination we can get these things to happen.

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That gives rise to another important aspect which is the transverse mode of this system which is get the lasing having a beam pattern which can be having this important transverse spatial profile which is due to the boundary conditions and wave equations of this beams being stabilized within the cavity or how they are forming the different modes within the cavity.

And, so we have both transverse as well as the other one was the longitudinal modes. And the transverse modes are the once which is we are after in terms of the laser is the TEM transverse electromagnetic 00 mode which is first one which is has the best Gaussian looking mode, and that is given as the intensity which is given by this form. And we are essentially interested to just consider this as our Gaussian intensity profile. And most cases when we talk about a laser we would be essentially taking the transverse electromagnetic 0 0 modes which are the Gaussian mode.

And if it is other modes then we would like to work it out to a point where it becomes is extremely high order wave which again sort how can we assume to be a Gaussian mode. That is how we get a way by almost always treating a Gaussian mode for the transpose beam.

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Negative Temperatures
To have $N_2 > N_1$ (Population Inversion):

$\frac{N_2}{N_1} = e^{\frac{-\Delta E}{k_B T}}$	$e^{\frac{-\Delta E}{k_B T}} = \frac{N_2}{N_1} > 1$	$\frac{\Delta E}{k_B T} < 0$ $T < 0$
	$\frac{-\Delta E}{k_B T} > \ln(1) = 0$	

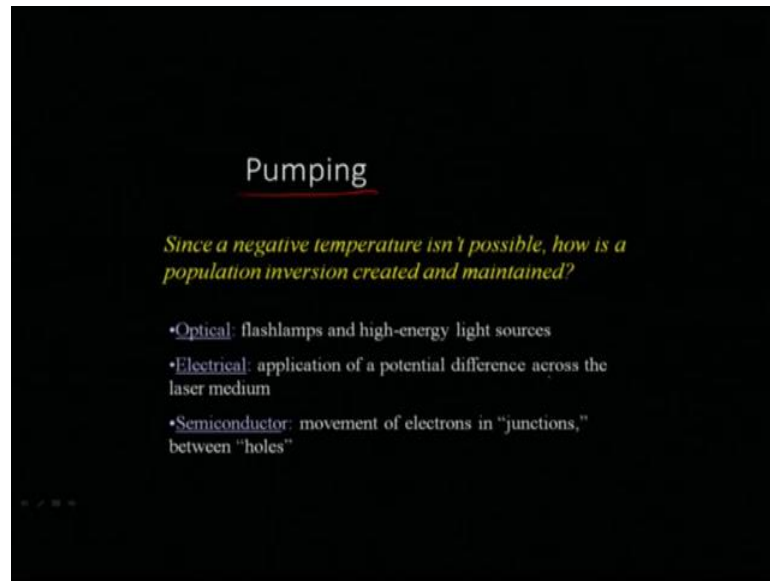
But this implies a negative temperature! Because of this, populations inversions were incorrectly referred to as negative temperatures.

The principle of the population inversion as we have been discussing for a 2 level system is not very exciting. In fact, in thermal equilibrium we know that we have this problem that we have to get the inverted system, and that is what we found that for a sample 2 level system very difficult task to do at room temperatures for achieving a lasing system, because as we know for a maser it's possible but to get that for a lasing system for a laser.

So, for the laser action to occur N_2 must be greater than N_1 . That is what we have looked at which is the population inversion conditions, which is not quite possible for a 2 level system in thermal equilibrium at room temperatures for a 2 level system that we have discussed. In fact, if we looked at it carefully what we find is that to achieve the population inversion for a 2 level system it implies a negative temperature as we show here, because of this population inversion where incorrectly refer to as negative temperatures and that was the reason why lasers were considered to be something which will not be possible when Maiman initially approached the problem.

So, this was one of the reasons why lasers were thought of as an impossible scenario even after the maser were discovered.

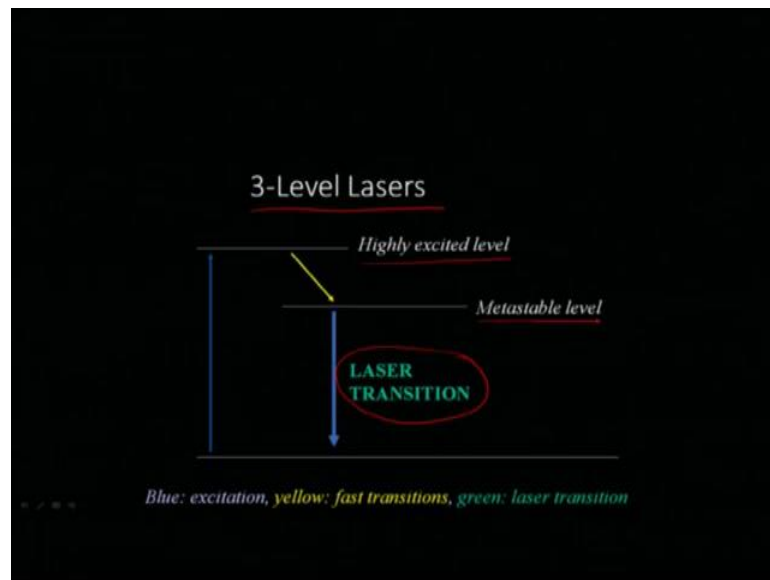
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So, the question is which Maiman looked at was to ask that if negative temperature is not possible how can population inversion can be created and maintained. And then he looked into this idea of pumping which is the principle of continuous process of providing energy; it could be terms of an optical, where it can be flash lamps and high energy light sources could be electrical, application of a potential difference across the laser medium, or as we know in present day semiconductors were we have movement of electrons in junctions between holes.

So, over the years this is become the process of pumping has become a more and more refined, but generally this was the initial principle behind it.

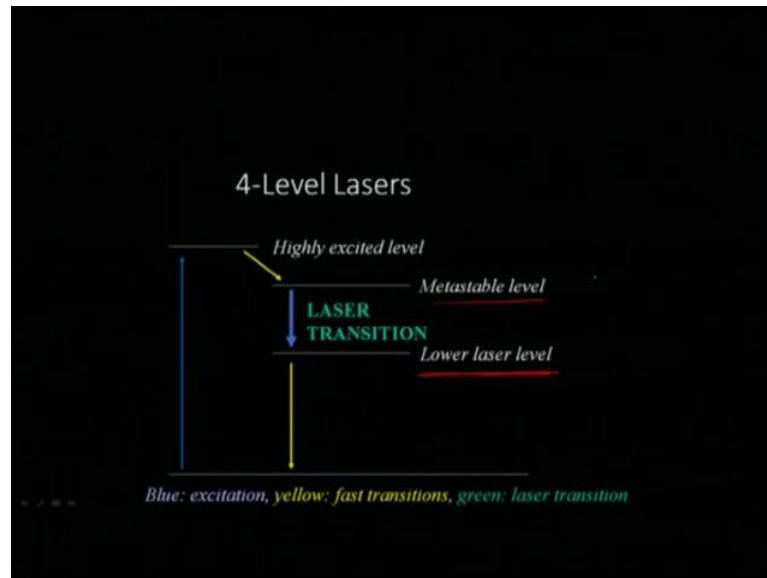
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And one of the things which Maiman finally was able to achieve was to use a medium which was just not simple 2 levels, but 3 levels. And now this class of lasers can also be defined in terms of the level structure that we use one is the principle of 3 level lasers where we have states which are more than 2. So, we have the excitation which goes to a highly excited state from which it has fast transition to some metastable state from where the laser transition occurs.

Now this is a possible scenario because the metastable state can be something where the population can come down after going through the highly excited level and so it can grow the population of the metastable state if it has stability to some level, so it does not decay quickly. Would mean that it will accumulate population and inversion of population with respect to the other state being looked into where it started from can get higher and so you can have an inversion. So, this is by using involving more than two states this can be looked at.

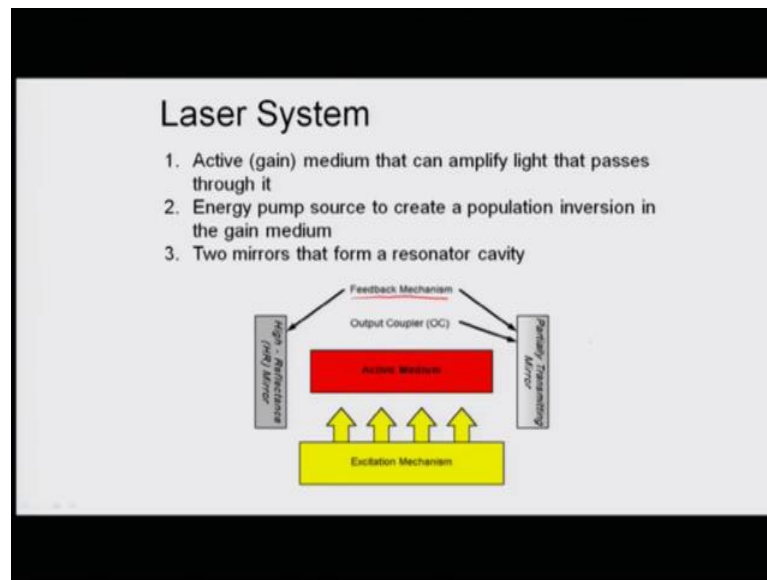
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You can also have a 4 level system for example, where the highly excited state comes to another before the metastable state and then there can be another lower laser level which is not exactly the state from where you started the whole process. And this is a in some sense is a much easier we are producing an inversion process, because the lower level state with respect to which the metastable state is being populated can easily achieve the inversion process, because it could be a situation where neither of them had enough population to start with. So, bringing in the population through the highly excited state to the metastable state can quickly achieve the principle of population inversions.

So, why going to higher number of states involved you can actually get the principle of inversion happen, population inversion happen even without the principle concept of negative temperatures. And that is one of the things which lead to the important aspect of the laser.

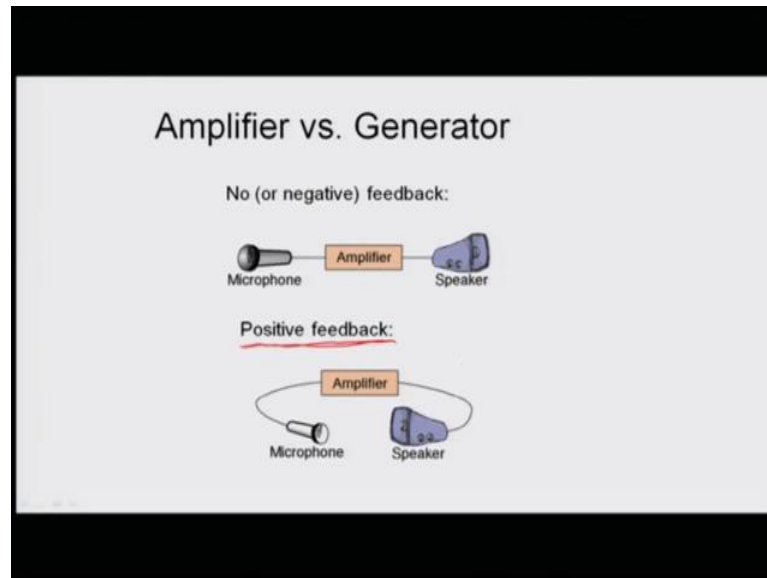
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In some sense the laser system can be lead once again we looked at as the active medium which is being excited or being pumped by the excitation mechanism, and which is being put inside a resonator one of the sight to have the feedback mechanism. The part, from which some part of the energy being taken out, technically is known as the output coupler. And the other sight from which almost no light or the no energy indication is happen is known as the high reflector.

