Radioactive Decay

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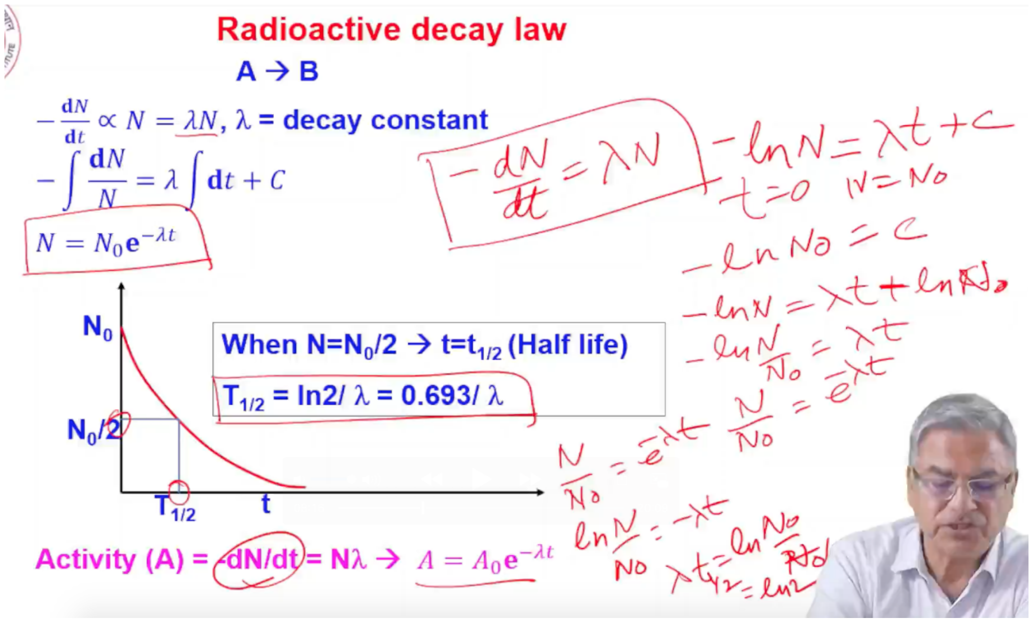
Lecture-1, Module-2

Hello everyone. In the first lecture, we discussed about the historical aspects of the phenomena of radioactivity and also how the different types of radio isotopes are there, whether they are occurring naturally or they are produced artificially and how they find their applications in different areas. Today, we will go into the fundamentals of radioactivity decay. And as usual, I will just briefly mention that at the end of this lecture, you should be able to answer these questions.

Questions

1. What is radioactive decay law?
2. What is half life and how to measure it ?
3. What is mean life of a radioisotope?
4. What are the units of radioactivity?
5. How does the activity of daughter nuclide depend upon that of parent activity?
6. How does the radioactivity decay in case of branching ?

For example, what is radioactivity decay law? What is half-life and how to measure it from the experimental data? What is mean life of a radioisotope? What are the units of radioactivity? How does the activity of a daughter nucleus depend upon that of the parent activity? And how does the radioactivity decay in the case of branching? So, we will discuss these details during the course of this lecture. Okay.



So, let us discuss first the radioactive decay law. We have just taken an example of a radioisotope A, which is undergoing some decay. It could be alpha or beta or gamma to B. So, the A is called the parent and B is called the daughter.

So, henceforth, once we say parent means it is the parent isotope which is decaying and whatever is formed, we call it daughter. Now the radioactive decay law is like chemical kinetics in chemistry. In chemistry, you have the order of a reaction, like if A goes B, you may say it is a first order reaction. Radioactive decay also follows the first order rate law. In the first order rate law, the rate of decay, that is if N is the number of atoms of the reactant at any point of time, then -dN/dt is the rate of decay of the atoms per unit time and is proportional to the number of atoms (N) at that point of time.

