Interaction of fast electrons with matter

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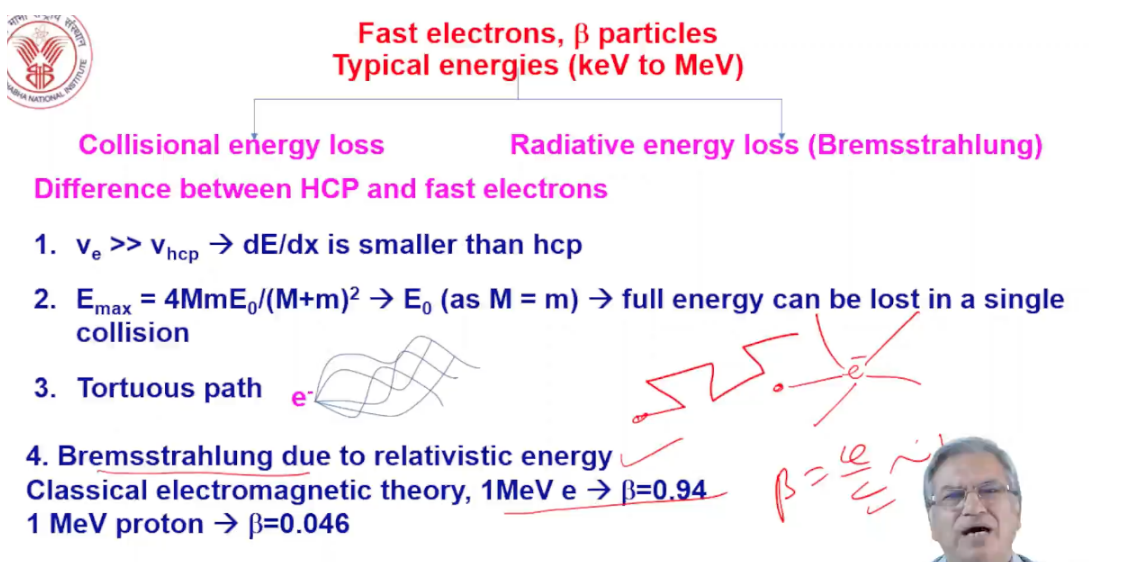
Lecture-8, module-1

Dear students, in the previous lecture, I discussed about the interaction of heavy charged particles with matter. And we found that the important properties of the heavy charged particles with regard to their interaction are the stopping power. The stopping power is nothing but the loss of energy in the unit thickness -dE/dx. And we also saw how that is related to the energy and charge of the ion. So there is a term depending upon the property of the ion. There is another term depending upon property of the absorber medium that is NZ, electron density, number of atoms per cc into the atomic number of the material. So it is essentially the electrons per cc.

So heavy charged particles are losing energy relatively much faster because they do not travel much because of the higher mass, higher charge. So mZ^2/E as you recall the previous lecture, as the energy decreases, the stopping power increases. As the Bragg curve signifies at the end of the journey, finally the velocity of the ion becomes less than velocity of electrons and the heavy charged particle picks up electrons and then stops. At the end of this track or the range, the stopping will be purely by the nucleus. But in the beginning it is electronics stopping wherein the ionization and excitation are the dominant modes of interaction.



Now we will discuss the interaction of fast electrons with the matter which also includes the beta particles, beta plus and beta minus. So the fast electrons or beta particles, the interaction mechanisms are same. Even positrons will behave the same way and the typical energies of this fast electrons or beta particles that we handle are in the range of few keV to few MeV.

Now in terms of the type of mechanism for interaction of these fast electrons with the matter, we can say that there are two types of processes that occur. First is the collisional energy loss. That means the electrons collide with the electrons of the medium through which they are passing and they can cause the ionization and excitation very much like in the case of heavy charged particles. But in addition to this, there is another mode of loss of energy and that is the radiative energy loss in which case the continuous radiation called Bremsstrahlung is emitted when the fast electrons are passing through the medium. We will discuss more on this subsequently.

