Nuclear structure and stability

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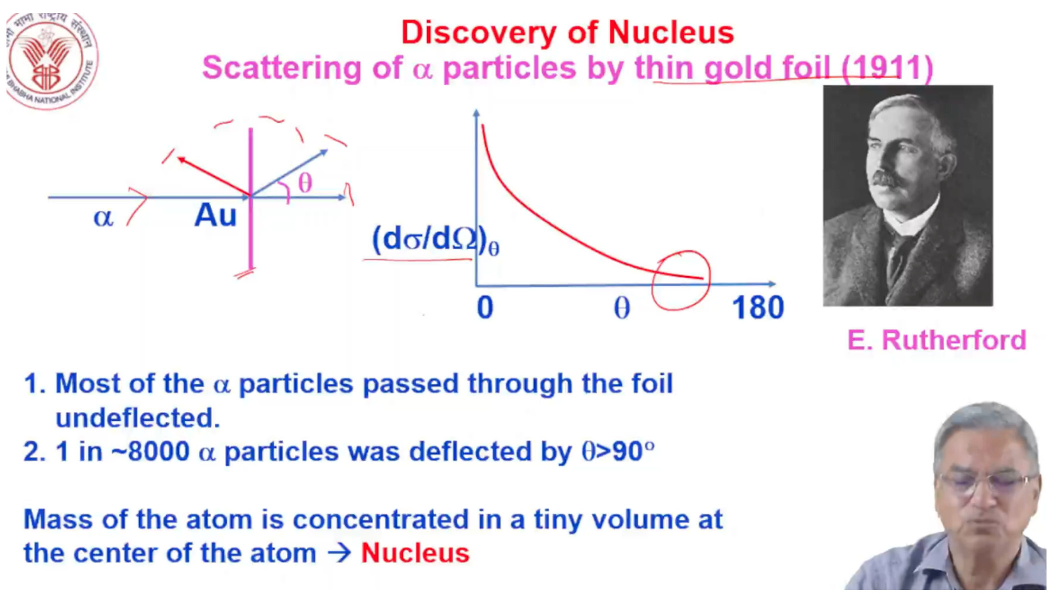
Lecture-3, Module-1

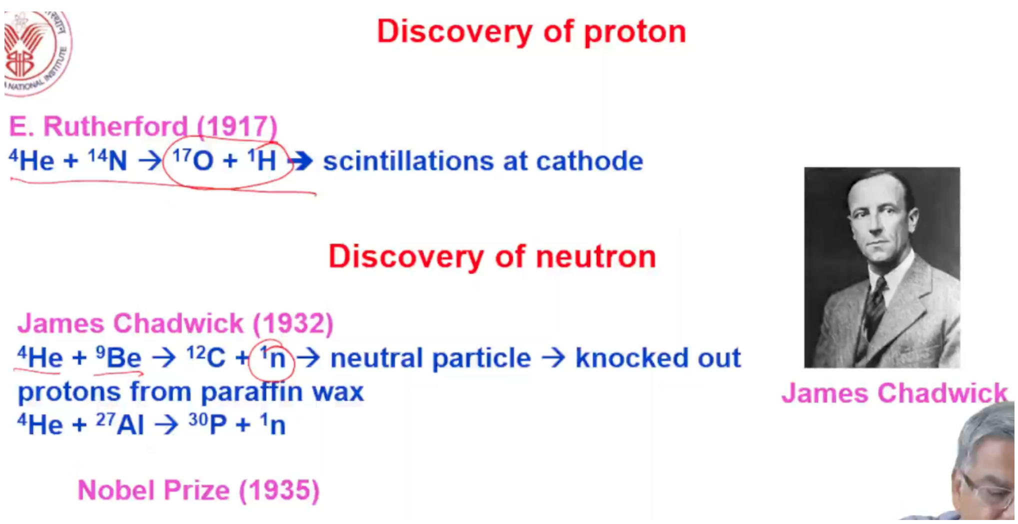
Hello everyone. In the last two lectures, we discussed about radioactive decay and the relationship between the activity of daughter atoms in terms of the initial activity of parent radioisotope and also some of the equilibria that are established depending upon the half life of the parent and daughter. In today's lecture, we will discuss the structure of nucleus and what are the factors that govern the stability of the nucleus. So, it will involve bit of the early history of nuclear science. Let us see right from the early 20th century, how the different discoveries contributed to understanding of the structure of matter. We discussed the radioactivity discovery in 1896 followed by the discovery of radium and polonium and then Ernest Rutherford came into picture and contributed in many ways in understanding the nuclear structure.