$-dN/dt \propto N$

And if you remove the proportionality sign, then lambda becomes the proportionality constant.

$-dN/dt = \lambda N$

This is the rate equation for radioactive decay, which is a first order rate equation. And if you want to find a solution of this equation, you integrate this equation. So, if it is an indefinite integral, you arrange the variables,

- $\int dN/N = \lambda \int dt + C$

Where C is the integration constant. The solution of this will become,

$-\ln N = \lambda t + C$

So, how do you find the C? when t = 0, N = N0. So, you put this condition,

$-\ln N_0 = C$.

So, you get

$-\ln N = \lambda t - \ln N_0$ or

$-\ln N/N_0 = \lambda t$

Which can be simplified to

$N = N_0 e^{-\lambda t}$

This is what is the expression for the decay of the radioisotope. So, N is the number of atoms decaying with time and the number of atoms decay exponentially with time, N_0 is the initial number of atoms. So, the exponential decay of radioactivity is the signature of first order rate law that governs the radioactive decay law. So, this I have shown here as a function of time, if you plot the number of atoms, then activity follows exponential decay. Now, from this graph, you can if you want to find out the term called half-life.

So, what is half-life? Half-life is the time when the number of atoms have become half. Suppose initially there were N_0 atoms, the time when the atoms left are $N_0/2$, we will call it the half-life. So, you can just draw on this graph, find out the time when the number of atoms have become half. So, from the exponential data, it may be difficult, graphically to find out the half-life, but we will discuss very soon how to make it more accurate. So, when $N=N_0/2$, t equal to $t_{1/2}$ and so you can substitute the value of t in this equation and find out what is the value of t half.

$\ln N/N_0 = -\lambda t$ □ when $N=N_0/2$, $t=t_{1/2}$

$\ln(1/2)= = -\lambda t_{1/2}$ □ $t_{1/2} = \ln 2/\lambda$

So, t half will become ln2 by lambda. How do you do that? We can see here that N upon N0 is equal to e raised to minus lambda t. So, take the logarithm ln N by N_0 equal to minus lambda t so lambda t equal to ln N0/N and substituting N as N0/2, and t by $1_{1/2}$ we get ln 2 equal to $\lambda t_{1/2}$. So, you can see here lambda becomes equal to ln $2/t_{1/2}$ or $t_{1/2}$= ln $2/\lambda$ where ln 2 is natural log of 2, that is, 0.693.

So, this is the relationship between half-life and decay constant. So, if you find out the half-life graphically from this exponential decay, you can find out the lambda and later on I will show you that if you find the lambda, then you can find out $t_{1/2}$ from another type of plot which I will show shortly. Now, most of the time, we do not determine the number of atoms. What we determine is the activity. So, the activity is nothing but the number of atoms decaying per unit time.

So, -dN/dt is nothing but activity, atoms decaying per second and it is given by N$\boldsymbol{\lambda}$. So, activity (A) is nothing but number of atoms into lambda. So,

$A = A_0\, e^{-\lambda t}$

If you plot activity as a function of time, this also will follow the exponential decay because lambda is a constant. When we do measurements, we do not measure the activity directly. We measure the counts. The detector system gives you counts and so if we say counts per second, we have to convert it into activity. There are some factors called detection efficiency and so on. So, what are the units of activity?

Units of Radioactivity

Activity (A) = -dN/dt = N.λ [Disintegrations per second]

1Bq = 1 Disintegration per second

1Curie = 3.7×10^{10} Bq

= Activity due to 1g of ^{226}Ra (Half life=1600y)

= (6.023×10^{23} atoms/226g)x0.693/(1600*365*24*3600s)

