Compound nucleus reactions

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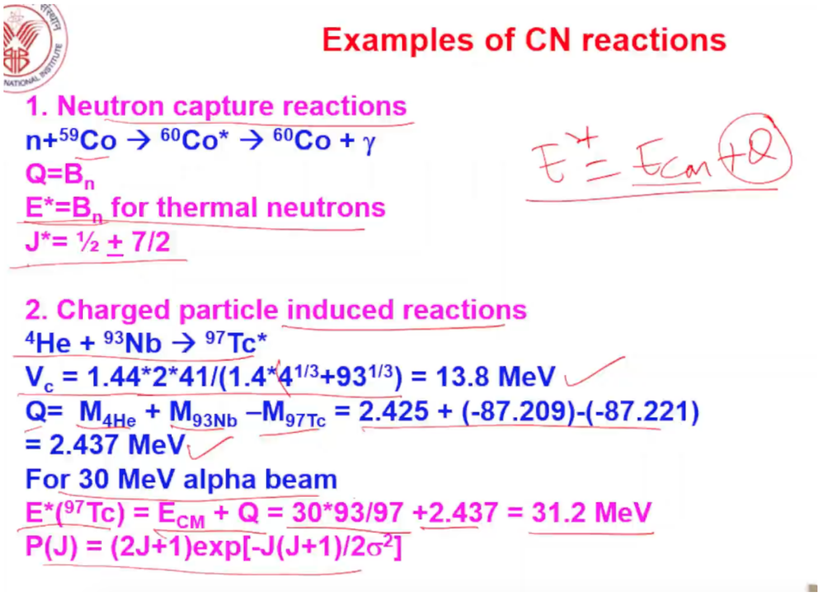
Lecture-13, module-2

Hello everyone. So, I start that next part of the lecture on nuclear reaction mechanism by discussing more details about the compound nuclear reaction.



Examples of CN reactions

1. Neutron capture reactions

$n+{}^{59}Co \rightarrow {}^{60}Co^* \rightarrow {}^{60}Co + \gamma$

$Q=B_n$

$E^*=B_n$ for thermal neutrons

$J^* = ½ \underline{+} 7/2$

$E^* = E_{cm} \boxed{+Q}$

2. Charged particle induced reactions

${}^{4}He + {}^{93}Nb \rightarrow {}^{97}Tc^*$

$V_c = 1.44*2*41/(1.4*4^{1/3}+93^{1/3}) = 13.8$ MeV ✓

$Q = M_{4He} + M_{93Nb} - M_{97Tc} = 2.425 + (-87.209)-(-87.221)$

$= 2.437$ MeV ✓

For 30 MeV alpha beam

$E^*({}^{97}Tc) = E_{CM} + Q = 30*93/97 + 2.437 = 31.2$ MeV

$P(J) = (2J+1)\exp[-J(J+1)/2\sigma^2]$

Let me give you examples of the compound nuclear reactions like neutron capture reactions are a very fine example of compound nuclear reactions. When the neutron is captured by the nucleus, it is amalgamated with the nucleus and the compound nucleus is excited to an excited state that excitation energy will be equivalent to the binding energy for the thermal neutrons. So essentially excitation energy is $E_{CM} + Q$. Q value is for the neutron capture reaction is the binding energy of the neutron in the compound nucleus.

And if you take thermal neutrons, then their kinetic energy is close to zero. That is why in the case of thermal neutrons, you can say excitation energy equal to the binding energy of neutron and the angular momentum that is given to the compound nucleus will be target nucleus plus the neutron spin. So, neutron spin is half and spin for target nucleus Co-59 is 7/2, 7/2 ±1/2. J of the compound nucleus can be 7/2 +1/2 or 7/2 -1/2.

Whatever is the state of the compound nucleus being available that will be populated because angular momentum has to be conserved. So we will discuss the decay of a compound nucleus in terms of the available excitation energy and the angular momentum. These are the two important properties which will govern the ultimate fate of the compound nucleus. In the case of charged particle induced reactions, as I discussed earlier also, the charged particle will bring in some energy because you have to cross the coulomb barrier of the target nucleus. So, the same example I discussed earlier, we calculate the coulomb barrier for this

