Nuclear reactions: Cross section

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Lecture-12, Module-1

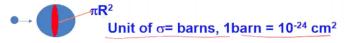
Hello everyone. In the previous lecture of nuclear reactions, we discussed the energetics of nuclear reactions. We discussed the Q value, the threshold for a nuclear reaction, whether it is for a charged particle reaction or neutron energy reaction. So, the energetics, the feasibility of the reaction was studied in that part, in this lecture, I will be talking about the cross section for the nuclear reaction, how to determine the probability of a nuclear reaction and what are the different types of reactions that take place when you use neutron and charged particle as projectiles.

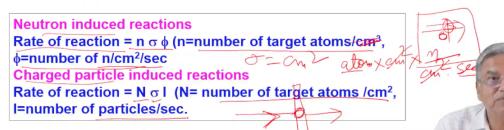
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Cross section of a reaction

Cross section (σ) is a measure of the probability of occurrence of a nuclear reaction.

It represents the cross sectional area offered by the target nucleus to the projectile for the reaction





So, let us talk about the cross section. The cross section which we denote as σ is a measure of the probability of occurrence of a nuclear reaction. So, that is the point to note that what is the probability of occurrence of a nuclear reaction which is represented by σ . However, in very simple terms, it is the cross sectional area that the target nucleus will offer to the projectile. It is a very simplistic way of telling the cross section. In fact, from that only the cross section derives its units of barn, one barn is 10^{-24} cm².

So, the nuclear dimensions are of the order of Fermis 10^{-13} cm and so the r_0 , the radius constant is $1.4*\ 10^{-13}$ cm and the radii of nuclei will be $r_0A^{1/3}$. So, that will be about 10^{-24} cm² or so. So, accordingly that nuclear area, the cross section of a nucleus, if you just cut the cross section, the circle πr^2 will be of the order of 10^{-24} cm² and the unit of cross

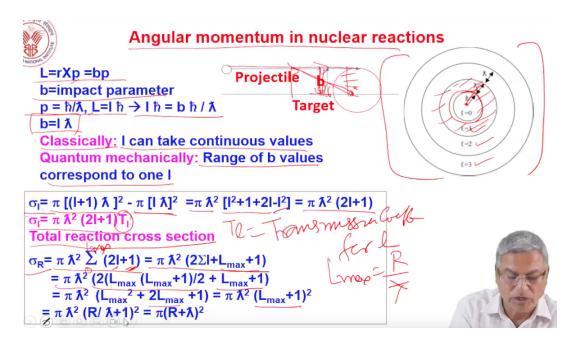
section that is why it is called as barn, where 1 barn is 10^{-24} cm². So, it is a very simple way of telling the cross section.

There are many more things which will come in actually defining the cross section of a nuclear reaction as we go along. So, how do we use the cross section in the nuclear reactions when we want to write the rate of the reaction? So, depending upon whether we are doing neutron induced reaction or charged particle induced reactions, the terms that come into the formulation of nuclear reaction rate will be slightly different which we will try to highlight here. So, the rate of a reaction for neutron induced reaction is called $n\sigma\phi$, where n is the total number of target atoms, n is the total number of target atoms because in case of reactor neutron, entire target is exposed. So, all the atoms in the target are exposed to neutrons.

And so $n\sigma\varphi$, n is the number of target atoms, σ is the cross section and φ is the flux. So, how many neutrons are going through a unit area per second? That will define the φ . So, $n\sigma\varphi$ is the rate of a reaction, that is the rate of reaction, so if you see here, the units will match atoms into centimeter square into neutrons per centimeter square per second. So, it will be per centimeter square into square, it will be atoms per second. So this many atoms are formed per second, that is the rate at which the reaction takes place.

In the case of charge particle induced reaction, you don't have a sea of charge particle but you will have a beam to bombard the target. So, in the target only a small area is exposed to the beam. And therefore, the number of target atoms is not defined by the total number of atoms but it is the target atoms per centimeter square. Unit area, how many atoms are present in the target? σ is the cross section and now the beam, whatever is falling on the target is now not per centimeter square per second but it's the number of particles falling on the target per second.