1 milli Curie = 10^{-3} Curie = 3.7×10^{7}Bq

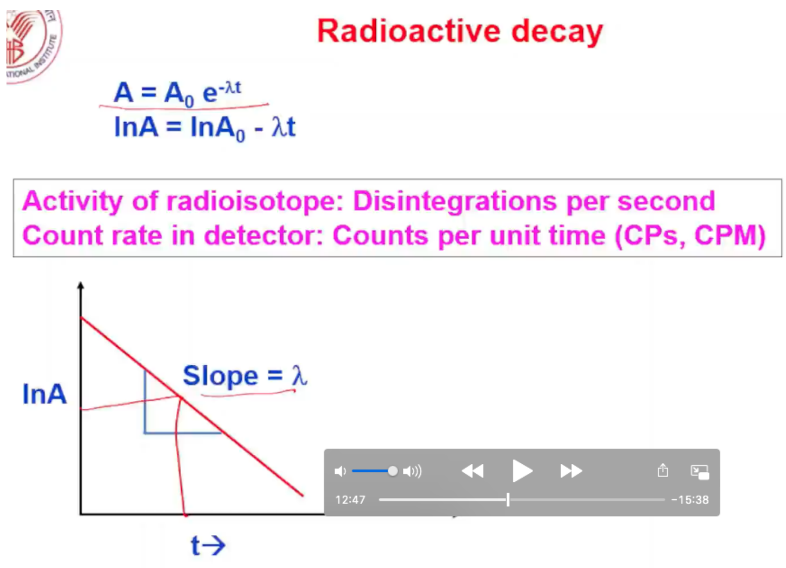
1 micro Curie = 10^{-6}Curie = 3.7×10^{4} Bq

Activity we will call as -dN/dt, number of atoms decaying per second, equal to N$\boldsymbol{\lambda}$. So, this is called disintegrations per second and to honor the discoverer of the phenomenon of radioactivity, we have a unit of Becquerel is equal to one disintegration per second.

Also, there is another unit of radioactivity called Curie in the name of Madame Curie and one Curie is 3.7 x10^{10} Becquerels abbreviated as Bq. So, you can see the Curie is a much bigger unit compared to Bq. So, how does this number come about? This 3.7 x10^{10} Bq actually is the activity due to one gram of radium-226. Radium was discovered by Madame Curie and therefore the activity of one gram of radium-226 has been defined as one Curie. Now, you can find out one Curie in terms of Becquerels. The half-life for radium-226 is 1600 years. So, you put in the equation activity equal to N$\boldsymbol{\lambda}$. Number of atoms in one gram of radium-226 will be Avogadro number (6.023×10^{23}) of atoms in 226 gram. This is the N in one gram into $0.693/t_{1/2}$ that is lambda. So, 1600 years into 365 days per year into 24 hours per day into 3600 seconds per hour. So, the units get cancelled and you have seconds. So, you have atoms per second and if you do the calculation it will come to 3.7 x10^{10} atoms per second or Bq.

Curie is a very very large unit. You can see that one gram of radium-226 is very difficult to handle. Normally we do not handle Curie level of activity We handle in fact even less than micro Curie level of activity. So, there are other units called milli Curie 10^{-3} Curie equal to 3.7 x10^7 Bq, micro Curie =10^{-6} Curie = 3.7 x 10^4 Bq. Sometimes we can have even nano Curie also.

So, for handling in the laboratory people use activity in few Becquerels. But if you are using for industrial applications like radiation source or irradiation you may be handling 100s of Curies. Now, let us go a little further.



