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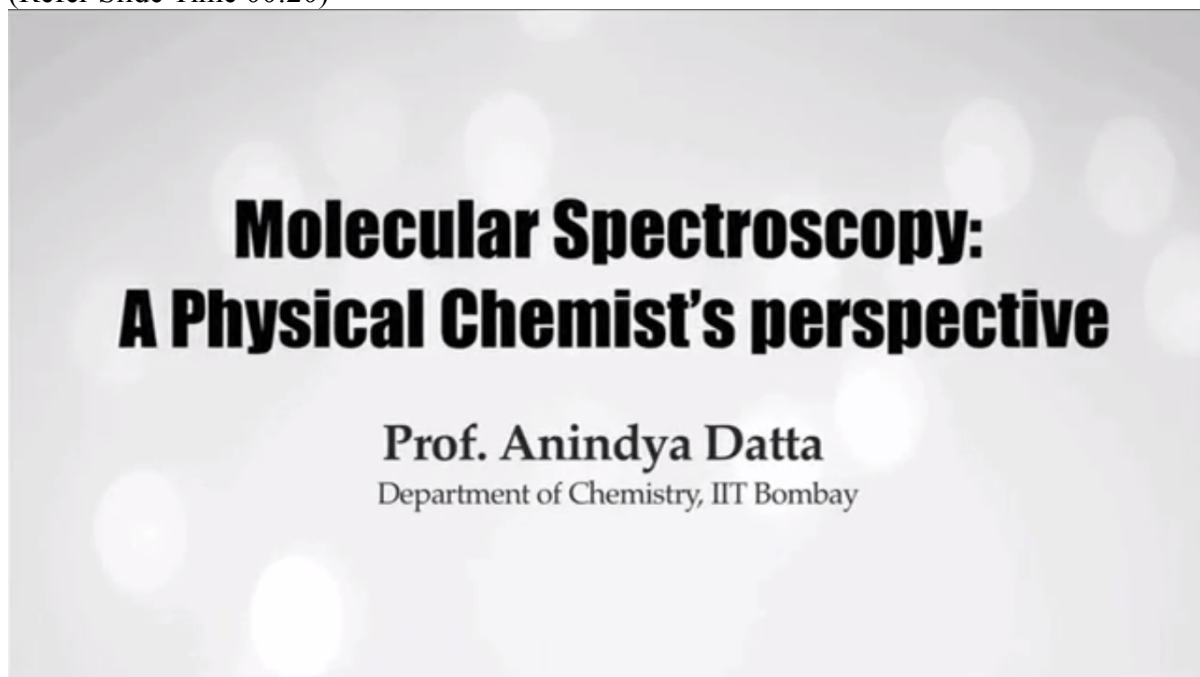
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Molecular Spectroscopy:  
A Physical Chemist's perspective

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## Lecture No. - 42

### Determination of Symmetries of Normal Modes of Vibration-II

Lecture No. - 42  
Determination of Symmetries of Normal Modes of Vibration - II

So we continue our discussion of this symmetry of normal modes of a D<sub>3h</sub> molecule with 4 atoms and so far we have reached this situation where for C<sub>3</sub> we have a 12 x 12 matrix and the character is 0.


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**C<sub>3</sub>**

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ x_2 \\ y_2 \\ z_2 \\ x_3 \\ y_3 \\ z_3 \\ x_4 \\ y_4 \\ z_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ x_2 \\ y_2 \\ z_2 \\ x_3 \\ y_3 \\ z_3 \\ x_4 \\ y_4 \\ z_4 \end{pmatrix}$$

Off-diagonal blocks: Atoms that change  $\Rightarrow$  No Contribution to  $\chi$

**$\chi(C_3) = 0$**



Okay. Why is it 0? Because this  $-\frac{1}{2}$ ,  $-\frac{1}{2}$  exactly offsets  $+1$ . But we have actually learned something very, very important from this exercise. What is it that we have learned?

You see these blue blocks are what we had worked out, right, from the co-ordinates initially. What is this one? This is the contribution of co-ordinates of atom 2 in atom 1. Okay. Only the co-ordinates of atom 2 make a non-zero contribution to the transformed co-ordinates of atom 1. Why? Because atom 1 has moved from its original position to the position of the original position of 2. Only the co-ordinates of atom 1 make a non-zero contribution to the transformed co-ordinates of atom 3. Why? That is how the transformation has taken place.

