Transport Processes I: Heat and Mass Transfer
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Lecture – 09
Dimensional analysis: Convection and diffusion

So, welcome to this lecture once again, we will go in through dimensional analysis in this course on transport phenomena. I had gone through in some detail trying to explain how we can use dimensional analysis to simplify the design of various systems which are used for transferring heat mass or momentum; we have done and example of a heat exchanger if you recall in the lecture before last, where we had the dependent quantity is an average heat flux and that depends upon the temperature deference as well as large, relatively large number of parameters.

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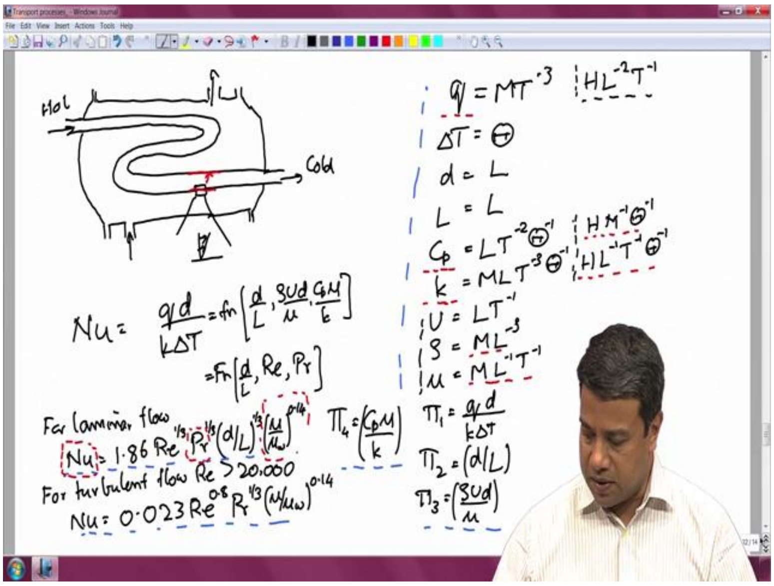

We had used dimensional analysis to consider please simplify the number of parameters and we had got dimension less groups, the dependent dimension less group the nusselt number as a function of independent dimension less groups. In this case there was a ratio of length now one that we called the Reynolds number and the number that we called the Prandtl number.

And I said beyond this you cannot do simply based upon dimensional analysis, you have to derive correlations based upon experiments or as we show see in this course, actually do calculations which will give you the forms of this dimension less numbers.

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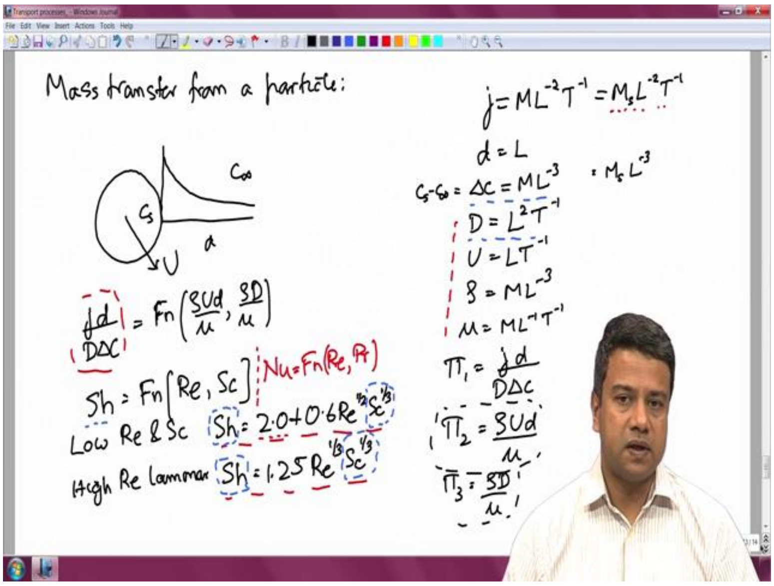

We have also looked at transfer form of particle, transfer of mass from a particle due to the concentration difference between the surface and the bulk fluid far from the particle. In this case mass transfer difference upon diffusion and we had once again to derive it correlation for the dependent dimension less group that contains the mass flux the average mass flux by units of his area, as a function of other dimension less numbers the Reynolds number and (Refer Time: 02:24) number in this case.

And I had told you that these dimension less groups for mass and heat transfer are analogues, if you know the dimension less the correlation for mass transfer in terms of the dimension less groups the (Refer Time: 02:45) number, the Reynolds number and the (Refer Time: 02:48) number.

