Beta decay

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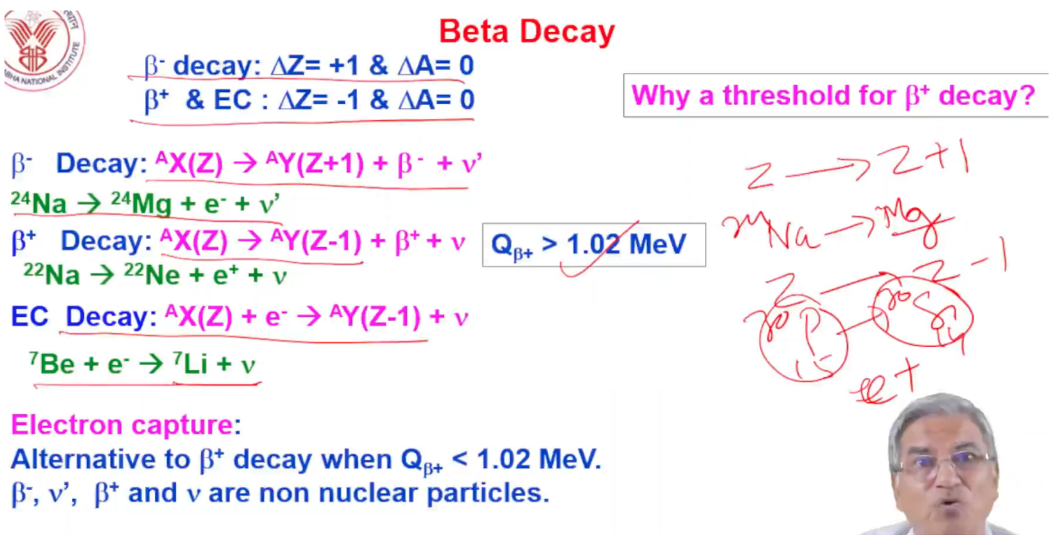
Lecture-6, module-2

Hello everyone. In the last lecture, we discussed the α decay and we observed that α decay is seen in the heavy nuclei, particularly the masses more than 200. But of course, up to 150 to 200 also α decay has been seen with very, very long half-lives. And then we also discussed that the half-lives for α decay could be explained by considering the barrier penetration formula. And also we discussed the systematics of α decay among the isotopes and the isobars of a particular element of isobaric chain. Today we will discuss the β decay.



As all of you know that β decay happens along an isobaric chain because the mass number does not change in the beta decay. β decay could be β- decay, wherein the atomic number increases by 1 or it could be β+ decay, wherein the atomic number decreases by 1. So I have just given the schematic for β decay. In β- decay, the atomic number increases by 1 and along with that a β-, which is an electron and an anti-neutron is emitted.

β- decay: $^{A}X(Z) \rightarrow {}^{A}Y(Z+1) + \beta^- + \nu'$

$^{24}Na \rightarrow {}^{24}Mg + e^- + \nu'$

For example, Na-24 emits a β- to give you Mg-24. In β+ decay, the atomic number decreases by 1, emitting a positron and a neutrino.

β+ decay: $^{A}X(Z) \rightarrow {}^{A}Y(Z-1) + \beta^{+} + \nu$

$^{22}Na \rightarrow {}^{22}Ne + e^{+} + \nu$

And this reaction has got a threshold of 1.02 MeV. So why this condition has come? Why is there a threshold for β+ decay? And why there is no threshold for β- decay? So as you can see here, when the atomic number increases from Z to Z+1 by β- decay, there is one extra electron required which is picked up from the neighboring atoms.

In the example of sodium-24 going to magnesium-24. When sodium converts to magnesium, it will be positively charged. It can easily pick up an electron from the surrounding. So one electron is emitted by sodium-24 and magnesium picks up an electron. So in terms of electron, there is no loss or gain. But in case β+ decay, when the atomic number decreases by 1, for example, phosphorus-30 goes to silicon-30. So Z from 15 becomes 14.

$^{30}P_{15} \rightarrow {}^{30}Si_{14}$

Then see here what is happening, a positron is coming out and so this atomic number has decreased. So it is one electron less than the parent. Because of that, the atom loses an electron from the atomic cells. So because of that, a pair, a positron is going out and electron from the atomic shells going out.

