Symmetry and Group Theory
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Lecture No. 15
Matrix Eigenvalue Equation: An Example

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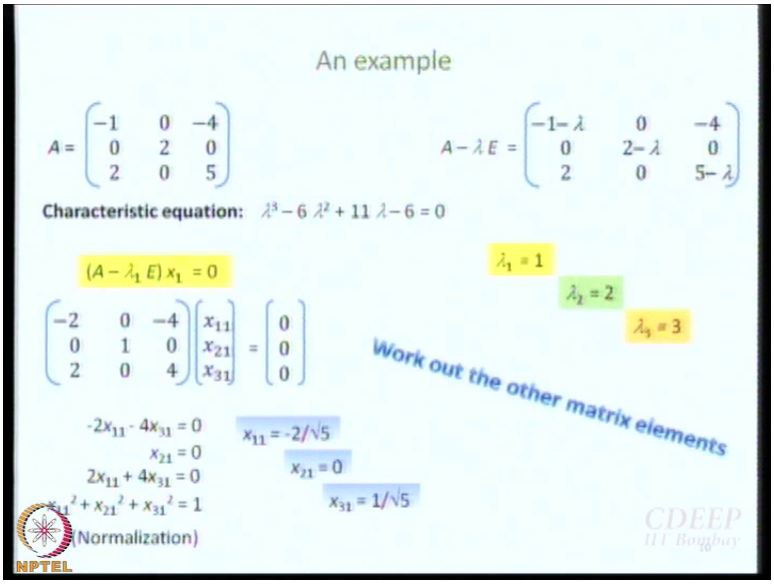

Now let us work out this example is given in this Bishops book ok this is a matrix I will work out another more interesting example after this. This is A ok, what is the first step what did I want to do? First step is we have to find that what is A - Lambda E right? What is A - Lambda E?-1-Lambda then 0 -4 and 0 2-Lambda 0 then 2 0 5-Lambda this is you are A - Lambda matrix very easy. What can I do when I have this matrix I can get the data of this and put it in the = 0 and what will I get then, no. What is that equation called? Characteristic equation of matrix A.

You see what we are talking about have talked about x yet, we are not even try to say what is x all I have done is I have written the matrix A from there I can write this characteristic equation, characteristics equation of the matrix A. What will be the characteristics equation be can you work it out expand the determinant and tell me what it is? Is the equation Lambda cube - 6 Lambda square + 11 Lambda - 6 =0 on the way she did it she did not even like the entire equation is it not is she just took the factors that is why it is much faster that is a smart way of doing this.

And the answer she gave also is correct there is an amazing matrix that lambdas are 1 2 and 3. Let us work it out and satisfy yourselves. So, will you; as everybody got 1 2 and 3 or do you have some other value? 1 2 and 3 right, now see we know the matrix A and we have determined the 3 eigenvalues. Do you expect the 3 eigenvalues are you get more or less, why? N, no forget about Lambda this is N by N Matrix 3 by 3 matrix N = 3 ok.

Now what we can do is for each of these eigenvalues I can find the eigenvectors, how will I do it? Just write the equation right. This equation A-Lambda e X1 =0, what is the meaning of X1, one is the number corresponding to the number of eigenvalue ok, i the second number alright. What is the; now we can put one here for Lambda 1 = 1 you just scoop on you what do you get? What is the matrix?-2 0-4 010 and 204 right.

Now see it is little clear now. Now see I am writing x11 x12 x31 Aachal; got it down working it for the first eigenvalue. So we are working with the first eigenvector of course what you have said that first eigenvector you could have taken the second or third it does not matter ok that is in your hands. But the point is available 3 eigenvectors and there will be 3 eigenvalues. So now once you have this; this is simple. How many equations do you get? 3 how many unknowns do you have? They are unknown's 3 unknowns. So you should be able to work it out let us see. What is the first equation -2 x 11-4 x 3 1 is 0 right.

What is the second equation x21 =0,what is the third equation 2 x11 + 4 x31 =0, now you have 3 equations 3 unknowns can you solve and get the values. I will give you the first value the most difficult one oops no I will give you the first letter what have given is the answer. You cannot, why cannot? You use these equations to get 3 values there are redundancy right there is redundancy right. It is necessary that we have redundancy no matter which matrix we use. So now what is the way out, where will the last equations comes from, yes.