The major differences between the interaction of heavy charged particles with fast electrons let us discuss this first. Firstly, the velocity of electrons is much, much higher than the velocity of heavy charged particles for the same energy that is few keV to few MeV. So because of that, you know, their velocities are very high, the $\Delta E/dx$, the stopping power for fast electrons is much lower than that for the heavy charged particles. If you recall the stopping power formula, $-dE/dx = mz^2/E$, in fact it does not remain the same when it comes to electrons, but the velocity comes in the denominator and so velocity is high, that means the stopping power is low, low SP.

The second important point is the energy transferred by electron to the electron in the medium in one collision. So considering the same equation as in case of heavy charged particles, the maximum energy an electron can gain when it is struck by the incident particle, may be alpha or electron given by $4MmE_0/(M+m)^2\cos\theta$. Maximum energy is lost when $\theta = 0$. Now if you substitute for M as mass of the electron, so both M and m are same in the case of electron interaction. And so this would become

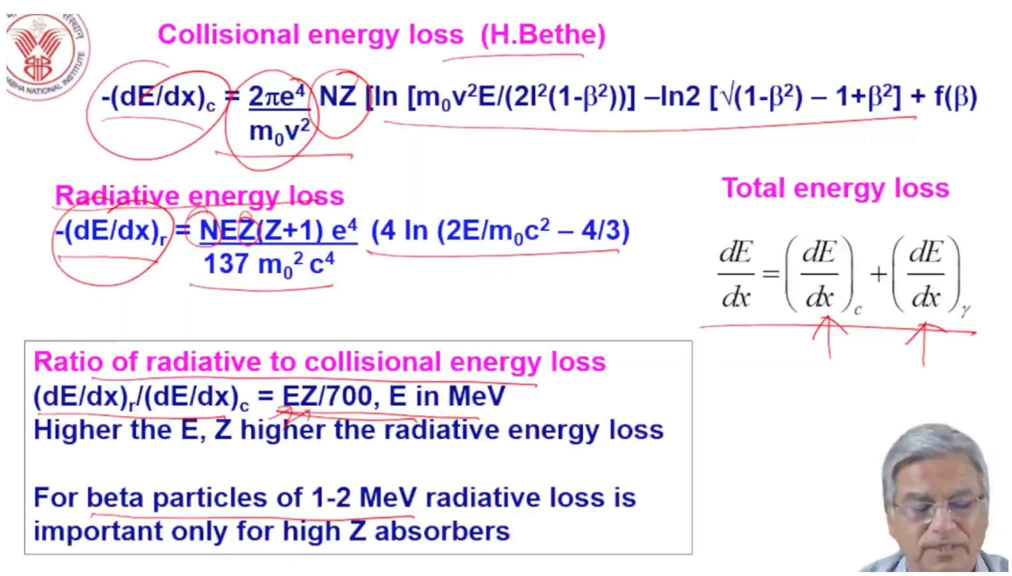
$E_{max} = 4mmE_0/(m+m)^2 = E_0$ as $M=m$

That means the electron can lose all its energy in single collision when it is colliding with the electron. Because of these processes you will find that the electron can get back scattered, scattered in all angles in a single collision. So this is a major difference when it comes to interaction of electrons.

The third difference is that the electrons move in a tortuous path. The reason for that is, second point, that is the electron can lose energy in a single collision. So because of that, suppose you have an electron here, which is colliding with an electron, this electron can get scattered in different directions. And so because of that, the way electron is going, it is scattered, it can further get scattered whenever it is interacting with an electron, it can go. So this electron can go in a zigzag path. And so you can say the different electrons will be moving in the tortuous path, like a tortoise in a zigzag fashion. So then you cannot define a well-defined range for the electrons because their motion is tortuous. That is because of the large energy transferred in single collision.