$Vc = 1.44*2*41/(1.4(4^{1/3}+93^{1/3}))=13.8$ MeV

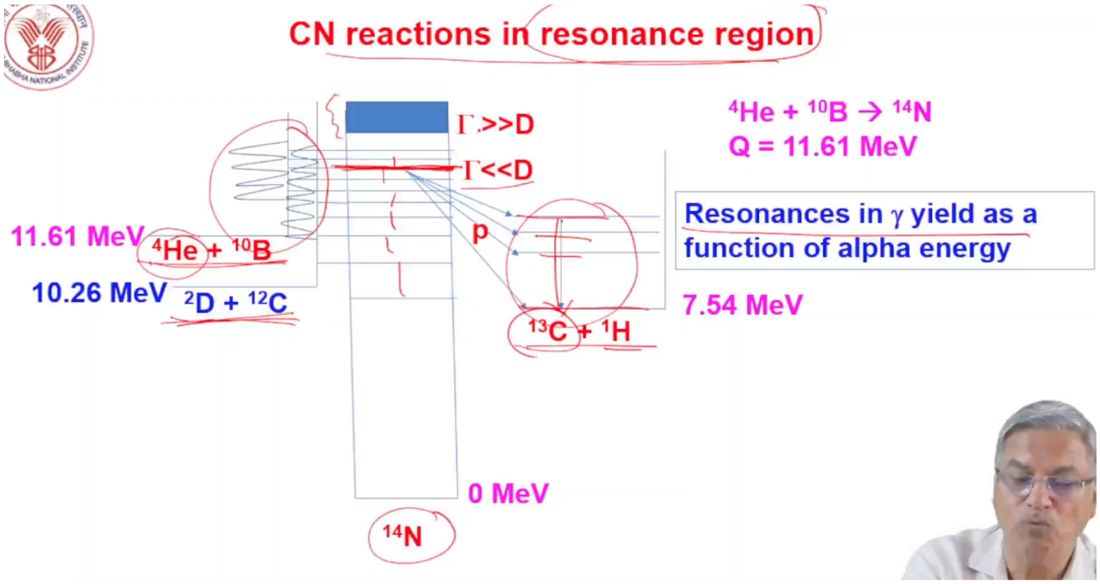
For alpha on niobium-93 coulomb barrier is 13.8 MeV. And the Q value for this reaction will be mass of alpha plus mass of niobium-93 minus mass of technetium-97. You can again put the value ΔM value and you get 2.437 MeV.

So, you have to have the energy, the center of mass energy of the alpha more than or equal to 13.8 MeV. Normally it will be much above the coulomb barrier. Let us say you take a 30 MeV alpha beam. And for a 30 MeV alpha beam, again calculate what would be the excitation energy and angular momentum. So, the excitation energy of the compound nucleus will be $E_{CM} + Q$. For 30 MeV E_{CM} will be $(30 \times 93/97)+2.437$. But here you take the masses, actual masses because it is not the difference in the masses that the A will get cancelled out. So when you are calculating center of mass energy or recoil energy, you take the mass number. Mass numbers are very close to the masses. So, you take here actual mass numbers plus Q value, that is 31.2 MeV.

And the angular momentum, now the projectile can bring in a large range of angular momentum from 0 to L_{max}. And so there is a distribution of angular momentum in the compound nucleus, unlike in the case of neutron induced reaction where we have only one spin populated. You have a distribution of angular momentum and that distribution is given by

$P(J) = (2J + 1) \exp(-J(J+1)/2\sigma^2]$.

So that is like a complete distribution of J values from 0 to some maximum value. These are the two properties we will be discussing more how the compound nucleus will decay.



CN reactions in resonance region

$^4He + ^{10}B \rightarrow ^{14}N$

Q = 11.61 MeV

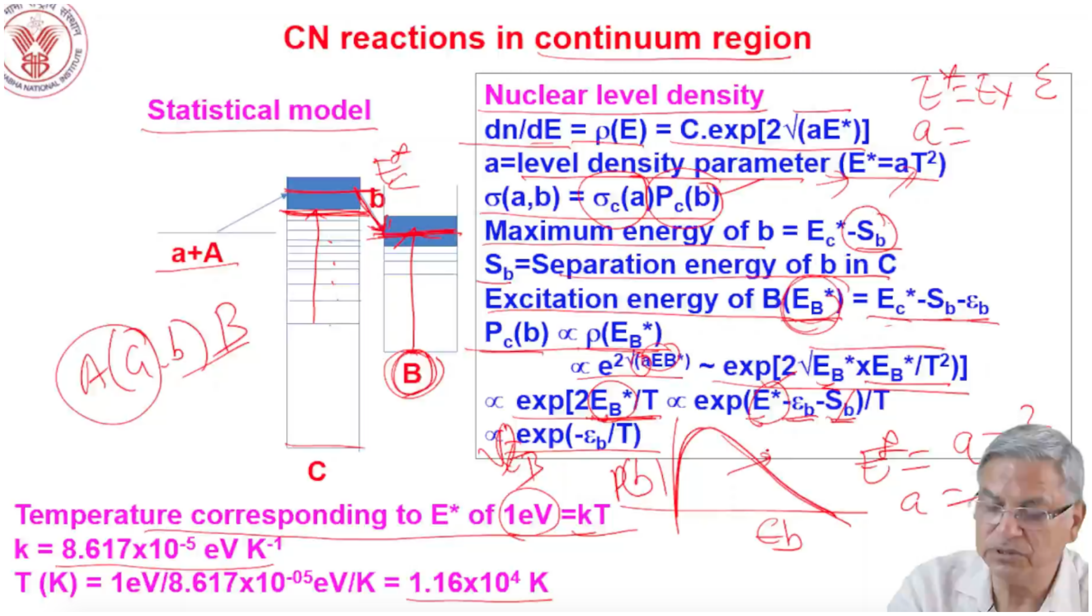
Resonances in γ yield as a function of alpha energy

So I will now discuss the compound nucleus behaviour in two regions. As you know, this is the compound nucleus. I have now again shown here the formation of the compound nucleus by two different reaction groups, alpha on boron-10 and deuterium on carbon-12 forming nitrogen-14 as the compound nucleus.

Now for the low mass projectiles and target, you will find that the excited states, this you can equate to the shell model states. There is a large gap in the excited state of this nuclei. So if you populate a compound nucleus when the excited states are discrete, they are well separated from each other, then that particular suppose you populate this state, then the subsequent decay of this excited compound nucleus will be governed by the width of this level and available states in the product nucleus. So it can emit a proton to form carbon-13. So, this when you populate the compound nucleus in discrete states, we call it as a resonance region.

