Mathematics for Chemistry
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Lecture - 1

Welcome to this course titled mathematics for chemistry.

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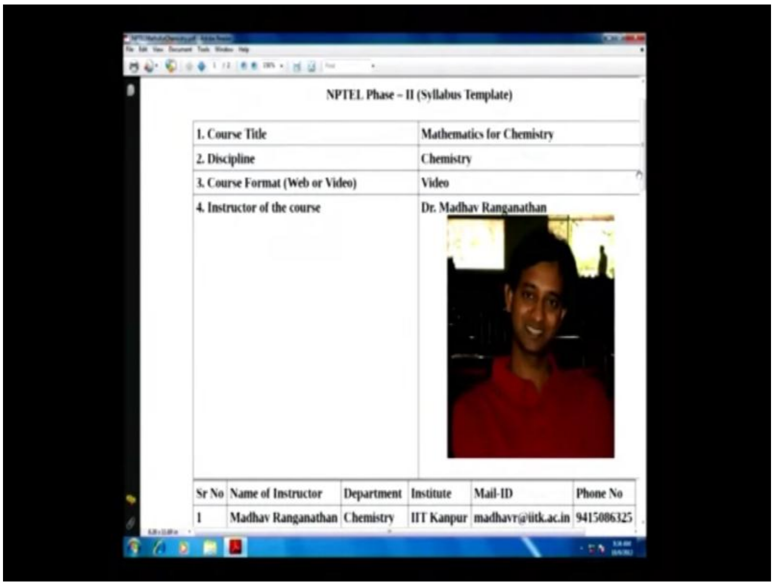

I am your instructor, Dr. Madhav Ranganathan and I am from IIT Kanpur. I will be going through this course consisting of 40 lectures. In a first, I will give a brief outline of the course and then, I will start going through the entire course. The title of this course is Mathematics for Chemistry and it is a typical course taught in the Chemistry Department. The format of this course will be video, I will be using primarily the black board behind me or you can call it a green board and all the lectures will be in video format. I am your instructor for this course and my name is Dr. Madhav Ranganathan, I am from IIT Kanpur and this is my mail and telephone number in case you need to talk to me.

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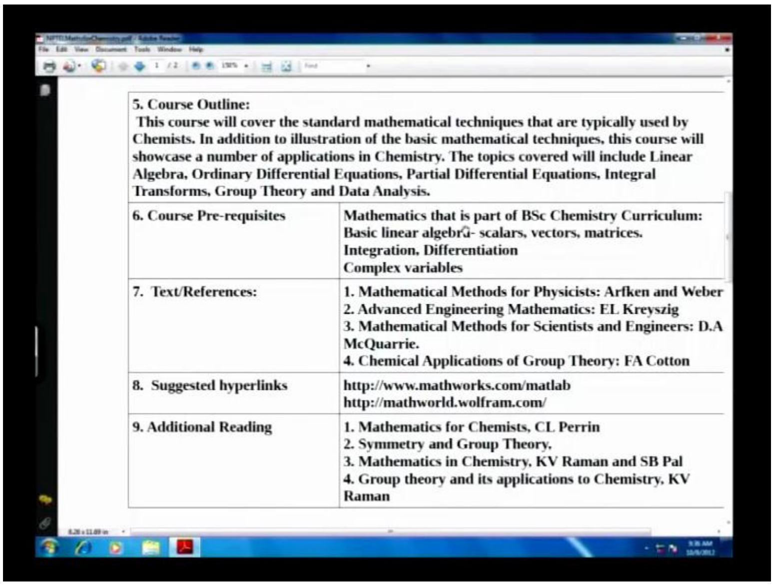

5. Course Outline:
This course will cover the standard mathematical techniques that are typically used by Chemists. In addition to illustration of the basic mathematical techniques, this course will showcase a number of applications in Chemistry. The topics covered will include Linear Algebra, Ordinary Differential Equations, Partial Differential Equations, Integral Transforms, Group Theory and Data Analysis.

6. Course Pre-requisites	Mathematics that is part of BSc Chemistry Curriculum: Basic linear algebra- scalars, vectors, matrices. Integration, Differentiation Complex variables
7. Text/References:	1. Mathematical Methods for Physicists: Arfken and Weber 2. Advanced Engineering Mathematics: EL Kreyszig 3. Mathematical Methods for Scientists and Engineers: D.A McQuarrie. 4. Chemical Applications of Group Theory: FA Cotton
8. Suggested hyperlinks	http://www.mathworks.com/matlab http://mathworld.wolfram.com/
9. Additional Reading	1. Mathematics for Chemists, CL Perrin 2. Symmetry and Group Theory, 3. Mathematics in Chemistry, KV Raman and SB Pal 4. Group theory and its applications to Chemistry, KV Raman

The course will cover all the standard mathematical techniques that are typically used by chemists. And in addition to doing the basic mathematical techniques, we will also cover lot of applications. The topics that will be covered they include linear algebra, differential equations of partial differential equations, integral transforms group theory and data analysis.

Now, it is very important that a student coming to this course has some basic pre-requisites and I will be assuming that you are familiar with all the Mathematics that is part of the standard B.Sc Chemistry Curriculum. And in particular, I will be assuming that you know some basic linear algebra, you know about scalars, vectors and matrices and some basic operations involving these objects. You will know techniques of integration and differentiation and you will have some familiarity with complex numbers.

So, this course is essentially an M.Sc level course. So, it is taught in M.Sc in most universities and IIT’s. So, I will be excepting that all the standard B.Sc Mathematics is familiar with you. There are some text books or reference books which I will be following and I would not be following a single text book throughout the course. I will be using various books, I will be taking from some internet source also and some other notes that I have prepared on my own.

