

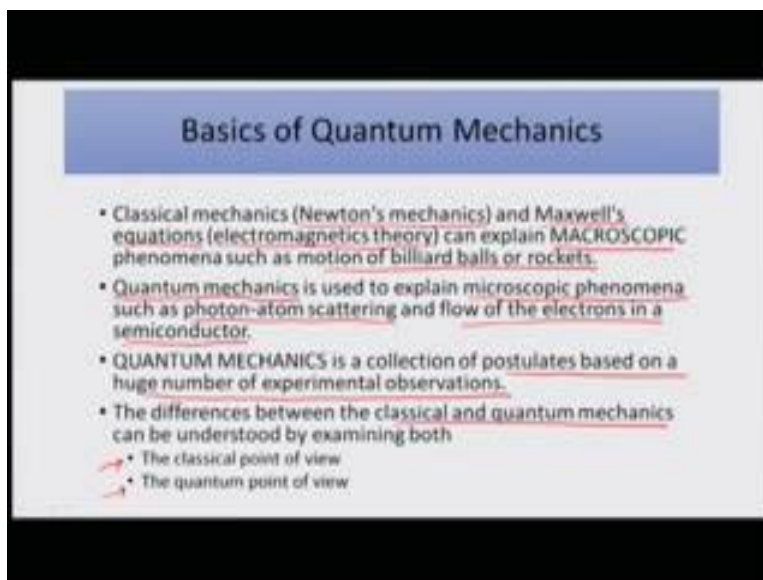
Implementation Aspects of Quantum Computing
Prof. Debabrata Goswami
Department of Chemistry
Indian Institute of Technology, Kanpur

Lecture - 11
Basics of Quantum Mechanics

We have been going over the details of quantum computing and the various aspects of bit. And we have also looked at some of the relevant considerations which are required for its implementation. However, one of the basic issues which have come about is that we have not really looked into some of the foundations or the basic aspects of quantum part of this problem.

So, I thought in today's lecture will back a little bit in looking into the Basics of Quantum Mechanics, and take it from there. So, this particular lecture set would be more on the basics of quantum mechanics and looking at some of the aspects related to the quantum phenomena.

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So, in terms of basics of quantum mechanics we have already realized that it is quite distinct and different from the classical mechanics which we label as the Newton's mechanics or the Newtonian mechanics following the laws of Newton, and Maxwell's equations which are for electromagnetic theory. So, when we consider Newtonian

mechanics and Maxwell's equations we can explain macroscopic phenomena such as motion of billiard balls or rockets.

However, when we are looking at microscopic phenomena such as photon-atom scattering and flow of electrons in a semiconductor or the kinds of discussions where we have been having until recently which is miniaturization of computing to try to do quantum computing and other things. Quantum mechanics is the main idea. So, quantum mechanics is a collection of postulates based on a huge number of experimental observations. The difference between classical and quantum mechanics can be understood by examining both, the classical point of view as well as the quantum point of view, so let us see.

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Classical mechanics: (Isaac Newton)

- In Newtonian mechanics, the laws are written in terms of **PARTICLE TRAJECTORIES**.
- A **PARTICLE** is an indivisible mass point object that has a variety of properties that can be measured, which we call **observables**. The observables specify the **STATE OF THE PARTICLE** (position and momentum).
- A **SYSTEM** is a collection of particles, which interact among themselves via **internal forces**, and can also interact with the **outside world** via **external forces**. The **STATE OF A SYSTEM** is a collection of the states of the particles that comprise the system.
- All properties of a particle can be known to infinite **precision**.
- **Conclusions:**
 - **TRAJECTORY** → state descriptor of Newtonian physics.
 - **EVOLUTION OF THE STATE** → Use Newton's second law.
 - **PRINCIPLE OF CAUSALITY** → Two identical systems, with the same initial conditions, subject to the same measurement will yield the same result.

Handwritten notes: "Classical point object" (next to the particle definition), "Journal" (next to the precision statement), and a diagram of a particle path between points A_1 and A_2 .