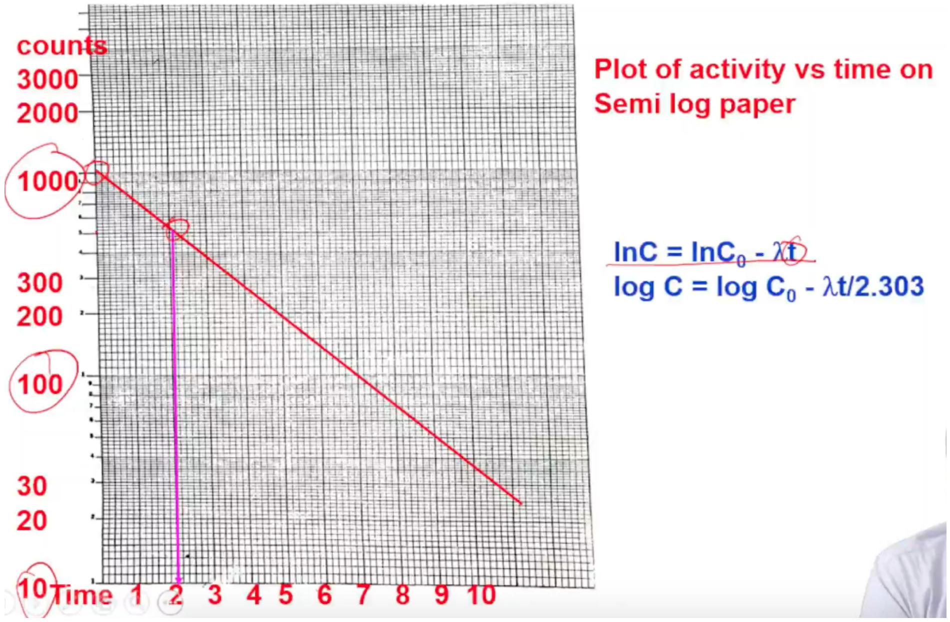
I was mentioning about determining the half-life from the decay data using the activity. So, activity is A = A0 $e^{-\lambda t}$, where A0 is the initial activity.

So, just a little bit of manipulation. If you see if I take the logarithm on both sides then,

$\ln A = \ln A_0 - \lambda t$

and now see this becomes a straight line of ln A versus t. So, you see here what I have plotted here is ln A on the y-axis and time on the x-axis and it becomes a straight line. So, it becomes easy to understand and handle also. So, the slope of this line is lambda and from the activity data also we can now find out the half-life using a paper which is called as semi-log paper because on the y-axis I have a logarithmic unit on the x-axis we have the linear unit.

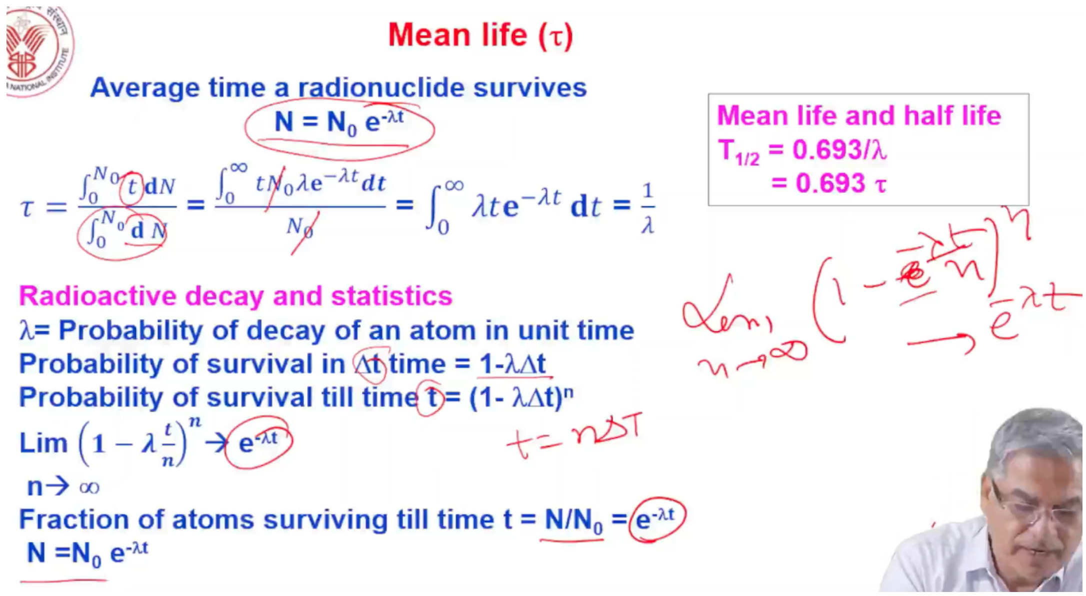
When we plot activity actually we will be plotting the count rate. So, the activity of radio isotope is disintegration per second, but when we have the experimental data we will have the count rate from the detector and the count rate can be counts per unit time, it could be counts per second, it could be counts per minute and so on. So, that I will discuss now in more detail and it would be very interesting if you understand to follow this semi-log paper.