So, this is a basically the principle, the energy pump source creates the population invention in the gain medium, and the two mirrors that are been referred to here are the resonator cavity.

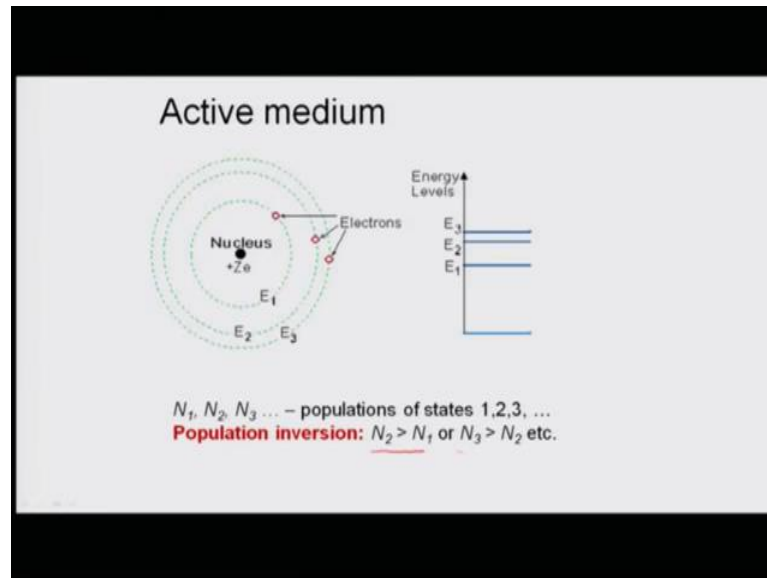
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So, this is the principle that we use here in which we more have the idea with positive feedback where the amplifier is being fed back into it and its cartoon wise it is like you know here is the picture we have the microphone face in the speaker and then it will be a lot of difficulty in this process where we know that here a high screech, because the microphone is feeding into the system and you have lot more amplification that you need.

Whereas, if they need looking at two different directions and then there is no feedback or negative feedback, so you do not see that. In our particular case the laser generally it s a positive feedback mechanism because the resonator which is being utilized for getting higher amplification.

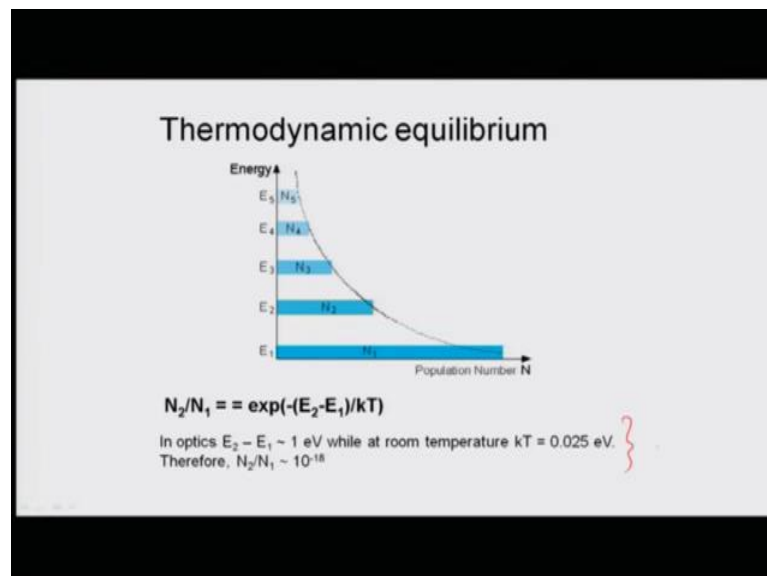
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So, the active media in these cases can be simple atom or molecule. In the case of an atom it is basically the electrons with respect to the nuclei which are residing in different electronic levels which are the population states and we are using the different states that we just mentioned.

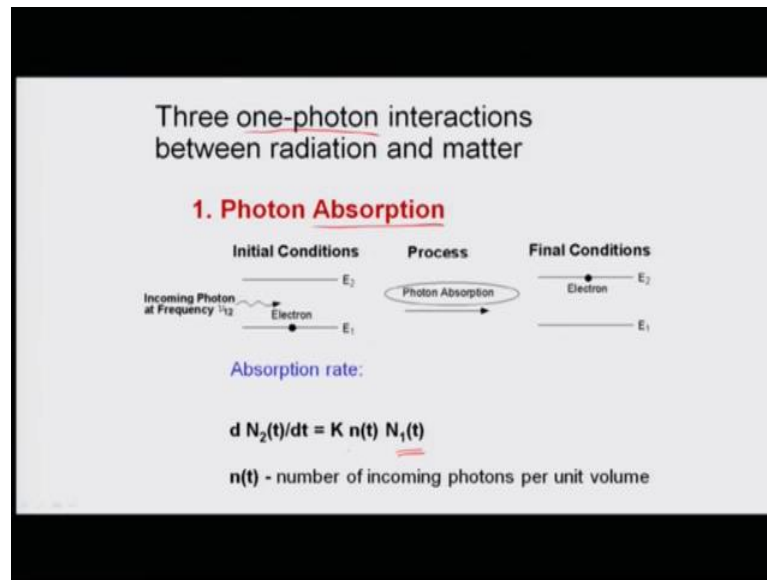
And we have to use more than two states as we know now. And we have to somehow generate a case where the state going to be higher than the other state. So, it could be 3 level systems or 4 level systems depending upon how we are achieving in that.

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In case of thermo dynamic equilibrium which you had seen before it was the case where knew that we have to use Boltzmann distribution so on and so forth. Then we had found out it was not possible to do things just simple two states which is right next to each other the ground and excite state.

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But we can go back and look at this same principle what happens when you go to the 3 level systems. So, generally speaking there one these are all single photon interactions between the radiation and matter which you are looking at. And there are three kinds we have as we discussed before; one is the absorption process, where the incoming photon of appropriate energy gap between the two takes the electron to the excited state for a specific case of an atom we are showing here.

So, we have this rate which can be related to the number of incoming photons with respect to the number of states of the kind that we are looking at; so N_1 atoms being interacting which are in the ground state with the number of photons coming in per unit volume.

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2. Spontaneous emission of a photon

Initial Conditions

Process

Final Conditions

Spontaneous decay rate:

$$d N_2(t)/dt = - g_{21} N_2(t) = - N_2(t)/ \tau_2$$

Solution: $N_2(t) = N_2(0) \exp(-g_{21}t) = N_2(0) \exp(-t/ \tau_2)$

Spontaneous photons are emitted randomly and in all directions

So, then it can have two different conditions, because now we have created initial condition where my electron is in the excited state so it could be a spontaneous process where it just comes out; outgoing frequency is equivalent to the ingoing because the energy gap is the same. However, it does not have any directionality so as per the solution we know that we will have that many numbers of outgoing photons with respect to incoming.

And they are emitted randomly and in all directions. This is the spontaneous decay.

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3. Stimulated emission of a photon

Initial Conditions

Process

Final Conditions

$$d N_2(t)/dt = - K n(t) N_2(t)$$

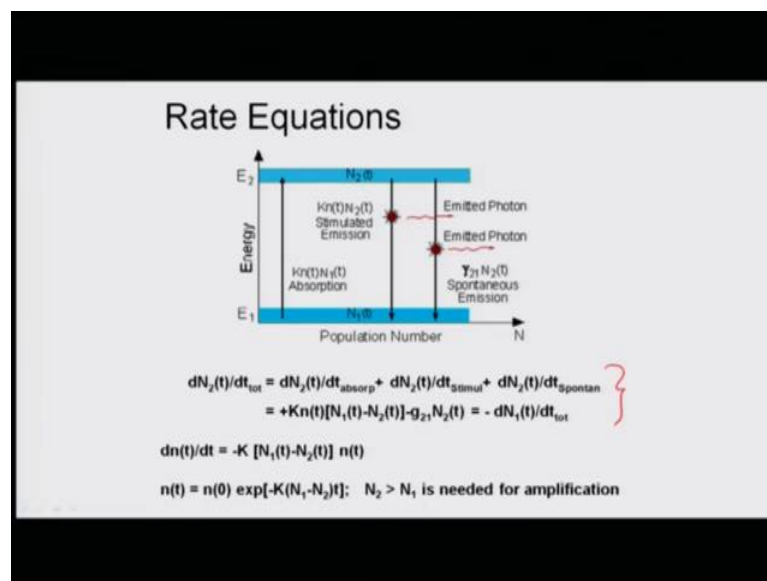
Proportionality constant (K) for stimulated emission and (stimulated) absorption are identical.

- Stimulated photons have the same frequency and direction.
- Stimulated emission is a result of resonance response of the atom to a forcing signal!

So, these are the rates equation that we have already looked at, but in a little bit more quantity manner. So, similarly here for the stimulated process we have this number of photons interacting with the number of states, number of excited systems and we can have the output shown. Proportionally constant for stimulated emission and stimulated absorption are identical. Basically, we call this; so the stimulated is in the bracket because we are providing photon in some sense stimulating the process of absorption versus the process of emission. And so this is how we have the word stimulated in both, and that is why I had started of all with that. But it is all absorption is in some sense stimulated, whereas emission can either be stimulated as spontaneous as we are saying here

So, the stimulated photons have the same energy, same frequency and direction, and stimulated emission is a result of resonance response of the atom to a forcing signal. That is the two different cases that we have here.

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So, in terms of the rate equation we have essentially written it down in terms of how we have written earlier, and we achieve the same study that we have shown before.

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Why inversion is possible in a three-level system

Assume we pump to a state 3 that rapidly decays to level 2.



$$\frac{dN_2}{dt} = BIN_1 - AN_2$$

$$\frac{dN_1}{dt} = -BIN_1 + AN_2$$

The total number of molecules is N :
 $N \equiv N_1 + N_2$
 $\Delta N \equiv N_1 - N_2$

Level 3 decays fast and so is zero.

$$\frac{d\Delta N}{dt} = -2BIN_1 + 2AN_2$$

$$2N_2 = N - \Delta N$$

$$2N_1 = N + \Delta N$$


$$\Rightarrow \frac{d\Delta N}{dt} = -BIN - BI\Delta N + AN - A\Delta N$$

Now the point is; why is inversion possible in a 3 level system. Say we pump the state three which rapidly decays to level 2. So, in terms of these expressions now we are going to look at the same idea that we talked about physically. And so we go through the same kind of expression that we had looked at before. And what we can see is given that we have a total number N for the molecular the atom that we are looking at.