So, there is a slight change in the units of flux or the particle intensity. In the case of neutrons, it was flux, neutrons per centimeter square per second. In the case of charge particles, it's the particles per second. So, both the reaction rate ultimately will come out to be atoms per second. You can see here $n\sigma I$ will be number of atoms per centimeter square into centimeter square into particles per second. It will be again atoms per second. In both the cases, whatever way the units of target atoms and the flux or the intensity, we always end up with the $n\sigma I$, rate of the reaction.



Now, as I mentioned that the cross sectional area offered by the nucleus to the projectile is a very simplistic way of defining the cross section. Essentially, if you see when the projectile hits the target, depending upon the distance from the center of the target nucleus, there will be different angular momenta are involved. So, actually when you define the nuclear reactions, we have particles carrying certain amount of angular momentum.

That angular momentum will be defined in terms of the

$$L = r \times p = bp$$
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where r is the radial distance of projectile from the target center. Let us say this is the target center and this is the projectile going. So, this is like the r. So, b is nothing but rsin θ . So, b is called the impact parameter and p is the linear momentum of the particle. So, r×p is the angular momentum. And angular momentum is a quantity in nuclear reactions, which dictates the different types of mechanisms of the nuclear reaction. b, the distance, the vertical distance from the target center to the wave at which the projectile is coming will call as the impact parameter. And this impact parameter essentially is the r sin θ .

So, this is like, you know, if it is here, then this is the r and this is θ . So, rsin θ is the impact parameter, the vertical distance from target center to the distance of the projectile from the center. Now the linear momentum can be written in terms of the de Broglie wavelength \hbar/λ cross, where λ -cross is the reduced wavelength of the projectile and the angular momentum can be written

So, essentially, angular momentum in terms of linear momentum can be now written as

 $lh = bh/\lambda$ -cross

and so the impact parameter

 $b = l\lambda$ -cross

This impact parameter b essentially is a classical quantity at what distance from the target center the projectile will hit the target that will be the impact parameter and L is the how much of the angular momentum the projectile will bring in the units of λ -cross.

So, classically you can see b can take any value and so the angular momentum can take continuous values. But then you know that the angular momentum is quantized in quantum systems and so quantum mechanically certain values of b will correspond to one value of L. So, L can vary from 0, 1, 2 and so on h. So, you try to have a correlation between the continuously varying impact parameter and the discrete values of L which I had tried to explain using this. So, suppose this is the target nucleus here cross section of the target and if that projectile hits at the center within this radius that is the radius of one λ -cross that we will call L value equal to 0 we call it as s wave particle.

If it is within the next one L λ -cross, L = 1, L = 2, L = 3 and so on. So, depending upon at what distance the projectile is hitting the target nucleus from the center it can bring in different amount of angular momentum and the one which is at the maximum of the target radius will be the L max. So, let us now try to calculate the cross section for a particular L value. So, it is like the annular area of this annulus. So, from L = 0 to L = 1 we will say L = 0, then 1λ -cross, to 2λ -cross, will have that L = 1, and so on.

So, this is what I try to calculate here the angular momentum dependent cross section σ_l . So, for a particular concentric ring the σ_l will be area of the outer circle upon area of the inner circle. So, taking λ -cross as Λ

$$\sigma_{l} = \pi[(l+1)\Lambda]^{2} - \pi[l\Lambda]^{2} = \pi\Lambda^{2}[l^{2}+1+2l-l^{2}] = \pi\Lambda^{2}(2l+1)$$

Since these charged particles have to penetrate the coulombic barrier. So, there is a transmission coefficient. Even for the neutron there is a change in potential and hence there is a transmission coefficient factor that is defined in terms of the transmission coefficients. T₁ is called the transmission coefficients for particular I value. So, now this is the actual definition of the cross section

$$\sigma_l = \pi \Lambda^2 (2l + 1) T_l$$

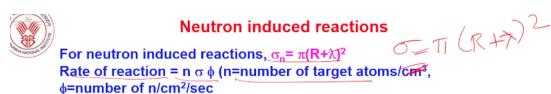
So, let us now try to see for the total reaction cross section will be the sum of the angular momentum dependent cross section for all L values up to L_{max} . So, L_{max} will be R/ Λ . So, we can sum the reaction cross section