In fact, the experiment of Rutherford involving the scattering of alpha particles by the thin gold foil in 1911 was a milestone discovery which ultimately led to the idea about the structure of atoms. So, what Rutherford did was alpha particles from a polonium isotope in vacuum were bombarded onto a very thin gold foil and then he put detectors at different angles with respect to the initial alpha beam direction and measured how the alpha particles are scattered at different angles. So, the angular distribution $d\sigma/d\Omega$, Ω is the solid angle, as a function of angle was found to have this type of behavior. What it says is that most of the alpha particles are passing through the foil undeflected.

You can see most of the alpha particles are concentrated at $\theta = 0$. So, majority of the alpha particles go in the forward direction. But there are a very few of them which are also backscattered close to 180 degree. And that is very, very small number, one in about 8000 alpha particles were backscattered at angle more than 90 degree. This led Rutherford to conclude, when the alpha particle has to be backscattered, it has to undergo a head-on collision with a massive body.

And since the backscattering is happening at a very small numbers, he concluded that the mass of the atom is concentrated in a very small volume and that small volume he called as the nucleus. So, the typical dimensions of the nucleus vis-a-vis that of the atom are that if the atom is of the order of the football field, then the nucleus is of the order of the football. So, that is the time order of magnitude difference between the atom and the nucleus. This was a very, very important discovery and I will come to this further. So, just before I discuss this backscattering further, what are the implications of that, I will also touch upon the discovery of proton and neutron, the constituent units of the nucleus.



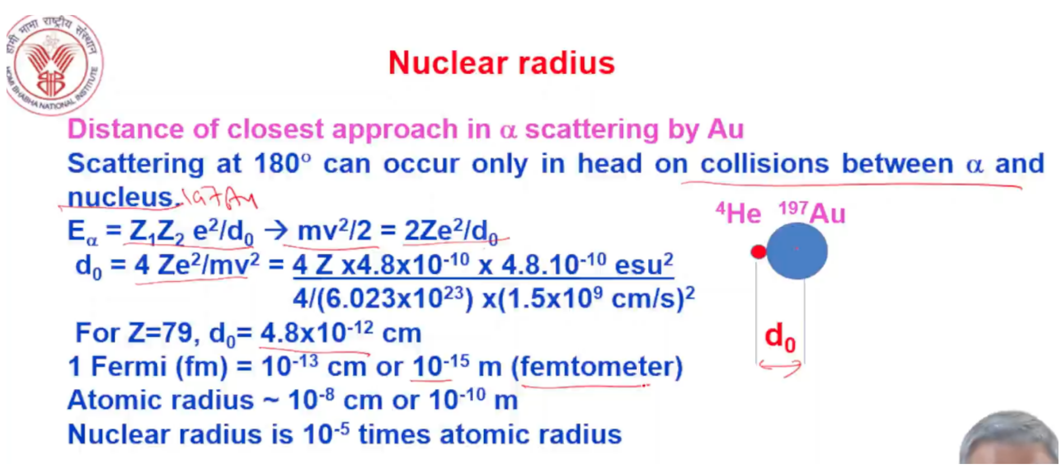
In fact, it was again Rutherford in 1917 at Manchester who studied the first nuclear reaction. The alpha particles from polonium source were made to bombard. In fact, when they are passing through air, what he observed that there was a scintillation produced at cathode. So, that means the bombardment of alpha on the air, the air is dominantly containing nitrogen gives rise to proton. This positive particle he identified as singly positively charged particles having the mass as of the order of hydrogen atom.