What I have shown here is a semi-logarithmic paper on the x-axis we have the time let us say it is hours. So, 1, 2, 3, 4 hours. So, this is larger graduations you can see there are the bold lines on the x-axis you will find vertical lines parallel to y-axis there 1, 2, 3 this is the time in linear scale and on the y-axis you see the scale already made in logarithmic units. So, it is in cycles you have 10, 100, 1000. So, it is called three cycle paper. You can plot the data on three orders of magnitude 10 to 100, 100 to 1000, 1000 to 10,000 and so on. So, now when you plot the counts on this scale you do not have to convert the counts into logarithm because the scale is adjusted like that.

So, when I say 100 next point is 200, 300, 400 so on when you have 1000 here then you have 2000, 3000 and so on. So, if you understand how to use a semi-log paper, activity data handling is very simple. So, what I have plotted here the decay of a radioisotope on a semi-log paper and as I discussed in the previous slide also even here also you can see the logarithm of the counts versus time will be a straight line. Now, instead of natural log you plot here the log base 10 and so logarithmic plot of activity will have a straight-line behavior. And now from here suppose initial counts was 1000 when counts become 500 so this is 500 at this time and you can read on the x axis the time is 2 hours.

So, straight away from the semi-log paper we can find out the half-life in a much simpler way. Once you find out the half-life you can find out the decay constant lambda equal to 0.693 upon t-half. Now, there is another quantity called mean life. So, what is mean life? Mean life is the average time for which a radionuclide survives.



Half-life is when half the atoms have decayed or half the atoms have survived. So, from the radioactive decay law N= N_0 $e^{-\lambda t}$ let us find out how many atoms survive or what is the time of the survival of atoms on the average. So, if you have a function and you want to find out the average how do you find out? So, you want to find out the mean value of t. So, you can take the t dN integrate over 0 to N0 divided by the total number of atoms that is integration of dN over 0 to N_0.

$$\tau = \frac{\int_0^{N_0} t\,dN}{\int_0^{N0} N} = \frac{\int_0^{\infty} t N_0 \lambda e^{-\lambda t} dt}{N_0} = \int_0^{\infty} \lambda t e^{-\lambda t} dt = \frac{1}{\lambda}$$

So, now you can write here now dN equal to the derivative of N here. So, dN will be N0 lambda e raised to minus lambda t upon N0. So, you can see now the variable has changed from N to t and so the limits of t will become 0 to infinity instead of 0 to N0 because the radioactive decay that follows exponential decay now that the curve will meet the x axis at infinite time and therefore the time can go from 0 to infinity upon the integral of this will become N0. So, what you have here is now N_0 will cancel. So, you will have lambda t e raised to minus lambda t dt and if you solve this integral you will find it will become 1 by lambda as shown above.

You can do it at your home find out whether it becomes equal to 1 by lambda. Now, there is one more way of deriving the expression N equal to N0 into e raised to minus lambda t based on statistics. In statistics we say lambda the decay constant is the probability of decay of an atom in unit time. So, if that is the definition of lambda then what is the probability of survival in unit time 1 minus lambda. Lambda is the probability of decay and the probability of survival is 1 minus lambda.