We have this different levels decaying and if level three decays fast and it essentially goes away because you do not see it to appear in the total number at the end of the day, then you have a situation where you can write the difference of the populations in this terms and so we can substitute them and we can come up with the expression which as the rate of the change of the number of systems one kind of with the other, in this kind with differential equation with respect to time.

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Why inversion is possible in a three-level system (cont'd)



$$\frac{d\Delta N}{dt} = -BIN - BI\Delta N + AN - A\Delta N$$

In steady-state: $0 = -BIN - BI\Delta N + AN - A\Delta N$

$$\Rightarrow (A + BI)\Delta N = (A - BI)N$$

$$\Rightarrow \Delta N = N(A - BI)/(A + BI)$$

$$\Rightarrow \Delta N = N \frac{1 - I/I_{sat}}{1 + I/I_{sat}}$$

where: $I_{sat} = A/B$
 I_{sat} is the **saturation intensity**.

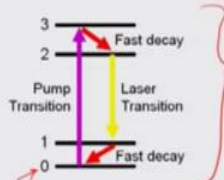
Now if $I > I_{sat}$, ΔN is negative!

And we can come up to the same condition with under the study state condition to this equation where, we will find that we generate the same kind of the saturation intensity that we discussed in the last lecture which is the ratio of between the two coefficients.

And we will find that we get to a situation that if our intensity is greater than the saturation intensity can get the delta N to be negative which is exactly we are looking for which is population inversion. And that is the reason why for a 3 level system we can generate population inversion.

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Why inversion is easy in a four-level system



Now assume the lower laser level 1 rapidly decays to the ground level 0.

As before: $\frac{dN_2}{dt} = BIN_0 - AN_2$

$$\frac{dN_2}{dt} = BI(N - N_2) - AN_2$$

Because $N_1 \approx 0$, $\Delta N \approx -N_2$

$$-\frac{d\Delta N}{dt} = BIN + BI\Delta N + A\Delta N$$

At steady state: $0 = BIN + BI\Delta N + A\Delta N$

The total number of molecules is N :
 $N = N_0 + N_2$
 $N_0 = N - N_2$

In fact, as I mentioned before population inversion is actually easier in a 4 level system. So, this is the similar condition except that the looking at a 4 level system where we have a fast decay and fast decay for the two states that we are going to consider for the lasing action. And this process of going up to another excite state the ground state availability is different. And, as before we can write these expressions with the same kind of idea that none of these states have enough to start with population, so we can write down these expressions in this form. And we can write N_0 now in terms of the population of the ground state. And we can write the total expression again in this form in the same spirit that we have been discussing.

Now, we have generally considered that our transient between states to have almost no initial population so we can get these kinds of expressions written. And we will be getting the steady state expression as before.

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Why inversion is easy in a four-level system (cont'd)

$$0 = BIN + BI\Delta N + A\Delta N$$

$$\Rightarrow (A + BI)\Delta N = -BIN$$

$$\Rightarrow \Delta N = -BIN / (A + BI)$$

$$\Rightarrow \Delta N = -(BIN / A) / (1 + BI / A)$$

$$\Rightarrow \Delta N = -N \frac{I / I_{sat}}{1 + I / I_{sat}} \quad \text{where: } I_{sat} = A / B$$

I_{sat} is the **saturation intensity**.

Now, ΔN is negative—always!

And going ahead with same kind of arrangements we find that will have this principle where ΔN is going to be this kind. And in this case now you find this is always negative. So, in some sense the 4 level system is a very conducive condition for producing lasing, because in this case just like the simple 2 level system where it was almost impossible to produce inversion it is just the opposite where we have almost always producing inversion.

So, that is the important part here.

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What about the saturation intensity?

$$I_{sat} = A/B$$

A is the excited-state relaxation rate: $1/\tau$

B is the absorption cross-section, σ , divided by the energy per photon, $h\nu$: $\sigma/h\nu$

Both σ and τ depend on the molecule, the frequency, and the various states involved.

$$I_{sat} = \frac{h\nu}{\sigma\tau}$$

$h\nu \sim 10^{-19}$ J for visible/near IR light

$\tau \sim 10^{-12}$ to 10^{-8} s for molecules

$\sigma \sim 10^{-20}$ to 10^{-16} cm² for molecules (on resonance)

10^8 to 10^{13} W/cm²

The saturation intensity plays a key role in laser theory.

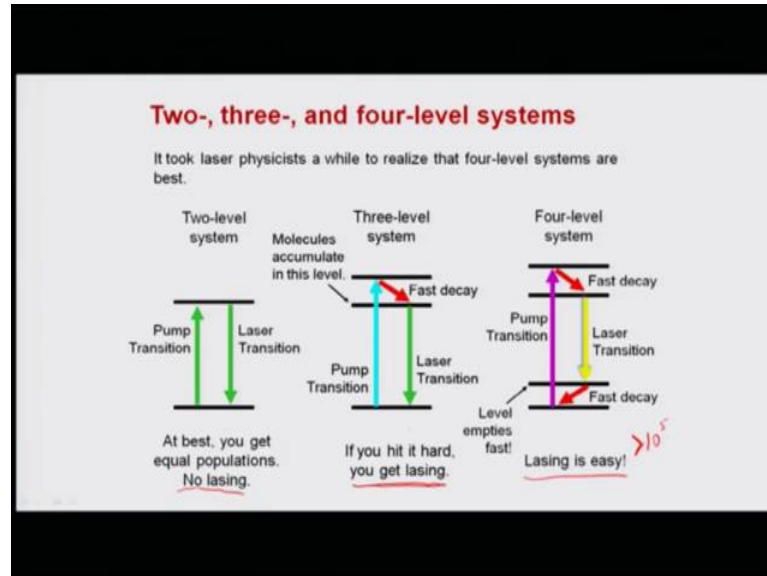
And the saturation intensities something which is as I mentioned in couple of times before it is the ratio between the excited state and the ground state in some sense. So, we can look at the relaxation rates and we can come up with how these can be quantified. And what we will find is that the B is absorption cross section sigma which is because A is appearing because of the time of the relaxation of the state that is being generated. So, as long as it is a time scale which is not going to be extremely short this will be something which you make some sense.

So, both sigma depend on molecule the frequency and various states involved depending on how we define it and so for here are some typical numbers. So, for the visible and near we have the $h\nu$ as this value and tau is roughly a peak second to some tens of nanoseconds for molecules. And so the cross section on resonance for molecules are quite high, are very are in the order of 10 to the power minus 20 to minus 16 square centimeter which gives rise to the values of the saturation to be in the range of 10 to the power 5 to 10 to the power 13 watts per centimeter square. And that is number density, the saturation in density which plays a key role in the laser theory.

You need to provide at least this amount of intensity to start out of the principle of lasing. So, for a laser which is under the very conducive condition which is a 4 level system you still have quite a bit of saturation intensity which is in the range of 10 to the power 5 to

the 10 to the power 13 watts per square centimeter. If you look for a similar number for a 3 level system this number turns to be a much larger, but still it is doable.

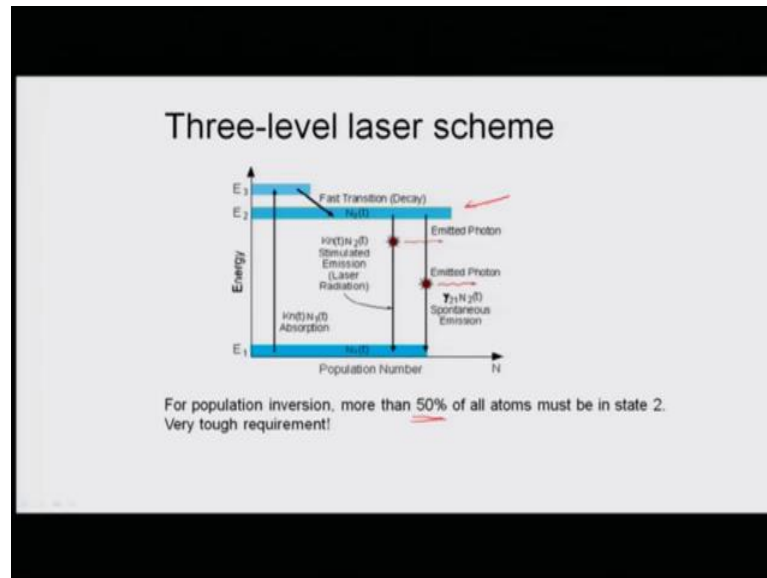
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So, here is a comparison now of the three different systems that we have talked about; 2, 3 and 4 level system. And as we have mentioned over a again there is a no lasing for 2 level system under the condition that we have look yet at least under normal condition, it is not possible. However, for 2 level system if you hit it hard you can get it lasing, as I just mentioned your I saturation becomes quite high in this case by using similar numbers that we used for the 4 level systems are the lasing essentially easy, because the number density is always in tour favor in this particular case your inversion is sort of very easy to generate in this case.

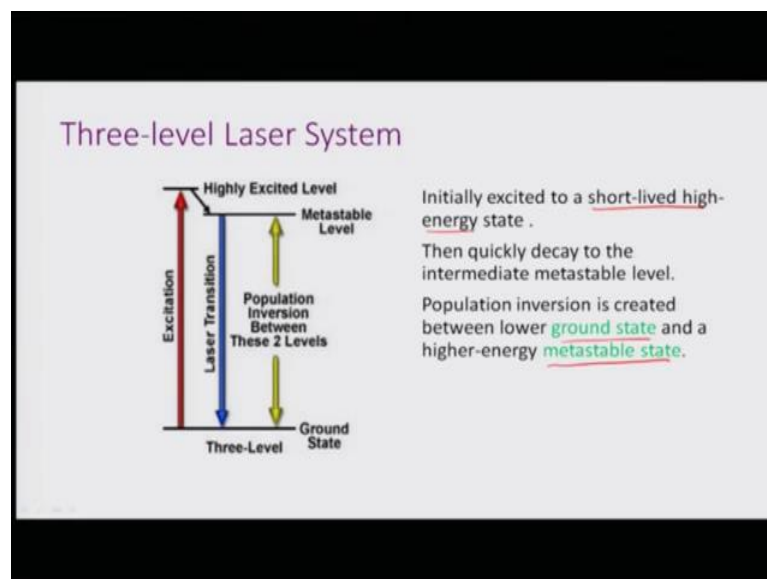
Therefore, easy case and yet you have to achieve a saturation intensity which is roughly for wave lengths in the visible infrared in the range of about 10 to the power of 5 or so. So, which is not difficult at all this are very nice number. And you need higher that is why if you actually can get to a point hitted hard and you can get lasing for 3 level, but in case of 2 level as we know it is not possible to get it under this situations.