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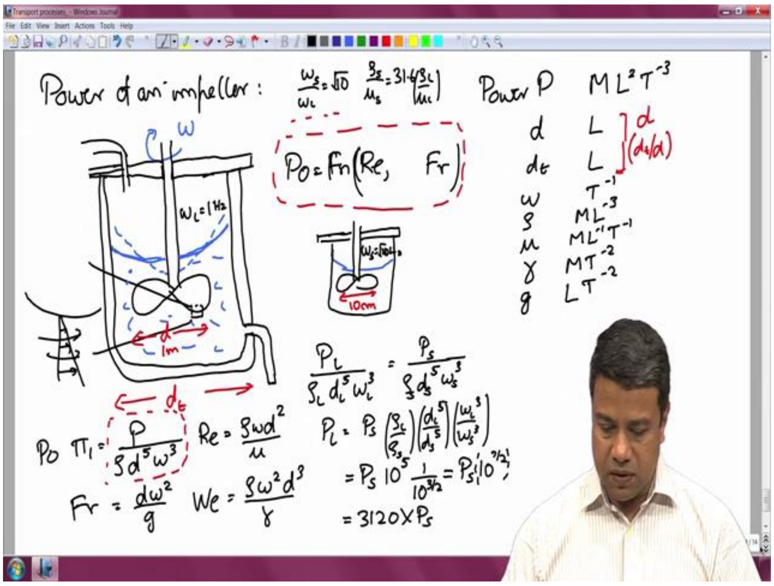

You can get an equivalent correlation for heat transfer, where the dependent dimension less group now is the nusselt number, the dimension less heat flux and the independent groups are the Reynolds number and the Prandtl number. How do these things come about and that is what we are going to study in the present class, and are also show the an example of momentum transfer, where we had looked at the power number for the power and an impeller. Of course, momentum transfer is a little different from heat and mass transfer for reasons that I will come to shortly, but let us first look at what is the meaning of this dimension less groups.

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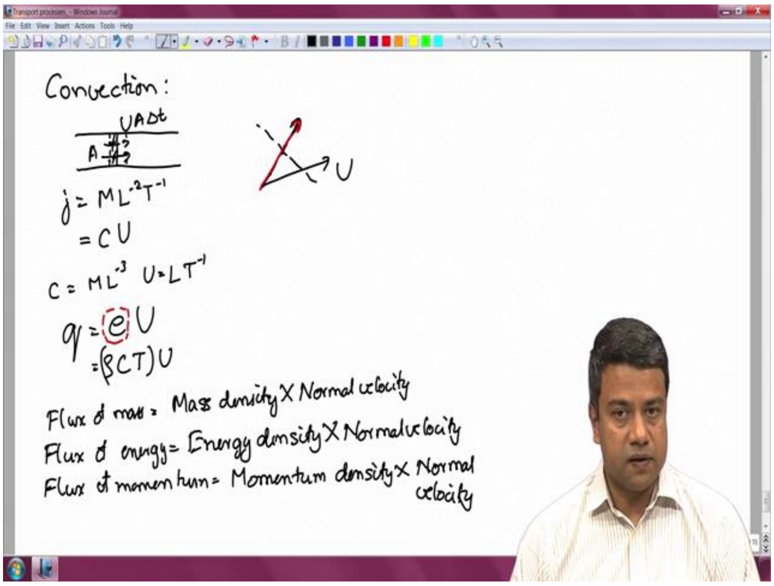

If you recall I have told you write in the beginning that there are two mechanisms of transport: one is convection, the transport of material carried along with the fluid, the transport of material because the fluid itself is flowing. So, if you had some kind of a tube for example, in their some fluid flow across some cross section and that fluid had with it a certain concentration of material now certain temperature, that would be carried along with the fluid because it is moving as a material and if the fluid velocity over you for example, what would be the flux of mass that was carried across the surface?

The flux as you know has dimensions of the mass carried per unit area per unit time. If there where fluid that was carrying the mass along with it, then the flux that is carried would simply be the concentration of mass in the fluid, times the velocity of the fluid. You can see that the concentration has dimensions of mass per length cubed, the velocity has dimensions of length per unit and therefore, if I multiply these two now you get the flux which is the mass per unit area per unit time. Basically whatever material volume element was here initially after time delta t it moves to a new location, that distance travel does u times delta t, the volume that is transfer does you into the cross sectional area into delta t.