And therefore, he concluded that these are the protons which are produced in the reaction of alpha particle with nitrogen-14. And this was the first nuclear reaction and you see here the products are not radioactive. So, this first reaction produced isotopes which were stable. So, that is how the proton was discovered. Though it was proposed that there is positive charge in the nucleus, the experimental evidence for this positive charge as proton came in this experiment.

Subsequently, in 1932, James Chadwick, again a student of Rutherford, discovered neutron wherein he bombarded beryllium9 with alpha particles from again that alpha source, polonium, radium. And there was an observation that there are some neutral particles or radiation. There was some confusion about this. In fact, Irene Curie and Frederick Joliot also observed what they called as gamma ray photons. So, there are some neutral particles being emitted, but it was not established whether they are photons or particles.

Then James Chadwick unambiguously confirmed that these neutral particles knocked out protons from paraffin wax, which is not possible using the gamma ray photons. So, he concluded that this reaction produces neutrons. So, that is how the neutrons were discovered. Incidentally, Irene Curie and Frederick Joliot also had found out neutrons, but they did not have the unambiguous proof. And in fact, when James Chadwick informed Rutherford that this group also are doing similar experiments and when it came to Nobel Prize, Rutherford proposed James Chadwick for Nobel Prize and he received the one and he told that this pair are smart enough to claim Nobel Prize for some other discovery.

And in 1934, Irene Curie and Frederick Joliot were awarded Nobel Prize for the same reaction, but for the discovery of artificial radioactivity. That are the kind of intense activities going on in the field of nuclear science in this early 20th century. So, I'll come back to the experiment by Rutherford, wherein he found out that the mass of the atom is confined in a very small volume at the center of the atom that is called a nucleus. And this very experiment, scattering experiment led him to find out the size of the nucleus.



And for that, he used the concept of the closest approach, the distance of closest approach, how close an alpha particle can come to the gold nucleus.

So, that he used this concept of head-on collision between alpha and gold nucleus, ^{197}Au nucleus and the Coulomb energy. So the alpha particle is going with a kinetic energy,

$$E = \frac{1}{2}mv^2$$

When it is in touch with the nucleus, all the energy becomes potential energy. So, the kinetic energy and potential energy are same. Potential energy is given by

$$PE = \frac{Z_1 Z_2 e^2}{d}$$

where D is the closest approach distance.

So, distance between the centers of the two nuclei that is $equal\ to\ \frac{1}{2}mv^2$. So, alpha particle Z =2. So, you can write

$\frac{1}{2}mv^2 = \frac{2Ze^2}{d_0}$.

What is $d_\circ$? d is the distance of the closest approach. the center to center distance.

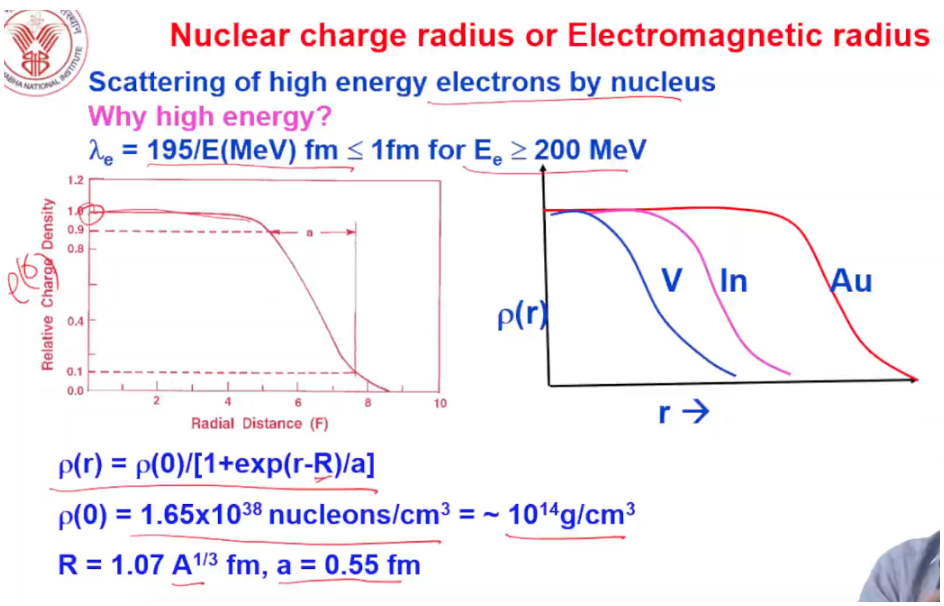
And if you neglect the radius of alpha particle with respect to the gold nucleus, then you can say d is the radius of ^{197}Au. So, that is how we found out that