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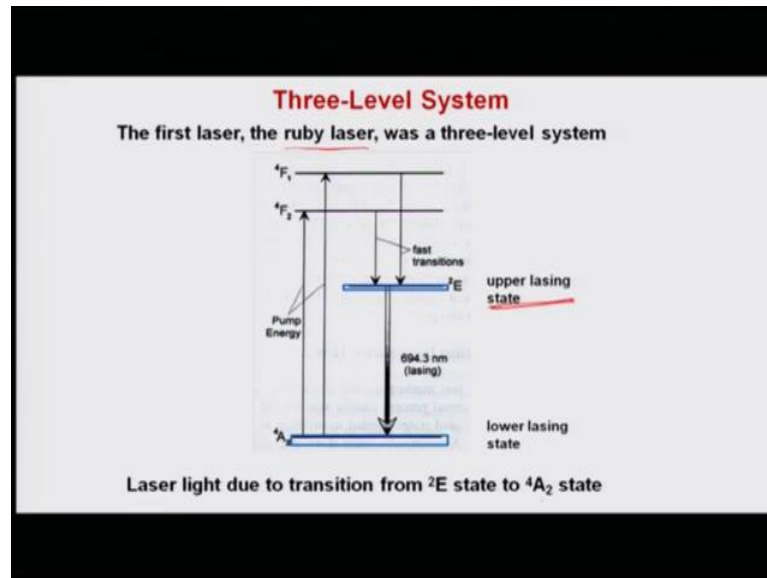
So, generally we have sort of understood the way of looking at it. So, roughly we are looking for more than 50 percent of all the atoms to be possible to be in the state two that is roughly how to look at in a 3 level system mode.

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And this are different representations of looking at the same condition, we call that metastable versus the ground most of the time, but it depends on how which molecule exactly what you look at. The most important thing is the short lived high energy state that we are looking at.

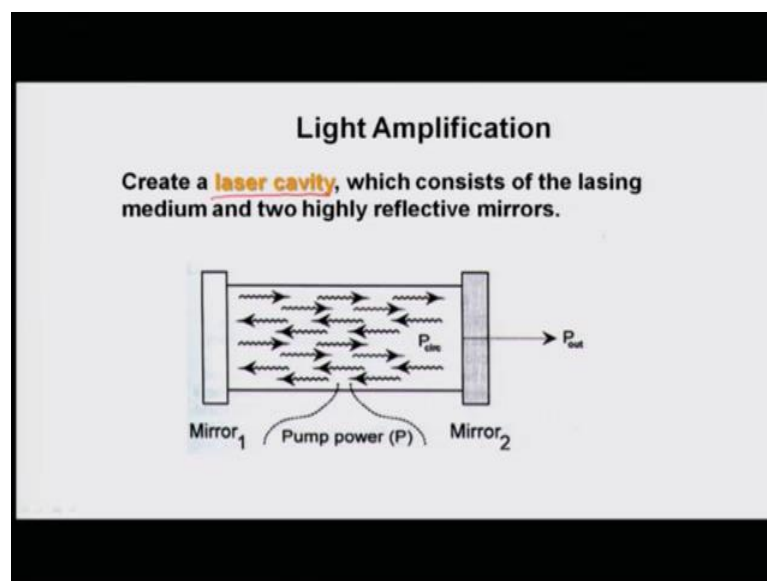
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The reason I went through the different pictures is also to make you realize that the first laser that was essentially shown by Maiman is the ruby laser which was a 3 level system.

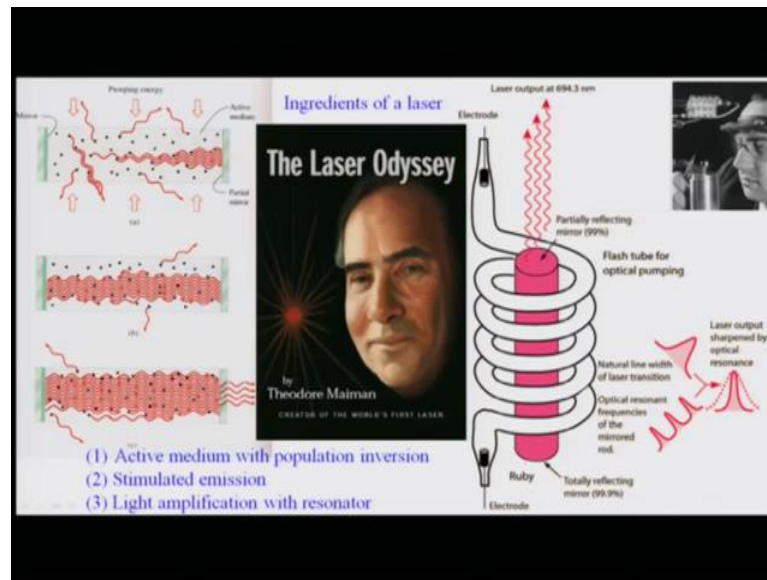
So, it had first transition from the steady states the ruby atom that it came down from the higher excite states down to the upper lasing state and then the laser essentially was possible to get it from the lower lasing. The laser light was due to the transition of E 2 to the 4 A 2 state.

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Once this idea was there it was basic concept of create a laser cavity which consists of lasing medium and two highly reflective medium as we said and providing an pump power to get the laser happen.

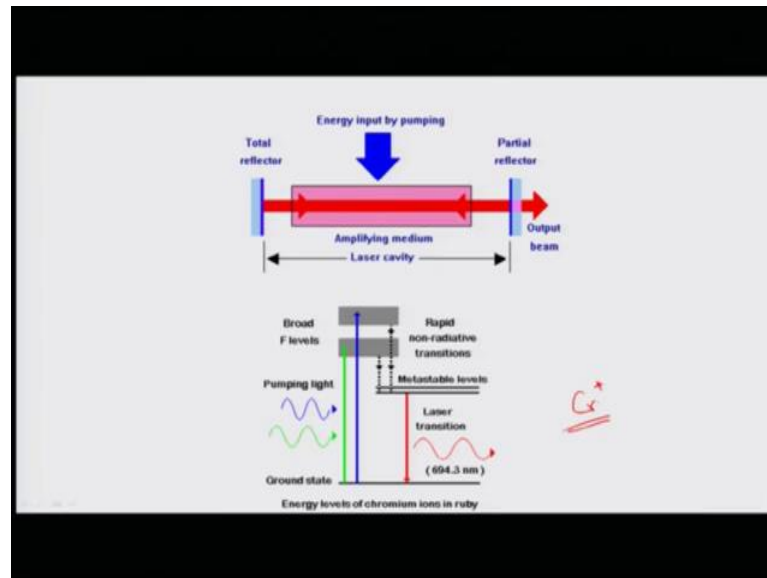
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And so here is the laser, this slide essentially tries to put together all the basic principle that went behind the demonstration of the first laser. It is a an extremely important step which I should mention because, when the first maser was demonstrated it was sort of also proved, almost immediately that point as we also shown over and over again that 2 level system would actually not be able to produce the concept of lasing at all. The population inversion concept is not going to happen there.

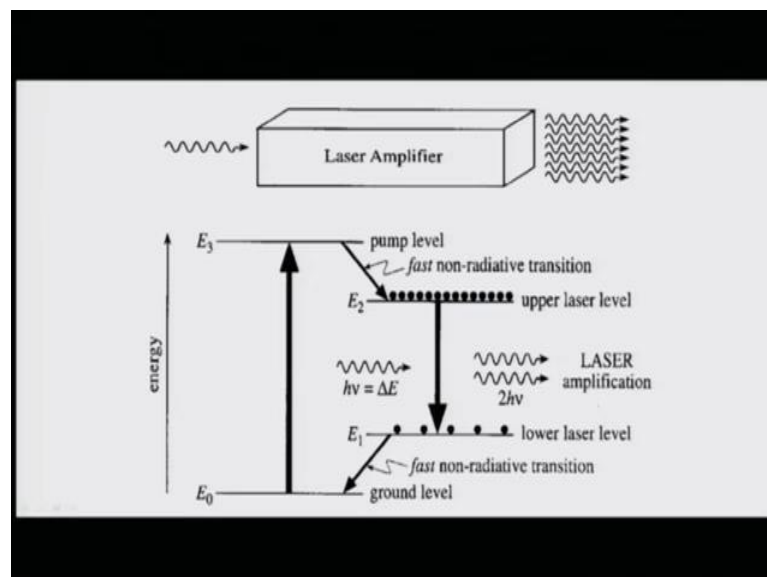
And, so it was almost like doctor Maiman was working against the all the odds in believing that a laser can actually works and so it took him lot of huge challenge that time where it was essentially believe that a principle of an optical laser would not be possible. And so it took him a lot effort to go through the process of providing the principle that it can happen. And he was able to use the idea amplification with the resonators which give rise to the lasing output at 694.3 nanometers.

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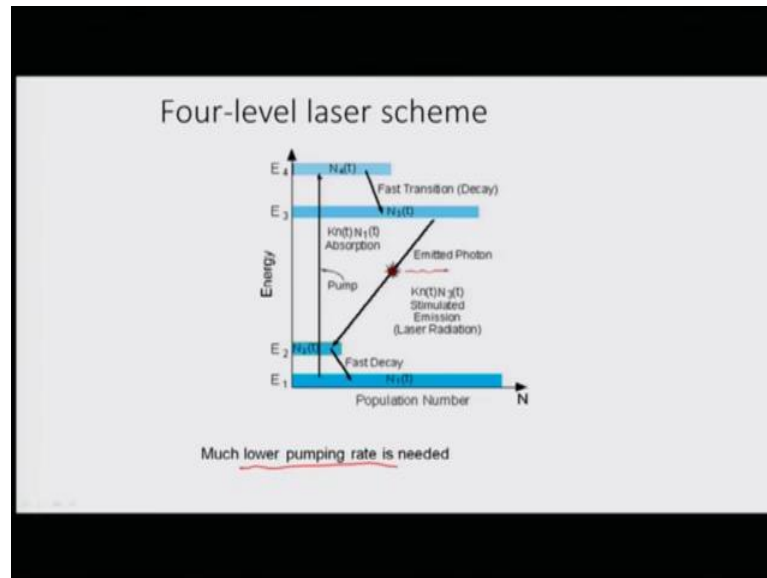
This is the basic principle; he used a electrical pumping by using the electrode to pump the laser system from the lasing medium, which is the ruby in the particular case. And that is how he got it the energy input was laterally provided which was the amplify medium and this is what the chromium ions; those where the once which are necessary in the ruby crystal which gave rise to the lasing material.

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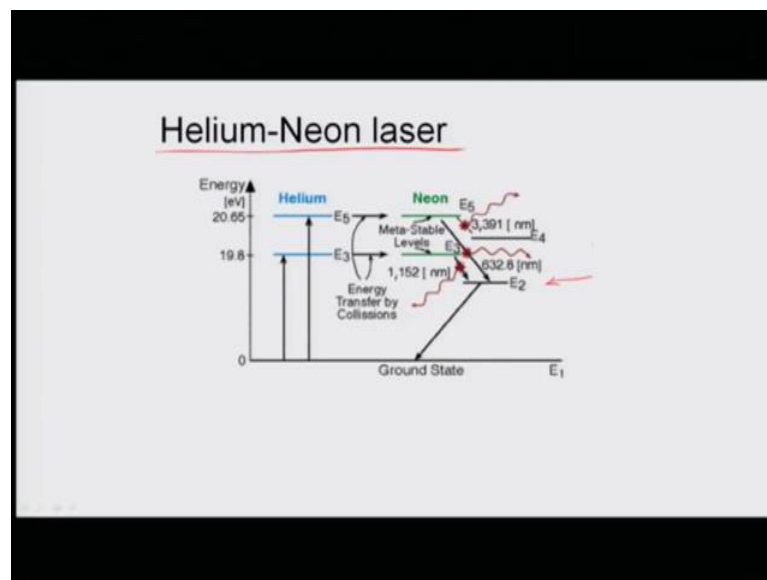
So, 4 level systems would have been much easier as we know, and as long as we use the 4 level systems it will be much easier to get amplification systems happening.