Another major difference is that when the electrons are interacting with the medium, particularly at energies which are of relativistic order, there is an emission of a continuous gamma radiation. Bremsstrahlung, that Bremsstrahlung is derived from the word breaking radiation. That means when the electron is accelerated or decelerated in the vicinity of a nucleus because of this electromagnetic interaction, the electron loses energy in the form of photons. That can be explained from electromagnetic theory.

This happens when the energy of the electron is of the order of relativistic energy, that means, β=v/c, v is the velocity of the electron, c is the speed of light. And when it is close to 1, then the electron is relativistic. And so that happens for the electron when the energy is of the order of 1 MeV. And therefore, the MeV order of energy electrons will undergo a large fraction of energy will be lost through the Bremsstrahlung emission. The same thing doesn't happen in case of protons of same energy because for 1 MeV proton, the beta value will be around 0.046, which is much, much less than 1. Therefore, protons are not relativistic energy at 1 MeV. The result of that, you don't see Bremsstrahlung in the case of the heavy charged particles. At high energy it may be possible, we will discuss that later on subsequently.



So let us first discuss the collisional energy loss and radiative energy loss with regard to the stopping power. We will not go into details of the derivation of this formula because they are quite complicated. And so here instead of one type of collisional stopping power, we have two types of losses. One is the loss in a collision with electrons. So they are called collisional energy loss. And this collisional energy loss again has the two terms, the electronic properties, the ion, single ion that is impinging

$$-\left(\frac{dE}{dx}\right)_c = \frac{2\pi e^4}{m_0 v^2} NZ[\ln ln\left[\frac{m_0 v^2 E}{2I^2(1-\beta^2)}\right] - ln2[\sqrt{1-\beta^2})\text{-}1\text{+}\beta^2] + f(\beta)$$

where v is the velocity of electron and NZ is the electron density in the medium. And this is term again, the velocity dependent, is a relativistic term. In the case of beta particles and fast electrons, relativistic term becomes important because beta is significant close to 1. But let us not go into too much details of the relativistic term. It is simply sufficient to say that in the collisional energy losses again depend upon the velocity of the electron and the electron density of the medium in the fashion similar to that of the heavy charged particles.

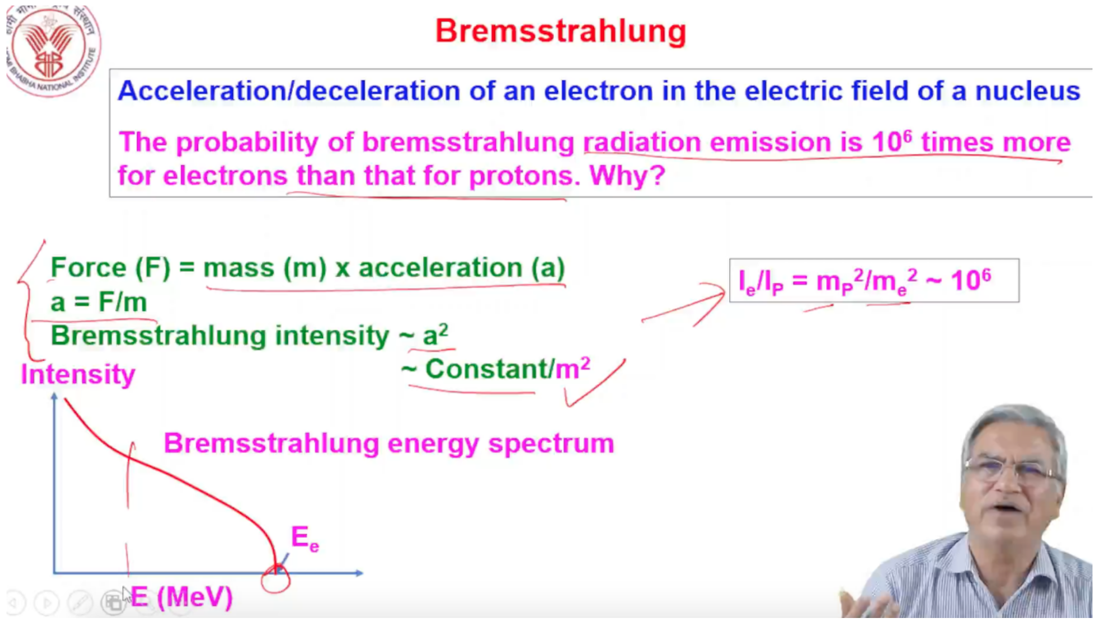
And the radiative energy loss, they are called radiative $-\left(\frac{dE}{dx}\right)_r$ is again given in terms of the electron, the atom density.