So, the classical point of view the Newtonian one they Isaac Newton's one is the one where the laws are written in terms of particle trajectories. A particle is considered as an indivisible mass point that has a variety of properties that can be measured, all of which we call observables. The observables specify the state of the particle, position and momentum. And here please note that we are mentioning both position and momentum. So, this is what we call the classical particle. And often it is represented in terms of point objects.

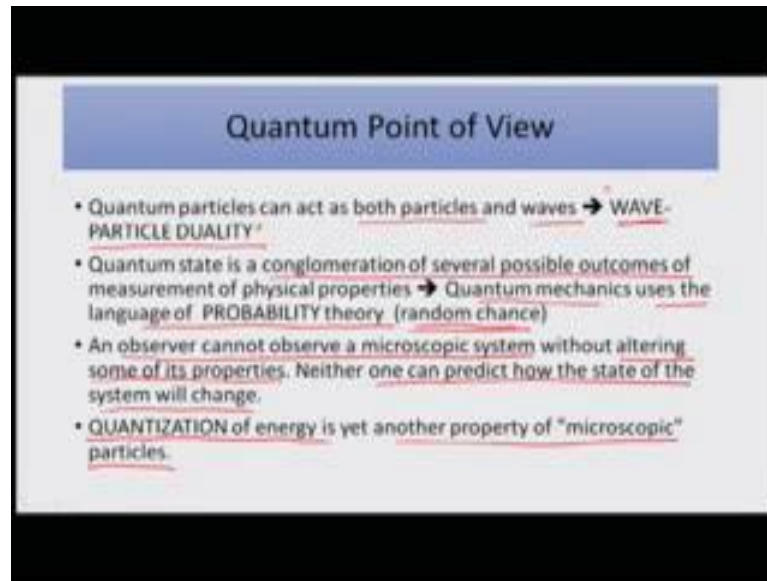
Just to make show things simple point objects allow drawing of path between difference points and that is why it is easier to mention in terms of classical point objects, and here

both the momentum as well as position. So, these like position x_1 , x_2 . This is the position as well as this motion associated would be a velocity and so both position and momentum can be specified and this is the classical object point. A system in this concept is a collection of particles which interact among themselves via internal forces and can also interact with the outside world via external forces.

So, the forces are of different kinds; one is the internal force and the other one is the external force. The state of a system is a collection of these states of the particles that comprise the system. All properties of a particle can be known to infinite precision. Now this is the part which is very important in classical mechanics, and we have been used to this principle. So, the conclusions from classical look of how you wanted to state are the trajectory is the state description of Newtonian physics. Evolution of the state is the Newton's second law; the principle of causality is the fact that two identical systems with the same initial conditions subject to the same measurement will yield the same result.

Now, these are the three important aspects of classical concept which give rise to or which we are used to in terms of classical mechanics. So, one of the important applications of classical mechanics we know is always in astronomical objects, like motion of stars, motion of planets, all of this work quite well, solar system. So, we defined them in terms of systems as well as individual objects. So, this is all what we described in terms of classical.

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The slide is titled "Quantum Point of View" in a blue header. It contains four bullet points:

- Quantum particles can act as both particles and waves → WAVE-PARTICLE DUALITY
- Quantum state is a conglomeration of several possible outcomes of measurement of physical properties → Quantum mechanics uses the language of PROBABILITY theory (random chance)
- An observer cannot observe a microscopic system without altering some of its properties. Neither one can predict how the state of the system will change.
- QUANTIZATION of energy is yet another property of "microscopic" particles.

On the other hand, in terms of quantum point of view quantum particles can act both as particles and waves. So, this is the first very important aspect of quantum mechanics which is it assumes to start with or it has built in wave particle duality. Quantum state is a conglomeration of several possible outcomes of measurement of physical properties, and quantum mechanics uses the language of probability theory. And therefore, a random chance is one of the important aspects that we have to built-in in quantum mechanics.

