Alpha decay

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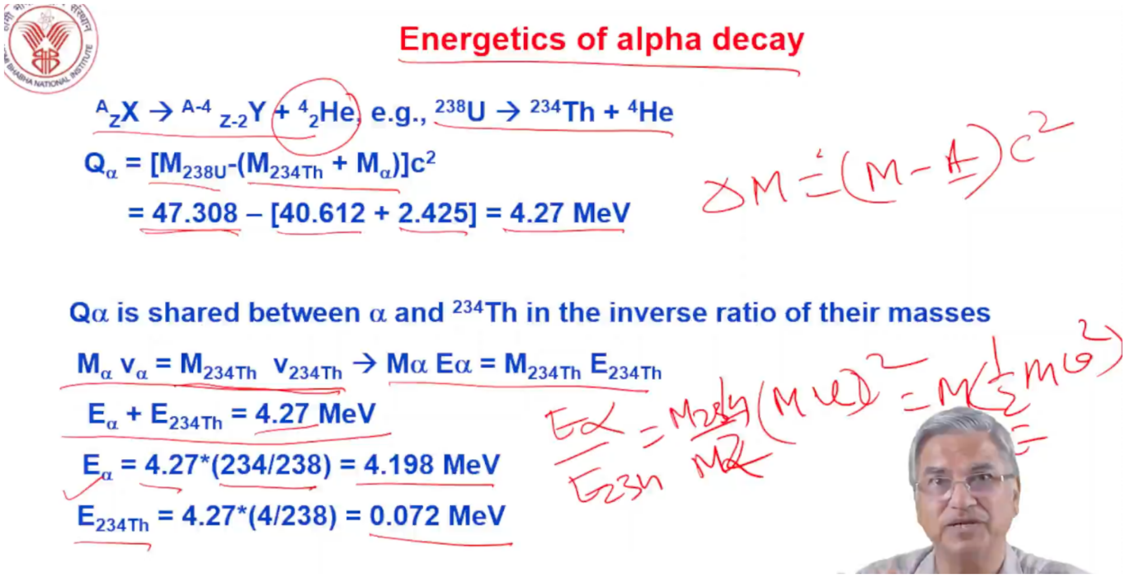
Lecture-6, module-1

Hello everyone. So far we have discussed the radioactivity, the different types of radioactivity decays, the nuclear structure, stability and the nuclear models, mainly liquid drop model and shell model. Today we will start the details of the nuclear decays like alpha, beta and gamma. Today's lecture I will discuss the alpha decay. So as we discussed in the introductory lecture, there are three types of decays, alpha, beta and gamma. There are others also like spontaneous fission and so on. But mainly we will be discussing the three types of decays like alpha, beta and gamma.



Energetics of alpha decay

${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$ e.g., ${}^{238}U \rightarrow {}^{234}Th + {}^{4}He$

$Q_\alpha = [M_{238U} - (M_{234Th} + M_\alpha)]c^2$

$= 47.308 - [40.612 + 2.425] = 4.27$ MeV

$\delta M = (M - A)c^2$

$Q\alpha$ is shared between α and ^{234}Th in the inverse ratio of their masses

$M_\alpha v_\alpha = M_{234Th}\ v_{234Th} \rightarrow M\alpha E\alpha = M_{234Th} E_{234Th}$

$E_\alpha + E_{234Th} = 4.27$ MeV

$E_\alpha = 4.27*(234/238) = 4.198$ MeV

$E_{234Th} = 4.27*(4/238) = 0.072$ MeV

$\frac{E\alpha}{E_{234}} = \frac{M_{234}}{M\alpha}$ $(Mv)^2 = M(\frac{1}{2}Mv^2)$

So let us see the energetics. What are the types of energies that are involved when a heavy nucleus undergoes alpha decay? So as you know already by the group displacement laws during alpha decay, the atomic number of the heavy nucleus is decreased by 2, whereas the mass number is decreased by 4 because the alpha particle is coming out of the nucleus. Just an example, U-238 undergoing alpha decay to thorium-234.

${}^{238}U \rightarrow {}^{234}Th + {}^{4}He$

To calculate the energy of the alpha particle that is emitted in alpha decay, we calculate the mass difference between the parent and the daughter products. So the mass of uranium-238 minus the sum of the masses of thorium-234 and alpha. And now if you write in terms of the atomic mass units, then you can multiply by 931 MeV or you can write in terms of the mass defect. So mass defect is nothing but

ΔM = [M-A]c^2

where M is the actual mass in atomic mass units and A is the mass number. So this into c square becomes actually the mass defect and the mass tables actually give the masses in terms of mass defect.

$$Q_{\alpha} = [M_{238U} - \left(M_{234Th} + M_{\alpha}\right)]c^2$$

$$Q_{\alpha} = 47.308 - [40.612 + 2.425] = 4.27\,MeV$$

So what you see here is the 47.308 MeV is the mass defect. That means (M-A)*c^2. So when you have alpha decay, since the mass number is conserved, essentially it will give you the difference in the masses of the parent and the daughter products. So ^{238}U, 238 this is the mass number, ^{234}Th and alpha particle. So if you see the difference between this is equal to 4.27 MeV that is the Q value of this alpha decay. Q value means the heat or the energy liberated in this process. It is a positive Q value, this much energy is emitted. This energy is now shared between the alpha particle and the daughter product that is thorium-234.