$$d_0 = \frac{4Ze^2}{mv^2}$$

from this formula. And we substitute the value of Z = 79, the charge of electron in electrostatic units is 4.8×10^{-10} esu and the mass of the alpha particle 4/ Avogadro number and the velocity of alpha particle is 1.5×10^9 of that order.

So, if you calculate this value of the d, you get 4.8×10^{-12} cm. So, that is how you got the idea that the radius of the nucleus is of the order of 10^{-12} centimeter. And to denote, to represent the radii of the nuclei in unit 1 Fermi has been defined that 10^{-13} centimeter. So, the radius of nucleus is in the order of Fermi's, which is also called as 10^{-15} meter or femtometer.

So, you can now compare the radius of an atom 10^{-8} centimeter or 10^{-10} meter, whereas the radius of the nucleus is of the order of 10^{-15} meter. So, the nuclear radius is roughly 10^{-5} times smaller than the atomic radius. Now, let us see actually whatever radius that we get from different experiments, there is a difference.



And so, there have been later on very refined experiments to find out the radius of the nucleus. One of the methodologies is to study the scattering of high energy electrons by nucleus.

And this gives you what is called as the nuclear charge radius. The question is why we need a very high energy electron to study the radius of nucleus. Electrons will interact with the nucleus by electromagnetic interaction or it is a coulombic interaction. So, if you want the electron to probe the inner dimension of the nucleus, the wavelength of the electron has to be less than the dimension of the nucleus. And therefore, you can use the de Broglie wavelength to find out what is the kind of energy the electrons will have to probe the structure of nucleus.

So, the wavelength of electron can be given by

$$\lambda_e = \frac{195}{E}$$

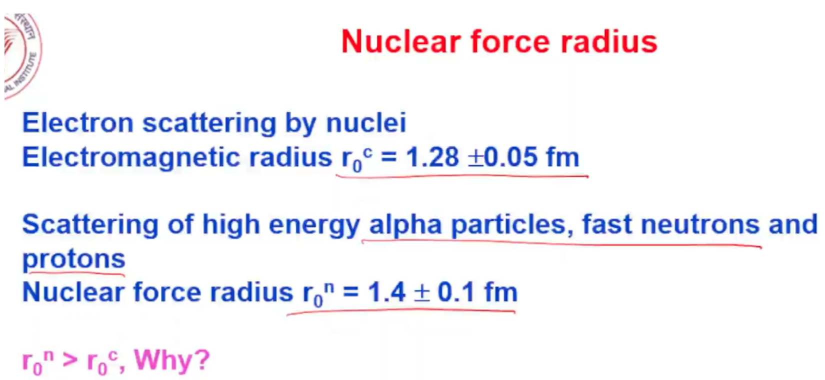
where E is in MeV and it becomes in Fermis. So, if wavelength of electron has to be less than 1 Fermi then the energy of electron has to be more than or equal to 200 MeV. So, that is how 100s of MeV electrons were bombarded onto different materials and the scattering functions were obtained. So, the net result of that was the distribution of charge in the nucleus. So, electron is only seeing the protons inside the nucleus.

It will not interact with the neutrons. So, the electrons are scattered from the protons inside the nucleus and so whatever radius you get is called the charge radius or electromagnetic radius of the nucleus. So, this data on the left hand side gives you as a function of distance from the new center of the nucleus how the charge density ρ is changing with the distance from the center of the nucleus. And you can see here that it is relative actually. So, it is flat up to certain distance and then sharply falls down following this relationship.