But, there are some very good books in this topic and you are well advised to read one or more of them. There is a book called Mathematical Methods for Physicists by Arfken and Weber and it is a really exhaustive book with very nice applications in all areas particularly pertaining to physics but, also they overlap with some of the applications that we described. Then there is a book called Advance Engineering Mathematics by EL Kreyszig, this is also a very good book and it is one of the book that is used in the first year engineering studies. Then, there is book called Mathematical Methods for Scientists and Engineers this is by D. A. McQuarrie and McQuarrie has written lot of fuel lot of books in physical chemistry and so, lot of the applications that he talks about are related to applications in chemistry.

So, these are general books on mathematics on the mathematical area especially, linear algebra, differential equations and integral transforms. Then, there is a book that is particular to group theory, it is called Chemical Applications of group theory by FA Cotton. It is really the bible for group theory in Chemistry. So, we will be spending nearly about 15 lectures on group theory. So, this book is an excellent book to have, whether you are a physical chemist, organic chemist or inorganic chemist, this is this is one of the standard books on group theory. In addition to these books, there are lot sources on the internet and I am pointing out 2 hyper links. These are: The first one is mathworks dotcom matlab.

So, matlab is a common mathematical... It is a common programming language which also has lot of in built mathematical operations and the reason I put this is that, lot of the things that you will learn in this course, you can actually implement some of them using matlab and that will strengthen your understanding.

Similarly, there is another program called Mathematica and that is then. So, this mathworld dot wolfram dot com is the site of that program mathematica and both these websites have a lot of information on the actual theory behind the mathematics. So, both these websites are very good to learn mathematics, in addition to actually doing small calculations. One of the nice things in mathematica is you can do what is called symbolic mathematics; you can actually give a function and ask it to calculate derivatives or integrals and so, this proves to be very useful when you want learn new mathematical methods.

There are additional readings; there is Mathematics for Chemistry by CL Perrin. There is a book on Symmetry and Group theory by Carter. Then, there is Mathematics in Chemistry by KV Raman and SB Pal. There is Group Theory and its applications to Chemistry by KV Raman and again, I want to emphasize that these techniques have been around for many years. So, you might find new books coming up all the time, you might find internet resources that are very helpful and sometimes you find, you learn these things from very unexpected sources.

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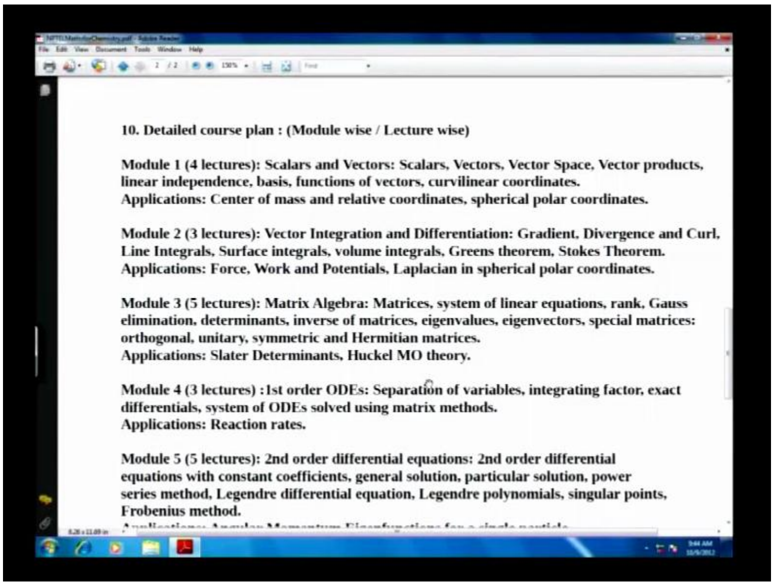

The detailed module wise plan is given below. So, these are the details of the contents that I will be covering. So, each lecture that I give will be approximately 50 minutes and 52 minutes to 1 hour and it will be in video format. So, at the beginning of each lecture, I tried to summarize some of the things that we have learnt in the previous lecture and I have divided this course into various modules. Actually, the whole course goes in one flow but, you can learn specific topics by going through specific modules. So, each module is in some sense self contained set of lectures.

Now, the first, second and third modules, they deal with linear algebra. The first module is it consists of 4 lectures, we will be talking about scalars and vectors and these are basics of vectors and I will introduce the abstract idea of a vector space. Then, we will talk about vector products, we will talk about linear independence basis, we will introduce functions of vectors and then, we will talk about curvilinear coordinates. I am introducing curvilinear coordinates in this topic because it is something that is used very widely in chemistry and this is a good place when we introduce functions of vectors, it is a nice place to introduce curvilinear coordinates. One application I will be doing is the use of center of mass and relative coordinates and I will be talking about spherical polar coordinates.

In module 2, which is 3 lectures, I will talk about advanced operations of vectors. We will be talking mostly about vector integration and differentiation. I will talk about three ways of taking derivates of vectors, gradient divergence in curve, we will talk about line integrals, surface integrals and volume integrals, greens theorem, stokes theorem and the applications that I will be dealing with are related to force, work and potentials.

So, there is a theory called potential theory which says how force can be calculated from the gradient of the potential and I will also talk about the Laplacian and spherical polar coordinates. So, the Laplacian is related to the second derivate and how what the expression for that in spherical polar coordinates and again this something that is seen a lot in advanced quantum chemistry courses.

In module 3, there will be 5 lectures. I will be talking about matrix algebra; I will be talking about matrices, system of linear equations, rank, gauss elimination, determinants, inverses of matrices, Eigen values, Eigen vectors. I will be taking about some special matrices like orthogonal, unitary, symmetric and Hermitian matrices. So, all the topics related to matrix algebra will be covered in these 5 lectures. I will be looking at applications, we will be looking at Slater Determinants, Huckel molecular orbital theory. So, the first 3 modules cover all the topics of linear algebra and this whole set of 12 lectures is complete set of lectures for somebody who wishes to learn all the techniques of linear algebra.