So how it is shared, how to calculate the energy of alpha? The basic concept is that since the ^{238}U is stationary, when alpha particle is emitted, then the momentum of this nucleus is zero. So when it is put split two particles, the net momentum should be again zero. And so the linear momentum is mass into velocity.

$$M_{\alpha}V_{\alpha} = M_{234Th}V_{234Th}$$

That means the momenta of the alpha particle and ^{234}Th are same.

They will fly in opposite direction of course, so that net momentum will be zero. Now MV if you can convert it to M_{α} and M_{α} E_{α}, if you take a square of this one, like for example, MV you take square and multiply by 1/2, so it will become M*(1/2MV^2). So essentially it becomes mass into energy. So you can convert this relationship between the momentum equal to relationship between the mass and energy.

$$M_{\alpha}E_{\alpha} = M_{234Th}E_{234Th}$$

And you can see here

$$\frac{E_{\alpha}}{E_{234Th}} = \frac{M_{234Th}}{M_{\alpha}}$$

That is what I was telling that the Q value is shared between the two particles in the inverse ratio of their masses. That means

$$\frac{E_1}{E_2} = \frac{M_2}{M_1}$$

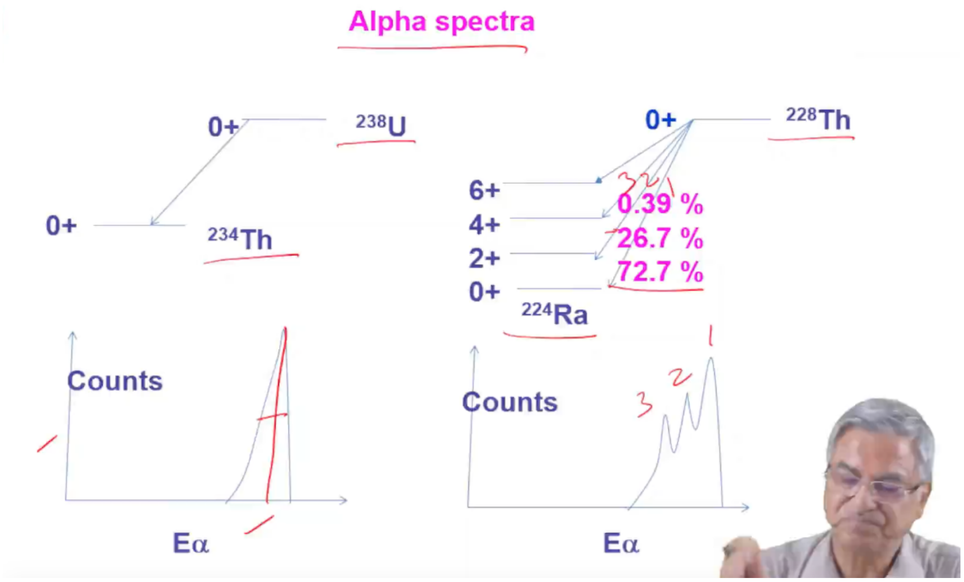
So you have two equations, E1 + E2= 4.27 MeV and the ratio between E1 and E2. So you can now calculate energy of alpha equal to Q value into the ratio of the masses 234 by 238 that is 4.198, energy of ^{234}Th, the other part is 0.072 M e V. So you can see here the lighter particle takes the major share of energy because energy is shared in the inverse ratio of their masses. So lighter particle gives major portion of energy and the heavier particle takes the smaller portion of the energy.

$$E_{\alpha} + E234Th = 4.27\ MeV$$

$$E_{\alpha} = 4.27 * \left(\frac{234}{238}\right) = 4.198\ MeV$$

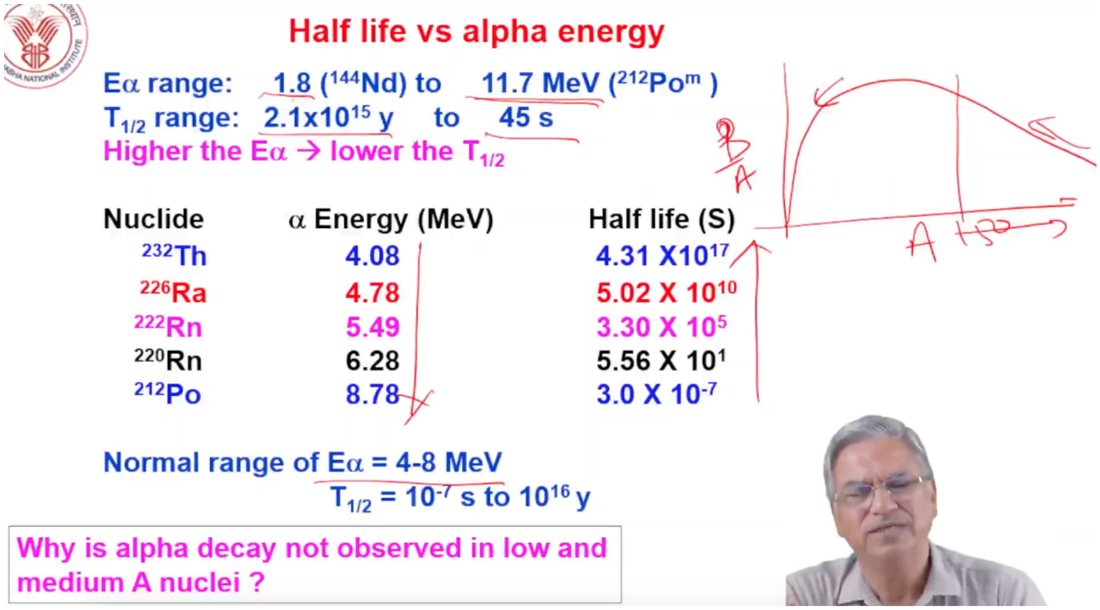
$$E_{234Th} = 4.27 * \left(\frac{4}{238}\right) = 0.072\ MeV$$

So that is why you will see that Q value is close to alpha because the Th^{234} has very small energy. So alpha particles that are emitted in alpha decay have the energies of the order of 4 to 8 MeV.