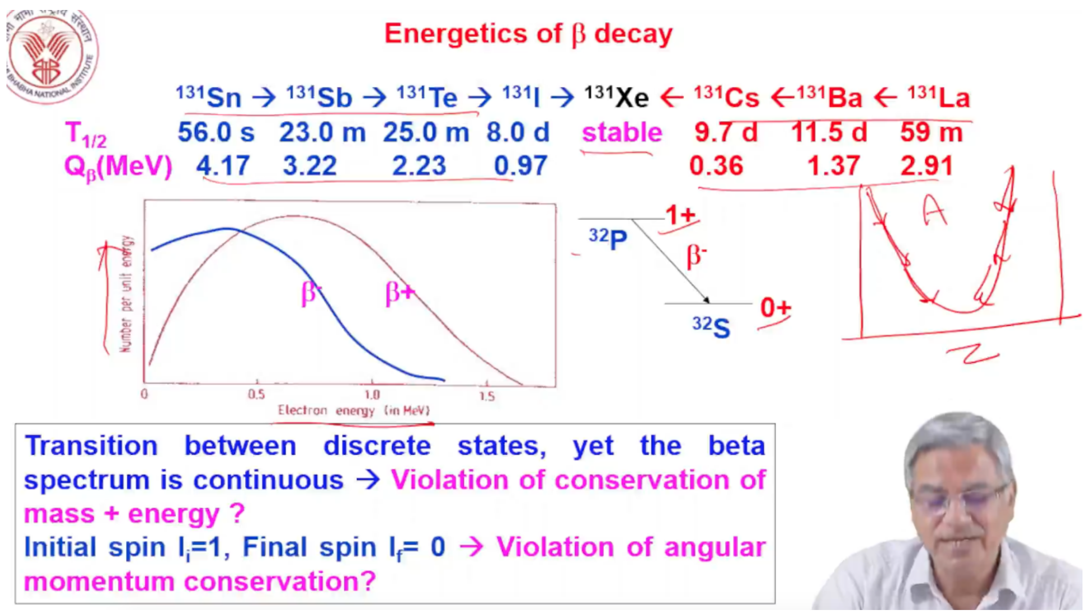
Because of this condition, the mass difference between the parent and daughter has to be more than the rest mass of a pair of electron positron that is 1.02 MeV. I hope it is clear. In beta plus decay, since the atomic number has decreased, the positron is in any way emitted, but the electron which is excess now in silicon 30, that electron also will be emitted and so the mass difference between the parent and daughter has to be more than or equal to the rest mass of a pair of electron positron that is 1.02 MeV. So whenever the Q_{β} value is more than 1.02, then only the β+ decay is possible. Otherwise β+ decay not possible. And in such situation, when $Q_{\beta+}$ is less than 1.02 MeV, electron capture is the mode of decay.

So electron capture competes with β+ decay when Q_{β} is less than 1.02 MeV. Just to give an example,

$^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu$

so it is an electron capture decay mode. What does it mean? The nucleus captures the electron from the atomic orbitals. So it could be K electron or L electron depending upon the Q value for the reaction. So electron is captured by the nucleus and you have here $^{7}Li + \nu$. So in this process, there are no β particles emitted, but when the electron is captured by the beryllium-7, lithium-7 will have a hole in the K shell and there will be emission of X-rays from the atoms. So electron capture is always accompanied by emission of

X-rays and also the Auger electrons. So these particles that are emitted in the beta decay, it could be electron or positron and also the neutrino and anti-neutrinos.



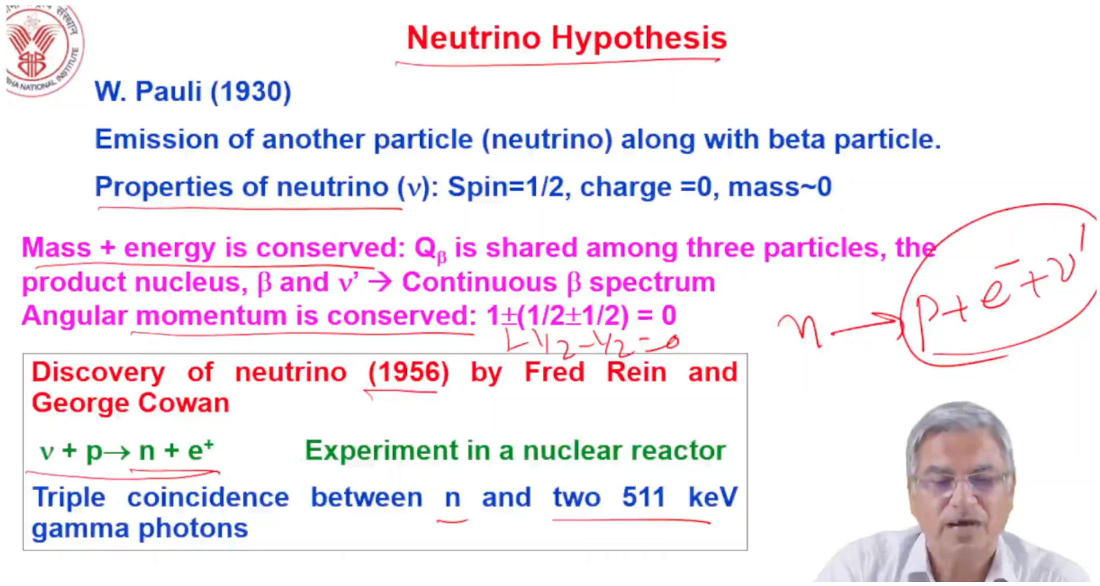
Let us first discuss the energetics of the beta decay. As we have seen previously, the beta decay occurs along the isobaric chain, say a particular A. For example, it is odd A. Then the lower Z like this and tin, antimony, tellurium, they are undergoing beta minus decay and the higher Zs are undergoing beta plus decay to stabilize at the most stable isobar. And as you can see here, the Q_β values are decreasing from both sides. So whatever nucleus is away from the stability has got a higher Q value.

One of the important observations of beta decay is that the beta spectrum is continuous. Just I have given an example here, the phosphorus-32 has a spin of 1+ undergoing β- emission to sulfur-32, which has a spin of 0+. So this beta decay taking place between two discrete states, ground state of phosphorus-32 to that of the sulfur-32. But then if you see the beta spectrum, that is this energy of electron versus the number of electron or dN/dE. The beta spectrum is continuous.