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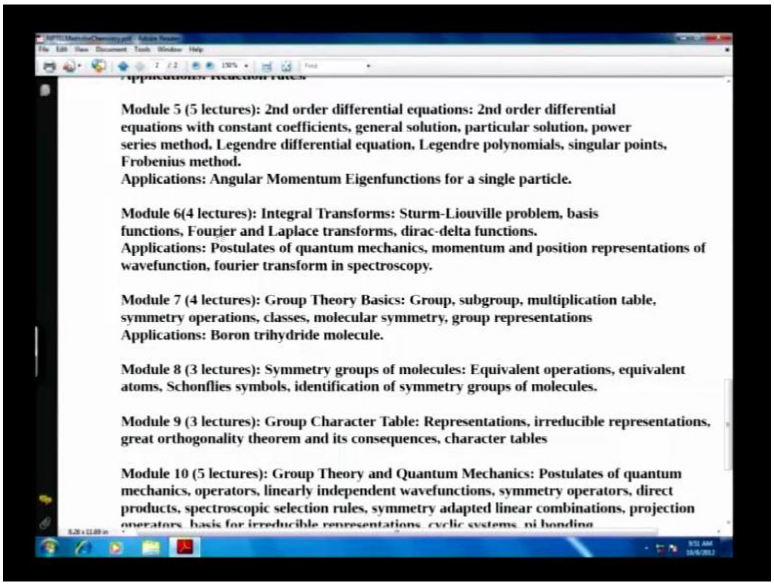

Then, we move to the next topic which is differential equation and so, modules 4 and 5, we will be mainly talking about differential equations. In module 4, I will have 3 lectures I will talk about first order differential equations, first order ODE stand for ordinary differential equations. I will be talking about separation of variables, integration factors, exact differentials, a system of ODE's solved using matrices methods and one application I will do is in reaction rate theory.

In module 5, there will be 5 lectures and this is the topic of second order differential equation and second order differential equations are extremely common and in chemistry particularly quantum mechanics and you need good techniques in order to solve them. So, we will be talking about second order differential equation with constant coefficients, general solution, particular solution, then, we will talk about the power series method, we will talk about an application of the power series method is in the Legendre differential equations and then we will talk about Legendre polynomials and then we will discuss about singular points and Frobenius method which is an advanced power series method.

So, this covers the set of techniques that are used to solve second order differential equation and again this is very widely used, especially in quantum mechanics. So, we will be talking about Angular Momentum Eigen functions of a single particle as an application of second order differential equation solution.

Then, module 6 is about Integral Transforms and this is a set of 4 lectures and I will be talking, I will first introduce the idea of orthogonal functions and this is the Sturm-Liouville problem and then I will be talking about bases functions and we will talk about Fourier and Laplace transforms and we will introduce dirac-delta functions.So, this set of topics is slightly more advanced but, an understanding of this topic will really make things a lot easier in quantum mechanics.

So, this topic in a sense connects differential equations and linear algebra in a very natural way and so, hence very nice way of looking at the Postulates of quantum mechanics and you can look at position and momentum representation of the wave function, we will look at the role of Fourier transformation and spectroscopy.

So, in these 12 lectures, I will be covering all the topics related to differential equations and then the next 15 lectures or so, we will be talking about group theory which is a completely different topic from linear algebra and differential equations and in fact you will need a different book in order to study group theory. So, module 7, 8, 9 and 10 are will be basically talking about group theory and its applications in chemistry.

Module 7 will consist of 4 lectures that will introduce the basics of group theory. We will talk about, what is a group? What is a sub-group? What is multiplication table? What is meant by symmetry operations? What are classes? Then, we will mention molecular symmetry and group representations and an application. We will see the Boron trihydride molecule. I want emphasis that in all of group theory, we will be constantly looking at molecules and seeing their symmetry. So, it is really not, we will be doing applications along the way and so, they would not be separately mentioned.

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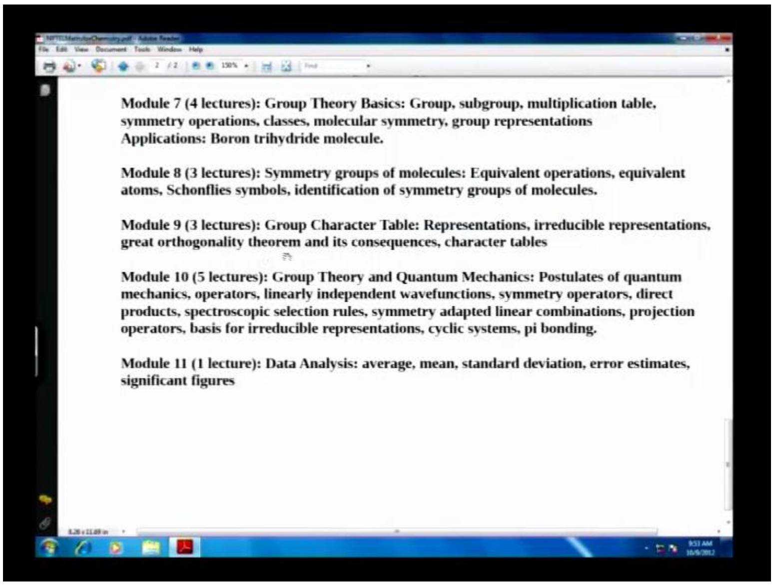

So, in module 7, we will talk about the basics of groups more from a mathematical point of view. In module 8, we will get into symmetric groups of molecules, we will talk about equivalent atoms, equivalent operations, the Schonflies symbols which is used to denote the symmetry group of molecules and we will mention how we identify the symmetry groups of molecules. So, suppose you had given a molecule with certain geometry, how do you indentify which symmetry group it belongs to? In module 9 which is 3 lectures, we will be going into the detail about of how we generate the group character table.