You have a unit time interval let us say in Δt what is the probability of survival of an atom it will be $1-\lambda\Delta t$. And now you want to find out what is the probability of survival till time t. Then this time t we can define in terms of n Δt that means you divide this time interval into a large number of Δt intervals, say, n intervals. So, it becomes $(1-\lambda\Delta t)^n$. You just multiply the probabilities over the n intervals. $\Delta t = 1/n$

And now when Δt becomes very small or n becomes very large so n tending to infinity,

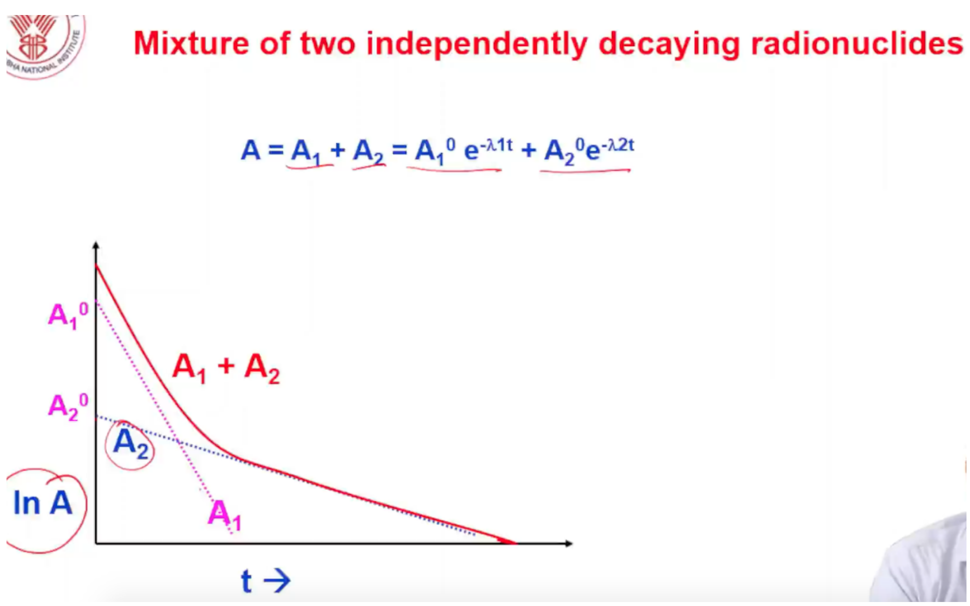
$\text{Lim}\left(1 - \lambda\frac{T}{n}\right)^n$ □ $e^{-\lambda T}$

1 minus lambda t upon n to the power n. So, this is actually you will see limit n tends to infinity 1 minus lambda t upon n to the power n will tend to e raised to minus lambda t. it will become e raised to minus lambda t. And so what you have here is the probability of survival till time t becomes e raised to minus lambda t when n becomes very large. So, if there are N_0 atoms in the beginning what is the fraction of atoms that survive till time t. So you have,

$N/N_0 = e^{-\lambda t}$.

So, that is how from the fundamental definition of lambda as a probability of decay of an atom per unit time also you could derive the expression for radioactive decay.

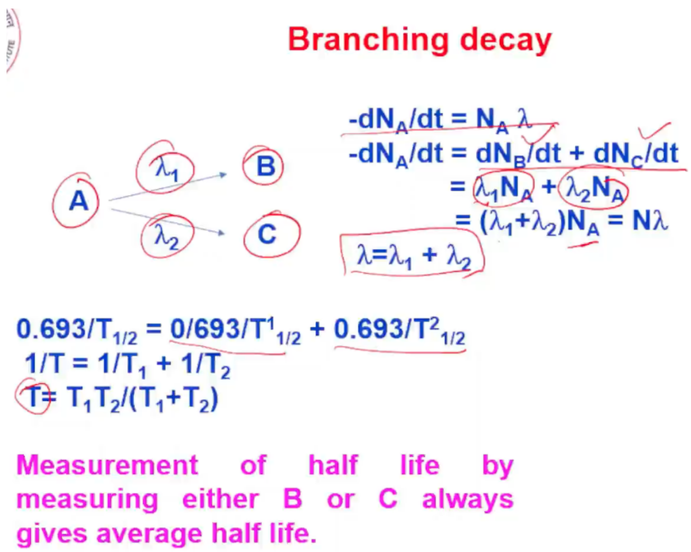
Now, there will be many situations when you will encounter two independently decaying radioisotopes.