The blue one is β- and the red one is the β+. So beta minus or beta plus both have continuous spectrum. So this cannot be explained simply if you consider the decay of a state of 1+ to other state as both of them are discrete. So apparently there looks to be a violation of conservation of mass and energy. How to explain the continuous beta spectrum we will see shortly.

Another violation that appears to be here is the angular momentum conservation. You see here, phosphorus-32 spin is 1+, sulfur-32 spin is 0. If only electron was emitted or then electron spin is half. So $1 \pm 1/2$ can be 1/2 or 3/2, but the daughter product spin is 0.

So again, the initial spin and final spin are not matching and apparently there is a violation of the conservation of angular momentum.



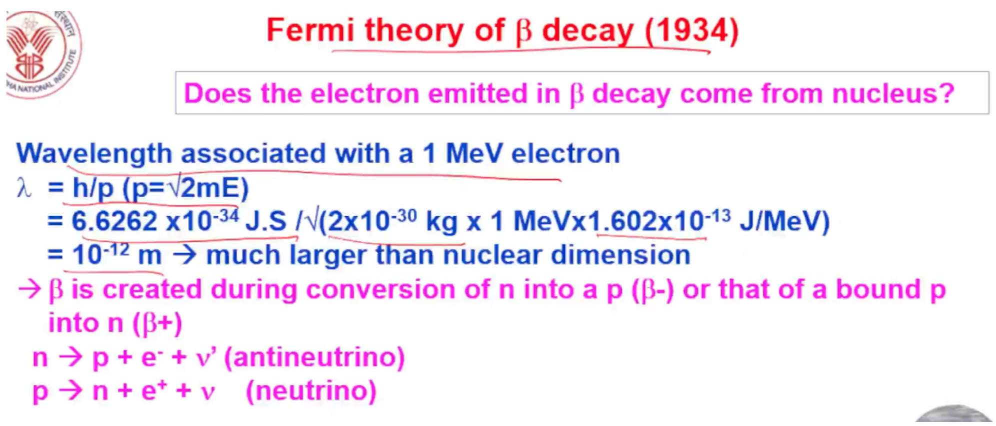
So this puzzle was actually solved by W. Pauli in 1930 when he proposed the neutrino hypothesis. You see, it was a hypothesis. W. Pauli proposed that there has to be one more particle emitted along with the beta particle in beta decay. And that particle he named as neutrino. So this is another particle accompanied with a beta particle.

So two particles are emitted along with the residual nucleus that is formed. And this, the properties that Pauli predicted, it has to have a spin of half, no charge and no mass. So once you include one more particle neutrino having a spin half, no charge and no mass, then you can explain the conservation of mass and energy as well as that of the angular momentum.

So you can see here, when you have another particle, so there is now three particles. So a neutron is getting converted into proton plus electron plus a neutrino. So the Q_β of this reaction is shared among the three particles. And when the Q is shared among three particles, there can be infinite solutions. Therefore, the energy of electron and neutrino can be varying continuously. So in this particular one, when the energy of electron is very high, then energy of neutrino will be low. When energy of electron is very low, energy of neutrino will be high. So that is how you can explain the continuous spectrum of beta.

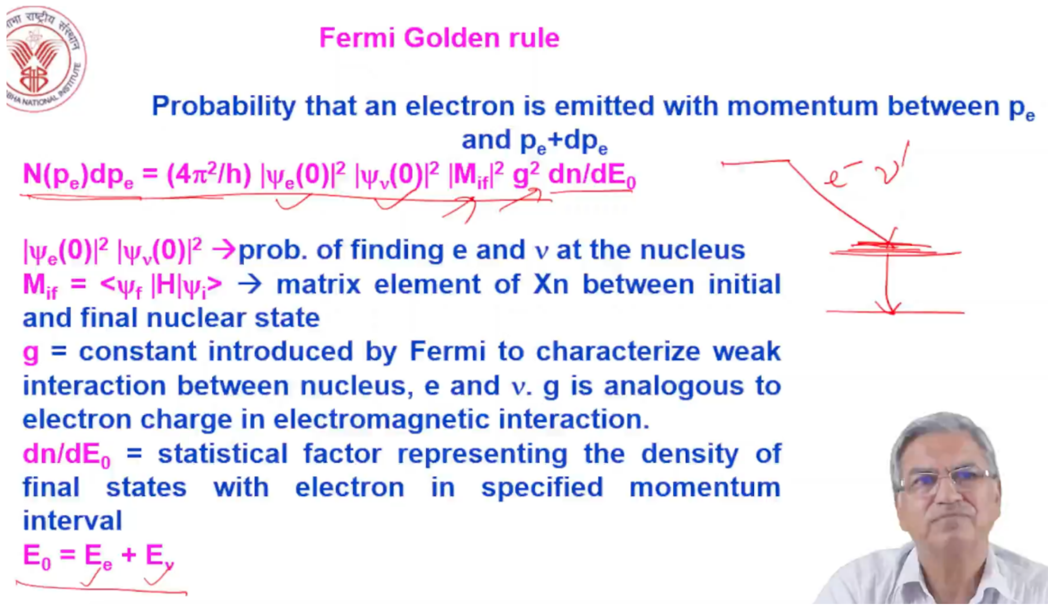
Similarly, the angular momentum conservation also can be explained now. So you have the phosphorus-32, 1+ spin state, an electron goes out and a neutrino also is emitted both having spin half. So the angular momenta can couple in a way, L ±1/2 and so there is a combination of L and S1, S2, here S1, S2 electron and neutrino can give you zero spin. For example, if you have 1 -1/2-1/2 can be zero. So it is now possible to explain the conservation of angular momentum by introducing spin half neutrino. And it is also possible to explain the continuous nature of the beta spectrum.