That means you can achieve resonances in the cross-sections whenever there is a population of a particular energy state of the compound nucleus. So this is what I have tried to explain here that for two different reactions, you can populate the same state of a compound nucleus, which can then decay by another channel to the residue and the particle that is emitted. There can be now even gamma emission from the excited states of carbon-13. So that will give you resonances in the gamma ray as a function of projectile energy. So, what you do, you measure the gamma ray yield, vary the alpha energy, you measure the gamma ray yield, vary the neutron energy, and you will see these gamma rays in the spectrum.

What these people found out is, in fact, it has been done quite early in the 1950s or 60s, it was found that the characteristic gamma ray spectrum and their yield are independent of the, which way the compound nucleus was formed. Both these reactions, alpha plus boron-10 and deuterium plus carbon-12, give rise to the same energy states. So again, in the resonance region, when you populate a nucleus in a discrete state, there are resonances in the cross section also of the compound nucleus and the gamma spectrum also shows the resonances, so this is the kind of experiment that have been done to prove the independence of the compound nucleus formation and decay. And subsequently, we will see even in the continuum region. So here, actually, I tried to explain here the width of this level is much less than the spacing between the levels.



CN reactions in continuum region

Statistical model

Nuclear level density

$dn/dE = \rho(E) = C.\exp[2\sqrt{(aE^*)}]$

a=level density parameter ($E^*=aT^2$)

$\sigma(a,b) = \sigma_c(a)P_c(b)$

Maximum energy of b = $E_c{}^*-S_b$

S_b=Separation energy of b in C

Excitation energy of B($E_B{}^*$) = $E_c{}^*-S_b-\varepsilon_b$

$P_c(b) \propto \rho(E_B{}^*)$

$\propto e^{2\sqrt{(aEB^*)}} \sim \exp[2\sqrt{E_B{}^* x E_B{}^*/T^2})]$

$\propto \exp[2E_B{}^*/T \propto \exp(E^*-\varepsilon_b-S_b)/T$

$\propto \exp(-\varepsilon_b/T)$

Temperature corresponding to E* of 1eV=kT

k = 8.617x10^{-5} eV K^{-1}

T (K) = 1eV/8.617x10^{-05}eV/K = 1.16x10^{4} K

You go to still high energy, the width of this level become more than the spacing between the levels. So the spacing between the levels are low, then they will start overlapping and you see what is called as the overlapping levels will be looking like a continuum. So the continuum, now we cannot apply the discrete states formalism here we apply there the statistical model formalism. So this was for the resonance region and now we come to the continuum region where the energy states, the gap between the excited states is decreasing as you go up and up. So finally, what will happen, the spacing between the states becomes so low that they will overlap and then you will see a continuum region, which is like a Fermi gas.

A Fermi gas where you have all the nucleons are populating states which are overlapping. So they give you the feeling of Fermions moving in a particular chamber. So the projectile plus target nucleus like charged particle induced reactions when you populate the nucleus, ideally you know the binding energy of the nucleon will be around 7 to 8 MeV. If you populate a nucleus more than that energy, invariably you will end up with the overlapping states and that will be the continuum region. So, for the continuum region compound nucleus, you apply a statistical model, you cannot count the individual levels.

So, what you talk about the density of the nucleus states. You don't count how many levels are there, like here we can count how many levels are there in one electron volt or one MeV. But in the case of continuum region, you apply a level density formalism, this is called number of levels in a particular unit MeV energy,

$dN/dE = \rho(E) = C \exp(2\sqrt{aE^*})$

$dN/dE = \rho(E)$, the level density and it is given by the Fermi gas model, a constant into exponential $2\sqrt{aE^*}$, where $E^*$is the excitation energy and a is called the level density parameter. So basically the level density parameter is a measure of what is the number of levels in one MeV. So that is a measure of that, it is always associated with the level density. So, this level density parameter is related to the excitation energy of the compound nucleus by a term called temperature, that is a nuclear temperature. So when we have an excited nucleus, we say it has got certain temperature and this temperature part becomes very important. We will see later on, we will try to get a value of temperature. So nucleus is hot, it is excited, it is hot, but the hot is so hot that the temperature you will get a feeling of the temperature. So, the temperature corresponding to 1 electron volt excitation, we can relate the temperature to kT in terms of Kelvin, k is the Boltzmann constant and this is given by 8.617×10^{-5} electron volt per Kelvin. So, if you calculate the temperature corresponding to 1 electron volt, 1 electron volt upon this much electron volt per Kelvin, 1.16×10^{4} K. So, if you have a nucleus and excitation energy of 1 electron volt, then it has got a temperature of 10^{4} K. So, 1 electron volt will be close to nearly ground state.

You populate at 1 MeV and straight away $10^{6} \times 10^{4} = 10^{10}$ Kelvin. So that if you recollect the nuclear fusion, we are talking about 10^{8} Kelvin that is about 10^{4} electron volts. So tens of keV. So this is where you try to correlate temperature of the nucleus which is behaving like a hot Fermi gas. So, this is like a pressure cooker where you have a gas molecule at very high temperature. So, we deal with the excitation of an excited nucleus as if it is a Fermi gas at high temperature. So we will try to see what are the signature of the decay of a compound nucleus which is having high temperature, which is having high angular momentum in the Fermi gas formalism.