So, the character table of various groups is found in most standard physical chemistry books and what we want to do in module 9 is to understand, how you work out the character table. So, what is the meaning of each of those things and how you work in each one of them out?

So, in order to do that, we will start with representations. We will talk about irreducible representations, the great orthogonality theorem and its consequences and then, we will show how we can use that to generate the character tables. So, these 3 modules 7, 8 and 9 they together cover the basics of group theory and how group theory can be used to generate what is called the character table.

So, in a sense, the goal of these 10 lectures is to help you generate the character table. Now, the next 5 lectures are actually trying to use the character table for various applications. So, these set of 5 lectures, we talking about Group Theory and Quantum Mechanics and in this, we will talk about the Postulates of quantum mechanics, we will talk about operators, linearly independent wave functions, symmetry operators. So, all the symmetry operations that are characteristics of the character table will be seen as operators, we will talk about direct product, spectroscopic selection rules, symmetry adapted linear combinations, projection operators, basis for irreducible representations, and then as an application we will look at pi bonding.

So, in these 15 lectures from module 7, 8, 9 and 10, I will be covering the entire significant part of group theory and its application in chemistry. Definitely, atleast at a substantial level so that you should you will able to use it in applications. Finally, in module 11 is just one lecture which is for completeness, I am giving this one lecture, this is on Data Analysis and this is actually, it is extremely important for all scientist whether be it a theorist or an experimentalist and what I will talk about is when you make various measurements, you get various data and what do you do with that data. How do you do proper statists on that?

So, we will talking about average means, standard deviation, error estimates and I will touch up on significant figure because it is a topic that is not very well understood because I will just spend one lecture on this just and that will complete the set of 40 lectures that constitute this course and once again I want say that, these are divided into modules. So, if you want if you have specific topic that you want to learn, you can just follow the lectures of those modules, there will be some overlap when in the sense, I will be referring to content from one modules when I am discussing another module but, for most part each of the modules are fairly self content.

So, let us get started with this course and before I start the topics, I want to make a few general comments on why we have a course called Mathematics for Chemistry. Traditionally, mathematics has been a part of chemistry and in particular, in physical chemistry but, the kind of mathematics that is been used in traditional chemistry has been very intuitive and fairly basic mathematics.

So, for example, you need some basic theory of differential equations, when you formulate kinetic rate lose or you are doing thermal dynamics and then and so on and lot of the lot of the applications of mathematics in chemistry have or things that you just learn as a part of your regular mathematics, as part of your regular chemistry curriculum and traditionally mathematics has been very intuitive and you never needed a course called Mathematics of Chemistry but, things change significant after in the early 20th century and when they were two new disciplines that were added to traditional chemistry, the first was quantum mechanics and the second was statistical mechanics and these two disciplines become an integral part of chemistry and these three disciplines in particular are characterized by mathematical basis at the very foundation of the discipline.

So, for example, if you want to learn Quantum mechanics, you need to know linear algebra, you need to know differential equations, you need to know fairly advanced techniques of solving differential equations, you need to know how to do matrix operations. So, it is not something that you learn during your regular high school or under graduate curriculum, this is something that you have to learn in the particular, you have to learn techniques how you solve complicated differential equations and some of these techniques are fairy involved, as you will see during this course. So, for this reason, it is now been realized that chemists need to know mathematical techniques and for this reason we have a course called mathematics for chemistry and this is now a standard part of almost all universities.

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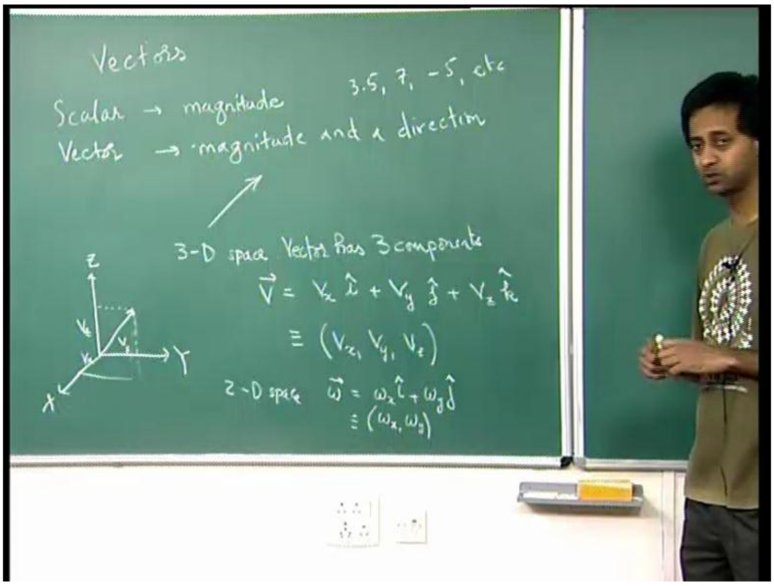

So, with this basic comment, I want move to the first topic for this course and that is vectors. So, will be talking about the vectors and now, you all have probably, many of you have heard about vectors, you have learnt that scalars are objects that have only a magnitude and vector are objects that have both magnitude and a direction.

So, the way you think of it as scalar as some number. So, scalar has only magnitude, where as vector is an object that has magnitude and a direction. So, a scalar can be a number like 3.5, 7, minus 5 etcetera. So, it is some number whereas a vector has both a magnitude and a direction and it is typically represented by arrows. So, you show arrows for a vector and typically, if you are dealing in a 3 dimensional vector space, then you say a vector has three components. So, you write a vector V in the form of which three components you can call it the x component of V times i plus the y component of V times j plus the z component of V times k.

So, you write vectors in terms of its components and what you have, these unit vectors, in each of the coordinate axis which tells you the direction of the overall vector. So then, what you do typically is you show a coordinate system where you will have the X axis, Y axis and Z axis and this vector V will have various components; this will be V z, this will be V y and you will have a component along V x, this is V x.