In fact, this neutrino was discovered in 1956, you can see here. And the reaction was that in the reactors, there are a lot of neutrinos emitted from fission products and there is hydrogen. So the neutrinos interact with the hydrogen to give you a neutron and a positron and a triple coincidence. So you detect the neutrino in a neutron detector and the positron will annihilate, give you two 511 keV gamma rays. So the triple coincidence between one neutron and two 511 keV gamma rays is an unambiguous proof of the existence of neutrino. So that is how the neutrino was discovered after almost 25 years of its prediction by Pauli. So we can explain the energetics.



Now let us see how to explain the decay constant. So the theory of the beta decay was in fact, given by Enrico Fermi in 1934. And the first question it comes to mind is whether this electron that is emitted, it comes from nucleus, how it comes. So that is the question. And we will see later on that actually electron is not present in the nucleus, but it is generated whenever there is a weak interaction.

We will see that. So let us see why it is not present in the nucleus because wavelength of the electron, if you have 1 MeV electron coming out in beta decay, and what is the wavelength? λ is given by h/p, momentum, the de Broglie wavelength or p can be written as $(2\ m\ e)^{1/2}$, mass and energy. And if you substitute the value of the Planck's constant in joule second, and then the mass of the electron, and then the conversion factor for joule to MeV, it comes to 10^{-13} meters. So the wavelength of the electron, having energy of about beta particles is much larger than the dimensions of the nucleus. Therefore, the hypothesis that the electron is present in the nucleus is not possible. So essentially, the beta particle is created during the conversion of a neutron into a proton or a beta plus is created if a bound proton is converted into neutron because the proton mass is less than that of neutron. So a free proton cannot decay into neutron. But a bound proton in the nucleus can be converted into a neutron by emission of a beta plus. So these are the expressions which give you how the conversion between neutron and proton happening in the nucleus. So these particles, electron and positron are created when one nucleon is converted into another in this process.



Now to calculate the decay constant, we will go a little bit into the more details though I will not go to complete theory or complete derivations of the expression for the decay constant. But I give you a flavor by considering what are the major factors that Fermi took in determining the λ for the beta decay. So Fermi golden rule for any transition given by this kind of expression, here it is only for the beta decay that is the probability that an electron is emitted with momentum between p_e and $p_e + dp_e$. That means p_e plus a small increment in a $p_e + \Delta p_E$, what is the probability that electron is emitted? And for the probability that electron and the neutrino also emitted. So when there is a dp_e, there will be dp_v also. So $N(p_e)\, dp_e$ is given by Fermi golden rule,

$$N(p_e)dp_e = (4\pi^2/h)\ |\psi_e(0)|^2\ |\psi_v(0)|^2\ |M_{if}|^2\, g^2\, dn/dE_0$$

the probability of finding electron at the nucleus ψ_e, ψ_v, probability of finding the neutrino at the nucleus, this is a matrix element between initial and final state for beta decay. g is the g factor which Fermi introduced to characterize the weak interaction between the three particles, nucleus, electron and the neutrino. And in fact, this g is analogous to electronic charge in electromagnetic interaction. So it is essentially a particle of weak interaction and then it is the density. Statistical factor, which is density of states. So when say for example, a particular isotope is decaying by beta, so it will be populating some states.

So what are the density of states in that nucleus? Suppose it emits gamma ray, so there is the density of states dN/dE. So when the electron is emitted and along with that neutrino is also emitted, then the density of states in the residual nucleus will be given by dN/dE_0.

And E_0, the total kinetic energy equal to energy of electron plus energy of the neutrino. So E_0 is here in the infinite number of ways between electron and neutrino. So let us see how we can derive the final expression for the decay constant.

Evaluation of dn/dE_0

$dn = dn_e\, dn_v = 4\pi p_e^2\, dp_e/h^3\; 4\pi p_v^2\, dp_v/h^3$

$= (16\pi^2/h^6)\, E_v^2/c^2\, dE_0/c\; p_e^2\, dp_e$

$E_v = E_0 - E_e$

$dE_v = dE_0$ (as KE of e is fixed)

p_v can be eliminated using $p_v = E_v/c = (E_0 - E_e)/c$ and $dp_v = dE_0/c$

$dn/dE_0 = 16\pi^2/h^6c^3\; p_e^2\, (E_0 - E_e)^2\, dp_e$

Substituting $\eta = p_e/m_0c$, $W = E/m_0c^2$, $W_0 = E_0/m_0c^2$

$dn/dE_0 = (16\,\pi^2\, m_0^5c^4/h^6)\, (W^2-1)\, (W_0-W)^2\, W\, dW$

$\eta^2 = W^2-1$

$dn/dE_0 \rightarrow 0$ at $W=1$ ($\eta=0$) and $W=W_0$

Explains the shape of beta spectrum

$p = E/c$

So a little bit more details, the dN, the density factor is nothing but the $dN_E \times dN_v$, density for the electron and neutrino.