Therefore the amount transferred per unit area per unit time will just be equal to the concentration times the velocity. To explain it again, if I had a volume here which has the sub some particular thickness and some cross sectional area A, after time delta t it has moved to the right, the amount that has moved to the right the volume that has moved to the right is going to be equal to U A times delta t therefore, the mass that has move to the right is going to be this volume times the concentration, the mass that has move to the right is going to be this volume times the concentration; is mass transferred is equal to the volume into the concentration.

Therefore the mass transferred per unit, area per unit time. We have to divide by the cross sectional area A, divide by time delta T and you just get U into the concentration. So, that is the flux that is the transport per unit area per unit time due to convection.

Similarly, this for mass transfer; for heat transfer the flux would be just the energy per unit volume times the velocity whereas; e is the thermal energy per unit volume. So, basically equal to mass times specific heat times a temperature divided by volume, which is equal to rho c times the temperature energy per unit volume times the velocity, that will be the heat flux there is transfer across the service.

The moment of flux of course, we will come to a little later so little slightly different; we will deal with at a little differently because the momentum itself is a vector; it is not a scalar like concentration or energy, so we will deal with it little different. So, the point are mechanises that are the fluid flows it carries along with it mass momentum or energy and the flux is just equal to the fluid velocity times the local concentration. So, the flux of mass is equal to the mass density; flux of mass across the surface is equal to the mass density times the normal velocity, there is a velocity perpendicular to the surface if I had some surface present the fluid and I wanted to find out what is the mass flux across that surface internal surface within the fluid and the fluid had some velocity in some direction U. This mass surface has to be equal to the component of U along this direction, the normal velocity to the surface times the concentration. So, normal velocity to the surface times the concentrate which is the mass density of the fluid, the mass per unit volume of the fluid.

Similarly flux of energy is equal to energy density there is the the energy per unit volume that I had over here, times the normal velocity and analogously you can always right the flux of momentum, momentum density times the normal velocity, everything that you to watch out for here is that momentum is also vector.

So, that is convection that the convector transports of material across the surface; this is equal to the normal velocity perpendicular to the surface times, the density of either mass momentum for energy.

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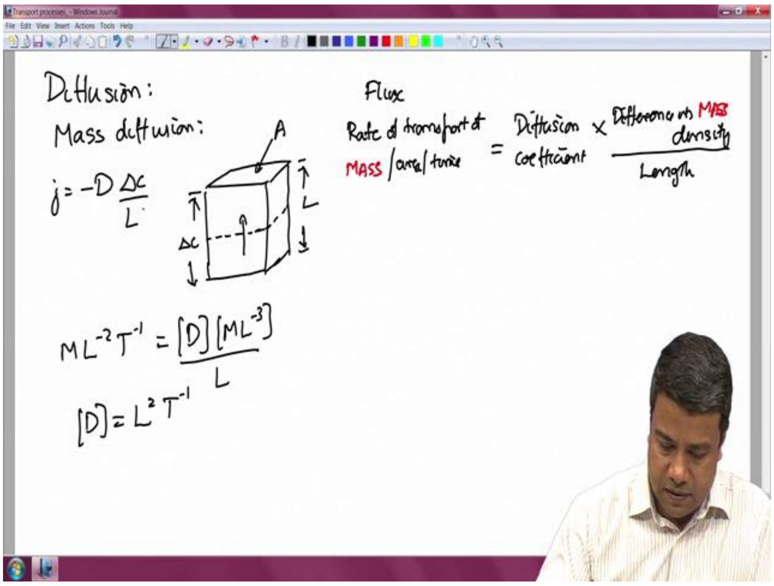

What about the transport due to diffusion? First let us take mass diffusion; this is given by Fick's law of diffusion. So, basically what it tells us that if I had some surface within a volume of fluid and I wanted to find out what is the flux across the surface, area of cross section is A, length is L, the flux of mass across the surface j will be equal to minus the diffusion coefficient, times the difference in concentration between these two, the difference in concentration between these two locations divided by L.

Recall the flux is mass per unit area per unit time; is equal to this diffusion coefficient, times concentration has dimensions of mass per unit volume divided by L and therefore, this diffusion coefficient has dimensions of length square T inverse. So, let us write it out in words, the flux which is the rate of transport of mass per unit area, per unit time is equal to diffusion coefficient times difference in mass divided by the length. So, that is the relationship for diffusion of mass and you can easily see there is you have one over area and one over time here and on the right side I have one over length.

So, inevitably diffusion coefficient is going to have should be carefully mass; difference in mass density concentration is the mass density mass per unit volume divided by per length. So, I have mass per unit area per unit time in the flux, one over length square one over time and mass density is one over length cubed and then this is other length here. So, therefore, the diffusion coefficient has dimensions of length square t inverse. So, there is a fundamental diffusion relationship for mass there is an analogues relationship for heat transfer is an analogous relationship for heat transfer and the way that works is if I had instead of mass, I had the heat flux; I wanted to know what is the heat flux q, due to a temperature deference delta T.