Another important point here to understand is that the observer in this case cannot observe a microscopic system without altering some of its properties. Neither one can predict how the state of the system will change in terms of absolute precision. And finally, quantization of energy is another property of this microscopic particle which is different from classical system, because in terms of classical mechanics energy can be continuous.

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Heisenberg Uncertainty Principle

- One cannot unambiguously specify the values of particle's position and its momentum for a microscopic particle, i.e.
- Position and momentum are, therefore, considered as incompatible variables.
- The Heisenberg uncertainty principle strikes at the very heart of the classical physics \Rightarrow the particle trajectory.

$$\Delta x(t_0) \cdot \Delta p_x(t_0) \geq \frac{1}{2} \hbar$$

$\hbar = \frac{h}{2\pi}$

So, one of the most important points mentioned in quantum way of looking at thing is the inability to be certain in measurements and that comes from the famous Heisenberg Uncertainty Principle which says that; one cannot unambiguously specify the values of particle's position and its momentum for a microscopic particle. And this is so different from the classical point of view.

So, this gives rise to the fact that there is always some error in measurement, at any point of time the measurement is being made. So, any measurement which is made will have some error associated with it either in space or in momentum and that product is always going to be act best equal to h cross which we know as h crosses h over 2π , which I clarify here. At best it is going to be h cross over 2 or more than that. Position and momentum are therefore considered as incompatible variables or in other words these are the ones which cannot be measured with absolute precision simultaneously.

The Heisenberg uncertainty principle strikes at the very heart of classical physics the particle trajectory, because if we cannot ascertain position and momentum simultaneously the entire picture of trying to draw a particles motion becomes difficult because neither the position nor the momentum can be certain to absolute precision. And that is the reason this becomes one of the biggest problems for classical physics or classical mechanics that is the Heisenberg Uncertainty Principle.

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The Correspondence Principle

- When Quantum mechanics is applied to macroscopic systems, it must reduce to the classical physics. Therefore, the non-classical phenomena, such as uncertainty and duality, must become undetectable. Niels Bohr codified this requirement into his Correspondence principle:

Diagram illustrating the Correspondence Principle:

A horizontal line with a double-headed arrow spans from 10^{-10} m to >0.01 m. Below the line, the left side is labeled "(atoms, molecules)" and the right side is labeled "(trees, buildings, planets)".

Below the diagram, the text reads:

Quantum Mechanics \longleftrightarrow Classical Mechanics

- Laws
- Physical Models
- Predictions

So, how do we apprehend this? In order to make sure that quantum mechanics can still be applied to macroscopic principle where is something which is sort of like known as the Correspondence principle; the idea being that quantum mechanics when applied to macroscopic systems it must reduce to the classical laws or must reduce to the case which the classical physics predicts. And therefore, the non classical phenomena such as uncertainty and duality must become undetectable. Niels Bohr codified this requirement into his Correspondence principle when he postulated his way of looking at quantum mechanics.

Now, Bohr's theory is something which is still considered as the old quantum mechanics. What we will now look at is how things have evolved. Now for him it was necessary to code this because he wanted the correspondent principle of looking at quantum mechanics or applying quantum mechanics to macroscopic systems. So, that is one wave of looking at it, but there are now the system the ideas have evolved to a wave that it need not be coded it comes automatically the wave the mathematics is done.

So, in principle we have the situation that when we are in the atoms and molecule range in size 10 to the power minus 10 or in that order then we are definitely in this quantum mechanics domain. And when we are in macroscopic domains when we are talking in terms of earth the trees buildings planets for example, then we are all in this classical

mechanics principle. And there are laws to connect them, and there are physical models which can lead us to the predictions that we talk about in both these domains.