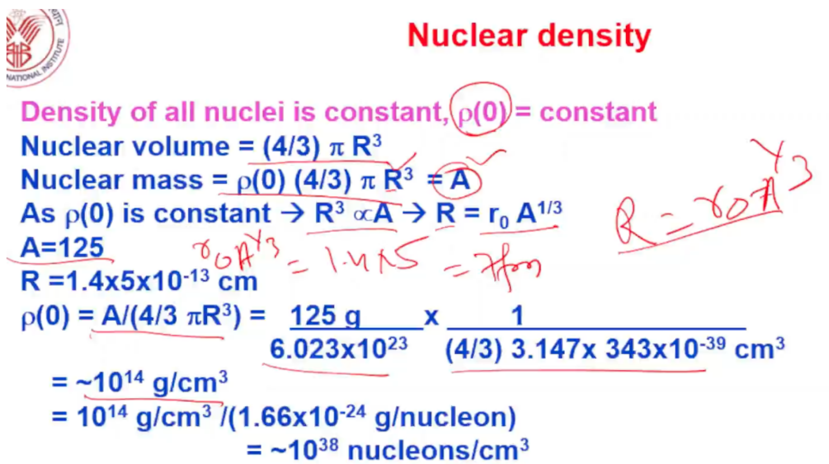
$$\rho(r) = \rho(0)/[1 + e^{\frac{(r-R)}{a}}$$

Where ρ(r), the charge density at any distance r, ρ(0), the central density R is the radius of the nucleus, a is the diffuseness of the nuclear charge distribution. And the central density at the center, density of the nuclear matter was of the order of 10^{38} nucleons cm^{-3} which comes to about 10^{14} grams per cc. And from this data also it was found out that the radius of the nucleus scales with the $A^{1/3}$ A is the mass number of the nucleus. And the skin thickness or the diffuseness of the nuclear charge density is 0.55 fm. So, this was very important observation which described how the nucleons are distributed in the nucleus. So, it is like you know you have a hard sphere but there is a diffuseness at the surface. And another important observation was that the central density was constant irrespective of the different materials that were studied. So, for example, vanadium, indium, gold, the nuclear density is same, only the radius will change. So, this is the radius of the nucleus for different nuclei and the diffuseness as you can see this is the diffuseness, but the central density is constant.

So, this essentially gave an idea that nuclear matter, first of all has very high density, so that is it is incompressible. And second is that all nuclei have same density in the central portion. So, these are the observations about the nuclear size. In fact, so apart from electrons, you can also study the scattering of high-energy alpha particles fast neutrons and protons and now these particles will interact with the nucleus by nuclear force. So, using the scattering of these particles, whatever value you get, that is called the nuclear force radius and whatever you get using electron scattering, you get the electromagnetic radius.



So, electromagnetic radius or the Coulombic radius is smaller than that of the nuclear force radius. Why it is so? Why the nuclear force radius is large than Coulombic radius? Because the proton distribution inside the nucleus, the nucleus has got lot of neutrons and so the neutron distribution is much wider than the proton distribution. And so in fact, there is something called neutron skin thickness, the outer periphery of the nucleus will have more of neutron. So, neutron distribution extends to larger radii compared to proton distribution and that is why the nuclear force radius is more than the electromagnetic radius. So, in the calculations whenever subsequently we will be using the nuclear force radius r_0 is1.4 fm.



Another important point we discussed is nuclear density and the scaling of the nuclear radius with the mass number. So, the density of all nuclei, ρ(o) is constant, we saw from the experiment of electron scattering. So, if the density is constant, we can relate the mass and the volume. So, the nuclear volume is 4/3 πR^3 and nuclear mass will be the density into the volume and this is nothing but proportional to the mass number. or you can also say this as, roughly you can say as a mass.