So, let us say, A_1 and A_2 are two isotopes decaying independently, they are independent of each other. So, A_1 is decaying with its own half -life, A_2 decaying with its own half-life, but it is a mixture of activity.

$A = A_1 + A_2 = A_1^0\, e^{-\lambda 1t} + A_2^0 e^{-\lambda 2t}$

So, when you plot such a activity on a semi-log paper. So, when I am saying lnA essentially I am plotting on a semi-log paper. So, you see here the total activity will be the red line. Now, how to resolve such a data on a semi-log paper when the half life of the two isotopes are different at a much later time when the short lived isotope has decayed you can extrapolate the linear portion to zero time and what you get is the activity of the long lived isotope and when you subtract the activity of this long lived isotope from the total activity you get this data that is the pure activity of short lived isotope. So, extrapolation to zero time gives you initial activity of the two isotopes and from the slope of these two you can find out their half-life. So using semi-log papers you can resolve the mixture of two activities into individual isotopes half lives and their initial activities.



There is another case of branching decay that means the same radioisotope decays by two modes. So, A going to B and C with different decay constants lambda 1 and lambda 2. Now, in such a case the decay of the parent we can say $-dN_A/dt$ equal to $N_A\boldsymbol{\lambda}_A$ which is equal to sum of the growth of B and C, that is $dN_B/dt + dN_C/dt$, as written below.

$dN_A/dt = N_A\, \lambda$

$-dN_A/dt = dN_B/dt + dN_C/dt$

$= \lambda_1 N_A + \lambda_2 N_A$

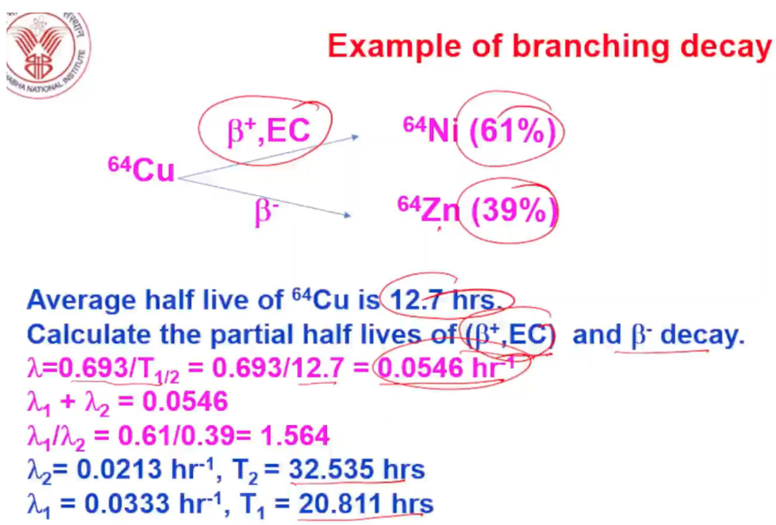
$= (\lambda_1+\lambda_2)N_A = N\lambda$ □ $\lambda=\lambda_1 + \lambda_2$

So, the decay to B and C can be written as $N_A\boldsymbol{\lambda}_1$ and $N_A\boldsymbol{\lambda}_2$, these are two branches of decay of A. So, this $\boldsymbol{\lambda}_1 N_A$ is first part $\boldsymbol{\lambda}_2 N_A$ is second part you take N_A as common which gives, $(\boldsymbol{\lambda}_1+\boldsymbol{\lambda}_2)N_A$. So, the the decay constant for the radio isotope A is the sum of the decay constant for the two branches it is like you know if you have a tank and it has got two outlets to empty it and you open both the taps then the tank will empty and depending upon the diameter of the two taps you know the flow will be different the time taken to empty will be different. So, it is similar to that. So, this equation, $\lambda=\lambda_1+\lambda_2$ and you can replace λ by $0.693/t_{1/2}$ as given below.