The cross section for a reaction A(a,b)B. So a and A form a compound nucleus and the compound nucleus is emitting by a particle b and is left with a residue B. So we have the formation of the compound nucleus and we have the decay of the compound nucleus, two steps which are independent of each other.

So let us try to see the probability of decay of the particle to the residue. Now depending upon the energy of the particle, it can populate different energy states of the residue. So what we essentially measure is the spectrum of this particle b and if it is a compound nucleus formation, what type of spectrum the particle will have that we are trying to see here. So, the maximum energy of the ejectile, that is b, is given by the excitation energy and let us say it is formed here, B* minus the binding energy of the particle in this nucleus. So that is called the separation energy or the binding energy because binding energy is the energy required to remove a nucleon from the nucleus.

So this much energy we have to supply. So excitation energy minus binding energy is the maximum kinetic energy of the particle.

Maximum energy of b = $E_c^*-S_b$

So here S_b is the separation energy of particle b in compound nucleus. So the excitation energy of this residual nucleus will be now, compound nucleus excitation energy minus the binding energy of particle in the compound nucleus minus the kinetic energy of the particle that is coming out.

$E(B^*) = E_c^* - S_b - \varepsilon_b$

these are the four energies involved.

When the particle is emitted from compound nucleus, energy equivalent to the binding energy is required and so this residual nucleus may be populated in the certain energy states and that excitation energy given by this formula. So the probability of decay to the particular energy state of the daughter residue depends upon this excitation energy. Higher the excitation energy, higher will be the level density. So basically, why it depends upon E* because level density depends upon E*. So, the probability of decay of B to a particular energy state depends upon the $\rho(E_B)$, ρ level density of the residual nucleus at a particular energy that is proportional to $e^{2\sqrt{(aEB^*)}}$.

And you can now explain this E_B, you can substitute this a in terms of E^*/T^2.

$$e^{2\sqrt{(aEB^*)}} \sim \exp[2\sqrt{E_B^* x E_B^*/T^2})]$$

$$\propto \exp[2E_B^*/T \propto \exp(E^*-\varepsilon_b-S_b)/T$$

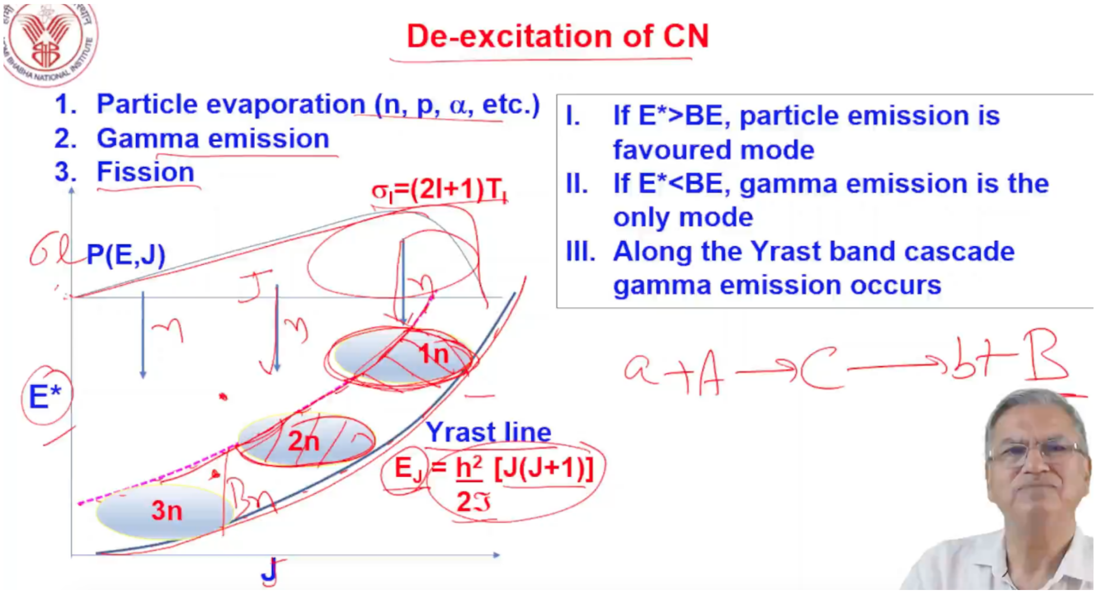
$$\propto \exp(-\varepsilon_b/T)$$

What we know is the binding energy of the particle in the nucleus, the kinetic energy that we measure and the initial energy. You can substitute E^*_B in terms of compound nucleus E* minus kinetic energy minus binding energy. So the excitation energy of the compound nucleus is known, the binding energy of the particle in the nucleus can be calculated. So they are constant terms. So it becomes variable term is the particle energy, particle energy by upon temperature.

So the probability of decay of the compound nucleus into the residual nucleus in certain states is equal to proportional to $e^{-\varepsilon_b/T}$. Where is ε_b? ε_b is the binding energy of the particle. So you will find that the part as a function of ε_b the probability of decay will be $e^{-\varepsilon b/T}$.

There is some exponential, pre-exponential term dependent on root of ε_b which we have not explained. So this term is $e^{-\varepsilon b/T}$.