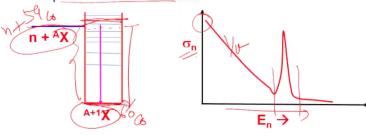
$$\sigma_{R} = \pi \Lambda^{2} \sum (2l + 1) = \pi \Lambda^{2} \left(2\Sigma l + L_{max} + 1 \right)$$

$$\sigma_{R} = \pi \Lambda^{2} \left(2\left(\frac{L_{max}(L_{max}+1)}{2} + L_{max} + 1 \right) \right)$$

$$\sigma_{R} = \pi \Lambda^{2} \left(L_{max}^{2} + 2L_{max} + 1 \right) = \pi \Lambda^{2} \left(L_{max} + 1 \right)^{2} = \pi \Lambda^{2} \left(\frac{R}{\Lambda} + 1 \right)^{2} = \pi (R + \Lambda)^{2}$$

Now, let us discuss the cross sections of neutrons and charged particles separately because the cross section varies in a different fashion for neutral particles like neutrons and the charged particles. So, just now we saw σ was nothing but $\pi(R + \Lambda)^2$. So, for the case of neutrons σ_n can be actually written as $\pi(R + \Lambda)^2$. So, here the rate of the reaction then will be in terms of $n\sigma\phi$ where n is the number of target atoms for neutrons and φ is the flux in terms of neutrons per centimeter square.





For low energy neutrons $\hbar >> R$ $\sigma_R = \pi \, \hbar^2 T_0$ as I=0 for low energy neutrons As E_n increases, \hbar decreases



Now, let us go a little bit deep into how the neutrons are captured by the nucleus. See for a neutron induced reaction which are always exoergic. Exoergic means whenever a neutron is captured by a nucleus energy is released equivalent to the binding energy of neutron in that nucleus. So, neutron plus target nucleus has higher mass compared to the compound that will be formed. So, suppose you write this the energy term.

So, this is the mass of the neutron plus target and this is the mass of the nucleus that is formed. For example, if you say neutron plus ⁵⁹Co then this is ⁶⁰Co in its ground state.

So when the neutron is captured by 59 Co this much energy is the binding energy that is released and so this 60 Co will be excited with this much energy of the order of 7 to 8 MeV and this excited nucleus then can emit gamma ray or it can have other channels also. So, neutron induced reactions, particularly the neutron capture reactions when a neutron is captured by a nucleus they are always exoergic and the energy released is equivalent to the binding energy of neutron in that nucleus. The cross-section for such reactions, neutron induced reactions you can see here σ_n is decreasing with the increasing energy of neutron, in fact which is called 1/v where v is the velocity of neutron.

So, neutron induced reaction cross-sections, the low energy neutrons are having higher cross-section I will be explaining this very soon and at certain intermediate energy there are resonances these resonances correspond to the nucleus populated in certain discrete energy states. So this is like a resonance when the neutron energy is such that the compound nucleus is populated in certain energy level then you would get a jump in the cross-section. When it is somewhere here where there is no level, cross-section will not be that much high. So now, the cross-section is falling with the increasing energy of neutron at low energy the wavelength of the neutron is Λ is much larger than the nuclear radius. You know that neutrons are used for neutron diffraction studies where the neutron diffraction means the crystals dimensions of the order of angstrom. So, low energy neutron, such as, thermal neutrons will have the wavelength of the order of dimensions of the atoms. So, this is much larger than nuclear dimensions and therefore, you will find that the energy of the neutron is increasing, wavelength is decreasing and so the cross-section is also decreasing to put it in a very simple way. So for low energy neutrons in fact the now you can see here this is the target nucleus then L equal to zero because first wavelength itself is more than the dimension of the nucleus and so you will have mostly zero L or S wave neutrons. So, cross-section can be written as

$$\sigma_n = \pi \Lambda^2 T_o$$

L equal to zero and as the energy of neutron increases Λ , the reduced wavelength, decreases, therefore the cross-section is decreasing with the energy of neutron.

Neutron induced reactions

Radiative Capture: (n,γ) Most common with thermal neutrons. Products are most often β - active.

197Au $(n,\gamma)^{198}$ Au $(\beta^-)^{198}$ Hg $(n,p), (n,\alpha)$ with lighter nuclei e.g. 32 S $(n,p)^{32}$ P, 35 CI $(n,\alpha)^{32}$ P (n,f): Common with actinides.