$$-\left(\frac{dE}{dx}\right)_r = \frac{NEZ(Z+1)e^4}{137 m_0^2 c^4}\left(4ln\left(\frac{2E}{m_0 c^2} - \frac{4}{3}\right)\right.$$

So electron is going to lose energy not only by collision with electrons, which leads to ionization and excitation, but also by loss in terms of emission of electromagnetic radiation that is Bremsstrahlung. And so the total energy loss now by the electron is sum of the collisional energy loss plus the radiative energy loss and for the higher energy electrons and particularly you know, if it is going to stop in a high Z medium, then the radiative losses become very very significant. Just to give you the feel of that, the dominant role of radiative energy losses, the ratio of radiative to collisional energy loss. So this upon this, radiative upon collisional losses, if you see these two terms, in the simple way, it can be written as EZ /700. We will not bother about the terms how it has been arrived at, but E is the energy in MeV, Z is the atomic number of the absorber upon 700. So it is a very simplified way of finding out the ratio of radiative to collisional energy losses. So what it tells is that higher is the energy of the ion, higher is the radiative energy loss, higher is the Z of the absorbing material, higher is the radiative energy loss. So if you are having a high Z material in the vicinity of a beta emitter, then you will have more radiations.

$$\left(\frac{dE}{dx}\right)_c / \left(\frac{dE}{dx}\right)_r = EZ/700$$

So that is why you know, when you are handling beta sources, you have to be very careful about nearby high Z materials. So for beta particles of the order of 1 to 2 MeV, in fact, radiative energy losses are not that high, but if it is a high Z material, then the radiative loss becomes significant for beta particles of this energy domain. So when you do beta counting, the absorber, the plate on which you make the source becomes important. You should not make the source on a high Z backing material. You can make on aluminium, not on stainless steel or any other high Z material.



So let's now discuss more the Bremsstrahlung. The Bremsstrahlung, as I mentioned, it derives from the name breaking radiation. That means suppose you apply a break. So when you apply a break on electrons, electron is losing energy. So how does it lose energy? Say if it is going in the vicinity of a nucleus, a nucleus has got charged particles like protons.

So electron can be accelerated in the vicinity of the nucleus or it can be decelerated. Then any accelerating or decelerating charged particle loses energy by means of emission of electromagnetic radiation. So essentially the electric field of the nucleus, if the electron is accelerating or decelerating, then it will emit electromagnetic radiation. And this electromagnetic radiation is in fact, in the case of electrons, it is 10^6 times more for electrons than that for protons.