And from the slope of this actually you can find out the temperature of the nucleus when it is hot. So this is called the Maxwellian nature of the particle spectra. So if a nuclear reaction happens and the particles that are being evaporated by the compound nucleus, they follow this pattern. We say this is a signature of compound nucleus. So one of the signatures of the compound nucleus is the Maxwellian nature of the particle spectra.



De-excitation of CN

1. Particle evaporation (n, p, α, etc.)
2. Gamma emission
3. Fission

I. If E*>BE, particle emission is favoured mode

II. If E*<BE, gamma emission is the only mode

III. Along the Yrast band cascade gamma emission occurs

So how does the compound nucleus decay? You have an excited nucleus having certain excitation energy and angular momentum. It has got different pathways for the decay like neutron, proton, alpha, even gamma ray emission can take place. And if it is a heavy nucleus it can even undergo fission. So there are different probabilities for the excitation of the compound nucleus. And as I mentioned previously, it depends upon the available channels.

So you have an excitation energy, you have an angular momentum. And that nucleus to decay you have different options, particle evaporation, gamma emission and fission. So whichever process can carry away the large amount of angular momentum and excitation will be more favored. I will try to explain here by this diagram here. So we if you recall, the compound nucleus has got two parts.

One is the excitation energy, second is the J here. So here the angular momentum population I have given here, $(2l+1)T_l$. That's what I have done here. So it is actually σ_l here, this is J.

And here it is E*. So it is a mixed diagram of excitation energy and angular momentum. Now this triangular distribution of the compound nucleus in excitation energy and angular momentum space will be now dissipating its energy. And what it will dissipate? By emission of particles, so it could be neutrons. For heavy nucleus proton and alpha emission is not favored much because there is an emission barrier. And so the particles, the residual nucleus that are formed you know, if you see a + A →C →b + B.

So these are the states of the B that is the residual nucleus will have certain angular momentum. And when you have a nucleus with some angular momentum, certain energy is tied up in its rotational energy. So the rotational energy is given by $\hbar^2/2I$ (J(J+1)), I is the moment J is the angular momentum. So this states of different E_J values. As a function of J, certain energy is tied up in rotation. This is called the YRAST line. So you cannot have a level below the YRAST line. So energy has to be more than the YRAST line. So if when it is populating, when emitting neutrons, let us say, then it will come to this energy state, it will populate in E_J plane. This is E_J plane. This population will lead to certain population in the residual nucleus.

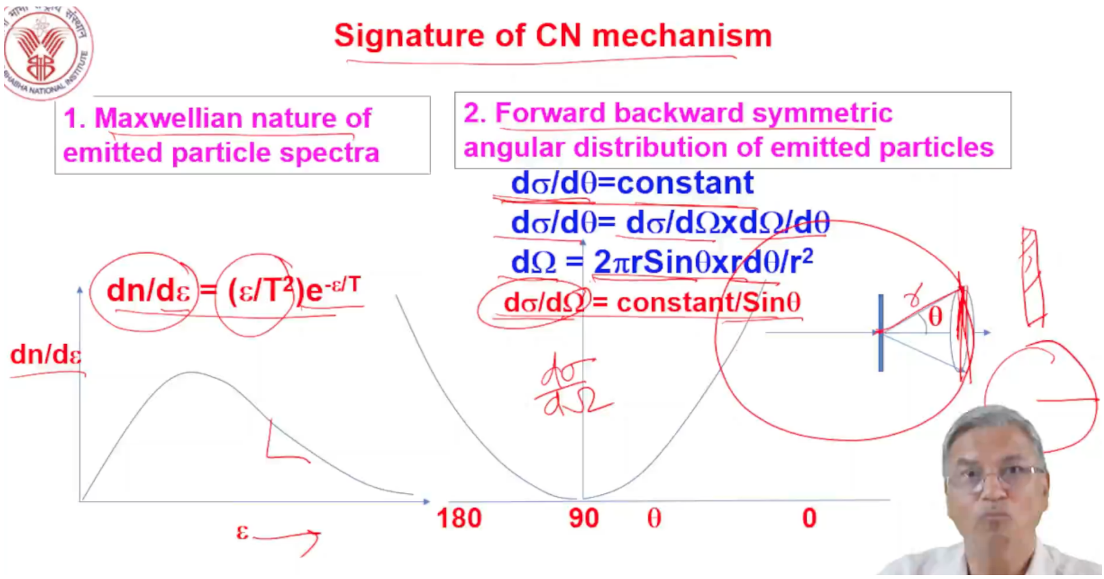
And this is called the drip line, this gap is called the binding energy. That means if you populate the nucleus here, then this cannot emit neutron. Whereas this if you populate here, it can emit neutron because its excitation energy is more than binding energy. And so any population which will come in this Yrast band will be now add to the particular product.

If you recall the excitation function 1n, 2n, 3n are formed, this population now is frozen to 1n product. Similarly, and this is emitting, this population is frozen to 2n product and so on. So in the angular momentum space, different product nuclei, the evaporation residues are formed as compound nucleus is emitting 1 neutron, this product is formed 2 neutron, this is formed 3 neutron, this is formed. And similarly protons and alpha emission can also take place. So once it is in this domain below the binding energy now, this nucleus is still excited, they will be emitting gamma rays.