So, this gives you an idea about relationship between radius and mass number. As ρ(o) is constant, we can say R^3 is proportional to mass number. And so

$$R = constant \times A^{\frac{1}{3}}$$

So, radius of a nucleus, you can write as

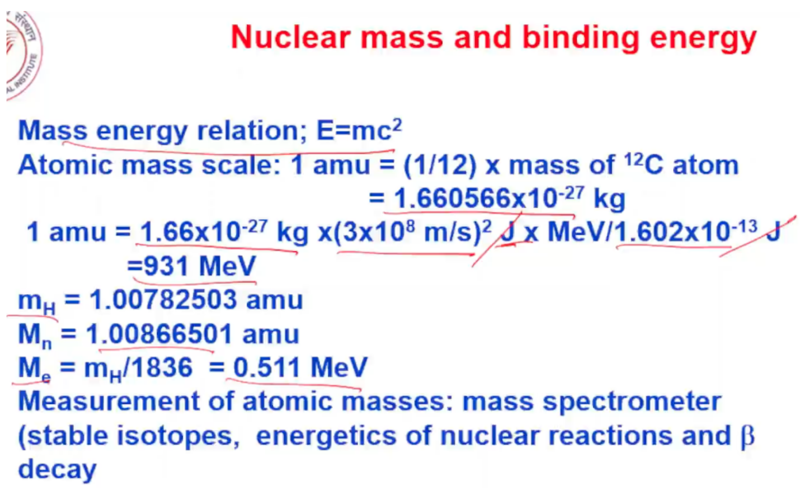
$$R = r_0 \times A^{\frac{1}{3}}$$

So, you can calculate the radius of any nucleus.

For example, if a nucleus of mass number 125, then radius will be $r_0 \times A^{\frac{1}{3}}$, that means 1.4 into 125 one third is 5. So, it is 7 Fermi. That is how we can calculate the radius of any nucleus using this formula. And also you can calculate the density from this mass upon volume.

So, you can just take 125 mass number, 125 grams will contain Avogadro number of nuclei and then you can convert into the volume of a nucleus for this one. So, you will get of the order of 10^{14} grams per cc. And in terms of nucleons, you can substitute mass of a nucleon 1.66×10^{-24} grams. So, in terms of nucleons per cc it would be 10^{38} nucleons per cc.

So, what did this experiments give us? Scattering of the particles, charged particles by nuclei gives you the radius of the nucleus, it gives the density of the nuclear matter. And so the radius of the nucleus scales with the mass number to the power one third.



Another important quantity, property of nuclei is nuclear mass and binding energy. So, the mass and energy are interconvertible. And so you will find that several times you can use mass and binding energy to replace each other as we see go along.

So, we use the famous relationship of Einstein,

$$E = mc^2$$

So, like if you burn one atomic mass unit, how much energy you will get? So, the atomic mass unit, the nuclei are written in terms of nuclear masses and in terms of atomic mass units. So, there is a scale called carbon scale. That means the mass of carbon 12 nucleus is 12.0000, exactly 12. And so using this information, you can find out the one atomic mass unit equal to $1.6660566 \times 10^{-27}$ kilogram. So, you can calculate from this because 12 gram of carbon will contain Avogadro number of atoms. So, you can calculate that mass of each atom is 12, exactly 12, you can find out one atomic mass unit. How to calculate the relationship between atomic mass unit and MeV? You can see here

$$E = mc^2$$

Mass of one nucleon, let us say, 1.66×10^{-27} kilogram or this is the 1/12 of the mass of carbon 12, the speed of light, c^2. And so kg x $(m/s)^2$ will become joule. So, if you use the units in proper way, you will get the proper units of the final answer. So, you can see this is joules and you want to convert into MeV.

So, 1.6×10^{-13} joules per MeV. So, joules will cancel with joule. So, you will be having 1 amu equal to 931 MeV. So, like that, the mass table of nuclei are if they are given in atomic mass unit, you can convert them into MeV by simply multiplying by 931. So,