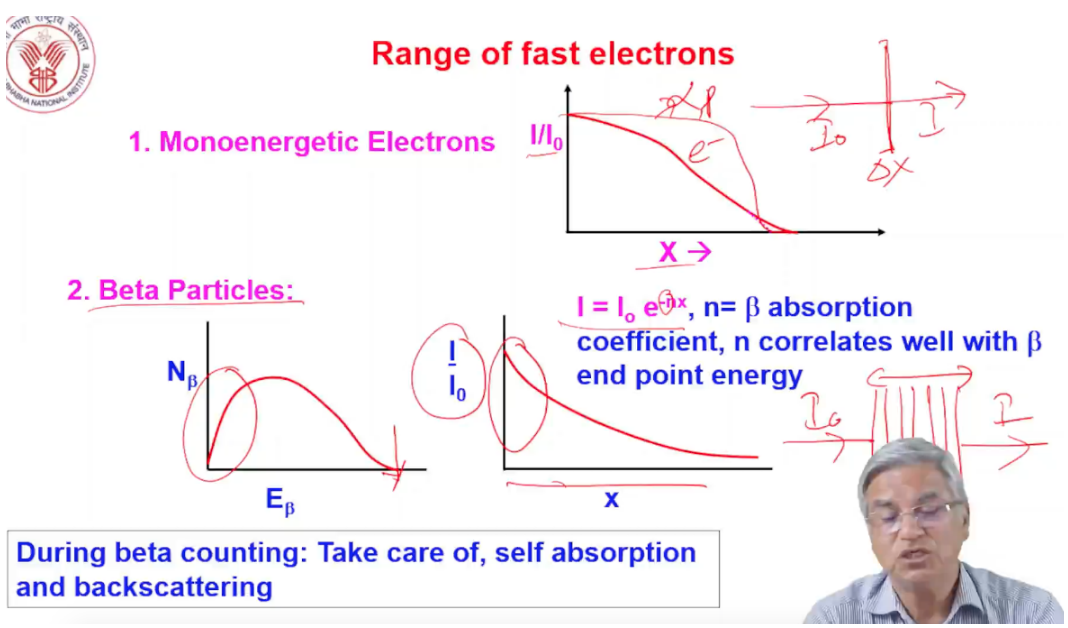
So why it is so that we did not have the Bremsstrahlung type of radiation in the case of protons and alpha particles. So the explanation lies here. That is the coulombic force between the electron and the nucleus, we can say mass into acceleration, $F=m \times a$. So the acceleration is $a = F/m$. And the Bremsstrahlung intensity is proportional to square of acceleration. So acceleration square, so if you assume that other things are constant, then it is $1/m^2$. So the Bremsstrahlung intensity is proportional to $1/m^2$, where m is the mass of the accelerating ion. So now based on this, you can say the intensity of Bremsstrahlung for electron divided by that for protons can be in terms of inverse ratio of their masses, that is the mass of the proton square upon mass of electron square.

$I_e/I_p = m_p^2/m_e^2 \sim 10^6$

And you know the ratio of mass of proton to mass of electron is 2000 and square of that is of the order of million times. So that explains the Bremsstrahlung intensity for fast electrons a million times more than that for the protons. So as a result of that you would find we do not have Bremsstrahlung for protons and other particles of energies of few MeV. But you can see that when the proton becomes relativistic, when the proton energy becomes GeV, giga electron volt. In fact, there are now machines which are running at 1000 GeV. So at 1000 GeV, proton becomes relativistic and you will find that the, you will have the high energy radiation emitted by protons. So only we are talking about the low energy at MeV range, protons and other particles, we do not talk about Bremsstrahlung. But if you have relativistic protons and other particles, they will also emit Bremsstrahlung.

Now this Bremsstrahlung has a continuous energy spectrum. So if you have electron of energy this much, then maximum energy of Bremsstrahlung can be energy of the electron. It is a continuous spectrum and the average energy will be somewhere one third of the maximum energy. So Bremsstrahlung gives you a continuous spectrum with average energy one third of the maximum energy of the fast electron. This is in fact a mechanism to produce gamma rays from electrons. These days you can easily have high electron, high energy electron accelerators.

So if you want to have an intense source of photons, you can use high energy electron sources and bombard them on high Z material like tantalum and you can get or tungsten, you can get photons of all energy spectrum.



So now I like to discuss the range of fast electrons. What I am showing here is that the intensity of the initial electron is I_0, it is passing through a thin slice of material and the transmitted intensity is I, this is Δx. Then I/I_0, that is called the transmitted intensity for fast electrons. If it is a monoenergetic electrons, then it will be just monotonically decreasing the intensity of this because the electrons get lost from the path in the beginning itself, unlike in the case of alpha particles. For alpha particles, we have like this, this is for alpha or protons.