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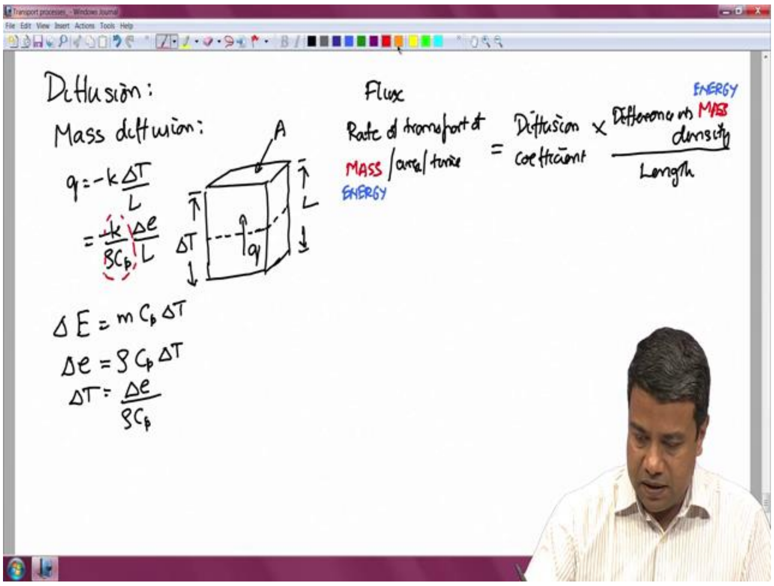

Rather than expressing the heat flux in terms of the temperature difference, I could also express the heat flux in terms of the difference in the energy density of the system; you could also express in terms of the energy density of the system, what is the energy density? Is the energy per unit volume of the system therefore, you know that the energy is equal to or the difference and energy between two locations will be equal to m C p times delta T therefore, the energy density which is the energy per unit volume, delta e that is in intensive variable will be equal to the mass density, mass per unit volume times C p times delta T.

Therefore I can as well write delta T is equal to the difference and energy density by rho C p. The expression for the heat flux q is equal to minus k delta T by L; q is equal to minus k delta T by L. If I were to write the temperature in terms of the energy density, I will get minus k by rho C p delta T e by L. So, what this expression is telling me is that the rate of transport of energy per unit time, when I have expressed the temperature difference in terms of the energy density is equal to diffusion coefficient times at difference in the energy density per unit length, same expression except that I have expressed at in terms of the instead of using the temperature difference, I use the difference in the energy density.

Since I have energy on the left side energy on the right side, this diffusion coefficient has to necessarily have dimensions of length square per time inverse. So, this is basically a diffusion coefficient for energy, a thermal diffusivity a fuel, it is often represented by the symbol alpha.

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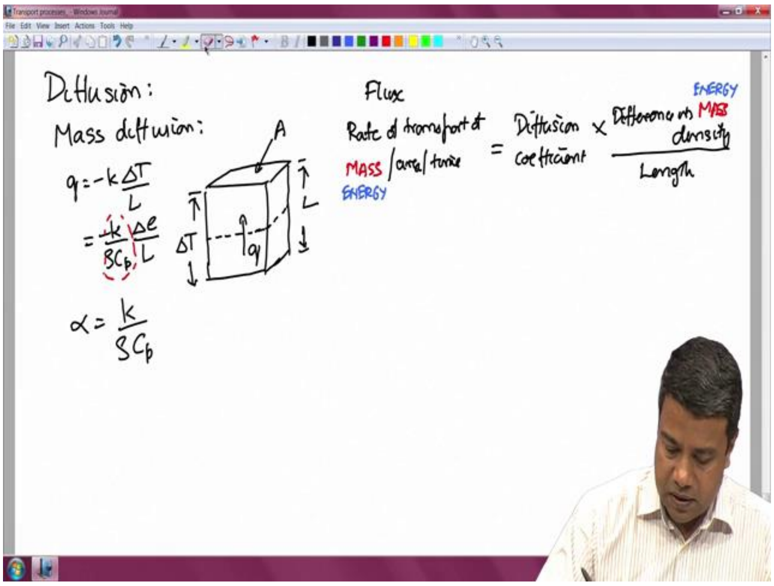

Alpha is equal to k by rho C p; this is a differential coefficient for (Refer Time: 17:57) it has dimensions of length square per unit time, you can verify that, you can quite easily to dimensional analysis and verify that this as dimensions of length square per unit time, this is a thermal differentiate coefficient. Now I can do the same thing for momentum as well.