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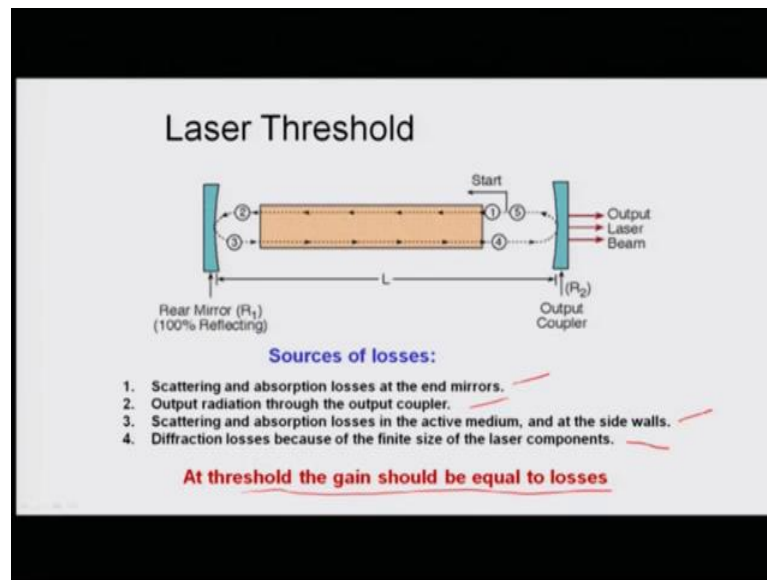
And, so a lot of work as then gone into the 4 level system which gives rise to laser switch required much lower pumping.

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And one of the lasers in this category is helium-neon laser where the helium and neon are put together inside the system, inside a gas cell. And the helium is excited which then transverse the energy by collisions to neon which as those exact met stable energy states where from the lasing can occur and to its higher excites states which are not the ground state. And therefore, lasing is much easier because a 4 level system.

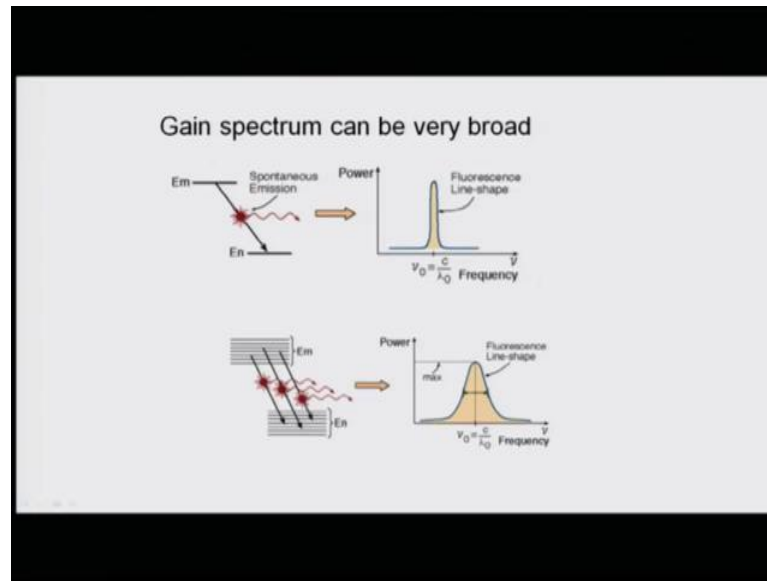
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So, this was the first demonstration of the 4 level systems some sense. The laser threshold issues can be looked at by calculating the losses, because as you go further into this idea of lasing you have to also take care of the principles of the thresholds. And in this case the sources of losses can be scattering and absorption losses by the mirrors, output radiation through the output coupler, scattering and absorption losses of the active medium and the side walls, diffraction losses because of the finite size of the laser components.

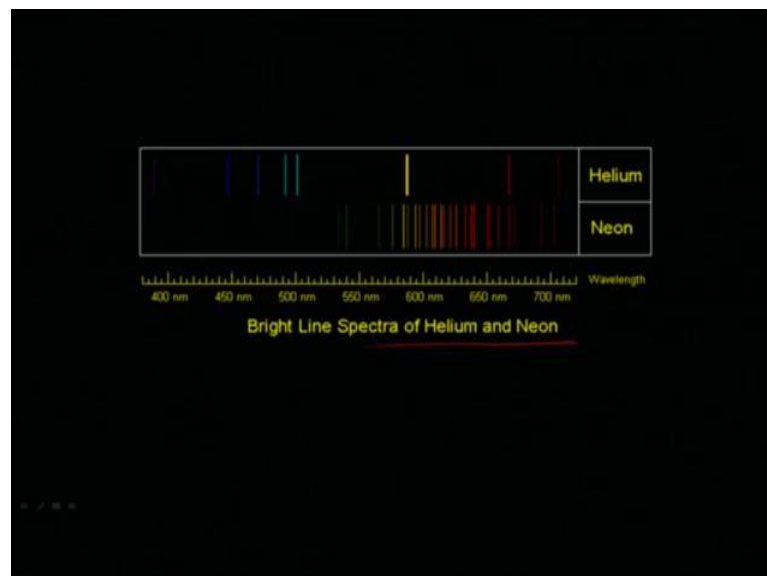
And at threshold the gain should be equal or higher than the losses. And that is why the threshold is important, because unless you are able to at least match the gain to the losses would not be able to see a lasing action.

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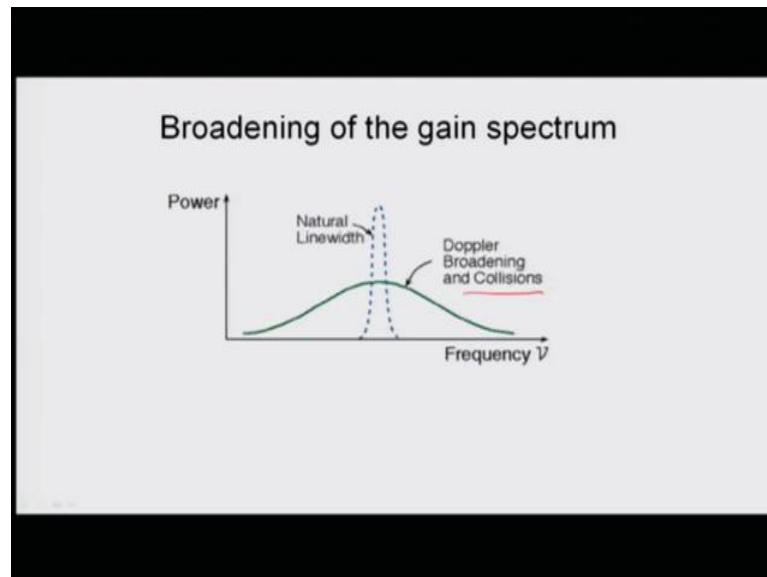
There are some certain other very important issues also, that the gain spectrum can become very broad because of this line shapes that we are looking at now due to many many photons coming out which are amplified. And the fluorescence line shapes can get broader due to the Doppler broadening and other process which can happen because of multiple photons coming out at same time. And that has its advantage as well as disadvantages.

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The bright line spectrum of helium and neon is sort of been shown here, which are at various different wave lengths. But lasing generally occurs at 633 nanometers, 632.8 nanometers as one of the popular wave lengths of the neon of the helium-neon; lasers were the transverse as occurred.

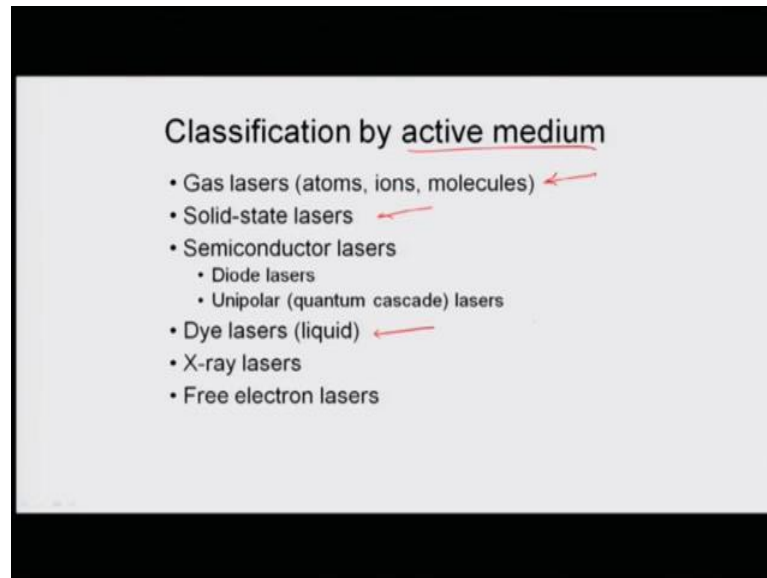
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The broadening of the gain spectrum is due to as I mentioned to the Doppler broadening and collisions with respect to the natural line width that we are considering. For instance, for the helium-neon case which is atomic line transitions which is extremely narrow and natural line width.

The laser line width would be larger due to the broadening and collisions.

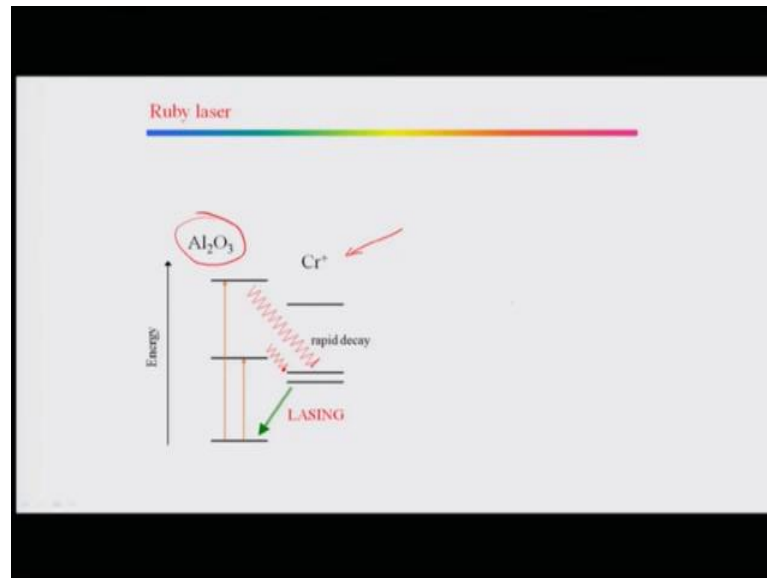
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At this point I like to just quickly go through some of the classification types of the active medium which are done by terms in terms of the active medium for these lasers. We have already looked at some of them; one of the gas lasers which are atoms, ions, molecules of all kinds. Solid state lasers, we already looked at ruby laser for instance. And there are semiconductor lasers which are diode lasers, unipolar; quantum, cascade, lasers and things which are more used recently.