And by emission of gamma ray, they come to their ground state and that's what we say when we measure the cross section for evaporation residue, this is formed by 1n. This nucleus could not further de-excite because this has come below the binding energy of neutron in that particular nucleus. So this is the schematic to explain how the evaporation of particles lead to different residues. And the competition, in fact, what is actually known is that if a nucleus can emit particle, it will favor particle emission than gamma emission.

Why? Because the gamma ray cannot carry much angular momentum. Gamma ray will carry one or two units. But the particles can carry much higher amount of angular momentum, so particle emission is favored for high angular momentum states provided, excitation energy is more than the binding energy of the particular particle in the compound nucleus. In fact, if the excitation energy is very high, fission is the first dominant reaction, but fission has got a barrier. So if fission is possible, first is the fission, then particle emission, and then the gamma emission.

That is the sequence. So if E* is more than binding energy, particle emission is favored. If E* is less than binding energy, gamma emission is the only mode. Along the Yrast band, now in this band, this is called the Yrast band. Along the Yrast band, only gamma emission is possible, and particle emission not possible. Only above the Yrast band, states can emit particles.



Signature of CN mechanism

1. Maxwellian nature of emitted particle spectra

2. Forward backward symmetric angular distribution of emitted particles

$d\sigma/d\theta = \text{constant}$

$d\sigma/d\theta = d\sigma/d\Omega \times d\Omega/d\theta$

$d\Omega = 2\pi r \sin\theta \times r d\theta / r^2$

$d\sigma/d\Omega = \text{constant}/\sin\theta$

$dn/d\varepsilon = (\varepsilon/T^2)e^{-\varepsilon/T}$

So what are the signature of the compound nucleus mechanism? I will try to explain. First one I already explained, the Maxwellian nature of the particle spectra,

$dn/d\varepsilon = (\varepsilon/T^2)\, e^{-\varepsilon/T}$

This is a typical signature of compound nucleus. ε/T^2 is the pre-exponential term, and $e^{-\varepsilon/t}$. So from the slope of this, in fact, we can find out what is the temperature of this compound nucleus that is emitting these particles.

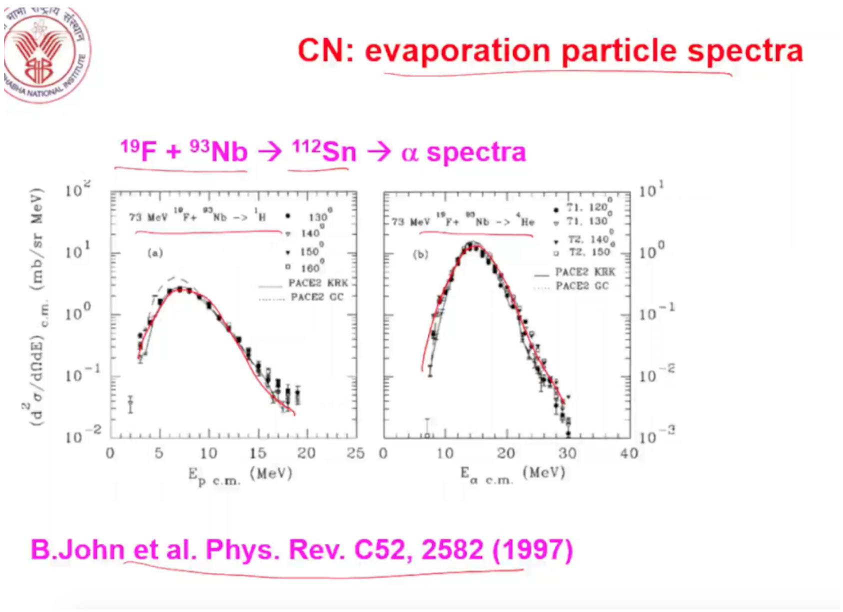
And the second important signature of the compound nucleus formation is forward-backward symmetry in angular distribution of particles. So as if you recall, the compound nucleus does not know what way it was formed. It loses the memory of its formation. Recall the experiment by Ghosal, that the two reaction channels, the same product formed by two reaction channels, cross-sections are same. And so if they do not know the memory of the entrance channel, then we can say that the emission of particle is isotropic, because there is no preferred direction.

So the angular distribution we call dσ/dΩ at a particular θ. At a particular θ, there is the same number of particles at all angles. So dσ/dθ is constant. And I try to explain what does it translate into in angular distribution. So I try to explain here. This is like a sphere. At any θ, the cross-section, dσ/dθ. is same So let us take a particular θ here. And at this θ, you have a disc. So that is the disc of same θ. So you try to find out the solid angle.

Because what you essentially measure is the dσ/dΩ. You are plotting here dσ/dΩ, where dΩ is the solid angle subtended by this strip. So it is a strip of the sphere, which is having the same angle θ. So how do you know the solid angle? Solid angle subtended by a sphere at this point is 4π. The surface area of the sphere is $4\pi r^2$, that gives the solid angle 4π. So this solid angle by this strip of the sphere will be area of this, so this is like a circle into a strip.