$dn = dn_e\, dn_v = 4\pi p_e^2 dp_e/h^3\; 4\pi p_v^2 dp_v/h^3$

$dn = (16\pi^2/h^6)\, E_v^2/c^2\, dE_0/cp_e^2\, dp_e$

So you can say p = E/c for neutrino and $dp = dE_0/c$.

So dE_0 because $E_v = E_0 - E_e$.

Since E_e is constant, we can say $dE_v = dE_0$. So you actually try to eliminate the term corresponding to the momentum and energy of neutrino. Because what we are observing is the electron. So p_v can be eliminated using

$p_v = E_v/c = (E_0 - E_e)/c$

accordingly, then $dp_v = dE_0/c$.

From here $dp_v = dE_0/c$. So you, so final expression will be

$dn/dE_0 = 16\pi^2/h^6c^3p_e^2\, (E_0 - E_e)^2\, dp_e$

So we have now the terms in energy and momentum of electron. And now we can make a small substitution that is called in the relativistic domain, you can write the relative momentum in terms of the momentum of electron and m_0c, relativistic momentum of electron.

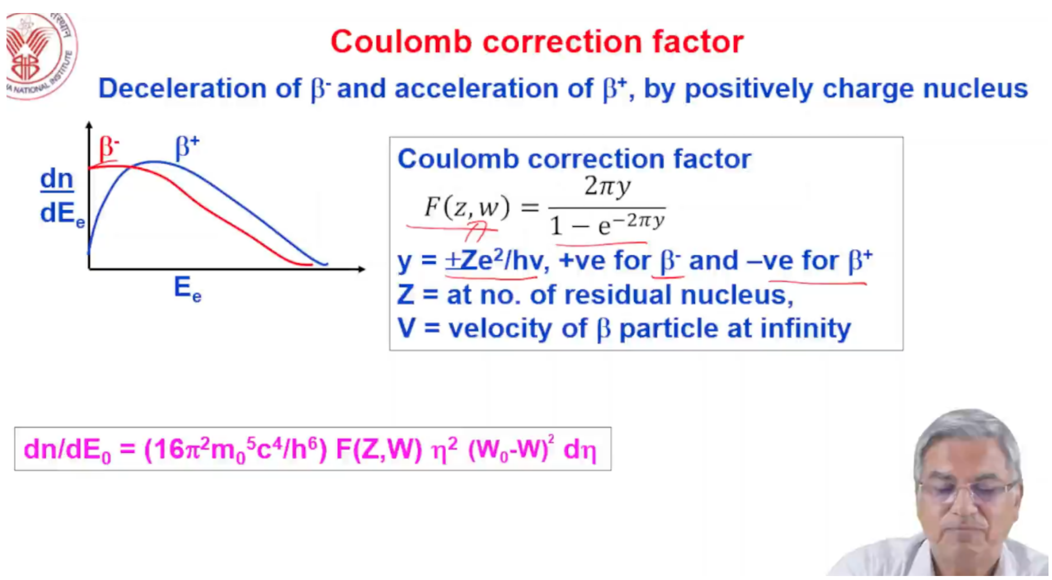
$\eta = p_e/m_0c$, $W = E/ m_0c^2$, $W_0 = E_0/ m_0c^2$

So it is the relativistic energy W. So these two terms η and W we introduce. To get this simplified version dn/dE_0 which is the term corresponding to this

$$dn/dE_0 = (16\pi^2\, m_0^5\, c^4/h^6)\, (W^2-1)(W_0-W)^2\, W\, dW$$

W is nothing but the relative energy and η is the relative momentum. And you can also find out that W^2 is nothing but $1+\eta^2$ or η^2 is W^2-1. So finally you can see this expression, I believe that density term is just explains the shape of the beta spectrum and W becomes 1 that means η equal to 0.

So that corresponds to the zero momentum of electron and W becomes W_0 then this term becomes 0. So at both ends, first end zero energy term end W =1 and the highest energy term W_0 =W. So the bell shape spectrum of beta can be explained by this Fermi-decay theory. But the observation is that you see the bell shape is observed for beta plus but for beta minus there is higher fraction of low energy. So more electrons of high low energy are observed than beta plus.