So, you show various components of the vector in this form and we say that the direction of the vector can be calculated if you know each of the components. So, there are two ways you can represent the vector, you can represent it as V x i plus V y j plus V z k or you write alternatively, you can write the same as V x comma V y comma V z and it is to be noted that V x, V y and V z are all scalars but, when you take a combination of them you get a vector an ordered set of them, you get a vector.

So, this is what you are probably familiar with how we understand vectors and what will see in this now, is that we want generalize this concept and there are many ways to generalize it and we can we can ask question, this is a vector in 3 dimension space, can you have a vector in 2 dimensional space? And the answer is yes, you can have a vector in 2 dimensional space. So, you can have a 2-D space and there you will have only 2 components. So for example, if you have w a vector in 2-D space, it will be have only 2 components. So, you can have w x i plus w i j or alternatively, you can write it as w x comma w i.

So, vectors can be either in 3 dimensional space or 2 dimensional space and the obvious question that arises is, does it have to be 2 or 3-D, can it be 4-D, 5-D,6-D and the answer is yes, there is nothing that says that you should have only 3 dimension you can have 4, 5, 6, you can have vectors in arbitrary dimensions and so, we are left with the idea that vectors can be 2-D, 3-D, 4-D and so on. A vector in one dimensional space is nothing but, scalar if there is 1 dimension, then you have only 1 component and it is just a scalar.

So, vectors can exist in all these dimensions. Now, this way of thinking of vectors turns out to be useful but, it is a little restrictive and what we want to do is to give the formal definition of a vector as an abstract object and once we do that then will see that it immediately gives many more objects that are also vectors. So, in order to motivate this, you ask a question, you ask yourself a question. Can I take a vector in 3-D space and add another vector to it? So, if I take a vector in 3 d space and another vector in 3 d space I will get another vector in 3-D space right. So, if you add these 2 vectors you get another vector.

Now, the question is if I take a vector in 3-D space and add a vector in 2-D space. Can I do that? And the answer is obviously, no you cannot do that because they are in different spaces and you cannot add them. So, this motivates us to give to write the axioms of vectors space.

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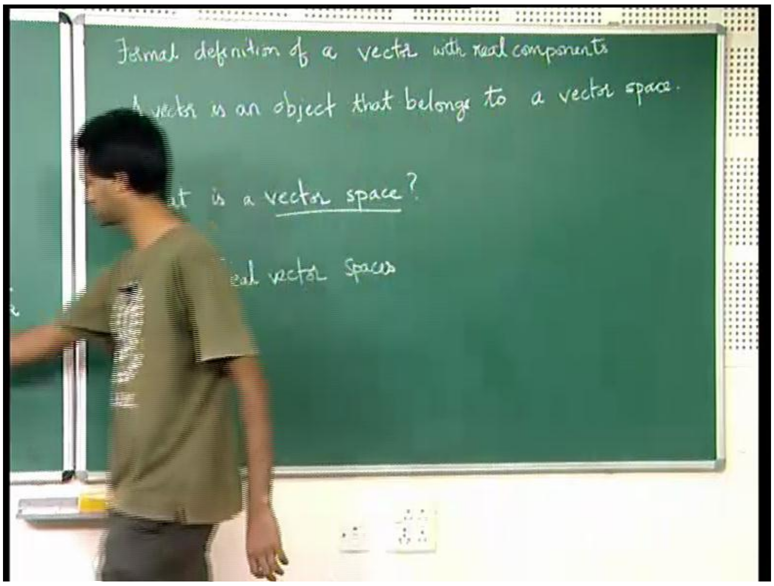

So, we want to give a formal definition of a vector. For initially, I will be focusing on vector with real components so, what is meant is that the components of these vectors V x, V y, V z etcetera are all real numbers. So, the formal definition of a vector with real components and the advantage of doing this is a definition that will hold for vectors in all spaces, it is a definition that will hold for all types of vectors also.

So, the formal definition is, a vector is an object that belongs to a vector space. So, a vector is an object that belongs to a vector space and clearly this formal definition immediately leads us to another question that is, what is a vector space. So, what is a vector space? A vector space is a space for example, the 3-D space what we talk about when we mention 3-D space we implicitly mention this 3-D space is actually a vector space. So, each object, each vector belongs to this 3-D vector space. So, now we want to come up with a formal definition of these vector space and in order to do this, we want to give a very very general definition that will hold for all kinds of vectors and it would not be restricted to 2 or 3 dimensions, you can use it for any dimensions and what will see is, we will try to find out, we will try to give some basic properties of vectors and if vector satisfy those object then, if any object stratifies those basic properties then it is a vector that belongs to a vector space.

I will be restricting to what are called as real vector spaces and in the real vector space, each of these components are said are real but, you can easily extended to vector spaces consisting of complex numbers. So, let us talk about the definition of a real vector space. So, now I am going to introduce the axioms of a real vector space and this is going sound a little abstract and it is abstract. So, we are going to generalize it. So, we are going to give a very formal definition and this definition might initially seem a little puzzling but, over time as you use it, you will become familiar with it, you will realize the advantage of having such a general definition.