So it is like this actually. And so you have this, this is a circular strip, to see the cross-section, this is this one, πr^2. But r is now r sinθ. So this is r, this r sinθ. So 2π r sinθ, which is the circumference of this strip into the thickness of the strip r dθ. So this is the area of this strip, if you see from this side. And so this strip is a part of that sphere, $2\pi r \sin\theta \times r\, d\theta / r^2$, the solid angle subtended by this strip at this point. And so it becomes, if you say this dσ/dΩ will be dσ/dΩ × dΩ/dθ will be 1/ sinθ. And so the angular distribution per unit solid angle follows, this is what is 1/ sinθ graph. So more than the 1/sinθ graph, because it is not always 1/ sinθ, it is more of forward, backward symmetry. So the angular momentum can change this 1/ sinθ dependence. Most important aspect is that it is forward backward symmetric. Whatever number of particles are limited in forward angle; same number of particles are limited in the backward angle. And so this forward backward symmetry is a signature of the compound.

So Maxwellian nature of the particle spectra and the forward backward angular distributions, these are the two proper signatures of compound nucleus. So I try to give you some examples of the compound nucleus evaporation particle spectra from our own paper of 1997.

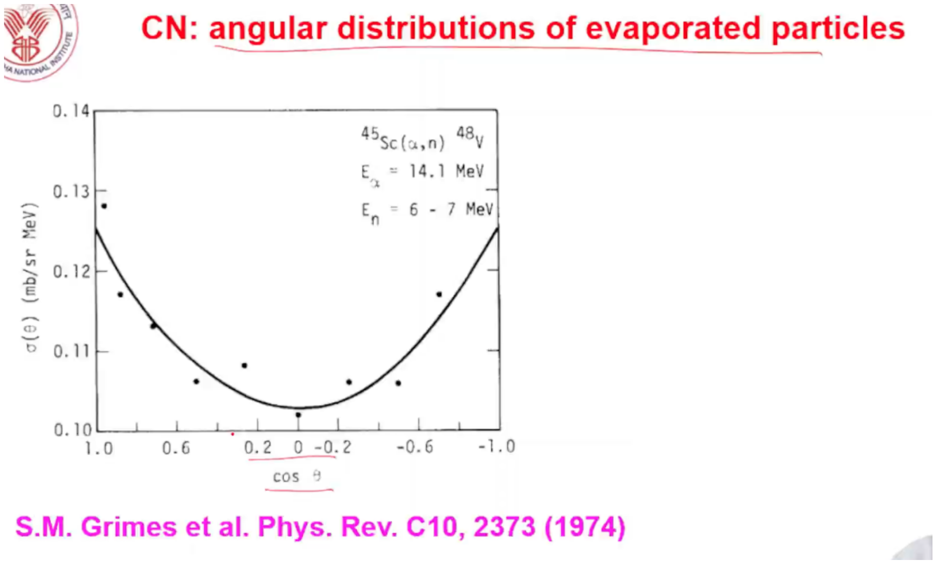


CN: evaporation particle spectra

$^{19}F + ^{93}Nb \rightarrow ^{112}Sn \rightarrow \alpha$ spectra

B.John et al. Phys. Rev. C52, 2582 (1997)

So $^{19}F + ^{93}Nb \rightarrow ^{112}Sn$, alpha particle spectra you see here, Maxwellian nature. This is for protons, this is for alpha. So this is a clear cut signature of a compound nucleus decay. This is how nuclear chemistry community will verify this particular reaction follow the compound nucleus mechanism if the particle spectra are Maxwellian in nature.



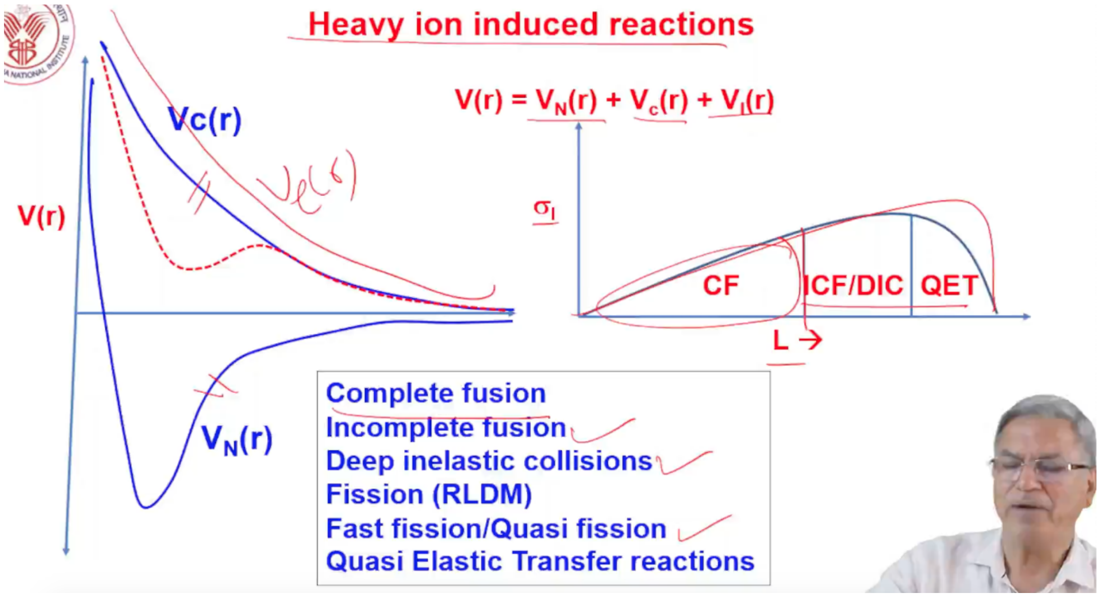
CN: angular distributions of evaporated particles

S.M. Grimes et al. Phys. Rev. C10, 2373 (1974)

Similarly, the angular distribution of evaporation particles will be forward backward symmetric as a function of θ or you can plot as a function of cos θ, then whatever particles are emitted they are forward backward symmetric.