Second important property of these alpha particles is the alpha spectra. So when a heavy nucleus undergoes alpha decay and if you detect these alpha particles in the detectors, then what type of spectra we will see in the alpha spectrum. So here the even-even nuclei that we are discussing like ^{238}U and ^{234}Th, they have their ground state spins 0. They are even-even nuclei and so mostly the alpha decay between the two even-even nuclei will go from ground state to ground state and so there will be only one alpha. So what you see here is that this is a typical alpha spectrum counts versus energy of alpha and you will get a single line in the spectrum. Actually you should have got a small, very thin line because there is only one energy but the detector had its own energy resolution which we will discuss in the radiation detection measurements and so because of that there is a broadening in the alpha spectrum. There will be in fact a left hand tail which happens because of the instrumentation aspect. In some cases you will find that apart from the ground state to ground state transitions, there are also transitions to the excited states.

Now radium-224, the daughter product of thorium-228 has the excited states, they are called the rotational states 0, 2, 4, 6. I have just touched upon the collective model. Mid shell nuclei can also have their collective motion of rotation and vibration and so apart from the ground to ground transition 72.7%, there can also be transitions to higher energy states but they are hindered with respect to the ground to ground conditions and in such cases there are multiple alpha particles emitted, the alpha spectrum will be something like this. So you have the main peak, this is the high energy peak, 72% and then you have the lower energy, this energy. So this will be 1, 2 and 3, alpha peaks and energy is decreasing this way. So from this data also we can make out how many alphas are being emitted in a particular radioactive decay.



Half life vs alpha energy

Eα range: 1.8 (^{144}Nd) to 11.7 MeV (^{212}Po^{m})

$T_{1/2}$ range: 2.1x10^{15} y to 45 s

Higher the Eα → lower the $T_{1/2}$

Nuclide	α Energy (MeV)	Half life (S)
^{232}Th	4.08	4.31 X10^{17}
^{226}Ra	4.78	5.02 X 10^{10}
^{222}Rn	5.49	3.30 X 10^{5}
^{220}Rn	6.28	5.56 X 10^{1}
^{212}Po	8.78	3.0 X 10^{-7}

Normal range of Eα = 4-8 MeV

$T_{1/2}$ = 10^{-7} s to 10^{16} y

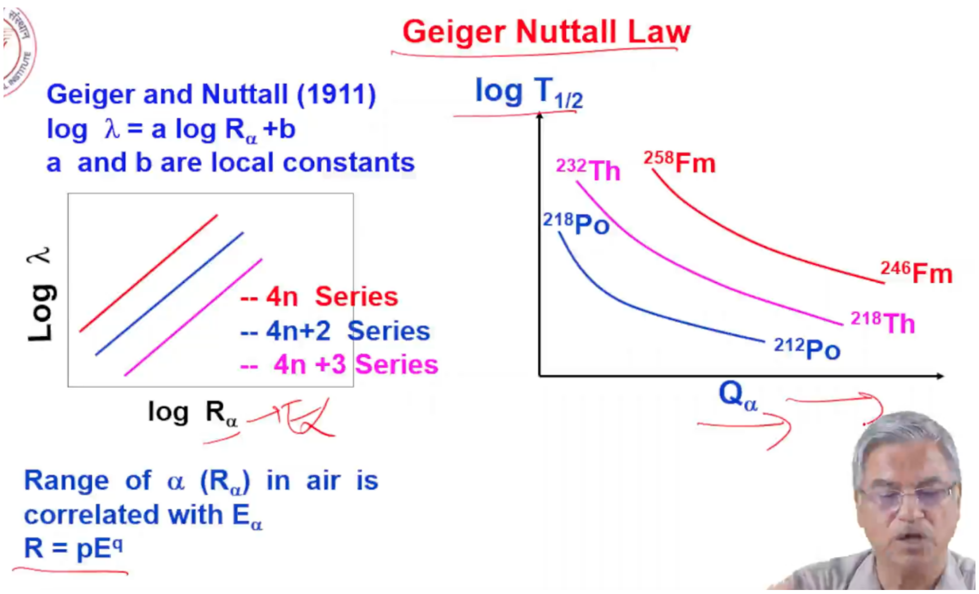
Why is alpha decay not observed in low and medium A nuclei ?

Now let us see how the half-lives of the heavy nuclei depend upon energy of the alpha particles. The typical values of alpha particle energy for heavy nuclei which are emitting alpha particles goes from 1.8 MeV e.g., for ^{144}Nd, one of the lightest nuclei to emit alpha particles and almost the highest energy alpha particle is emitted is 11.7 MeV for ^{212}Po which is a metastable state of Po^{212}. And you can see here the corresponding half-lives.

If you have the alpha energy very low, then the half-lives become very high, 10^{15} years and when we have the alpha energy very high, half-lives become very small and we will discuss this part in more details when we see how to explain the decay of alpha particle for heavy nuclei in terms of the penetration of a coulomb barrier. So the higher the energy of the alpha, lower the half-life for the alpha decay. Just to see an example, see different examples here, all the even-even nuclei of thorium, radium, radon, again Rn^{220}, Po^{212} and you can see the alpha energy is increasing whereas the half-life is decreasing from here to here.