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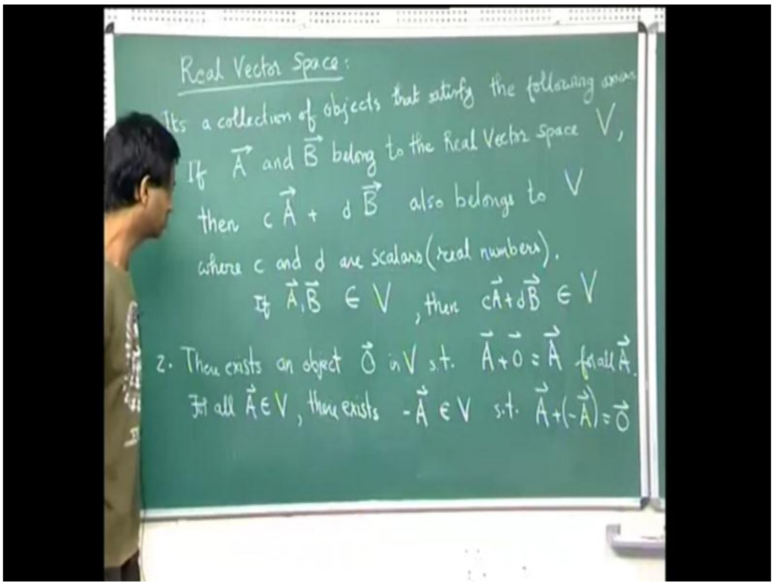

So, a real vector space, it is a collection of objects and later on, we will see that these objects are nothing but, vectors. So, it is a collection of vectors that satisfy the following axioms. So, the axioms they satisfy are: first if A and B belong to the real vector space V, it should be V then, I will c A plus d B also belongs to V, where c and d are scalars or real numbers. So, let me go thought this again, if A and B I have denoted them by arrow. So, A and B are objects that belong to the vector space and later on we will cal this object vectors. So, if A and B belong to this real vector space V then, any linear combination of A and B also belongs to V. So, you take any 2 scalars c and d and you multiply A by c and d by B then this object also belongs to V. We will use a short notation for this, we will say that if A comma B are belong to V, then c A plus d B also belongs to V.

So, this is the short notation for this long statement and we will be using short notations from now on. So, this is one axiom you can easily verify this for the 3-D space that you are familiar with. So, if you take any two vectors, you multiply 1 vector by a number multiply another vector by another number and you add them with up you always get another vector. So, this is the first axiom the second axiom, there exists an object which we will denote as a 0 vector in V such that A plus 0 equal to A for all A, also for all A belonging to V there exists minus A belong to V such that A plus minus A equal to 0.

So, the second axiom says that there exists an object which we call the 0 vector. So, 0 is a vector which is also belonging to the vector space such that you add this 0 vector to any other vector, you will get the same vector and for all the vectors belonging to V, there exists another vector called minus A vector which you add to a vector you will get 0.

So, in other words you can take any vector, you can always find a vector such that when you add those 2 vectors you will get the 0 vectors. So, this is the second axiom. The third axiom actually, this is a set of axioms, you know in different books, sometimes books break up these axioms into various axioms but, I will just show this as 3 axiom, the third axiom is a set of axioms that have to do with properties of this addition and multiplication.

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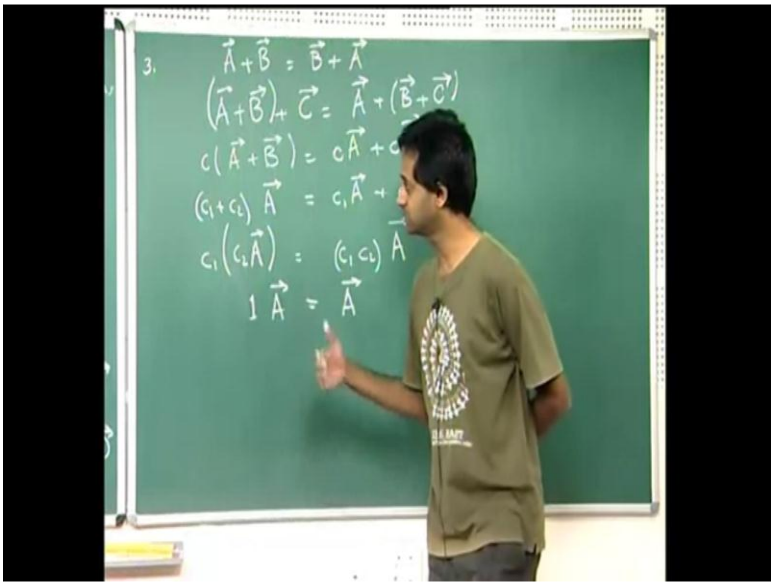

So, these have to do with commutativity, associativity and distributivity. So, I will just write them down and they are fairly obvious.So, suppose you take A plus B, that is equal to B plus A. So, in other words addition of 2 vectors is commutative. Then if you take A plus B and you add C to it, this is a same as taking A and adding B plus C. Then, you take a scalar and multiply it to A plus B; this is same as C A plus C B. Then, you can take C 1 plus C 2 added to A, you will get C 1 A plus C 2 A.

Finally, you can take C 1 into C 2 A which is same as C 1 C 2 into A. Finally, you take a number 1, multiply it by A you get back A. So, 1 is a scalar 1. So, these are the other axiom of a vector space and these are also fairly obvious and they are satisfied by most vectors spaces. So, these are the other axioms of a real vector space. Notice, that the space is actually the collection of all vectors. So, if you take, if you add two vectors you will get a third vector and if you multiply by different scalars, you get all possible vectors. So, it is the space really refers to the set of all possible vectors. So, that is what is the vector space and that is how we understand the 3-D vector space, we think of all possible vectors and combination of all of them forms the Vector space. These axioms can be understood in fairly in a very intuitive manner.

So, it looks a little abstract but, really all that we are saying is that the first axiom is saying that you take 2 vectors, you add them, you will get a third vector, you multiply a vector by a scalar you will get another vector. So, basically these axiom is saying that the vector space is closed to addition and scalar multiplication and this in some sense, this is the most important axiom of the vector space that you add 2 vectors you get another vectors, you multiply it by scalar you get another vector and this is nothing but, a combination of those two conditions. Then, axiom 2 talks about the 0 vector which should also be part of the vector space and the inverse or the negative vector. So, corresponding to every vector, there is a negative of that vector such that you add them and you get 0.