For electrons, you will find electrons start getting lost from the path from the beginning itself. And that is why we cannot define, we do not have a well defined range for fast electrons. So normally, there is no point doing an experiment to determine the range of electrons for monoenergetic electrons. But for beta particles, what happens? The beta particles, it has been found experimentally that the transmitted intensity follows an exponential decay that is given by this expression

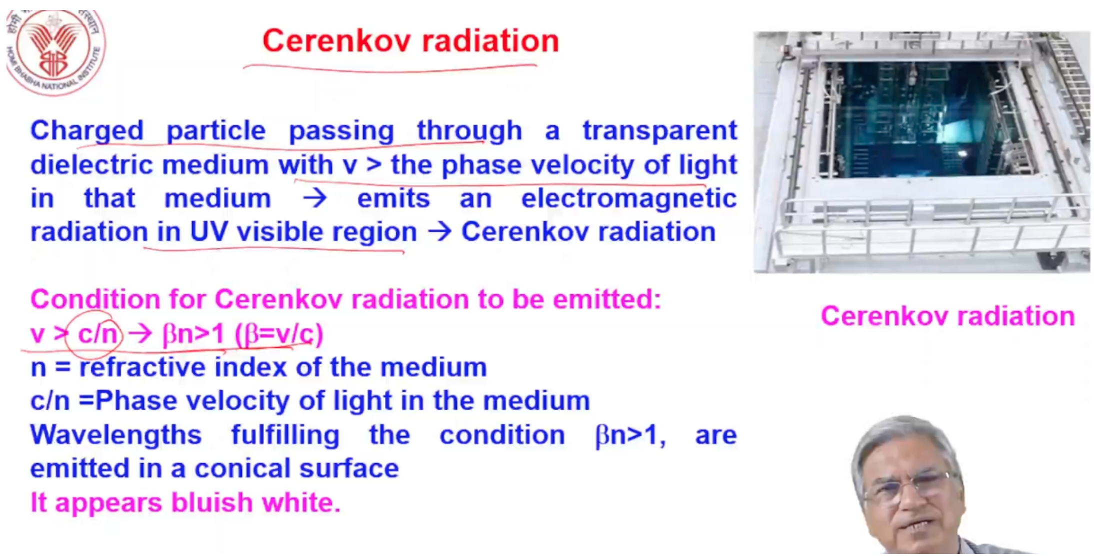
$$I = I_0 e^{-nx}$$

where n is called the beta absorption coefficient. And this n, the coefficient correlates well with the beta end point energy. What is the end point energy? The maximum energy of beta particle. Now you know that the beta spectrum is a continuous spectrum. Why it is continuous spectrum? We discussed in the beta decay that because of the emission of three particles, So you have an electron, you have a neutrino and you have the daughter nucleus. So the energy, the Q_β, is shared in infinite number of ways among the electron, neutrino and heavy residue. And therefore, this continuous spectrum of beta is responsible for the exponential decay. So it does not have any phenomenological explanation why beta decay transmission graph shows an exponential decay. So what essentially happens is that the low energy part of the beta spectrum gives rise to the initial part of the exponential decay and the high energy component of beta spectrum gives you the later part. So it is a fictitious type of correlation. It does not have any particular basis based on a particular concept. But it became very handy in the earlier times when we did not have the advanced detector systems. Then people are determining what you do, you take a source of beta and you put foils of different thickness, different foils. You are measuring the loss in the intensity as a function of thickness, keep on putting different foils and what fraction is going out. So this is the experiment, this is the so many files to put, you generate this graph.

So I and I_0. And what they found is an experimental finding that the beta decay attenuation, intensities attenuate in an exponential fashion and this attenuation factor absorption coefficient n, it has got an excellent correlation. And so from the n value, one could identify the end point energy. At that time, the radio chemistry people did not have access to high resolution equipment, like a mass spectrometer. Now you can determine this energy spectrum by a mass spectrometer.