One of the other popular lasers which we have been around for a while is the liquid laser or the dye lasers where a dye molecule is being used as very nice absorber and having a wide absorption region where you can actually the wave lengths to the certain energies that you need. And there are X-rays lasers and free electron lasers many other different kinds of lasers.

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So, we already know of the chromium active ion which is essential for the ruby laser for instance. The host is the alumina which is the reason of the ruby crystal it acts as a host, whereas the chromium is the active medium which leads to the lasing for the system.

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Solid-state Laser

- Example: Ruby Laser
- Operation wavelength: 694.3 nm (IR)
- 3 level system: absorbs green/blue

• Gain Medium: crystal of aluminum oxide (Al_2O_3) with small part of atoms of aluminum is replaced with Cr^{3+} ions.

• Pump source: flash lamp

• The ends of ruby rod serve as laser mirrors.

So, the 3 level systems as we mentioned before also, but this is the level structure. And this is how the solid state laser as we shown before was shown to work operation wave length and the exact 694.3 nanometers. It is actually still red very near to infrared part around 700, but it still quite bright red. So, 3 level system absorbs green blue radiation to

gives rise to this, gain medium is the crystal of aluminum oxide with small parts of atoms of aluminum is replaced by the chromium three plus ions. So, that is the active source that we are looking at.

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Gas Lasers

The laser active medium is a gas at a low pressure (A few milli-torr).

The main reasons for using low pressure are:

- To enable an **electric discharge in a long path**, while the electrodes are at both ends of a long tube.
- To obtain narrow spectral width not expanded by collisions between atoms.

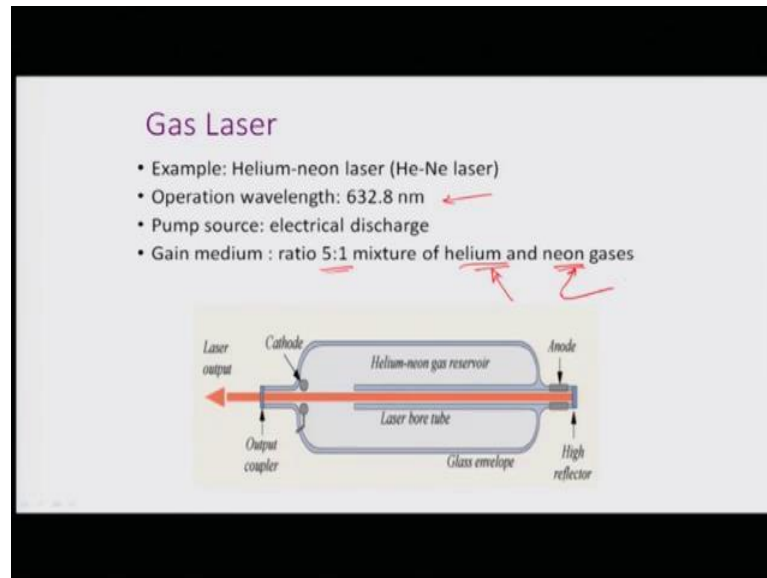
The **first gas laser** was operated by **T. H. Maiman in 1961**, one year after the first laser (**Ruby**) was demonstrated.

The first gas laser was a **Helium-Neon laser**, operating at a wavelength of 1152.27 [nm] (Near Infra-Red).

The pump source is a flash lamp in this case and then the ends of the ruby rod serve as laser mirrors the basic idea of this. Again for gas lasers where the example a laser active medium is the gas at low pressure roughly a few milli-torr, the main reason for using the low pressures are to enable an electric discharge.

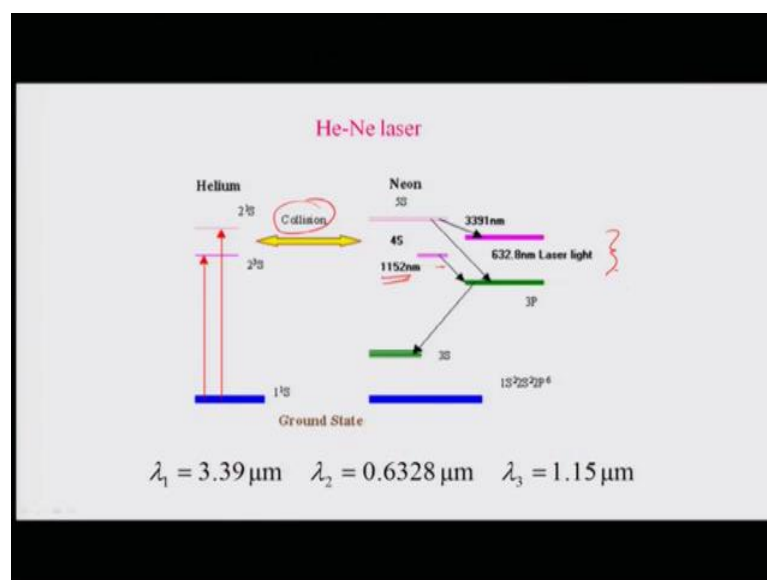
So, most of the gas lasers are based on electric discharged principle, with a long path while the electrodes are at both ends of a long tube to obtain narrow spectral width not expanded by collisions between the atoms. That is another important way to reduce the line broadening. The first gas laser was again operated by Maiman in 1961; one year after the first ruby laser was demonstrated. And the first gas laser was a helium-neon laser operating at the infrared at 1152.27 nanometers.

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The gas laser typically once that they use now days are in the visible region of 632.8 nanometers, so electric discharge, and the gain medium is the ratio of 5 is to 1 mixture of helium and neon gases. The helium is been excited and been passed on to the neon gas energy process goes inter conversion into the neon gas which is the one which is our active place medium in this particular case.

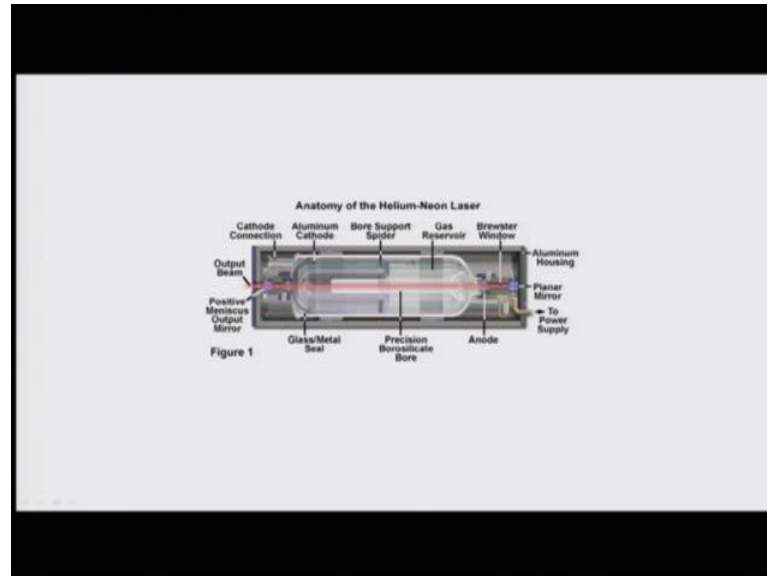
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Here, it goes through the collisions into the neon system and it goes through to the principle where able to get the lasing in this particular case. The first lasing how was

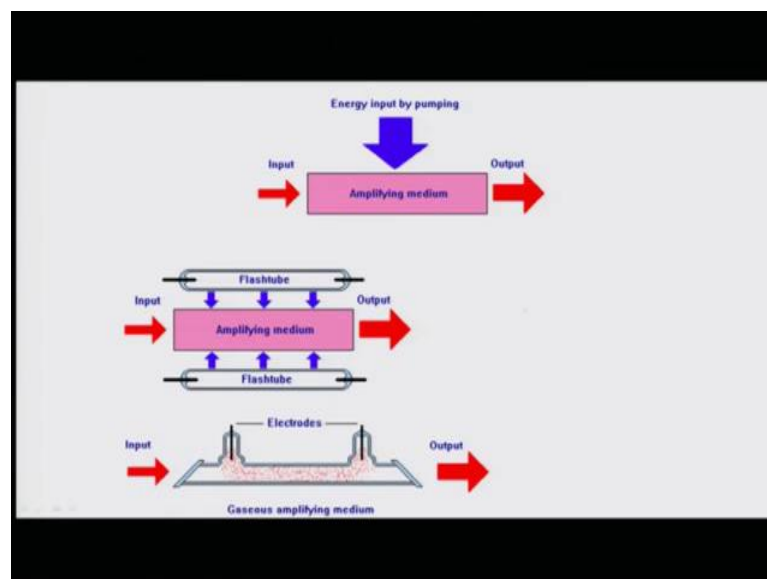
shown at 1152 by Maiman in this particular region, whereas today this is the one which is more typically used which is 632.

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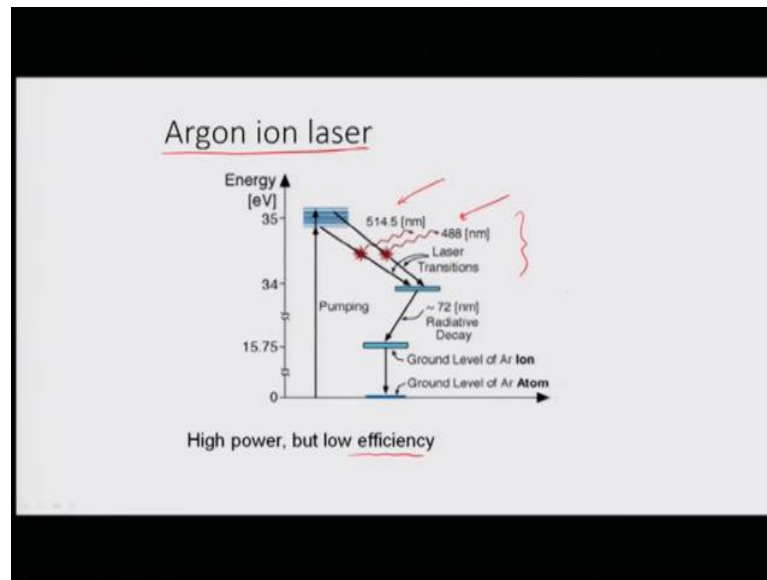
So, there are many ways of looking into these kinds of lasers, but we do not want to really spend much time on this anymore, these are glass chamber in which the gas is main stored and all that.

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Roughly speaking we have the electrodes and where the gas amplified mixtures are put. If we want to look at as high voltage which is creating the discharge, and that is what is shown here amplifying medium the flash tube is doing that and we get the laser tube.