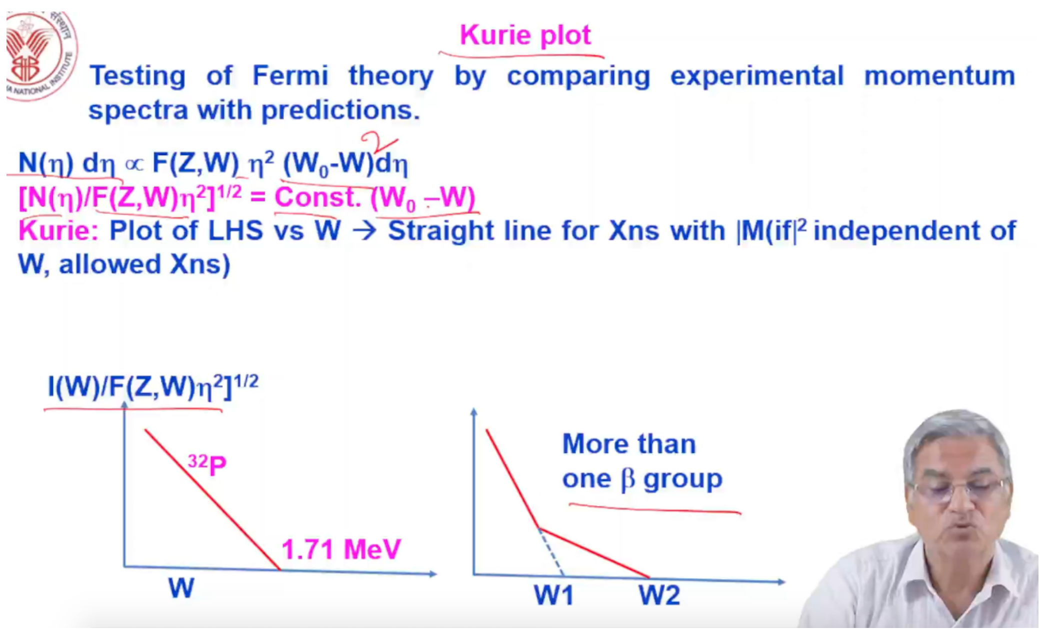
So this difference is there. This is explained by a Coulomb correction factor. That means when the electron or positron is coming out of the nucleus, electron is attracted by the nucleus whereas positron is repelled by the nucleus in simple terms to explain. So there are more high energy positrons than electrons, or there are more low energy electrons than positrons. That is why the electron spectrum has a lower energy component more than the positron.

So this Coulomb correction factor was given by Fermi. It depends upon the atomic number of the nucleus and the energy; the energy W is the relative energy and it is given by

$$F(Z, W) = \frac{2\pi y}{1-e^{-2\pi y}}$$

where $y = \pm Ze^2/hv$

where Z is the atomic number of the residual nucleus and v is the velocity of beta particle at infinity. And this term y is actually positive for electrons and negative for positrons. And if you substitute this value, this $\pm Ze^2/hv$, this factor here, then we can explain the shape of the beta minus spectrum. Beta minus spectrum is more having high lower energy component, beta plus spectrum has got high, higher energy components. So that is how the Coulomb correction term can explain the shape of the beta minus and beta plus.



The Fermi theory in fact got a boost when it was successfully validated by Kurie plot. So Kurie in fact came out with an excellent idea of plotting what is called as this factor, let us see what is this factor. So again when I am saying momenta, it is the relative momentum η. So

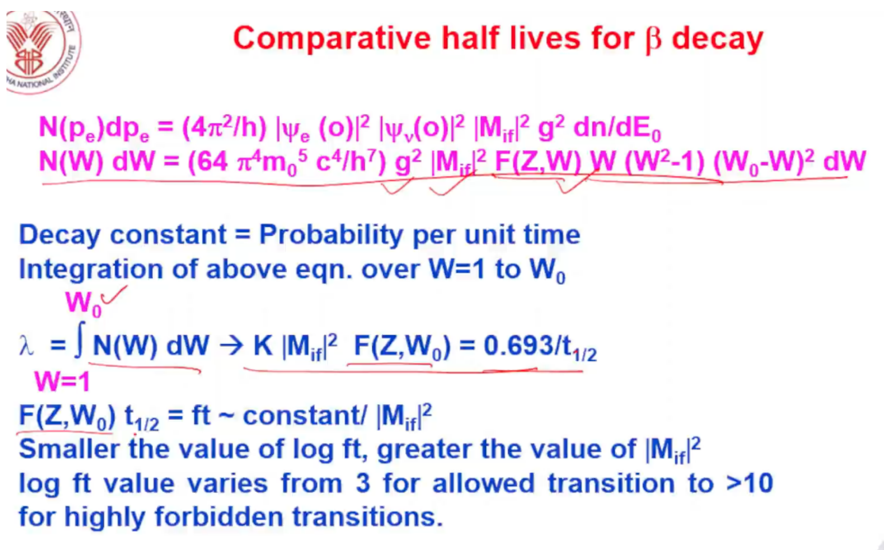
$$dn/dE_0 = (16\pi^2 m_0^5 c^4/h^6)\, F(Z, W)\eta^2 (W_0-W)^2\, d\eta$$

we can write it from the previous expressions. And then you can rearrange this equation,

$$[N(\eta)/F(Z, W)\, \eta^2]^{1/2} = \text{const.}\, (W_0-W)$$

So actually it has to be square term here. So this term now if you plot this left hand side against W, then you get this term. So I(W) is nothing but N(η), we can write in terms of W or η, $W^2 = 1 + \eta^2$ upon the Coulomb factor $[\eta^2]^{1/2}$ is found to follow a straight line with a negative slope because of this negative term. And very nicely these plots, Kurie plots in fact were used to identify if there is a single beta or there are multiple beta particles. So like if there are more than one beta group, then you have these two linear paths. There are two, more than two, you have three linear paths and so on.

So the Kurie plot was very handy when the mass spectrometers were utilized to determine the electron spectrum, the momentum space. Mass spectra will resolve them as per their momentum. And when you plot the data, this particular expression IW upon F (Z,W), then you find that each beta group followed a straight line with the slope that you can find from this equation. So this was the success of Kurie plot that validated the Fermi theory of beta decay.