(I) Thermal neutrons: $\epsilon_n < 0.5 \text{ eV} < \epsilon_n < 1 \text{ keV}$ (III) Fast neutrons: $\epsilon_n > 1 \text{ keV}$

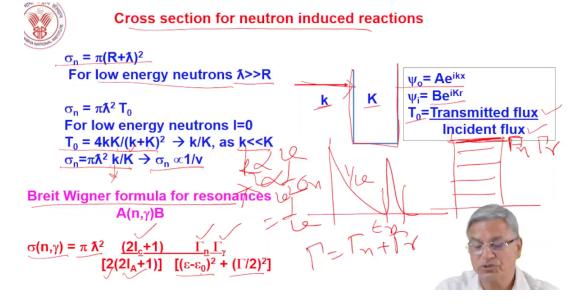
So, what are the different types of reactions that can occur with the neutrons? Most important reaction that take place is called the neutron capture or it is called the radiative capture.

Radiative capture means when the neutron is captured by the nucleus, the nucleus that is formed, the compound nucleus will emit gamma rays. So, that is the gamma radiation and so neutron capture followed by emission of gamma rays called radiative capture and this kind of reactions are the most common with the thermal neutrons. And as you will always see that when you add a neutron to a stable nucleus we are increasing the neutron to proton ratio and therefore most of the products of n,γ reactions are beta minus emitters because we are producing a neutron rich isotope. For example, 197 Au $(n,\gamma)^{198}$ Au and this 198 Au will be beta minus active and will decay to 198 Hg. So, with the heavier nuclei like cobalt, gold, uranium, tin, lead the (n,γ) is the most common reaction but when it comes to low Z target nuclei, sulphur, silicon, magnesium then you will find the reactions where charged particles are emitted, (n,p) reaction, neutron capture followed by proton emission, neutron capture followed by alpha emission, etc.

So, this kind of reactions are possible with the low Z target nuclei because the coulomb barrier for emission of protons and alpha are not very high. But for heavy nuclei the coulomb barrier for emission of alpha and protons are quite high and so such heavy nuclei only undergo n,y type of reaction but the light nuclei can undergo (n,α) and (n,p) type of reactions. When you go to still heavier elements like actinides then the thermal neutrons can induce even fission because fission barriers are low for actinides and so upon capture of a neutron like 235 U even thermal neutrons can induce fission in the heavy nuclei. Now when you see depending upon the energy the type of reaction that take place with neutrons, neutrons are classified into thermal neutrons having energy less than 0.5

eV. Epithermal neutrons, epithermal means the region where it is slightly higher than thermal and there are resonances in the cross sections as a function of energy of neutron, 0.5 eV to about the keV of energy and then more than one keV are called the fast neutrons. So you can see here that the thermal neutrons are most reactive and this classification actually has come from cadmium. Cadmium is absorbing all the thermal neutrons. So if you wrap a sample with the cadmium foil all thermal neutrons are captured and then the sample will not see the thermal neutrons.

From that this classification has come that if it is less than 0.5 eV then the neutrons are called as thermal neutrons. So we classify it into three zones, low energy zone where it is 1/v law, the resonance region and the fast neutron energy region. Thus to explain why cross section for neutron interaction follows 1/v if you recall we said 1/v law and then the resonances.



So we try to explain using the transmission of neutron through the potential well. The total cross section for the neutron is $\pi(R + \Lambda)^2$ and for low energy neutrons we said that Λ is much larger than the radius of the nucleus.

So you can actually set up an equation for transmission of the neutron. Neutron is like a wave and going to hit the nuclear potential of the nucleus. So initially you have a plane wave e^{ikx} and once it is captured the energy of neutron is much higher. So inside the nucleus is called e^{iKx} . So the wave number of the neutron is changing from k to K as it is transmitted through the potential well and the transmission coefficient is written as transmitted flux upon the incident flux. So general scattering theory of neutron actually can be used to find out the transmission coefficient and this transmission coefficient can be in fact if you solve the scattering equation for the S wave neutrons because for low energy neutrons the angular momentum is zero it can be written as

$$T_0 = \frac{4kK}{(k+K)^2}$$

and since K is much larger than k inside the nucleus neutron energy is almost 20 to 30 MeV and outside it will be thermal neutron. So you can neglect the small k inside the nucleus. So this k can be neglected with respect to K and so it becomes

$$\sigma_n = \frac{\pi \Lambda^2 k}{K}$$

So k is actually proportional to velocity Λ^2 proportional to $1/v^2$ lambda cross is h cross by μv and so net result is 1/v.