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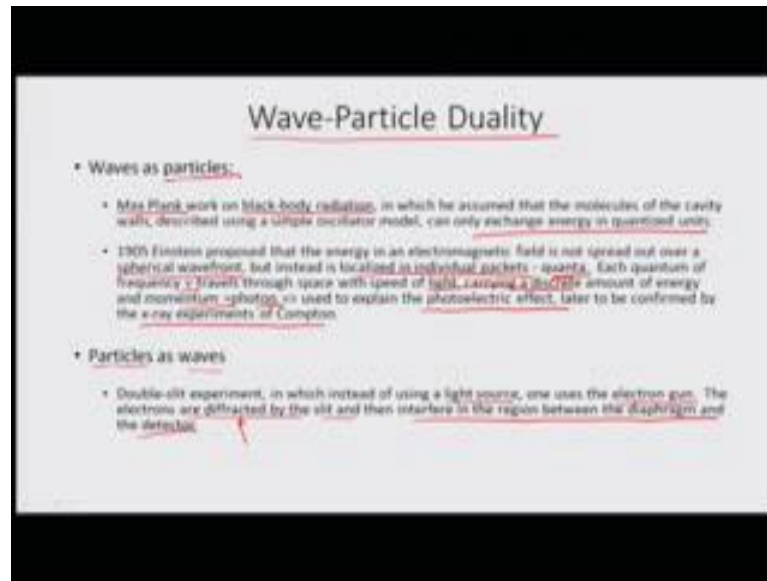
Particle-Wave Duality (Light circled in red)

- The behavior of a "microscopic" particle is very different from that of a classical particle:
 - → in some experiments it resembles the behavior of a classical wave (not localized in space) *particle*
 - → in other experiments it behaves as a classical particle (localized in space)
- Corpuscular theories of light treat light as though it were composed of particles, but can not explain DIFFRACTION and INTERFERENCE.
- Maxwell's theory of electromagnetic radiation can explain these two phenomena, which was the reason why the corpuscular theory of light was abandoned.

So, the wave particle duality which is the other main aspect of quantum mechanics gives rise to the very different behavior of microscopic particles from that of classical particles. In a sense in some experiments we have a situation where the behavior of classical wave is shown by particles, so the particle shows resembles the behavior of classical wave that is it is not localized in space. In other experiments the particle behaves like a classical particle localized in space. So, in some sense it is showing a duality between the particle and the wave like picture. Corpuscular theories of light treat light as though it were composed of particles, but cannot explain diffraction and interference.

On the other hand Maxwell's theory of electromagnetic radiation can explain diffraction and interference phenomena, and that was the reason why corpuscular theory was abandoned earlier. So, this is from earlier times. In quantum mechanics we are apprehensive of the fact that both can coexist and that is the reason why the property of light used to have both wave like in terms of Maxwell's phenomena or particle like as for as the corpuscular theory.

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So, in terms of wave particle duality we have specific examples where waves as particles exist Max Planck work on black body radiation in which he assumed that the molecules of the cavity walls described using simple oscillator model can only exchange energy in quantized units. So, that was one place where Max Planck introduced the concept when he used to study the black body radiation.

So, this is the case when particle nature was used. In 1905 Einstein proposed that energy of an electromagnetic field is not spread out over the entire wave front, but localized in individual packets or quanta. Each quantum of frequency new travel through space with speed of light carrying discrete amounts of energy and momentum, which are known as photons which are needs to explain the photoelectric effect. And this was later on confirmed through x-ray experiments of Compton.

So, these are the two cases where for example we have waves to be treated as particles; black body radiation and photo electric effect. Similarly, for particles as wave we have double slit experiments where instead of using a light source if one uses an electro gun; the electrons are diffracted by slit and then interfere in the region between the diaphragm and the detector to give rise to the same kind of pattern which looks like the case of interfering waves.

So, the principle of diffraction and interference are shown by electrons. And therefore, electron particles behave as wave under these conditions.