The third axiom or the set of axiom refers to the properties of the addition so, addition should be commutatively, it should be distributive, then the scalar multiplication should be distributive and similarly, when you add 2 scalars and multiply it to a vector is same as multiplying by each of the scalars and adding 2 vectors and similarly, scalar multiplication is also distributive and there is a scalar called 1, 1 is a scalar, it is not a vector, 1 is a scalar that you multiply by any vector you get the same vector.

So, these are in some senses are very imitative and very obvious. So, really the crucial part of definition of the vector space is that it should be close to addition and scalar multiplication. Now, what we do next is to look at various examples of vector spaces and we will immediately see that the example that we consider, they satisfy all these axioms.

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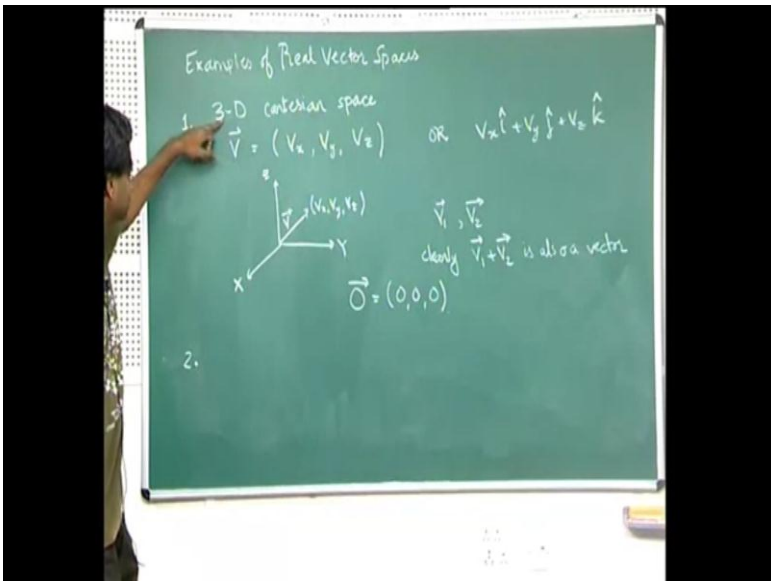

So, now let us look at some common examples of vector spaces. So, the first example is the usual 3-D Cartesian space. So, actually it is a 3-D space represented by Cartesian coordinates. So, any vector is so, the vector V has 3 components V x, V y and V z. So, it can be expressed as these 3 components or alternatively it can be written as V x i plus V y j plus V z k, these are essentially the same vectors but, you just write it in different symbols and you show it as an arrow going from the origin to that point. So, as we had seen before x x, we have 3 axis we call X, Y and Z and you have an arrow this V x, V y, V z this is a vector V and these are the components along each of the axis. So, this is an example of a 3 dimensional vector space and it is very easy to see that if you take 2 vectors and you add them you will get another vector. Similarly, if you multiply a vector by a scalar you will get another vector. So, for example, if you add 2 vectors V 1 and V 2 and clearly V 1 plus V 2 is also a vector. Similarly, the 0 vector is nothing but, 0 0 0, it is a vector of length 0 and that is a member of the space and similarly, for any vector you can define the negative of that vector.

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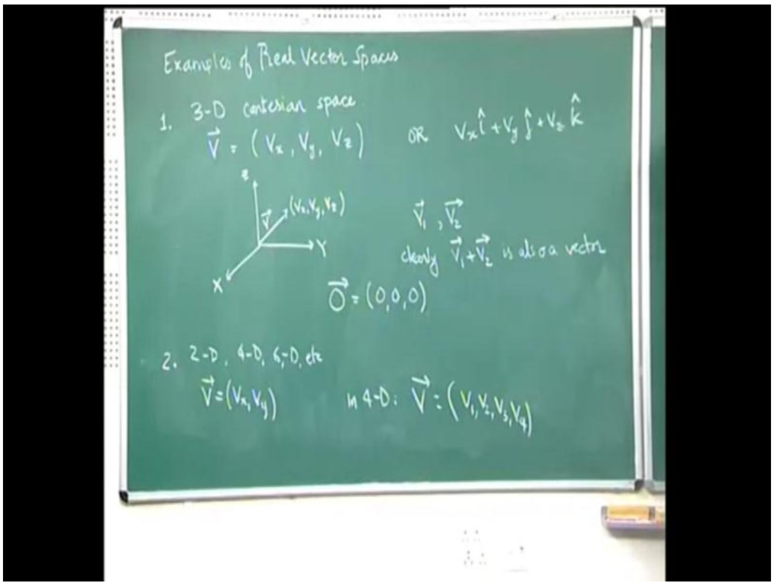

So, clearly this satisfies all the axioms of a vector space. So, the set of all possible vectors forms a vector space second example, is that we said it is 3-D Cartesian space. So, there is nothing special about 3 so, you can have 2-D or you can have 4-D, 6-D etcetera. You can have any dimensions 7-D, 8-D. So, for example, in 2-D space you have vector V would have only 2 components V x, V y where as vector in 4 dimensional space in 4-D this vector would have 4 components, just for convenience I will call it V 1, V 2, V 3, V 4 and so on and so, you can write vector space in any number of dimensions, remember all these components are scalars and this is just a way of writing the vector. So, vector is an object in this 4 dimensional space and it is typically written in this form and you can easily show that these also satisfy the axiom of a vector space.