So as the alpha energy increases, the half-life decreases. And the normal range for the alpha energy is 4 to 8 MeV, though there are exceptions like 11.7 MeV or 1.8 MeV, but by and large in the mass region of 200 to 250, you will see the alpha energies are of the order of 4 to 8 MeV and the half-lives accordingly go from microseconds to 10^{15} years or so. Now, you must be observing here that there are no alpha emitters having masses below 144 or in fact, there are none.

So the question I would just put here, why is alpha decay not observed in low and medium A nuclei ? This is a question which must be coming into the mind of the students. So the reason is that as you go down in terms of the mass number, the Q value becomes very small. If you recall the binding energy curve as a function of mass number B/A. So when we are here, heavy mass region, let us say 150 may be somewhere here, then alpha decay is happening like this. A heavy nucleus is going to a lighter nucleus, slightly lighter nucleus during alpha particle emission. So the binding energy is increasing. So when the binding energy is going up means mass is coming down and so Q value will be positive. If you see in this region, when there is alpha decay, the binding energy is decreasing. So Q values are negative. So below mass number 150, you will find the Q values become negative or very low and the coulomb barriers are high. So because of this reason, very low Q value or negative Q value and the very high coulomb barrier leads to no alpha decay for lighter nuclei, masses more than 150. In fact, already around mass 150, the half-lives are in the range of 10 to the power 15 or 16 years.



Geiger Nuttall Law

Geiger and Nuttall (1911)

$\log \lambda = a \log R_\alpha + b$

a and b are local constants

Range of α (R_α) in air is correlated with E_α

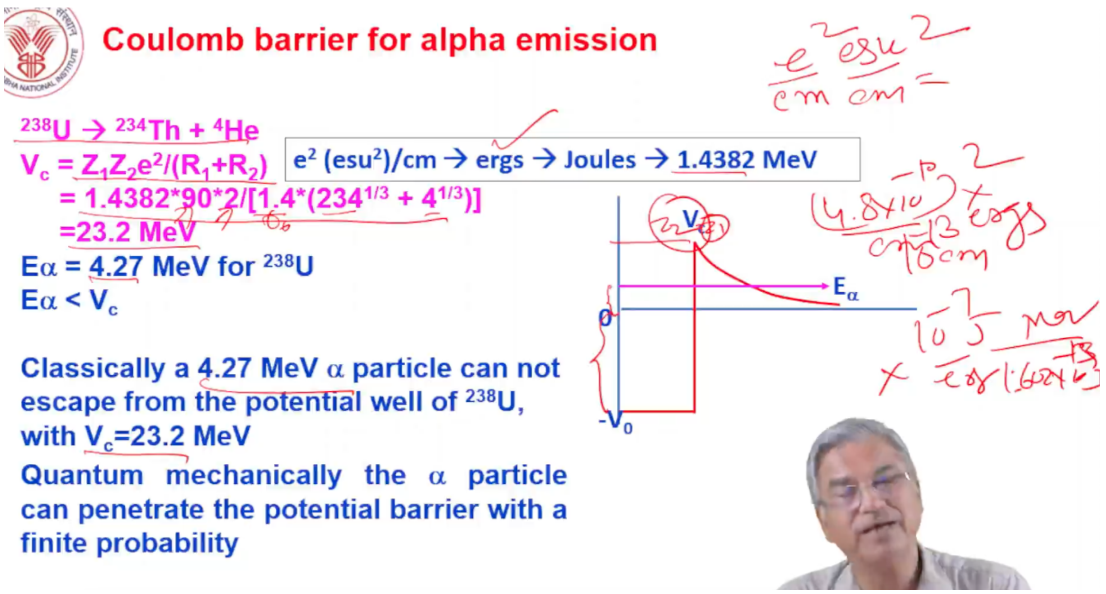
$R = pE^q$

In fact, very interesting correlations were obtained by Geiger and Nuttall in 1911, they called Geiger-Nuttall law. At that time, they did not have surface barrier detectors to detect the energies of alpha particles. Instead, they were measuring the range of alpha particles in different materials. And what they found, the range actually is how much distance the alpha particle will travel in a material. So that's what they call the range. And this range is actually related to the energy of the alpha.

$R=pE^q$

By, let us say, p into E to the power q, where E is the energy, R is the range. Like alpha particle travel in two centimeters in air. So what they found that the for different natural radioactive series, that the λ is increasing with the range. That means as the rate increases, the decay constant increases or the half-life decreases and this range can be correlated with the E_α. So as the energy of the alpha particle increases, the decay constant increases. That means the half-life goes down. So the same thing is shown here for different isotopes of heavy elements like polonium, thorium and fermium, the half-life and Q_α are inversely correlated.

As the Q_α increases, half-life goes down. Same correlation you can see from the Geiger-Nuttall law. So the half-life of an alpha emitter will increase if the energy of the alpha particle decreases. Higher the $Q\alpha$, shorter the half life. Now let us discuss why the nuclei cannot emit the alpha particles instantly? So as you know, alpha particle is a charged particle and so heavy nuclei offer a potential. So they are all in the attractive potential of the heavy nucleus, alpha particle is formed inside the nucleus. To come out of the nucleus, alpha particle has to cross a Coulomb barrier.