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Blackbody Radiation

Known since centuries that when a material is heated, it radiates heat and its color depends on its temperature

- Example: heating elements of a stove:
 - Dark red: 550°C
 - Bright red: 700°C
 - Then: orange, yellow and finally white (really hot !)
- The emission spectrum depends on the material

BLACK BODY

A material is constantly exchanging heat with its surrounding (to remain at a constant temperature)

- It absorbs and emits radiations

Problem: it can reflect incoming radiations, which makes a theoretical description more difficult (depends on the environment)

- A blackbody is a perfect absorber:
 - Incoming radiations is totally absorbed and none is reflected

Max Planck
1858-1947
Nobel Prize in Physics, 1918

So, little bit more on Blackbody Radiation because this was one of the first important points which gave rise to the idea of quantum mechanics; if this kind of radiation has been known since centuries, since whenever a material is radiated. This kind of radiation has been known for centuries since when a material is heated it radiates heat and its color depends on its temperature. For example, heating elements of a stove turned from dark red to bright red when the temperature goes from 550 degree centigrade to 700 degree centigrade. And it can even further go in to orange and yellow, and finally white when the temperature goes really hot.


So, depending on the temperature the radiation and its color changes, so the emission spectrum as we call it as a result of this change depends on the material heating as well as on the material property. Blackbody is defined as a material that is constantly exchanging heat with its surroundings to remain at a constant temperature. It observes and emits radiation. The problem is it can reflect incoming radiations which makes a theoretical description much more difficult depending on the environment and so on and so forth.

However, a blackbody is a perfect absorber incoming radiations are totally absorbed and none is reflected. So, that is the ideal definition of blackbody.

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Another Example of Blackbody

- Blackbody = a cavity, such as a metal box with a small hole drilled into it.
 - Incoming radiations entering the hole keep bouncing around inside the box with a negligible change of escaping again through the hole => Absorbed.
 - The hole is the perfect absorber, e.g. the blackbody Radiation emission does not depend on the material the box is made of => Universal in nature

A diagram illustrating a blackbody cavity. It shows a square box with a small hole on the right side. A red arrow points towards the hole from the left, representing incoming radiation. Inside the box, several blue lines represent radiation rays bouncing off the walls. One blue line is shown exiting the hole to the right, representing radiation emission.

Another example of blackbody can be something like a cavity which is someone shown here, such that a metal box with a small hold drilled in it the incoming radiation enters the hole keep bouncing around inside the box with negligible change of escaping again through the hole, that means it gets absorbed completely.

The hole is the perfect absorber for example, blackbody radiation emission does not depend on the material the box is made of; this is generally universal in nature. So, there are many ways blackbody and blackbody radiations can be designed and can be checked and it has been there for several. This concept has been around for very long.

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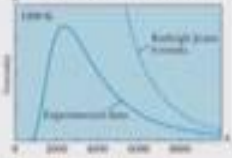
Classical physics:

Emission spectrum: a superposition of electromagnetic waves of different frequencies. Frequencies allowed: standing waves inside the cavity.

Equipartition of the energy: Every standing wave carries kT of energy. Flaw: when $\lambda \rightarrow 0$, the number of standing waves \uparrow , leading to $E \rightarrow \infty$.

Understanding the blackbody radiation spectrum

- Attempts to fit the low and high wavelength part of the spectrum.
- Using classical theory of electromagnetism and thermodynamics, Lord Rayleigh comes up with
$$j(\lambda, T) = \frac{2\pi^5 k^4 T^4}{15 \lambda^4} \text{ Rayleigh-Jeans formula}$$
- Major flaw at short wavelength ("Ultraviolet catastrophe")



Ultraviolet Catastrophe failure of classical theories.

The work of Rayleigh-Jeans was considered as state-of-the-art, using well tested theories, which were in very good agreement with experimental results in many other circumstances. Need for a new theory...

Deciphering the blackbody emission spectra: one of the outstanding problems at the beginning of the 20th century

However, the difficulty this principle faced and it was labeled as one of the outstanding problems at beginning of the 20th Century was to describe the blackbody emission spectra. It was difficult because classical physical principles could not quite explain what was going on here. So, when an attempt to fit the low and the high wave length part of this spectrum was tried simultaneously, the theory always failed. The theory worked quite well for the high temperature zone, but it always had a problem when it was trying to fit both low and high wave length part of the spectrum.