$$\sigma_n \propto \frac{1}{v}$$

That explains the σ_n varying as 1/v at low energy neutrons from the transmission coefficient. Then for the resonance region, Breit and Wigner gave a formula for resonances for (n,γ) reaction

$$\sigma(n, \gamma) = \frac{\pi \Lambda^2 (2I_c + 1) \Gamma_n \Gamma_{\gamma}}{\left[2(2I_A + 1)\right] \left[(\Sigma - \Sigma_0)^2 + \left(\frac{\Gamma}{2}\right)^2\right]}$$

So these Γ s are the widths of the nuclear levels that are called Γ n for neutron decay and $\Gamma\gamma$ for gamma decay they are the widths of the nuclear levels. So the resonance formula essentially quite accurately predicts the cross sections for the resonances for neutron induced reactions. So we will not go into details but I thought it is good to tell you that the resonances also can be explained by Breit-Wigner formula and the 1/v also can be explained by the transmission coefficient formula.

Charged particle induced reactions As projectile approaches target nucleus, projectile momentum decreases At distance of closest approach $p=\sqrt{2\mu(E_{CM}-V_C)}=(2\mu E_{CM})^{1/2}(1-V_C/E_{CM})^{1/2}$ $C_{max}=R(2\mu E_{CM})^{1/2}(1-V_C/E_{CM})^{1/2}$ $\sigma_R=\pi \, \hbar^2(Lmax+1)^2\to\pi \, \hbar^2Lmax^2$ $=\pi \, \hbar^2(Rp/\hbar)^2=\pi \, \hbar^2 \, R^2[(2\mu E_{CM})/\hbar^2](1-V_C/E_{CM})$ $=\pi \, R^2(1-V_C/E_{CM})$ $1/\sqrt{\lambda^2}$

Now let us come to the charge particle induced reaction. The scenario is different in the case of charge particles because there is a Coulomb potential here and so you try to consider a charge particle coming to a target nucleus. So there will be different types of potentials playing; one in the nuclear potential which is attractive at short distances and there is a coulombic potential which is repulsive all over. So when the projectile is coming close to the target nucleus it will experience the coulombic repulsion of target nucleus which starts becoming positive at a longer distance and as it is coming closer it will experience the attractive potential due to nucleus potential. So the sum total of the repulsive and attractive potential will be this dashed line and this shows a barrier which is called as the fusion barrier.

So for the light charge particle like proton and alpha mostly we can say that coulomb barrier this can be called as a coulomb barrier but when you take heavy ions into picture then the nuclear potential becomes significant and so we call it fusion barrier. So right now we will use this as a coulomb barrier. So this is the barrier the projectile has to cross. So now you see from infinity the charge particle is coming and as it is coming down it is momentum. So this is the center of mass energy, momentum is decreasing and at contact the momentum may become zero.

So this is what I tried to show here at distance approaches the momentum is

$$p = \sqrt{2\mu(E_{CM} - V_{C})} = (2\mu E_{CM})^{\frac{1}{2}} (1 - \frac{V_{C}}{E_{CM}})^{\frac{1}{2}}$$

So this is the momentum at infinity and this is the momentum at contact. The angular momentum at infinity and at contact has to be conserved and you take the help of this formalism to calculate L_{max} at contact

$$L_{max} = R \left(2\mu E_{CM} \right)^{\frac{1}{2}} \left(1 - \frac{V_C}{E_{CM}} \right)^{\frac{1}{2}}$$

So $R \times P$ is the angular momentum. So I have replaced linear momentum at contact with L_{max} equal to R into that momentum. And so sigma becomes

$$\sigma_R = \pi \Lambda^2 (L_{max} + 1)^2 \rightarrow \pi \Lambda^2 L_{max}^2$$

and since for charged particles you will find Lmax will be quite high so you can say $\pi\Lambda^2(L_{max} + 1)^2$, 1 can be neglected with respect to Lmax.

$$\sigma_R = \pi \Lambda^2 (Rp/\hbar)^2 = \pi \Lambda^2 R^2 \left[\frac{2\mu E_{CM}}{\hbar^2} \right] \left[(1 - \frac{V_C}{E_{CM}}) \right]$$

This P is nothing but the momentum at contact so momentum at infinity into the $1 - \frac{V_c}{E_{CM}}$ and this is nothing but 1 by lambda cross square momentum at infinity so this will cancel with this.