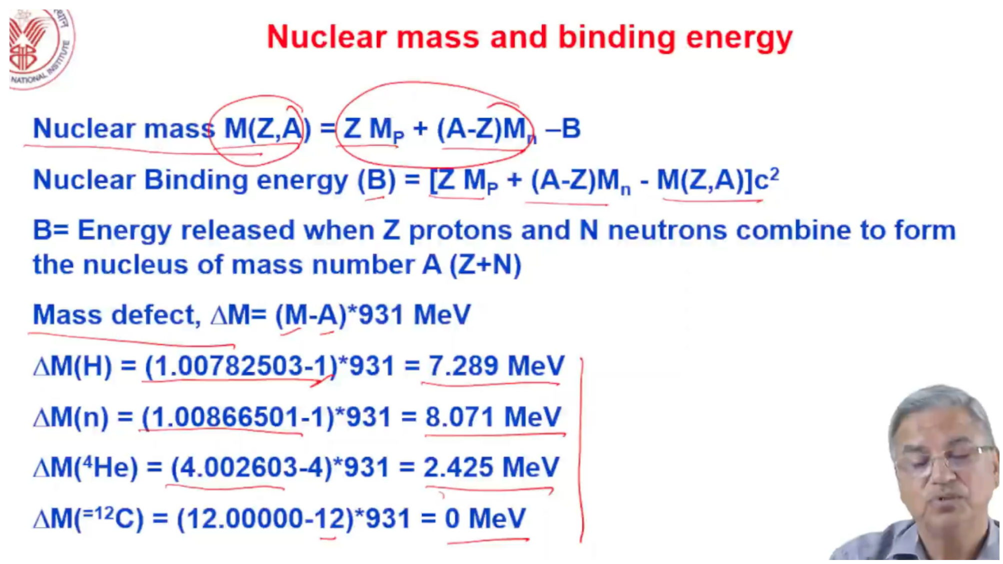
the mass of proton = m_H = 1.00782503 amu

mass of neutron = m_n = 1.00866501 amu

mass of electron = m_e = m_H/1836 = 0.511 MeV

And because the mass of electron is very small compared to that of proton and neutron, many times instead of proton mass, we write atomic masses as we see later on, we will be writing in terms of mass of proton, we can write as atomic mass of hydrogen atom. And this masses of nuclei are measured by mass spectrometers or there are other ways like energetics of nuclear reactions, beta decay, etc. By several ways, you can find out the masses of the nuclei. So, this is an important relationship we will use in our calculations.

Another aspect is nuclear mass and binding energy. So, as I mentioned that mass and energy are interconvertible. So, when the particular nucleus is formed from constituent nucleons, certain energy is released. That is what we call as the binding energy. In other words, if you want to break a nucleus into its constituent nucleons, then we need to supply energy that is called the binding energy. So, the nuclear mass, a nucleus of mass number A, atomic number Z, we say M(Z,A).



When we combine Z protons and A-Z neutrons to give you this nucleus, certain energy is released. That means the mass of the nucleus is less than the sum of the masses of constituent protons and neutrons. And the difference is binding energy. So, we can write the binding energy B equal to mass of the protons plus mass of neutrons minus mass of the nucleus.

$$B = [ZM_p + (A - Z)M_n - M(Z, A)]c^2$$

So, the binding energy B is the energy released when Z protons and N neutrons combine to form nucleus of mass number A.

Many a times instead of writing the binding energy, we write as mass defect. So, that is how we can relate the mass with the binding energy. The mass defect, that means you have mass number, which is an integral number, mass number, means number of protons plus number of neutrons and you have actual mass of the nucleus. So, actual mass minus mass number is called as the mass defect.

$$Mass\ defect = \Delta M = (M - A)\times 931 MeV$$

So, like for 12 -carbon, mass is 12.000 and the mass number is 12. So, mass defect is 0. But for other nuclei, mass defect is not 0.