That is because as per classical physics emission spectrum is a super position of electromagnetic waves of different frequencies, and the frequencies allowed where the standing waves inside the cavity. Again the equipartition of energy would give rise to the fact that every standing wave carries kT of energy, Boltzmann constant and temperature T of the energy. The flaw is in this approach that when the wave length is approaching 0 the number of standing waves keeps on increasing leading to an energy which goes to infinity. And that is why almost all the laws sort of show the theory in terms of classical physics showed that the energy keeps on increasing as the wave length goes lower.

This came to be known as the ultra of highlight catastrophe, and that led to the failure of the classical theories. One of the most celebrated work in this the work of Rayleigh Jeans was considered as state of the art using well tested theories which have very good agreement with many experimental results in many other circumstances. And even in this

case wave length part was well explained only when the wavelengths went to lower numbers it failed. Thus, there was a need of a new theory.

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Planck's radiation law

- Planck assumed that the radiation in the cavity was emitted (and absorbed) by some sort of "oscillators" contained in the walls. He used Boltzmann's statistical methods to arrive at the following formula:

$$j(\lambda, T) = \frac{2\pi^5 k^4}{15} \frac{1}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$
- Planck made two modifications to the classical theory:
 - The oscillators (of electromagnetic origin) can only have certain discrete energies determined by $E = nh\nu$, where n is an integer, ν is the frequency, and h is called Planck's constant.

$h = 6.6261 \times 10^{-34} \text{ J.s}$
 - The oscillators can absorb or emit energy in discrete multiples of the fundamental quantum of energy given by $\Delta E = h\nu$.

$E = nh\nu$

So when Planck looked at this problem, he was also an expert in thermo dynamics, he assumed that the radiation of the cavity was emitted and observed by some sort of oscillators contained in the walls, and this concept is something which he introduced. He uses the Boltzmann statistical methods to arrive at the formula which is the Planck's radiation law.

Now, Planck made two modifications to the classical theory in coming to this form. The oscillators of electromagnetic origin can only have certain discrete energies determined by $E = nh\nu$; where n is an integer, ν is the frequency, and h is called the Planck's constant which was given by a number which was very small 6.6 times 10 to power minus 34, however with the units of joule second.

When there the oscillators can absorb or emit energy in discrete multiples of the fundamental quantum of energy given by $\Delta E = h\nu$. And that was one of the most important points of the Planck's radiation law which showed that energy was in packets.

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Quantization I

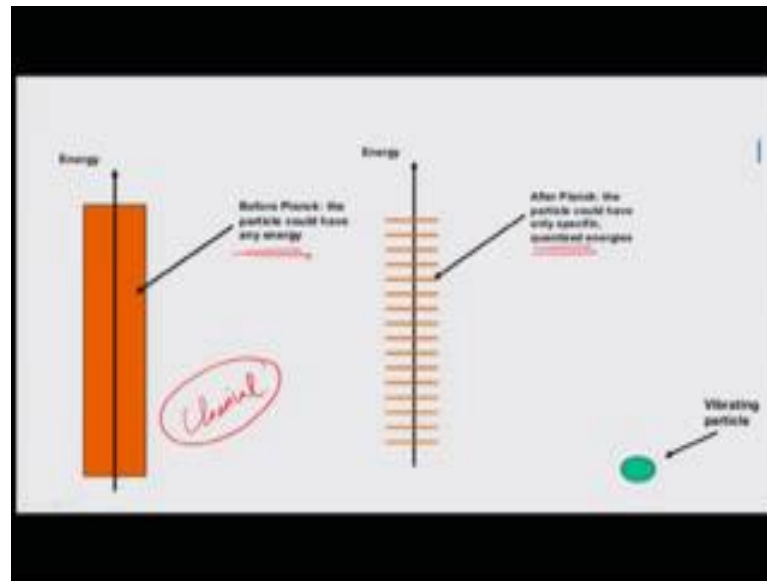
- Blackbody emission spectrum explained by introducing quantization of energy transfers, resolves the ultraviolet catastrophe
 - Low wavelength \Rightarrow high frequency ($\nu = c/\lambda$)
 - At small λ , the energy $E = h\nu$ needed to fill up the "oscillator" states increases. Their probability to be occupied decreases rapidly, e.g. faster than the rate found in the Rayleigh-Jeans formula so ultraviolet catastrophe.
- Very disputed
- Planck himself looked for a few years in ways to get $h \rightarrow 0$ without success