Coulomb barrier for alpha emission

$^{238}U \rightarrow {}^{234}Th + {}^{4}He$

$V_c = Z_1Z_2e^2/(R_1+R_2)$

$= 1.4382*90*2/[1.4*(234^{1/3} + 4^{1/3})]$

$=23.2$ MeV

$E\alpha = 4.27$ MeV for ^{238}U

$E\alpha < V_c$

e^2 (esu^2)/cm → ergs → Joules → 1.4382 MeV

Classically a 4.27 MeV α particle can not escape from the potential well of ^{238}U, with $V_c=23.2$ MeV

Quantum mechanically the α particle can penetrate the potential barrier with a finite probability

So this is a typical example that we have the attractive potential well for the nucleons or if there is a preformed alpha particle, the alpha particle is sitting inside the nucleus in an attractive potential well. And so suppose the energy of the alpha particle is this order, it is coming out with this energy, but then there is a Coulomb barrier. The Coulomb barrier and alpha energy as we discussed of the order of 4 to 8 MeV for ^{238}U alpha decay, we found it is 4.27 MeV. Now let us try to calculate the Coulomb barrier. The Coulomb barrier when they are like here, it is like the heavy nucleus and the small nucleus. When they are in contact, it will say that Z_1, Z_2. So at contact the Coulomb energy of this system can be calculated by

$$V_c = \frac{Z_1 Z_2 e^2}{R_1 + R_2}$$

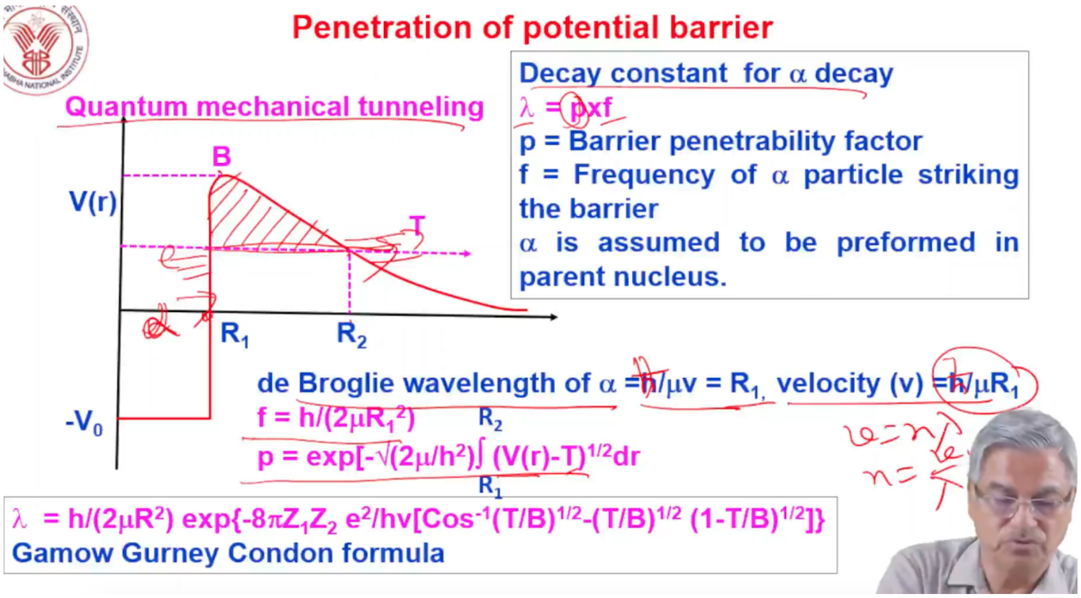
$$V_c = \frac{1.4382*90*2}{1.4(234^{\frac{1}{3}}+4^{\frac{1}{3}})} = 23.2 \text{ MeV}$$

And so you can calculate this by this formula 1.4382, this is a term arises from e^2/r_0, Z_1, the atomic number of alpha particle, Z_2 atomic number of thorium and into, so this is R_0 $(A_1^{1/3} + A_2^{1/3})$. Now why this 1.4382 has come, this comes in terms of MeV. So what I will try to see here that if you see e squared, so only terms here are the charge of the electron square upon the radius in centimeter e squared by centimeter.

So e^2(esu^2)/cm→ergs→joules→1.4382 MeV

e^2(esu^2), electrostatic upon centimeter and 1 esu, so it will become $(4.8*10^{-10})^2$ /cm, it will become ergs. So this is what I have done here. Then if you put this value, it will come in ergs. And now you can put this in terms of, this you can write 10^{-13} centimeter, this is like the radius of a nucleus. And then you convert ergs into joule, so 10^{-7} joule per ergs, multiply by that and joule to MeV, it will be 1.602* 10^{-13} joule per MeV, 1 MeV 1.602* 10^{-13} joules. So like that, if you do this, put the different units and their factors, the whole thing will come to 1.4382. So instead of, every time putting these numbers, you straight away put 1.44 or you can even make 1.44 into charge atomic numbers of the alpha and the thorium upon 1.4 is $R_0(A_1^{1/3}+A_2^{1/3})$.

So this will come out to be 23.2 MeV. So you can see here, a alpha particle of energy 4.27 MeV has to come out of a coulomb barrier, which is in the potential well where the coulomb barrier is 23.2 MeV.