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The other important gas laser is Argon ion, where the pumping is being done by again by plasma is being created by electric discharge and then we get different laser transitions. Again a 3 level system and so go to that excite state when it come to the metastable states and then you see the laser transition at different wave lengths 514.5, 488 nanometers and which is at the higher plasmaonic transitions which have been used.

So, that is why it is an argon ion laser is been used. It can provide high power, but it again as low efficiency because it is a 3 level system.

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Liquid Laser

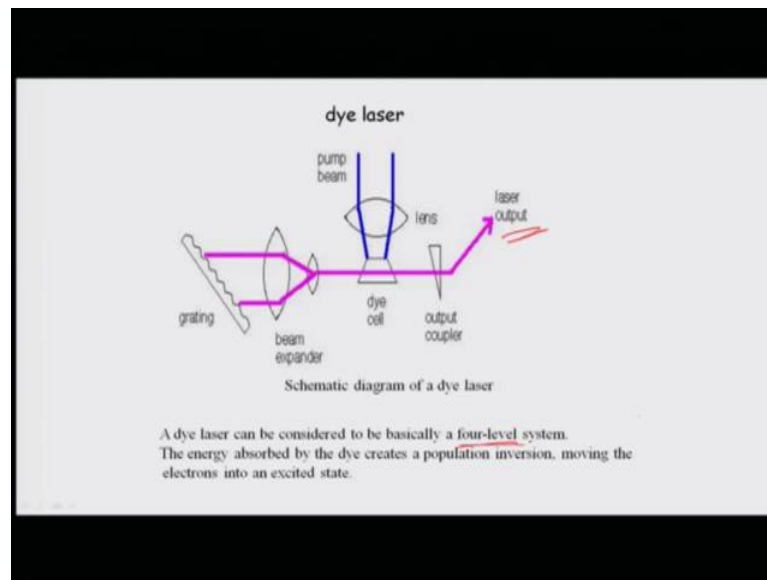
- Example: dye laser
- Gain medium: complex organic dyes, such as rhodamine 6G, in liquid solution or suspension.
- Pump source: other lasers or flashlamp.
- Can be used for a wide range of wavelengths as the tuning range of the laser depends on the exact dye used.
- Suitable for tunable lasers.

The other important laser system which is been used for a while now is the liquid laser system. It was popular until about 5-10 years back, but due to many other issues dye lasers and not that extremely popular nowadays. However, it is important to provide the details on this very important class of lasers where it is the complex organic dyes such as rhodamine 6G, liquid solution or suspension which are been utilized.

The pump sources can be other lasers or flash lamps can be used with a wide range of wavelengths that is the key issue why the dye lasers became very very important as they can be having a huge tuning range off the laser depending on the exact dye used. So, change the dye and you can get a new laser. And that is the reason why this tunable lasers sources it was one of the biggest source of this.

(Refer Time: 53:20) was one of the first persons who involved in this process of this dye lasers, and it has been lot of has gone through lot of development and its very useful lasing system because many spectroscopic application need to tunable lasers where the dye lasers played a very important role.

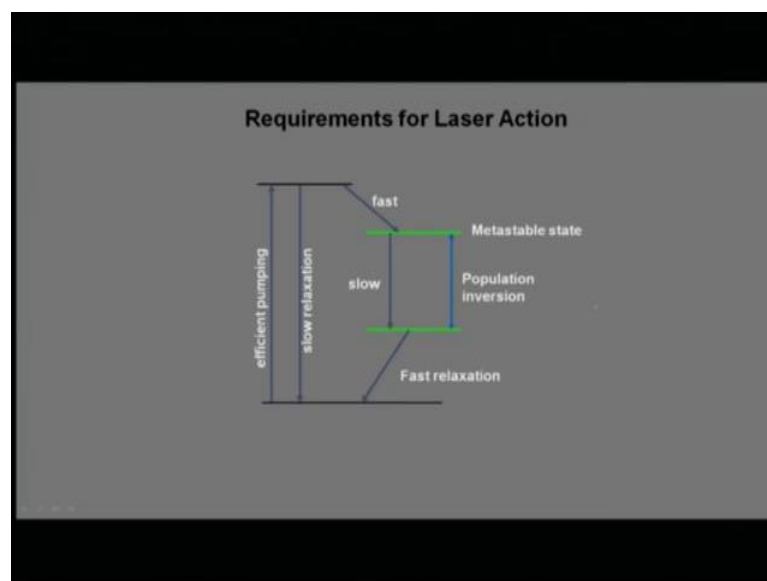
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Typically, in this process there either a dye cell or a dye jet into which the pump is being focused and that sets of the cavity where there can be different ways of selecting the wave length by using a grating or prism, and then there is this cavity which is inside which the laser is been put so that you can get the output.

And it is a 4 level system, and the energy absorbed in the dye creates a population inversion moving the electrons into excites state.

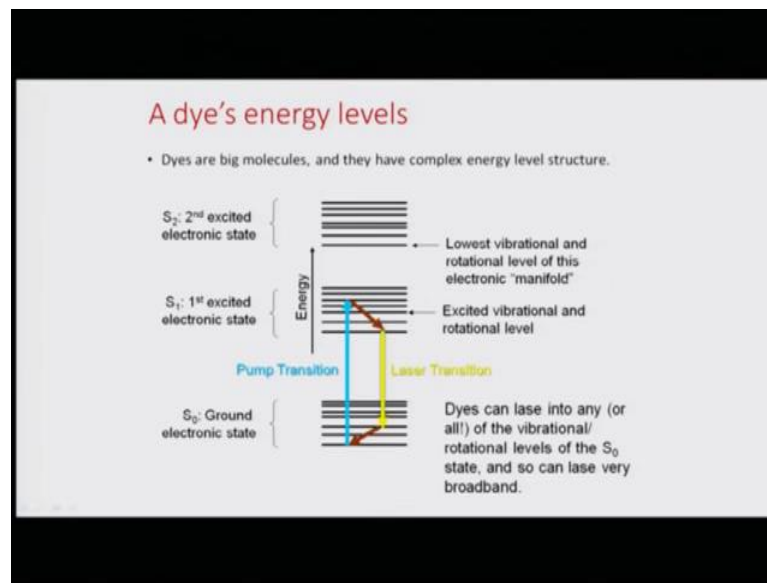
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In this case this is sort of like this the 4 level system that we can see in a dye laser where we have the efficient pumping to the excite state, and this excite state it as very low relaxation to the place where it came from ground state. Instead, quickly goes into a metastable state of which is sort of stable, because it is a relaxation into the other possible state down there is the low, creating a population inversion with respect to another metastable state which can again undergo fast relaxation when it is going there.

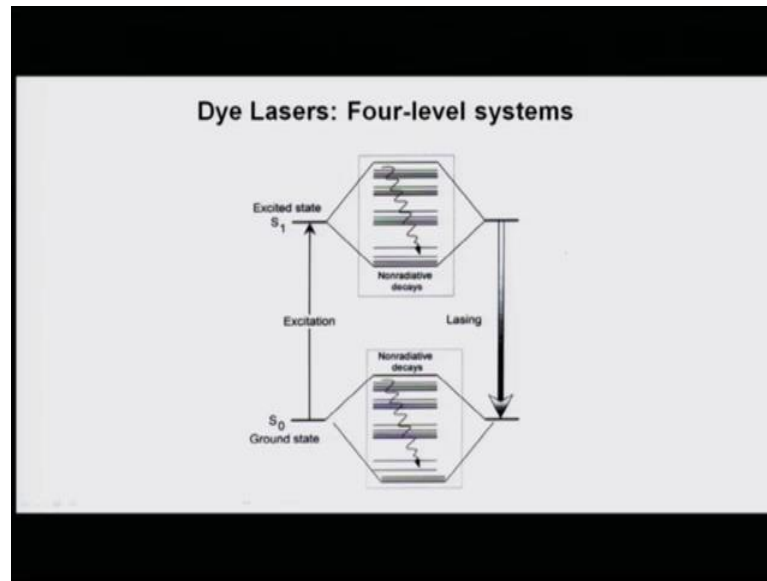
So, that is why this is a very important 4 level system which is been used.

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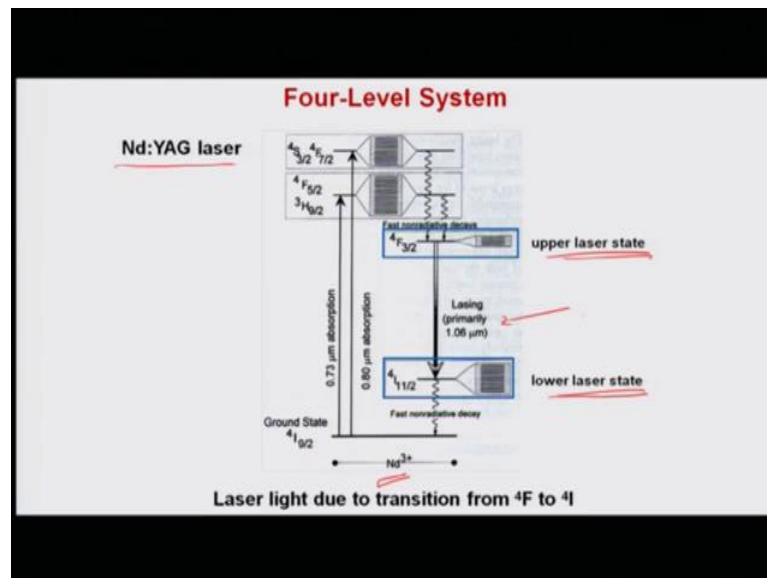
And in case of the dye this is a big molecule where the complex energy structures and the transitions are going from different ground to excited level. And the vibration states of these different levels can be utilized to get the transitions. And, you can look this one of them many different kinds of dyes which you can use them so on and so forth.

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It is technically a 4 level system as we have been saying, and the time scales involved are important because that can give rise to different kinds of things.

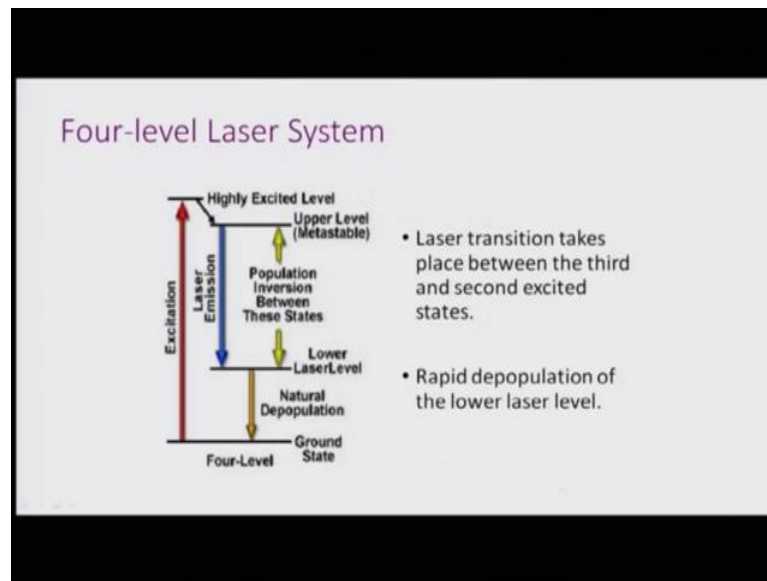
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The other one important 4 level which I showed you laser so I thought I will just mention before closing the lecture today; is a neodymium YAG laser which is again a solid state laser. Is the YAG crystal in which neodymium aluminum (Refer Time: 56:06) that is the crystal in which the neodymium ion is being used as the.