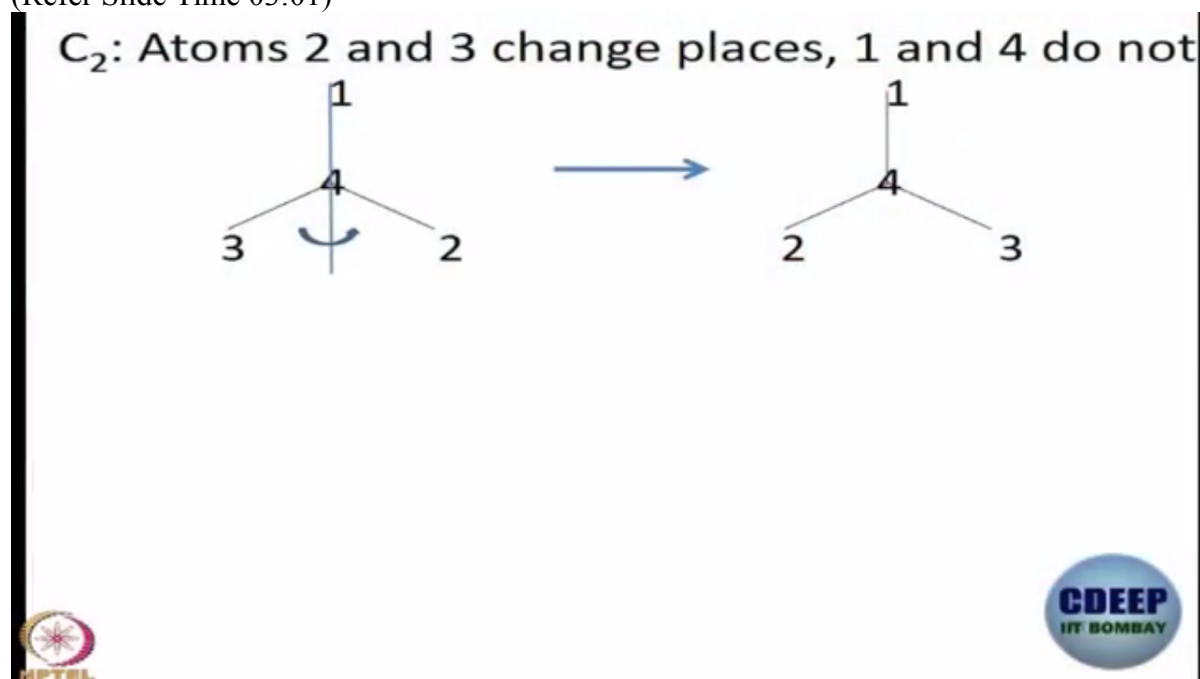
So what we see is that if atoms change place, then the non-zero blocks actually go off-diagonal. Right? What is the only atom that has made some non-zero contribution to the character? Atom 4, the only atom that has not moved from its own position as a result of the transformation. Right? This is something that is very, very important and you see to get this one matrix, we have spent like half an hour. Now we are going to get the other matrices in a matter of 2, 3 minutes each. Okay.

So the important lesson that we have learned is that the off-diagonal blocks, non-zero off-diagonal blocks are for atoms that change places, and they do not contribute to  $\chi$ . Right?

So if you remember the old English proverb, rolling stone gathers no moss, right, if atoms move from their own position as a result of a symmetry transformation, symmetry operation, then they do not contribute to the character. This is something that we learned, and this is something that we'll use now. This will become very, very simple.

Okay. Let's go to the next symmetry operation. Before that, is there any question at this stage? Let's move.

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$C_2$ . Remember where  $C_2$  is? Where is  $C_2$ ? This is one, this is one, this is one. Do they belong to the same class?  $C_2$ 's, do they belong to the same class? Yes. So it is enough if you work with any one  $C_2$ . Okay. We choose to work with this  $C_2$  that goes to atoms 1 and 4. All right.

Now as a result of this C<sub>2</sub>, what will happen? Which atoms move from their original position? Yes. 3 and 2 will interchange. Right? 1 and 4 remain in their own position. This is how the picture is going to be. So I hope it is not very difficult to understand that 3 and 4 will make no contribution to the character. Only 1 and 4 will make contributions. First. Second point is 1 and 4 will make equal contributions. Are we clear?

2 and 4 move from their original position. Therefore, they do not contribute to the character. 1 and 4 remain in their position. Therefore, they contribute equally to the character. Okay. So see now the problem has become so simple. You don't have to work with 12 co-ordinates anymore. You only have to work with 3 co-ordinates. That's all.

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$C_2: 1 \text{ and } 4: x \rightarrow x, y \rightarrow -y, z \rightarrow -z$

$\chi(C_2) = -2$

Off-diagonal blocks: Atoms that change  $\Rightarrow$  No Contribution to  $\chi$

These are the co-ordinates, right? I hope you remember this is X, this is Y, this is Z. So tell me then what happens to X<sub>1</sub> as a result of C<sub>2</sub>? What is X<sub>1</sub>'? Same. What is Y<sub>1</sub>' as a result of C<sub>2</sub> operation? -1 is a character. What about Z<sub>1</sub>'? It becomes minus, right?