Penetration of potential barrier

Quantum mechanical tunneling

Decay constant for α decay

$\lambda = p \times f$

p = Barrier penetrability factor

f = Frequency of α particle striking the barrier

α is assumed to be preformed in parent nucleus.

de Broglie wavelength of $\alpha = h/\mu v = R_1$, velocity (v) $= h/\mu R_1$

$f = h/(2\mu R_1^2)$

$p = \exp[-\sqrt{(2\mu/h^2)}\int_{R_1}^{R_2} (V(r)-T)^{1/2}dr$

$\lambda = h/(2\mu R^2) \exp\{-8\pi Z_1Z_2\, e^2/hv[\mathrm{Cos}^{-1}(T/B)^{1/2}-(T/B)^{1/2}\,(1-T/B)^{1/2}]\}$

Gamow Gurney Condon formula

So classically, it's not possible for a 4.27 MeV alpha particles to escape from a potential well of 23.2 MeV. But quantum mechanically, yes, alpha particle can penetrate this coulomb barrier and come out with a finite probability. That is what is the problem of barrier penetration. So by quantum mechanical treatment one can in fact calculate the decay constant for the alpha decay of heavy nuclei. So the decay constant for the alpha decay lambda is equal to the penetration probability for alpha decay into the frequency of the alpha particle. So you see, the alpha particle is formed here, and it is striking the coulomb barrier.

$$\lambda = p * f$$

So there is a frequency of striking at the coulomb barrier that is f and there is a penetration probability. So this is alpha here, it is trying to come out of this. What is the probability that this particle will penetrate? And it is assumed that the alpha particle is already preformed inside. So that means inside the nucleus, there are clusters of alphas, two protons and two neutrons combined inside the nucleus to form an alpha particle. So the probability is assumed to be one for even in the nuclei.

So it is easily possible. Now, how to calculate this one, these two factors? So the frequency of striking the barrier, you can calculate based on the de Broglie wavelength ħ/mv where m is the reduced mass, alpha is the velocity. So de Broglie wavelength is close to the radius of the nucleus. So because the distance that alpha has to travel inside the potential well will be close to the radius of the nucleus. So this is the radius of the heavy nucleus. So de Broglie wavelength equal to R_1 velocity. So you can see velocity of alpha particle in the nucleus will be, you can put it from here, $\hbar/\mu R_1$. So this is, now you can find out the frequency ν = n λ, n is the frequency. So λ is given and v is given, you can find out the n. So n, the frequency equal to, frequency ν = n λ.

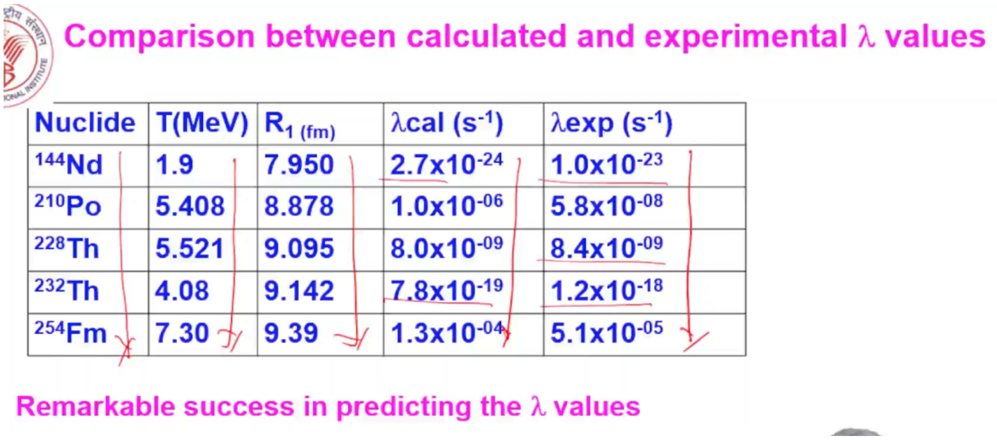
So ν = n λ. So v is the velocity with terms of ħ/μR, λ is equal to 2R. So the frequency (n) becomes $\hbar/ 2\mu R^2$ So this is the frequency with which the alpha is striking the barrier and the penetration probability. So the barrier penetration can be obtained by considering the wave function of alpha inside the nucleus in this region and outside the nucleus by calculating a transmission coefficient that is nothing but the transmitted flux upon the incident flux. So there is a flux of alpha striking the barrier, and the flux of alpha going out.

So there are three areas. There are different wave functions in these three areas. You can calculate the transmission coefficient. We will not go into details, but the product of these two quantities will be, so there is an integration here in fact from r1 to r2, V(r) potential as a function of r minus T, the kinetic energy to the power half. But essentially it is the area under this this curve. So the curvature, the height of the Coulomb barrier and the thickness of this barrier determines the lambda.

So the integral can be solved. Here is the formula

$$\lambda = \frac{h}{2\mu R^2} exp(- 8\pi Z_1 Z_2 e^2/h\nu(cos^{-1}\left(\frac{T}{B}\right)^{\frac{1}{2}} - \left(\frac{T}{B}\right)^{\frac{1}{2}}\left(1 - \frac{T}{B})^{\frac{1}{2}}\right))$$

Where T is the kinetic energy, B is the barrier. So this is called the Gamow-Gurney Condon formula and this formula is found to be reasonably successful in predicting the half-lives or the λ values of nuclei.