So you are left with

$$\sigma = \pi R^2 (1 - V_c / E_{cm})$$

So the cross section for charged particle induced reaction you can see here with E_{cm} as the energy of the projectile is increasing it will go up like this. So this is the V_c . Below V_c the cross section is 0, above V_c the cross section will increase with increasing energy in the center of mass system.

So the cross section for charged particle induced reactions are increasing beyond Vc, cross section for neutron induced reactions decreases as 1/v. So the charged particle induced reactions typically when the projectile is coming it can bombard with target form a compound nucleus and let's try to see what is the excitation energy of the compound nucleus



Charged particle induced reactions

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<sup>4</sup>He + <sup>93</sup>Nb → <sup>97</sup>Tc* (Compound nucleus)
Q = 2.425 + .87.209 - (-87.221) = 2.437 \text{ MeV}
V_c = 1.44*2*43/(1.4*(4^{1/3} + 93^{1/3})) = 14.05 \text{ MeV}
E(^4\text{He}) \ge 14.05*97/93 = 14.65 \text{ MeV}
For 30 MeV alpha
E^*(^{97}\text{Tc}) = E_{\text{CM}} + Q = 30*93/97 + 2.437 = 31.2 \text{ MeV}
Deexcitation of compound nucleus → emission of neutrons, protons, gamma rays, etc.
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so calculate Q value for ⁹³Nb+ ⁴He \square ⁹⁷Tc

mass of alpha + mass of niobium 93 - mass of niobium 97 this will be plus 2.437 MeV and you can calculate the coulomb barrier by

 $Vc = 1.4382 Z_1 Z_2 / [r_0 (A^{1/3} + A_2^{1/3})]$ you get 14.05 MeV. So let us try to calculate what is the minimum energy of the alpha that will be inducing the reaction or the threshold for the coulomb barrier. So 14.05 should be the energy available in the central mass

$$E_{lab} = E_{cm} *97/93 = 14.05 *97/93 = 14.65$$

So alpha particle should have 14.65 MeV energy to induce this reaction. This is the threshold for coulombic barrier. Let us take a case of 30 MeV alpha particle which is higher than the coulomb barrier. So in such a case excitation energy of the compound nucleus will be

$$E^* = E_{CM} + Q = 30*93/97 + 2.437 = 31.2$$

So the compound nucleus ⁹⁷Tc will be formed with an excitation energy of 31 MeV and then this excited compound nucleus can emit neutrons, protons, gamma rays depending upon the emission barrier and the angular momentum states available.



Charged particle induced reactions

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Proton induced reactions: {}^{18}O(p,n){}^{18}F, {}^{111}Cd(p,n){}^{111}In, {}^{68}Zn(p,2n){}^{67}Ga, {}^{201}TI(p,3n){}^{201}Pb, {}^{127}I(p,3n){}^{125}Xe, {}^{127}I(p,5n){}^{123}Xe Deuterium induced reactions: (d,p), (d,\alpha), (d,n), etc.

Alpha induced reactions: {}^{109}Ag(\alpha,2n){}^{111}In, {}^{93}Nb(\alpha,2n){}^{95}Tc, {}^{238}U(\alpha,4n){}^{238}Pu, etc.
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In case of charged particle induced reactions a hot excited compound nucleus can emit gamma ray, neutron, proton. There is a competition between emission of different types of particles. So now you can have proton induced reactions like 18O(p,n) ¹⁸F, ¹¹¹Cd(p,n) ¹¹¹In. So these are the reactions used to produce useful isotopes like ¹⁸F as a positron emitter, ¹¹¹In in nuclear medicine, ⁶⁸Zn(p,2n), (p,3n), etc.

These are all some of the useful isotopes how they are produced by bombarding proton on the target and that compound plus emitting neutrons. Deuteron induced reactions you can have (d,p), (d, alpha), (d,n) and alpha induced reactions you can have (alpha,2n), (alpha,3n) types of reactions. So these are the kinds of reactions that can happen with the charged particles. The charged particle is now forming a compound nucleus and the compound nucleus can emit neutrons, protons, alpha depending upon the excitation energy and the emission barriers for different particles. So I will stop here and then discuss next lecture how to determine the cross sections using experiment. Thank you.