Of course I played spoil sports here by giving away the answer by mistake I forgot the sequence of my animation. But the point is whenever you use eigenvectors you always want them to normalise ok. Otherwise you would have lot of redundancy just multiply by 3 by 4 that does not make any sense. They are always normalised like this ok x11 square + x21 square + x31 square is =1. Normalising means setting the value to some particular value setting it to some particular value ok, so you have encountered this condition of normalisation in many places right where?

Orbital's fine, so now can you work out the values. I will give you the most difficult one x21 =0, now just work out the easier ones 1 by root 2, 1 by root 5 - 2 by root 5 and 1 by root 5 alright.

So these are there 3, x values what is the eigenvector then. What is the first eigenvector -2 by root 5 0 1 by root 5 normalised fine. Now I want you to work out Lambda 2 = 2 or Lambda 3 = 3. Maybe let us split Lambda 2 = 2 this Lambda 3 = 3 Lambda 2 = 2 Lambda 2 = 3 let us work it out. So for Lambda 2 = 2 what is the matrix? What is the A - Lambda 2 matrix? -30-4 000 203 now work it out what are the equations -3 x12 -4 x32 is = 0.

Second equation is fantastic 0 = 0 better be; then third what is the third one 2 x12 + 3 x32 is = 0 4th x12 square + x32 square = 1, work it out what is the answer you get 0, 010 is the answer whatever Lambda 3 is = 3 what is the matrix -40-4 then 0-10 202. So, what is the first equation -4 x13 then -4 x33 is = 0 second one -x23 is = 0 notations are all problematic third one you get redundant equation and 4th is there normalisation condition and what is x13 then x13 there is no minus, x33 is -1 by root 2, x13 is 1 by root 2 yeah not a problem how they are written that is all.

So now see what we have been able to do it using a given 3 by 3 matrix we are being able to work out the eigenvalues as well as eigenvectors right. Now the question is does it works only for this matrix for certain matrices or should it work for any matrices or any square matrix. Should it work for any matrix any square matrix or should it work for only some matrices. Let us take group force approach and then we figure out. You are 22 people present today. So I so what I want you to do today write down matrix 2 by 2 matrixes each.

And this 2 by 2 Matrix cannot be null matrix cannot be a diagonal matrix cannot have the same elements and for your convenience keep the number less than 10 you can use minus there is no problem like 2 8 -1 5 something like that. So just write down matrices any matrix that you want we will find out now, done stage on matrix anyway the magic tricks that you have used to write name the number and add 5 multiplied by 3 and so on and so forth finally I will tell you what the answer is?

Have you written the matrix right 2 by 2 is fine do not write 1 by 1 that will be problematic, 2 by 2 matrix. For this 2 by 2 matrix can you work out the characteristic equation? Do it. This is very simple characteristic equation will be of which order? Second order, do it. Write down the characteristic equation, solve it and get the roots and tell me if you encounter any problem. Write

down the characteristic equation, how will I solve it, second order equation. What is that formula that everybody learnt in class 11 maybe earlier of quadratic equation -B + or - root over V square -4 AC divided by 2A.

So what you need for this. What will happen if B square is less than 4AC if that is possible then, we get a complex number that is all. It does not mean that it is not an eigenvalue work it out. Hari, what are your roots. I mean what are the roots of the equation 5 and -1 lucky Chap is not got any imaginary number. Tell me if there is any imaginary number, you get any imaginary number somebody, good that is also good you know I think it is enough if you just walk out the roots. If you get the roots are you convince them that the rest will work out.

You get the roots means you get the eigenvalues right, so if you have the eigenvalue better you have some eigenvector. It is just that might be complicated, Aachal, one of the roots that you got I am sorry root 7 and 9 2 minus root 7 right. If you have eigenvalues like that life will be little complicated that is all but then you are going to going to have corresponding eigenvectors is it not. So this exercise tells you what you are talking about is just not something in the abstract mathematics you can put actually numbers in it.

You take any square matrix you can find out the eigenvalues and eigenvectors. Like many things that we do find the values first and vectors later. Maybe formulated the problem we have the matrix we have the vectors and then we find the eigenvalue but we all love that calculation right ok can we move on.