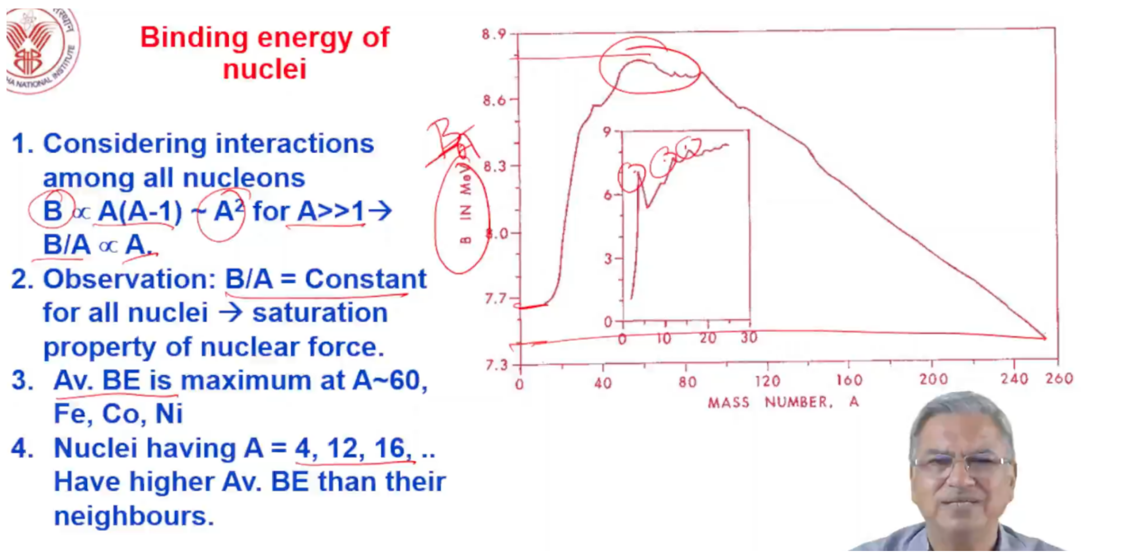
So, for protons, $\Delta M_H = (1.00782503 - 1)\times 931 = 7.289 MeV$

For neutrons, $\Delta M_n = (1.00866501 - 1)\times 931 = 8.071 MeV$

So, in fact, the nuclear mass table contains this data.

For helium nucleus, $\Delta M_{He^4} = (4.002603 - 4)\times 931 = 2.425 MeV$

So, when you use the nuclear mass table, you will have this data as written as ΔM value. So, it is important to remember the mass number is nothing but the number of protons plus number of neutrons, whereas the mass which is given in the nuclear mass tables is nuclear mass minus mass number into 931.



So, now let us discuss the binding energy of the nuclei. How does the binding energy of nuclei vary with the mass number? What I have shown here on the right-hand side, the data of average binding energy.

This is nothing but B/A. The total binding energy divided by the mass number average binding energy is plotted as a function of mass number starting from let us say close to 10 or so up to mass number more than 240. And you see here that the binding energy, the lowest binding energy point you see here 7.6 or so, highest it goes to 8.8 or so. So, from low mass to up to mass 60, it rises from 7 to 8 and then it falls down to close to let us say 7.4 or so. So, you can say that the binding energy per nucleon or average binding energy is fairly constant over all mass numbers. But if you consider the binding energy is nothing but you know, see it is like the bonds are formed between nucleons inside the nucleus. So, if each nucleon was going to interact with all the nucleons, then there should have been A ×(A-1) interactions. If every nucleon interacts with all nucleons and so the binding energy should be proportional to A^2 because if you take A much larger than 1, then it is proportional to A^2.

And in that case, B/A should be proportional to A. But actually what we get B/A is constant, assuming that 7 to 8 is fairly constant. So, this is a very important observation that average binding energy of all nuclei is really constant. That tells about the saturation property of the nuclear force. Another important observation is that the average binding energy is maximum at about mass number 60 like iron, cobalt, nickel.

These are the nuclei having highest binding energies. And if you see the inset here, the insets from 0 to mass number 30, there are sharp peaks in the binding energy curve corresponding to 4, 12, 16 and so on. So, that means these nuclear mass numbers have higher binding energy than their neighbors. So, essentially we start getting an idea about the how the nucleons are bound inside the nucleus. So, the constant value of binding energy per nucleon gives an indication that each nucleon is not interacting with all nucleons in the nucleus. Suppose there is a nucleus of gold 197, there are 197 nucleons, protons plus neutrons. A particular nucleon may be interacting with the nucleons only in its nearest immediate vicinity, not with all. Like one nucleon at the center may not interact with those at the surface. And this is what is called as the saturation property of the nuclear force. We will use this later on when we discuss the nuclear force and also when we discuss the nuclear models, what are the ways, how we can explain different properties of nuclei for that we need nuclear models.

So, I will stop here and subsequently I will discuss the stability of nuclei and the nuclear force. Thank you very much.