So again, the forward backward symmetric angular distributions and Maxwellian nature of particle spectra are signature of compound nucleus reaction.

Okay, so lastly, I do not have time to discuss the compound nuclear reaction mechanism, but just I wanted to tell you that this is not the complete story. Other than compound nucleus formation, there are many other mechanisms, particularly when you go to heavy projectiles like carbon-12, oxygen-16, neon-20 and so on. So when you have a heavy ion as a projectile, the heavy ion induced reactions, the nuclear potential plays an important role.



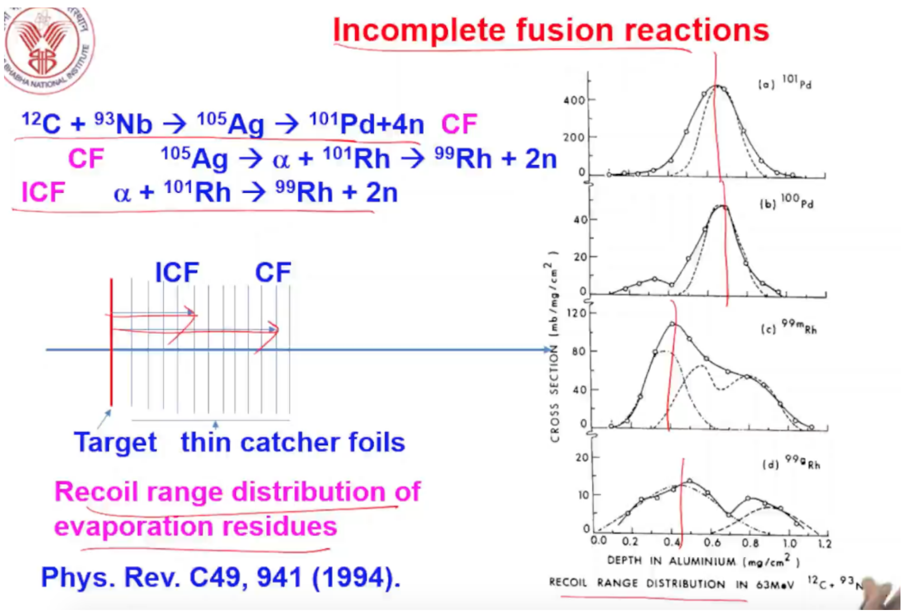
Heavy ion induced reactions

$V(r) = V_N(r) + V_c(r) + V_l(r)$

- Complete fusion
- Incomplete fusion
- Deep inelastic collisions
- Fission (RLDM)
- Fast fission/Quasi fission
- Quasi Elastic Transfer reactions

The angular momentum plays an important role apart from the coulomb barrier. And so you will see the nuclear potential, the coulomb potential, the coulomb potential is anyway there, but there will be one more potential called centrifugal centrifugal barrier (V_l) also will contribute. And so the fusion of the projectile target gets limited. So in addition to the complete fusion, there are non-compound nuclear reactions like incomplete fusion, deep inelastic collision, fast fission and so many other reactions open up. So heavy ion reactions is a domain of research, study, what are the reactions that do not follow complete fusion. So that 2l+1 distribution, if you recall, σ_l vs l. complete fusion is this compound nucleus part.

In addition to that, there are competing modes of incomplete fusion, deep inelastic collision and quasi-elastic transfer. They all encompass non compound nuclear reactions. And there is so much of research actually open for the nuclear chemistry or nuclear physics community to study the limitations of nuclear compound nuclear fusion reactions.



Incomplete fusion reactions

$^{12}C + ^{93}Nb \rightarrow {}^{105}Ag \rightarrow {}^{101}Pd + 4n$ CF

CF $^{105}Ag \rightarrow \alpha + {}^{101}Rh \rightarrow {}^{99}Rh + 2n$

ICF $\alpha + {}^{101}Rh \rightarrow {}^{99}Rh + 2n$

Recoil range distribution of evaporation residues

Phys. Rev. C49, 941 (1994).

We have done some experiments on incomplete fusion reactions. That means the same compound, same product can be formed by complete fusion as well as incomplete fusion reactions. So these incomplete fusion reactions are similar to complete fusion reactions, but the momentum transfer is different. Linear momentum transfer in the formation of the same product is different. That we have tried to find out by measuring the range of the recoils. So this is the recoil range distribution the evaporation residues formed in the complete and incomplete fusion reactions. So the complete fusion products are stopped at much later part of the recoil stack than the incomplete fusion. It means the kinetic energy of the recoil products that are formed is much higher for the complete fusion than for incomplete fusion that is reflected in there. So this is the momentum, this is essentially the range of the particles in the aluminum. These are the compound nucleus products, but the non-compound nucleus products have got lower range. So the momentum transfer is not complete for incomplete fusion reactions that is revealed in the required distribution of the evaporation residues.

So I wanted to just give you a glimpse of what are the different reaction mechanisms other than compound nucleus. You see them all opening up in heavy ion induced reactions. So I will stop here and then discuss the later part subsequently in the next lecture about the nuclear fission and nuclear fusion reactions. Thank you very much. Thank you.