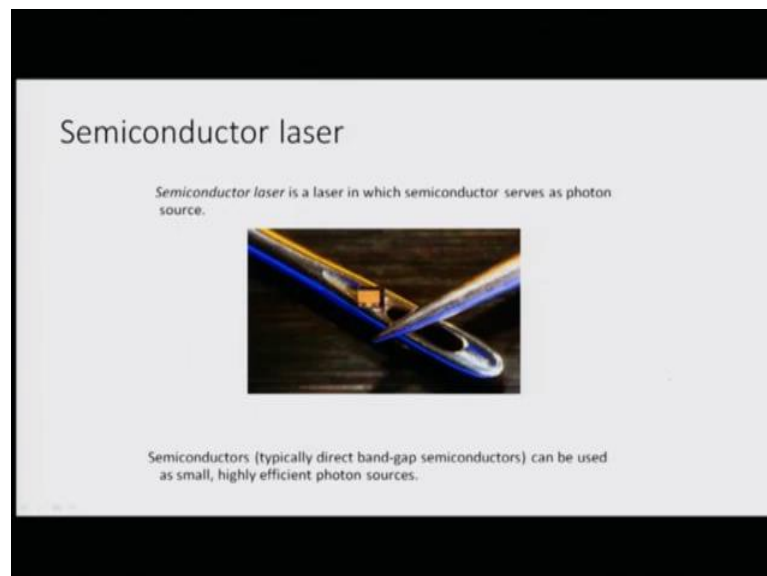
So, our acting medium in this case neodymium 3 plus which is been shown, and that is the one which is getting pumped into the excited states. Which then goes into the upper lasing state which is one of the metastable state and then there is a lower state available. So, it is 4 level system where you can get efficient lasing at 1064 nanometers, it is a primary lasing wavelength of this and turns out to be a very effective and useful lasing and the transition which we look at.

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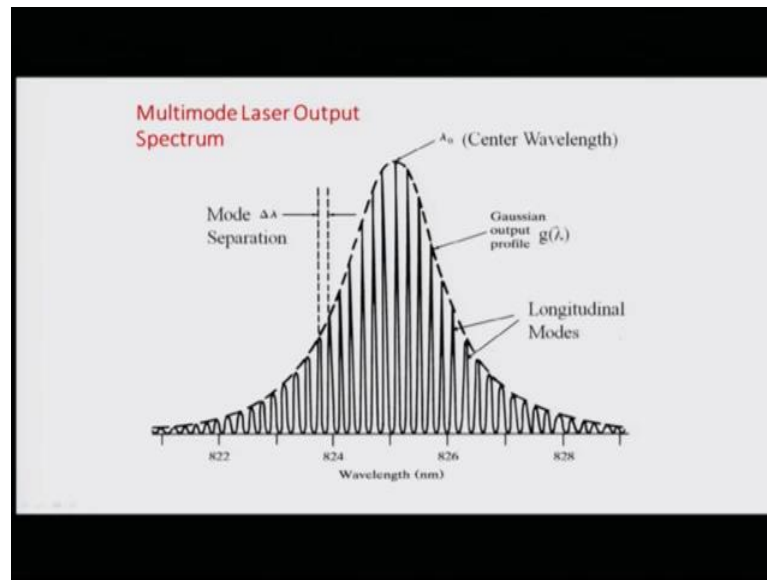
So, this is the population inversion looking at for all the system which is very effective.

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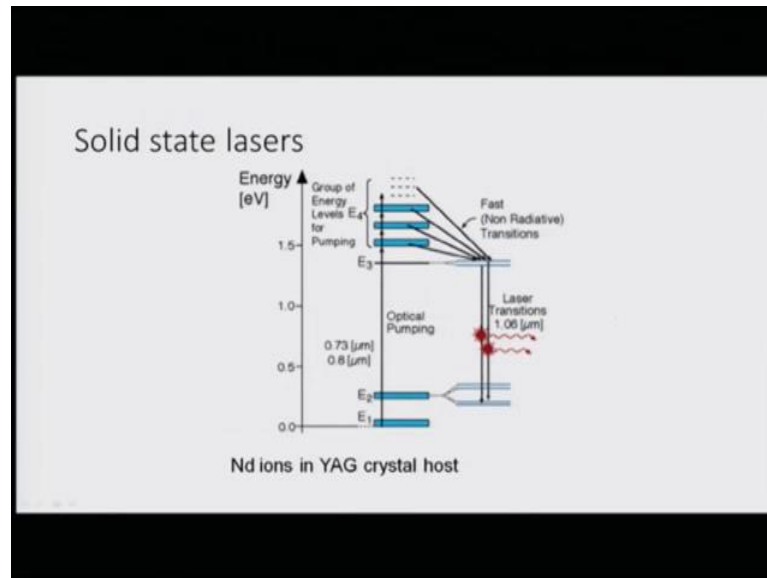
And finally, I would not mention much about it but semiconductor laser is a laser in which semiconductor serves as a photon source. And typically this are direct band gap semiconductors which can be used as a small highly efficient photon sources, is also the fiber lasers where you have doping which can create lasing and other kind of things.

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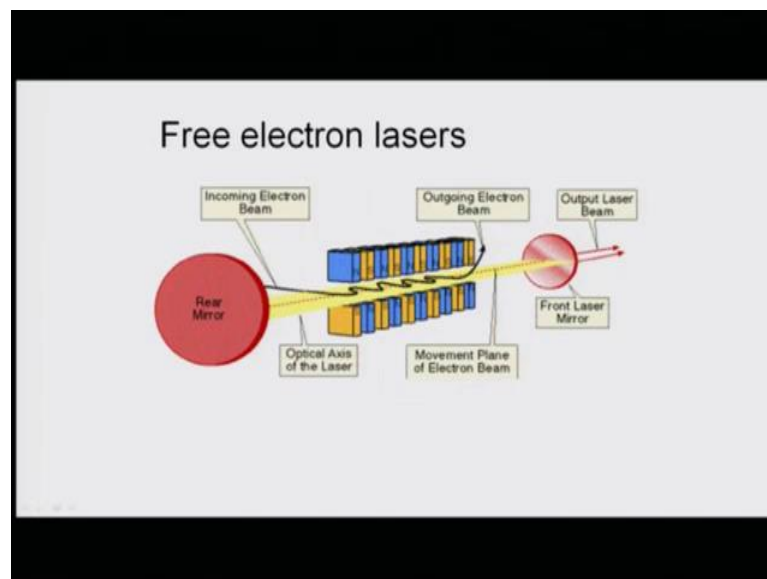
We will not do too much of about it now because we will have to move on to other aspects. So, most of the lasers that we are talked about have multi mode laser output, and where there are mode separations, center wave length, and the typical Gaussian output is there, but then there are this longitudinal modes, and this is a typical laser output.

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And call this is the energy diagram; again we have already gone through this.

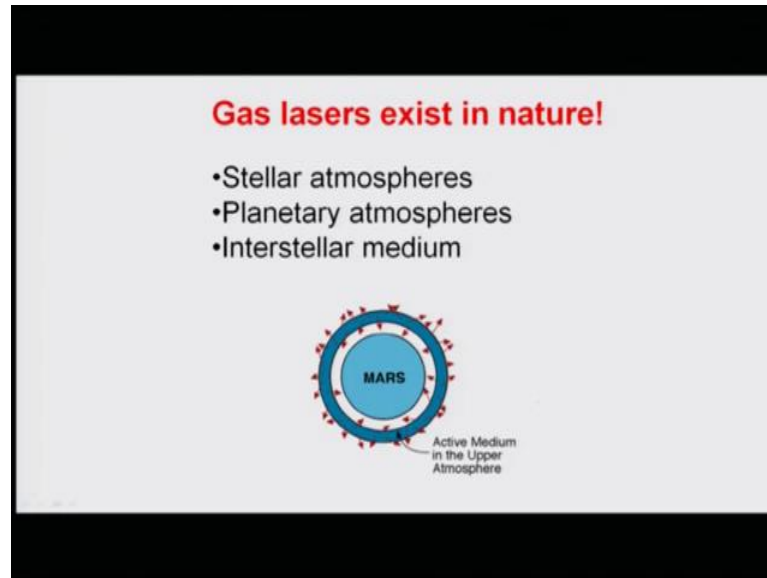
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And there is a another small quick example of the free electron laser which has also as a very high tenability, but it requires a lot more energy to start with the system. But this works with the principle of electron beam, and output laser beam is visible or it as wide range of spectrum that you can get into the infrared and many other regions.

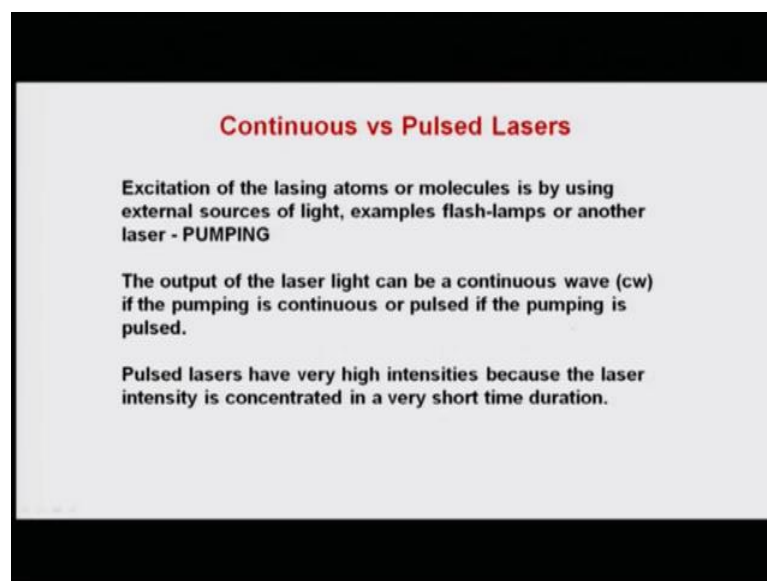
And here the electron beams are being oscillated and amplified and result of that light and there the associated electromagnetic radiation gets amplified and so you get the lasing output.

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In fact, lasers exist in nature also. So, here is an example of gas laser in nature. In the stellar atmosphere in the planetary atmospheres interstellar mirrors and there are active regions there are gas layers which can exist which are known to be exist in nature.

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With this I would like to actually close this lecture because in the upcoming lecture we would be looking at the pulse laser. I just alluded to the idea that will be having various modes, so you want to put them together and then will have a time spectra. So, in the next lecture we will be looking at pulse lecture. With this I would like to close.

Thank you.