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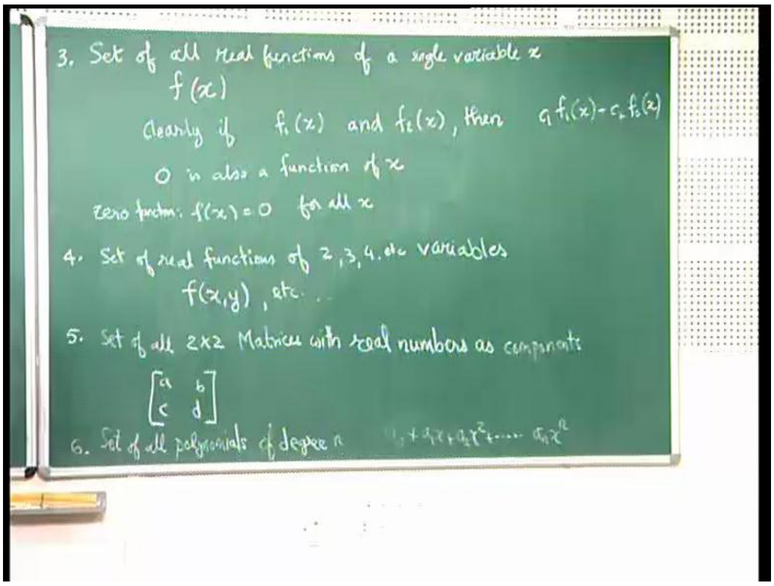

The third example, that we are going to talk about is set of all, let us say real function, set of all real functions of a single variable x. So, suppose you have a variable x, then a function of x is denoted by f of x and f of x and if you take the set of all possible functions, any possible function of a single variable then that will form a vector space. So, you can easily show that by showing that clearly if f 1 is a function of x and f 2 is a function of x then c 1 f 1 of xplus c 2 f 2 of x is another function of x. So, f 1 and f 2 if these are functions of x then, clearly c 1 f 1 plus c 2 f 2 is another function of x and you can take any 2 functions, you can take, you can construct such combination of them and you will get another function.

Similarly, 0 is also a function of x and what does function does is, you can have a function f of x is equal to 0 for all x. So, this is the 0 function, just to remind you function you should think of function as some sort of machine, you give a value of x and out you will get a number and the 0 function basically, you give any value of x you will return 0. So, this is also a function and then similarly, you can define the inverse and you can show that it satisfies all the additive, associativity and distributive properties and now.

So, here is an example of a very different set of objects that also forms a vector space, see you are used to and this sort of explains why we have to have a general definition of a vector space, if you had just thought of arrows and pointing in some direction the magnitude and a direction then we would not be able to identify the set of all functions as a vector space but, that the axioms of vector space allow us to treat this set of all functions also as a Vector space.

Now, you can extend this, you can say set of real functions of 2 variables 2, 3, 4 etcetera variables. So, for example, f of x y the set of all possible functions of 2 variables that also forms a vector space and so on. So, again this is another example of a vector space which is more general than the conventional idea of vectors as objects having magnitude and a direction.

So, we can immediately see the connection between the vectors and you know how to think of functions as vectors in some sense. So, let us look at one more example, this is set of all just for a start, I will start with 2 by 2 matrices, with real numbers as components. So, set of all 2 by 2 matrices with real numbers as their components or elements so for example, you can have a matrix a b c d.

Now, if you consider the set of all possible matrices, all possible 2 by 2 matrices that will also form a vector space because you add any 2 matrices you will get another matrix, you can always define a 0 matrix and so on. And again there is nothing special about 2 by 2, you can consider 3 by 3, 4 by 4 but, you take the same, you keep the same dimension then the set of all matrices of particular dimensions will form a Vector space.

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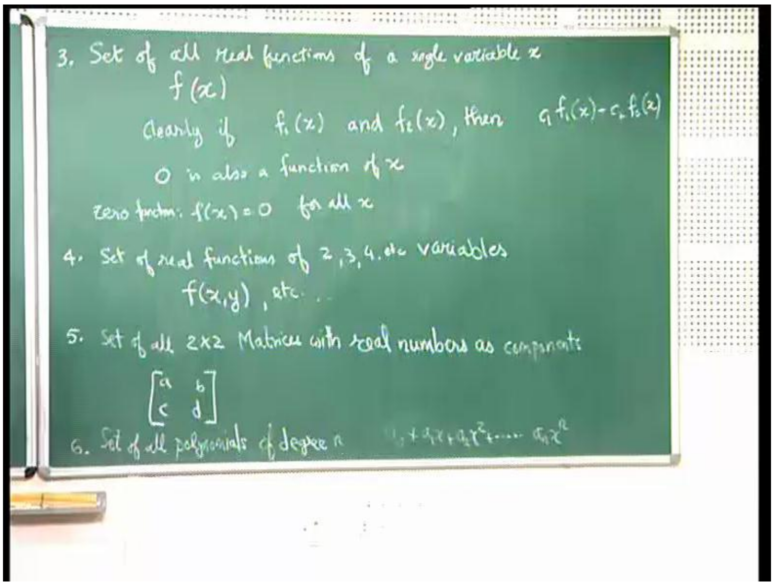

The last, I will give you one more example and then we will stop for today, this is the set of all polynomials of degree n, where n can be 2, 3, 4, 5 anything. So, a polynomial of degree n is typically written as a 0 plus a 1 x plusa 2 x square plus a n x raised to n. So, polynomials of degree n means, you can go only upto n you do not have any terms that are greater than n. Now, if you take the set of all possible such polynomials, they will also form a vector space. So, if you take any 2 polynomials and you add them, you will get another polynomial and so on. And n can be any number, you can start at 1, 2, 3 anywhere, polynomial of degree 0 is a scalar. So, that is typically not called a vector but, from degree 1 onwards you can take any number and you will get a Vector space.

So, these are some examples of vector spaces and what I hope I have convinced you that is that you know generalizing the definition of vectors and using a rather abstract definition, we are able to bring a lot more objects that we do not usually think of as vectors into this same description. Now, notice that the axioms of a vector space do not say anything about multiplication of vectors. So, indeed in order to define something as a vector space, you do not need to define multiplication, you did not need to say what happens when you multiply 2 vectors and what you know from your vector analysis is that there are many ways to take products of vectors, there is a dot product or the cross product or triple products and so on. So, there is no unique definition of the product of vectors and so, this actually have is what we will start discussing in the next class. We will look at different ways to look at products of vectors.