So, this gives rise to the principle of quantization. Blackboard emission spectrum explain by introducing quantization of energy transfers resolved the ultraviolet catastrophe. So, that was one of the most important contributions at that time in this filed by Max Planck. Even at low wave lengths and at high frequencies this particular theory worked. At small wave length the energy $h\nu$ needed to fill up the oscillator states the probability of the occupied oscillators decrease rapidly, that is faster than the rate found in the Rayleigh Jeans formula so no ultra violet catastrophe happened.

So, at small λ the energy $h\nu$ needs to fill up, the oscillators straights increase their probability to be occupied then decreases rapidly. For example, faster than the rate in the Rayleigh Jeans formula, so no ultra violet catastrophe happens. However, it became very highly disputed. Planck himself looked for a few years find whether h could be made to go to 0; the Planck constant without any success.

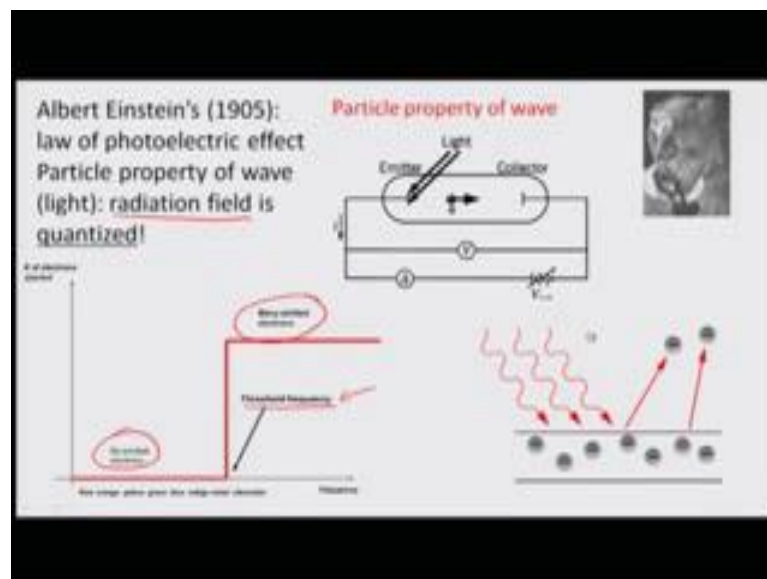
So, that was the basic principle behind quantization where even Planck who discovered it had difficulty in apprehended.

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So, as a result of Planck's law the energy which is the classical one continuous, any energy could be there became discrete. The particle could have only specific quantized energies and this was in conjunction with idea that there was some sort of oscillating vibrating particle inside this entire picture which was being shown to think in terms of blackbody radiation.

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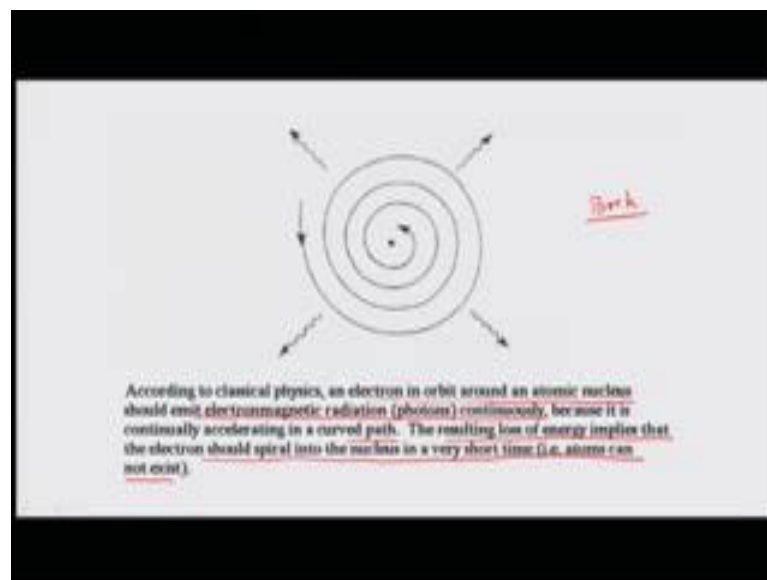