So you want to find out the end point energy of beta, you just do a simple attenuation experiment like this one, you can find out what is the energy of this beta. So beta counting in fact requires that you need to really take care of this attenuation. So if you have a thick absorber, for example, you are doing beta counting. How do you do beta counting? You have a sample on a plate, you put this precipitate and you put it in a, you have a detector. So beta will go from here to here, you detect the beta.

Now if this absorber material is having high Z, like lead or stainless steel or tantalum, then there will be backscattering, one is backscattering and second is the self-absorption. So this precipitate will be absorbing, it will absorb and third may have the radiative losses. So if it is a high Z material, Bremsstrahlung can also take place. So when you do beta counting, one has to be very careful not to use high Z backing material for beta.



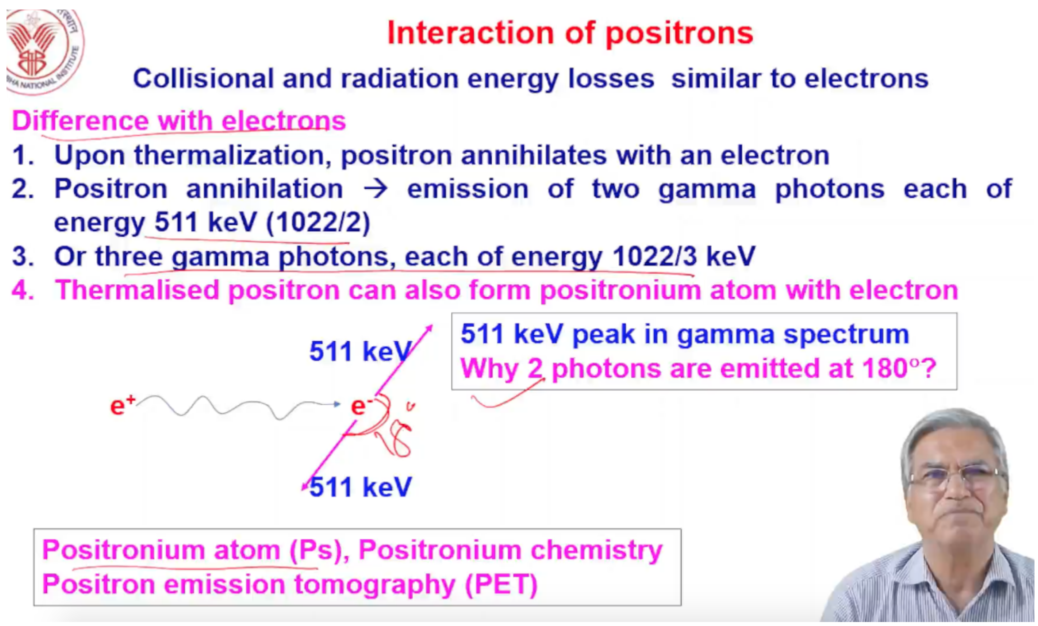
I did not discuss this part in the beginning, one of the dominant mode of interaction of half electron because the energy lost by this mode is a very small fraction. So it does not really matter when it comes to loss of energy by electron, but it is a very fascinating topic to discuss. And in fact, it is also used in the detection of electrons and this is called the Cerenkov radiation. Some people call it Cherenkov, I will say, you can use this term Cerenkov. So what is Cerenkov radiation? So the blue light that is emitted in a swimming pool type of reactor, so that is a manifestation of the Cerenkov radiation that is emitted and the beta particle is passing through a dielectric media. So what is Cerenkov radiation? When a charged particle passes through a transparent dielectric media with velocity more than the phase velocity of light in the medium, then this charged particle emits electromagnetic radiation, which is in the UV visible range.