So this is the transformation matrix I get. I don't worry about anything else. See the only thing I worked out is this. For X<sub>1</sub> we said there is no change. For Y<sub>1</sub> it becomes -Y<sub>1</sub>. For Z<sub>1</sub> it becomes -Z<sub>1</sub>. So that block is 1 0 0, 0 -1 0, 0 0 -1 and you will have an identical block for atom 4 because atom 4 also doesn't move.

Have we understood this matrix? Okay. Why do I not write the off-diagonal blocks? Of course, they will be a similar block for 2 and 3 as well, but they will be off-diagonal. So I don't even write. The only thing I want is character. What is a character then? 2.  $\chi(C_2)$  is -2.

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$\sigma_h (xy) : \text{All atoms: } x \rightarrow x, y \rightarrow y, z \rightarrow -z$



$$\begin{pmatrix} x_1' \\ y_1' \\ z_1' \\ \dots \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ \dots \end{pmatrix}$$



$\chi(\sigma_h) = 4$



All right.  $C_2$  is done. What is next? Let's do  $\sigma_h$ . For  $\sigma_h$ , which are the atoms that move from their position? None. Does that mean we have to work with 12 co-ordinates? No. Why not? Yes, because all 4 will make identical contribution. Just work out any one. Okay. So  $\sigma_h$  is very easy. Isn't it?

$X_1$ . What does  $X_1$  become? What is  $\sigma_h$ ? It is the XY plane. Z is your  $C_3$  axis. So what happens to X? Same. What happens to Y? What about Z? -1. So what is the contribution to character of each atom?  $1 + 1 - 1$ . So it is 1. Maybe you are giving me the answer when you said 1. I thought you are giving me the answer for X only anyway, and how many such blocks are there? So what is the character? Character of  $\sigma_h$  is 4.

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$S_3 : 1, 2, 3$  change places, 4 does not



$$\begin{pmatrix} \dots \\ x_4' \\ y_4' \\ z_4' \end{pmatrix} = \begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} z \\ \dots \\ x_4 \end{pmatrix}$$

$\chi(S_3) = -2$



$S_3$ . What happens when I do  $S_3$ ?  $S_3$  is similar to  $C_3$  in that 1, 2 and 3 interchange places. 4 remains in its own place. What is the only difference? You are doing an additional reflection. So Z will become -Z. That is all. Okay.

So what is it then? This is the only block you have to worry about. From X and Y, you know already you will get -1/2, -1/2. For Z you are earlier getting +1. Now you will get -1. So -1/2, -1/2, -1. Okay.

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$\sigma_v (zx) : 2 \text{ and } 3 \text{ change places, } 1 \text{ and } 4 \text{ do not}$

1 and 4:  $x \rightarrow x, y \rightarrow -y, z \rightarrow z$       Two groups of diagonal blocks

$$\begin{pmatrix} x_1' \\ y_1' \\ z_1' \\ \dots \\ x_4' \\ y_4' \\ z_4' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ \dots \\ x_4 \\ y_4 \\ z_4 \end{pmatrix}$$

$\chi(\sigma_v) = 2$

$\sigma_v$ .  $\sigma_v$  is ZX, right?  $\sigma_v$  is ZX. Of course, I could have taken this one, but it is more convenient if I take this. So I will take this.  $\sigma_v$  is ZX. So what will happen to X? No, first of all, wait. When I apply  $\sigma_v$ , which atoms remain in their place? Which atoms move from their place? Again, 2 and 4 interchange places, so they don't matter anymore. 1 and 4 will only matter and they will make identical contributions. Okay.

So what will happen? What will happen to X? This is X. This is  $\sigma_v$ . Will it change? No. So character is 1. Sorry, coefficient is 1. Why? Will it change? (inaudible 08:14) Yeah. Y will become -Y. Yeah. And Z same. So what will the character be for each atom? 1, 1, -1. That is again +1, right? See X remains same. ZX. Z also remains same. 1 + 1, 2 and Y becomes -Y. Yeah. I didn't even watch. So 1 + 1 + -1, that is 1. How many atoms contribute? Two. So 2 into 1 is 2. This. Simple.

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## Reducible representation containing the 3N (=12) co-ordinates of $\text{CO}_3^-$

E	$2C_3$	$3C_2$	$\sigma_h$	$2S_3$	$3\sigma_v$
12	0	-2	4	-2	2

- 3 translational co-ordinates
- 3 rotational co-ordinates
- 6 vibrational co-ordinates

How can we reduce this RR into constituent IRs?



So, finally, after doing all this, this is where we are. This is the representation we have got, right, using the 3N Cartesian co-ordinates.

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