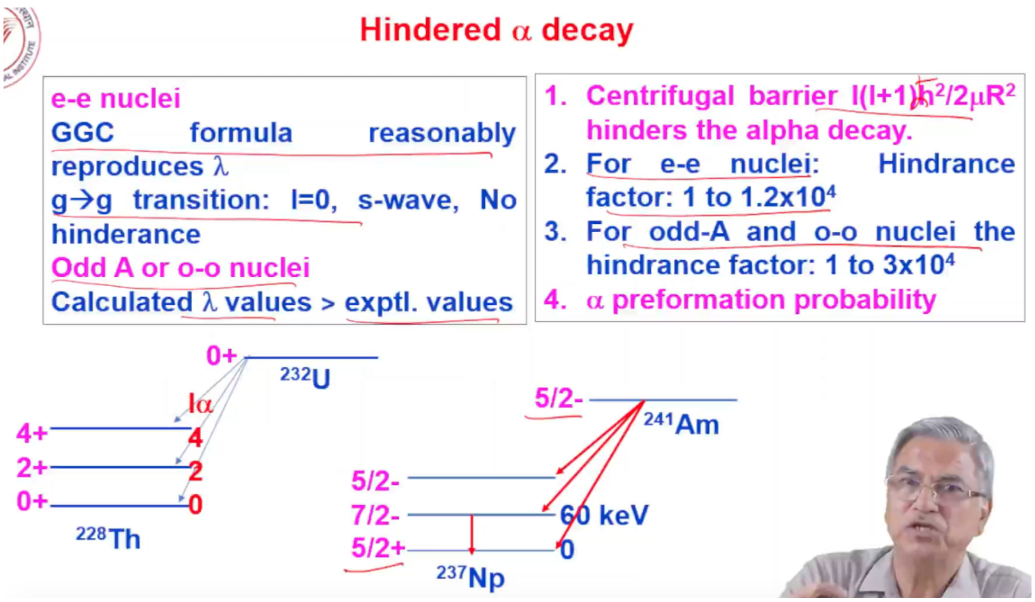
Comparison between calculated and experimental λ values

Nuclide	T(MeV)	R_1 (fm)	λcal (s^{-1})	λexp (s^{-1})
^{144}Nd	1.9	7.950	2.7×10^{-24}	1.0×10^{-23}
^{210}Po	5.408	8.878	1.0×10^{-06}	5.8×10^{-08}
^{228}Th	5.521	9.095	8.0×10^{-09}	8.4×10^{-09}
^{232}Th	4.08	9.142	7.8×10^{-19}	1.2×10^{-18}
^{254}Fm	7.30	9.39	1.3×10^{-04}	5.1×10^{-05}

Remarkable success in predicting the λ values

We will just see here the decay constant values λ calculated and the λ experimental value in second inverse. So right from Nd-144 to Fm-254 experimental and calculated alpha decay constants are given here which are experimentally known, their radii are known and you can see here in terms of the order of magnitude 2.7×10^{-24}, 1.0×10^{-23}, 1.0×10^{-6}, 5.8×10^{-8}, 8.0×10^{-9}, 8.4×10^{-9}, 7.8×10^{-19}, 1.2×10^{-18}.

So you will find that this the barrier penetration formula of Gamow-Gurney and Condon reasonably gives the prediction about the decay constant and therefore the half-lives which are very close to the experimentally determined half-lives. So now this is very fine for even-even nuclei the formula just now we derived or just now we saw the final form of it gives the values close to the experimental values that is possible for the even-even nuclei. See very close to that so why it is because the success of it lies because alpha particle is carrying no angular momentum. So l value is 0 because alpha particle spin is 0 and the two states like 0+ to 0+ of uranium-232 to thorium-228. So 0 to 0 transitions and with alpha having 0 spin easily you can explain it based on that formula just now we saw.



Hindered α decay

e-e nuclei

GGC formula reasonably reproduces λ

g→g transition: l=0, s-wave, No hinderance

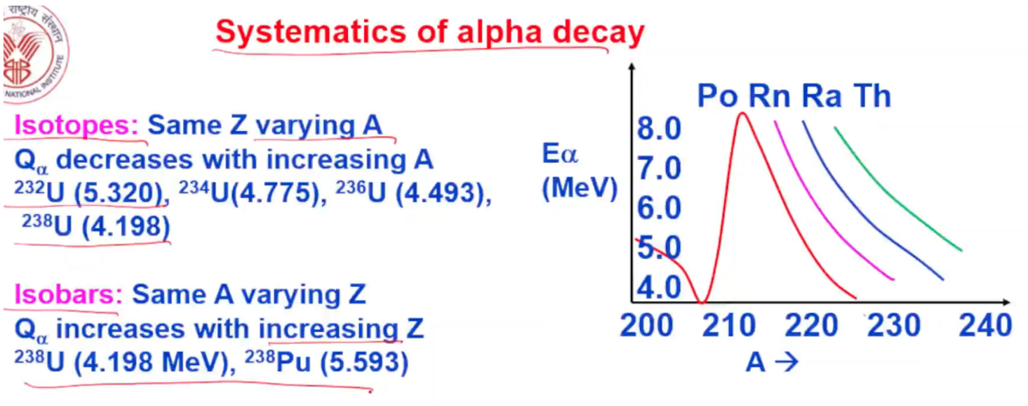
Odd A or o-o nuclei

Calculated λ values > exptl. values

1. Centrifugal barrier $l(l+1)\hbar^2/2\mu R^2$ hinders the alpha decay.
2. For e-e nuclei: Hindrance factor: 1 to 1.2×10^4
3. For odd-A and o-o nuclei the hindrance factor: 1 to 3×10^4
4. α preformation probability

So S wave alpha particles there is no problem there is no hindrance. Once we come to odd A or o-o nuclei then what was found that the calculated lambda values are much more than the experimental values because there is a hindrance. What is the hindrance? One is the centrifugal barrier. If the alpha particle has to carry certain angular momentum then there is an additional barrier apart from the coulomb barrier. There is a centrifugal barrier that is $l(l+1)h^2/2\mu R^2$. It is like rotational energy of the system. So that hinders the alpha decay so alpha decay half-lives are much longer or λ values are shorter. So because of this for example here apart from the ground to ground transition there are decays to excited states 2+, 4+. So the probability to populate 2+ , 4+ state will be much lower than 0+ state because now alpha has to carry 2 units of angular momentum, here 4 units of angular momentum and whenever alpha has to carry angular momentum then there is a centrifugal barrier and therefore even for even even nuclei the hindrance factor go from up right up to 10^4 because of the requirement of carrying angular momentum.