So the energy of the photons that are coming out is very small. So a tiny fraction of the electron energy is utilized in Cerenkov radiation. But it is very interesting to see, even this one, if you put appropriate photo-multiplier tube, you can detect the electrons energy, electron intensity by means of this Cerenkov detector. So what exactly happens? What is the mechanism of this one? So what happens that, first of all, the condition for this Cerenkov radiation is that the velocity of electron has to be more than c/n. What is c? c is the speed of light, which is univrsal constant. n is the refractive index of the material through which the particle is passing through. Now if you see the v/c, velocity of the electron upon c velocity of light, called β, then it becomes βn> 1. So the condition for Cerenkov radiation is βn is more than 1.

Now how can the βn be more than 1? The velocity of the ion can be close to c at the most. So it cannot be more than c. Relativistic ion we just now saw, 1 MeV electron has a beta value 0.94, but it is not more than c. So if you have a medium of high refractive index, like glass n=1.5, there are certain materials, dielectric materials which are transparent and have the refractive index close to 1.5, then c/1.5. So the phase velocity of that light in that medium becomes much less than c. And therefore, velocity of the electron becomes higher than the velocity of, phase velocity of light in that medium. And as the electron is moving, essentially the concept behind that, that electromagnetic field associated with the electron, electron is moving in the direction, it has got electromagnetic field associated with, that field moves with velocity c/n. And so the electron is moving at much higher speed than the velocity of the field associated with it in that medium. And it is like, you know, the field is left behind, the electrons.

And so that light is emitted because of the field leaving behind the ion, it is emitted in a cone. And this condition is made for a very small time, because the once the electrons loses energy in the medium, that condition will not be met. So till that condition is met, which happens in only in the high energy part of the beta, then you will find for that particular time, in the initial phase of this journey, it will emit that light. And that light happens to be in the blue region.

That is why the Cerenkov radiation is having colour in blue range. So this is a typical swimming pool reactor, Apsara-U in Mumbai. And you can see the beautiful blue color in the swimming pool. So any swimming pool type of reactor, if there is fission taking place, then the fission products are emitting beta particles of high energy and you will see blue light. So Cerenkov radiation, in terms of energy loss is not much, but it is important in terms of the beauty of the radiation and the detection of the beta by Cerenkov radiation.



Lastly, I will discuss the interaction of positrons. Positrons are also anti-electrons in terms of collisional energy loss, radiation energy loss, they are similar to electrons, there is no difference, but there is a subtle difference between electrons and positrons. That is positrons, when they thermalize in the medium, the positron comes close to an electron and annihilates with an electron. Electron-positron-annihilate and you get two photons, the rest mass energy of electron-positron is 1.022 MeV. That energy is now emitted in the form of photons. And since the electron and positron as they thermalize the momentum is zero, the two 511keV photons are emitted at 180 degree. So this positron annihilation gives you two 511keV gamma rays which are emitted at 180 degree. Why at 180 degree? Because the momentum is zero. When the positron is thermalized, its momentum is zero, electron is assumed to be already stationary or very low energy. So at that time when the system is annihilating, mass is converted into energy and hence two photons have to have zero momentum and that is possible when it is emitted at 180 degrees.

In fact, there is another mechanism of three gamma emission. When the three gamma emission happens, then they have to be 120 degree each. So, but the three gamma probability is much smaller than the two gamma and in fact, very rarely you see three gamma. So the reason for the two photons emitted at 180 degree is because of the zero momentum.

Another very interesting aspect is the positronium atom formation. In fact, the positron and electron can form an atom like hydrogen atom, proton and electron, hydrogen atom, positron and electron, positronium atom. And the positronium atom has a beautiful chemistry. So there is a subject of positronium chemistry itself.

And lastly, the positron emission tomography. The fact that the two photons are emitted 180 degree is utilized in detecting the tumors in the human body by injecting a positron emitter like Fluorine-18, carbon-11, oxygen-15 and so on. So many positron emitters when they are put in the body and they emit positron, the positron will annihilate and give two 511 keV gamma rays and they can be used to detect the position of the tumors in the human body.

So we have a lot of applications of this positron. I thought I would touch upon this aspect, this particular. So I will stop here. Thank you very much. Next time I will discuss the gamma ray interaction. Thank you.