$0.693/T_{1/2}(A) = 0/693/T_{1/2}\,(1) + 0.693/T_{1/2}(2)$
$1/T = 1/T_1 + 1/T_2$
$T= T_1T_2/(T_1+T_2)$

This give the relationship between the half partial half lives for the two branches to the parent half lives. But when you measure the activity of either B or C the half life that you will get that of the parent because if you measure the rate of water emptying from a tap with both taps open, using one tap or another, the tank is getting empty anyway it is decaying by both the taps. So, the half life that you will get while measuring either B or C will be the average half life of the radioisotope. So, this is important.

I will give you an example where this is the isotope copper-64. The copper-64 can decay by both β+ and β-.



So, in β+ decay it decays to nickel-64 and in β- decay it decays to zinc-64. Now, when you do the measurement of half life of copper-64 you get 12.7 hours. So how do you calculate the partial half-lives for decay of copper-64 by beta plus decay or beta minus decay. I am writing here for beta plus electron capture (EC) that we will discuss in the nuclear decay. But right now we will assume that this is the one channel β+/EC as one channel. So, when beta plus decay can not occur then there is a competing mode of decay to beta plus decay that is called electron capture. We will discuss this more details in the nuclear decay. So, now decay constant for copper-64 is $0.693/T_{1/2}$, $T_{1/2}$ is 12.7 hours. So, you can write it equal to 0.0564 $hour^{-1}$. Now, how do we get the partial half life for decay into nickel-64 and zinc-64? We set up two equations, as given below.

$\lambda=0.693/T_{1/2} = 0.693/12.7 = 0.0546\ hr^{-1}$
$\lambda_1 + \lambda_2 = 0.0546$

$\lambda_1/\lambda_2 = 0.61/0.39$, $\lambda_1 = 1.564*\lambda_2$
$1.564\lambda_2 + \lambda_2 = 0.0564$, $2.564\lambda_2 = 0.0564$,
$\lambda_2 = 0.0546/2.564 = 0.0213\ hr^{-1}$, $T_2 = 0.693/0.0213 = 32.535$ hrs
$\lambda_1 = 0.0546-0.0213 = 0.0333\ hr^{-1}$, $T_1 = 0.693/0.0333 = 20.811$ hrs

So, here the branching intensities are given. 61 percent of the time nickel-64 is formed and 39 percent of the time zinc-64 is formed. So, I can write lambda 1 upon lambda 2 is 0.61 upon 0.39 and that is 1.564. So, I have two equations lambda 1 plus lambda 2 equal to 0.0546 lambda 1 by lambda 2 0.61 by 0.39, that is, 1.564. So, I can solve these two equations to find out lambda 1 and lambda 2. So, lambda 2 is equal to 0.0213 hour inverse lambda 1 equal to 0.0633 hour inverse and now you can convert this lambda into half life they are called as the partial half-life.

You see here the partial half life for decay to nickel-64 is 20.8 hours the partial half life for decay to zinc-64 is 32 hours and the average half-life is 12 hours. So, you just see here the partial half lives are more than the average half-life because it is decaying by both routes. So, it is less than both the half lives. So, what we discussed today was the radioactive decay and different types of decays like the decay of a mixture of radio isotopes or the branching decay of a radio isotope into different modes like beta plus, beta minus and so on. And so in the next lecture I will discuss the radioactive decay chain and the growth and decay of the daughter products. So, I will stop here. Thank you very much.