After this Einstein in 1905 found out that the same principle of quantization was necessary when he was studying the photoelectric effect. What he found that, his law

required that the radiation field be quantized; his observations were that, when the light fell on the emitter and photo electrons were being generated which showed as a potential in the circuit there was a threshold frequency until which there were no emitted electrons.

However, once a threshold was reached many emitted electrons came out. So, essentially there was a threshold in terms of the energy which was necessary for the electrons to come out showing that it had a particle property rather than a continuous property. So, this threshold frequency or the threshold energy gave rise to the idea of quantization again.

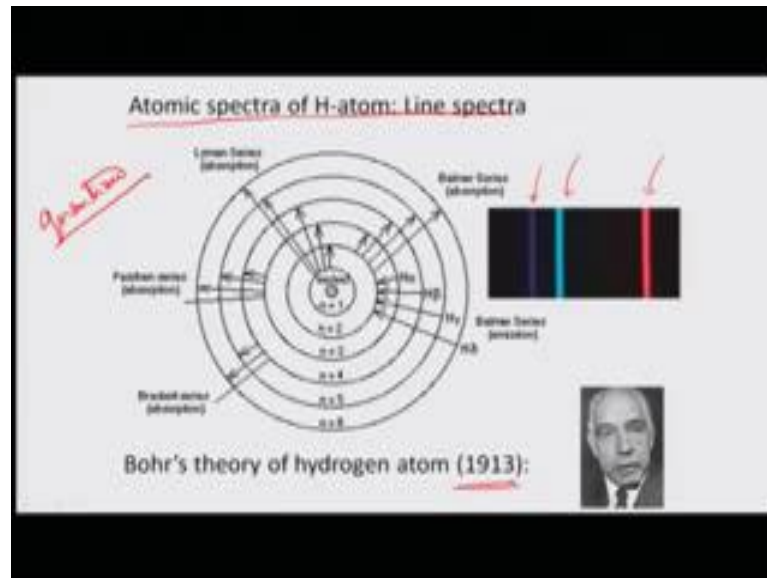
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This led to the other part which was creating a problem in terms of classical mechanics to look at an atom. In terms of classical mechanics an electron in orbit around an atomic nucleolus should emit electromagnetic radiation or photons continuously as per Maxwell equation, because it is continuously accelerating in curved part. The resulting laws in energy imply that the electron should spiral into the nucleolus in a very short time; that is an atom cannot exist.

So, this was one of the issues which the Bohr model was able to handle, because once this concept of classical nature of radiation being emitted at any energy was possible to be looked at differently, he was able to come up with the first quantized atom picture.

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And that immediately explained the atomic spectra of the hydrogen atom the line spectra and all that. So, his theory in 1913 was the one where these energies of these electrons around the nucleolus were considered as quantized so that the transition of an electron could only happen at discrete intervals, and so only discrete line spectra which was always observed was immediately explainable because of this principle. So, the atomic spectra of the hydrogen atom line spectra became understood from this picture introduced by Bohr.

And so that is the first Bohr's theory on the hydrogen atom in terms of quantized aspect which was given. So, will get more into quantum mechanics after this lecture in the next one because we have just come to the point where the first Bohr's model of quantization and the old quantum mechanics as we call it is justified by using the principle of packets or discrete energies which was obvious from the previous experiments and ideas which were necessary to explain this kind of observations that we are observing.

So, both Max Planck and Einstein quantization principle of very small energy packets which were given based on the Planck's constant were being utilized to give rise to this principle of quantum mechanics, more on this in the next lecture.