Whereas for odd A nuclei even the decay to the ground state is hindered, because now the spins of the parent and daughter nucleus ground states are different like here 5/2- and 5/2+ and the excited states are also there. So in the case of odd mass or odd o nuclei not only that the there is a centrifugal barrier even the preformation probability. Now the probability that the alpha particle will be formed inside the nucleus will be much less because for alpha to form a pair may have to break. So there is an unpaired nucleon. So because of that the nucleus is left in an excited state. So invariably you will find that when there is an alpha decay from a odd mass nucleus or odd-odd nucleus, population of excited state is more than the ground state. Like Am-241 probability is more to these excited states and which undergoes gamma decay of 60 keV to the ground state of Np-237. So what we saw that for the even-even nuclei the penetration formula can reasonably explain the half lives of several nuclei. But for odd mass and odd-odd nuclei there are hindrances due to the reduced preformation of alpha and the centrifugal barrier. Therefore, experimental half lives are much longer than the calculated ones.



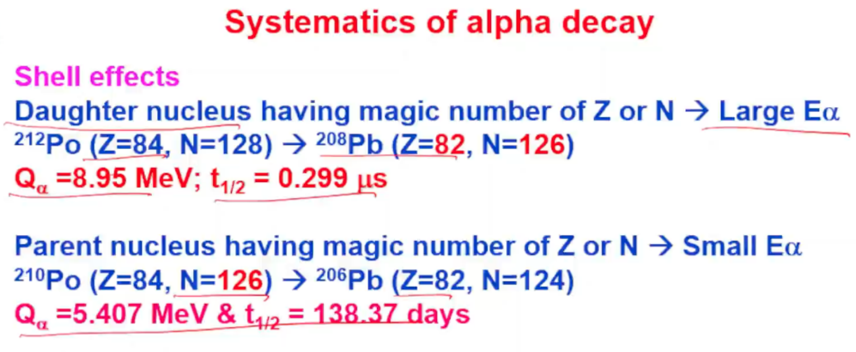
Systematics of alpha decay

Isotopes: Same Z varying A
Q_α decreases with increasing A
^{232}U (5.320), ^{234}U(4.775), ^{236}U (4.493), ^{238}U (4.198)

Isobars: Same A varying Z
Q_α increases with increasing Z
^{238}U (4.198 MeV), ^{238}Pu (5.593)

Now lastly we see the systematics of alpha decay because they are very useful and we need to know what is the reason. So among the isotopes of an element having the same atomic number but varying mass number the Q value decreases with the increasing mass number. You can see here for the same element like polonium as the mass number is increasing the Q value of the alpha decay or the energy of alpha is decreasing and this trend is seen for all elements.

So you can see here also, uranium-232 to uranijm-238 alpha energy is decreasing. And this can be explained even by the liquid drop mass formula. Later on we will see similar trend in fissility parameter. The same happens with the fission half life. In isobars same mass number and varying atomic number Q alpha increases with increasing Z. You can see here ^{238}U, ^{238}Pu are isobars, with increasing Z the alpha energy is increasing.



Systematics of alpha decay

Shell effects
Daughter nucleus having magic number of Z or N → Large Eα
^{212}Po (Z=84, N=128) → ^{208}Pb (Z=82, N=126)
Q_α =8.95 MeV; $t_{1/2}$ = 0.299 μs

Parent nucleus having magic number of Z or N → Small Eα
^{210}Po (Z=84, N=126) → ^{206}Pb (Z=82, N=124)
Q_α =5.407 MeV & $t_{1/2}$ = 138.37 days

And now shell effects. The nuclear shell effects influence the half lives. So if the daughter product, is having a magic number of protons or neutrons then because daughter has got a lower mass. When we say magic number means that nuclei are having more stability so masses are low. So there the alpha energy is high and therefore the half-lives are short. Typical example, polonium-212 Z = 84 goes to lead-208. In fact, it is a doubly magic number so the mass of the lead-208 is much smaller and therefore Q_α is very high correspondingly the half-life is very short. And other aspect is if the parent is having magic number so the parent mass is low like N=126 though here Z = 82 but the neutron cell effects are more dominant here because it is a higher l value. And so you will find that the again the half-lives are long because the Q_α is short.

So if the parent is a magic number then the parent mass is low and therefore the Q_α is low and hence the half-life is high. So just see polonium-210, half-life 138 days, polonium-212 microseconds. So this is the systematics, the shell effects influence the half-life of alpha decay and also the among the isobars and isotopes how the half-life changes we have just now seen. So this is all about alpha decay. I hope I could give you a flavor of the alpha decay, why alpha decay is not seen in the lower mass, is seen in the heavy nuclei all that and the systematics you can explain using the simple concepts.

So I will stop here. In the next lecture I will take the beta decay. Thank you very much.