Now let us see how to compare the half-lives for beta decay. So again the same expression I am writing again in more details,

$$N(W)\, dW = (64\pi^4\, m_0^5\, c^4/h^7)\, g^2\, |M_{if}|^2\, F(Z, W)\, W\, (W^2-1)\, (W_0-W)^2\, dW$$

N(W) dW, this is the term which came from the momentum space, the g factor, the transition matrix is square, Coulomb correction factor and the W energy factor. Now the decay constant essentially is as we discussed previously, the decay constant is nothing but the probability of decay per unit time, probability of decay of an atom per unit time. So if you integrate this equation over from W=1, that means energy from zero to highest energy, then you integrate this N(W) dW, this expression. So you get, it can be final expression, I am giving K, a constant term,

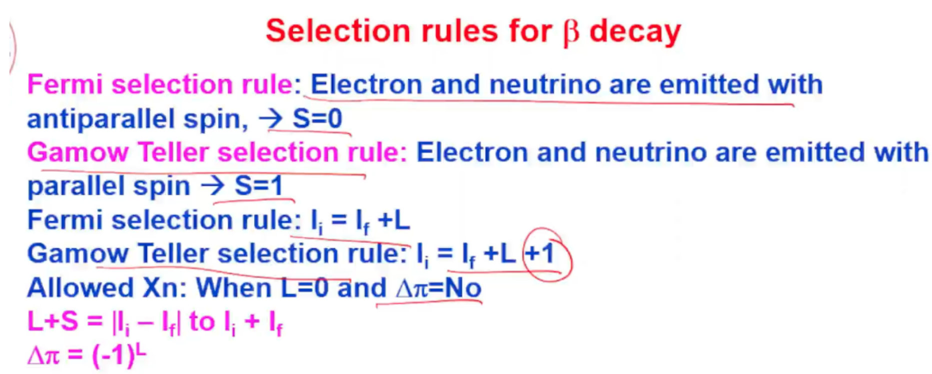
$$\lambda = \int_{W=1}^{W_0} N(W)dW \rightarrow K\left|M_{if}\right|^2 F\left(Z, W_0\right) = 0.693/t_{1/2}$$

$\left|M_{if}\right|^2$,the transition matrix square, $F\left(Z, W_0\right)$ and this λ you can mention it, $0.693/t_{1/2}$ or

$$F\left(Z, W_0\right)t_{\frac{1}{2}} = ft = constant/\left|M_{if}\right|^2$$

So you can see here, ft is inversely proportional to $\left|M_{if}\right|^2$. Lower the value of log ft, greater the value of $\left|M_{if}\right|^2$. Essentially it means that if a transition is allowed, it will have higher value of $\left|M_{if}\right|^2$.

So lower value of log ft is an indication of an allowedness of this. So for allowed transitions, log of T is varying from three and it goes on to higher values. We will see this soon.



So the selection rules for beta decay are in fact there were two selection rules, one by Fermi depending upon the spins of electron and neutrino, whether they are parallel or anti-parallel. In the Fermi selection rule, electron and neutrino are emitted anti-parallel to each other, so the spin component is zero. Whereas in the Gamow-Teller selection rule, electron and neutrino are emitted with parallel spin, so the spin factor is one.

So Fermi selection rule says

$I_i = I_f + L$

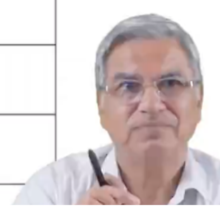
L is the angular momentum carried away by the electron and so S is zero. But Gamow-Teller selection rule says that

$I_i = I_f + L + 1$

one is due to the spins of electron and neutrino which add up to one. For allowed transitions we say L = zero and there is no change in the parity because this is the S equal to zero for alpha particles. So we can say L+ S equal to I_i - I_f to I_i + I_f where I_i and I_f are the spins of the parent and daughter isotopes.

Type	Decay	Orbitals involved	ΔI	L	$\Delta\pi$	Log ft
Super allowed	$n \rightarrow p$	$s_{1/2} \rightarrow s_{1/2}$	0	0	No	3.1
	$^{3}H \rightarrow {}^{3}He$	$s_{1/2} \rightarrow s_{1/2}$	0	0	No	3
	$^{17}F \rightarrow {}^{17}O$	$d_{5/2} \rightarrow d_{5/2}$	0	0	No	3
Allowed	$^{35}S \rightarrow {}^{35}Cl$	$d_{5/2} \rightarrow d_{5/2}$	0	0	No	4 to 7
1st forbidden	$^{139}Ba \rightarrow {}^{139}La$	$f_{7/2} \rightarrow g_{7/2}$	0	1	Yes	~7
	$^{87}Kr \rightarrow {}^{87}Rb$	$d_{5/2} \rightarrow p_{3/2}$	1	1	Yes	~7
	$^{89}Sr \rightarrow {}^{89}Y$	$d_{5/2} \rightarrow p_{1/2}$	2	1	Yes	~8
2nd forbidden	$^{137}Cs \rightarrow {}^{137}Ba$	$g_{7/2} \rightarrow d_{3/2}$	2	2	No	10-13
3rd forbidden			3	3	No	>15

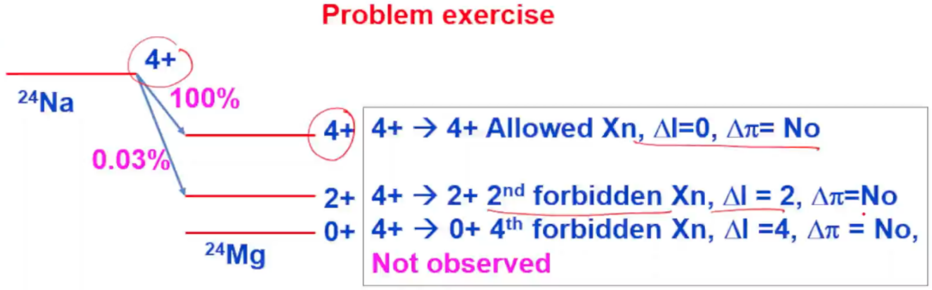
Xn between pairs of mirror nuclei



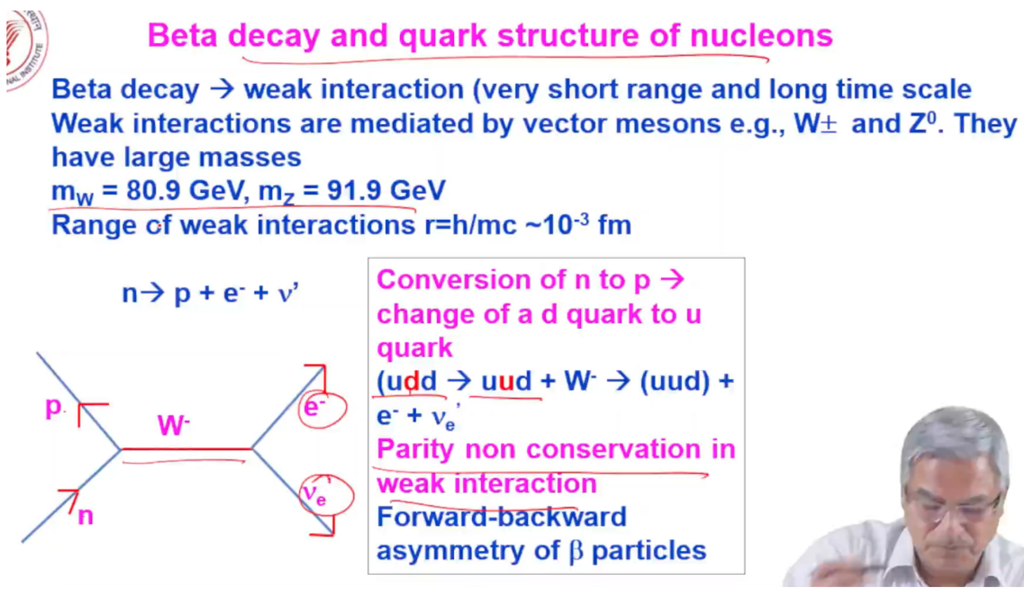
$\Delta\pi$, the change in parity is given by $(-1)^L$. These are the selection rules. Super allowed transitions occur between the pairs of mirror nuclei. You can see from neutron to proton or tritium to helium-3 or fluorine-17 to oxygen-17. You can see here that the orbitals of proton and neutron in the two nuclei are exactly same and the result of that ΔI is zero, ΔL is zero and $\Delta\pi$ is zero. That means there is no change in parity and the log of T values are of the order of three.

Whereas in the case of allowed transitions, the ΔI is zero but the proton and neutron need not necessarily occupy the same orbit. Therefore, ΔI is zero, L = 0 and no change in parity but the log ft values are higher than 3 in the range of 4 to 7. First forbidden, now you can see ΔI 0,1, 2, the L value 1,1,1 and there is a change in parity. So log ft values are higher and second forbidden again, ΔI is two, L is two, no change in parity 10 to 13 log ft, third forbidden and so on. So as you go to higher and higher forbiddenness, there is a rise in the L value.

Problem exercise



Just to give an example of this decay, sodium-24 having spin state 4+ decays to magnesium-24 and you see the ground to ground transition is forbidden because it is ΔI = 4. Whereas the 100% transition goes to 4+ state, the 4+ to 4+ is allowed transition, ΔI is zero, Δπ is no, no change in parity. 4 to 2, see this second forbidden transition, ΔI is two and Δπ is no change in parity whereas there is no ground to ground transition. So that is how we can explain that transition.



And lastly, just before I conclude, I was telling about how electron and neutrino are coming out. So from the quark structure of nucleons, a neutron is udd, one up quark and two down quarks. A proton is uud, two up quark and one down quark. And so conversion of a d quark into u quark happens during beta minus decay. So this is actually nicely explained by this Feynman diagram.

A neutron is converted into proton. In the process, a W boson is emitted and then this W boson decays to a electron and a anti-neutrino. So that is how in fact the advanced theory of beta decay in terms of the weak interaction whereby when the neutron is converted into proton, then a W boson is emitted and in the process the W boson breaks into electron and a neutrino. And the forward-backward asymmetry of beta particles in fact was observed that explain that in the beta decay or in the weak interactions parity is not concerned. So this bosons, W bosons and Z bosons in fact, they are actually the particles that are mediating the weak interactions in beta decay.

And you can see the masses of these particles, they are very high, of the order of GeV. So this is in fact ultimately all these interactions, beta decay, weak interactions are explained in terms of the weak interaction wherein the W bosons or the Z bosons are involved in the process. So I will stop here and next I will take the gamma decay. Thank you very much.