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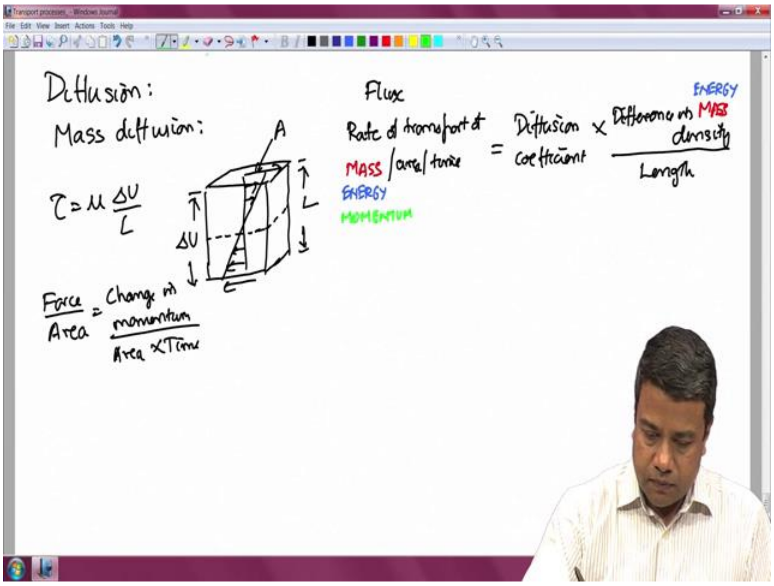

You know that the Newton’s law of a viscosity can be written as tau is equal to mu delta U by L. So, at the same system here rather than the difference in temperature, I have a difference in the velocity; rather than difference in temperature, I have a difference in the velocity. So, the top surface is moving at a different velocity then the bottom surface and because of that if I plot the velocity across, I will get a velocity difference across this surface, this is how the shear stress is let me. Shear stress is the force exhausted per unit area.

Force is the rate of change of momentum, changing momentum per unit area per unit time. So, is equal to per unit area into time. So, this is effectively flux of momentum, the change in momentum of the volume below is because that momentum has been transferred to the volume above this surface and that change in momentum per unit area per unit time it is a momentum flux you can change in momentum per unit area per unit time is a momentum flux.

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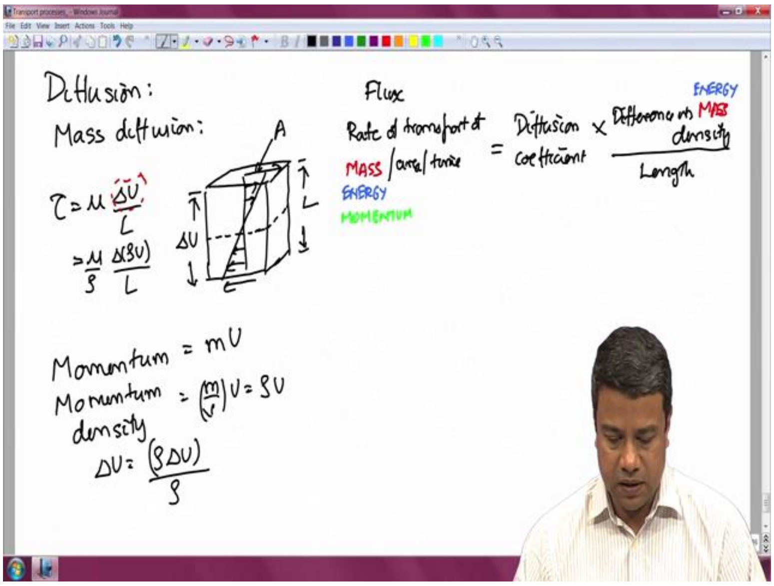

So, this momentum flux I have to expressive in terms of the difference in the momentum density; the momentum as you know is equal to the mass times the velocity.

Therefore the momentum density is equal to mass per unit volume into the velocity which is equal to rho times U. So, therefore I should express the difference in velocity in my expression here in terms of the momentum density and that is quite easy to do. So, the difference velocity delta U is equal to the momentum density which is rho delta U by rho. So, therefore, instead of U I write down the momentum density as rho delta U by rho, then my expression becomes U by rho into delta of the momentum density by L. For the present of will assume that the mass density does not change, those are called in compressible fluids in most fluids so long as the velocity is smaller than the speed of sound, you can assume that it is incompressible without any loss of generality.

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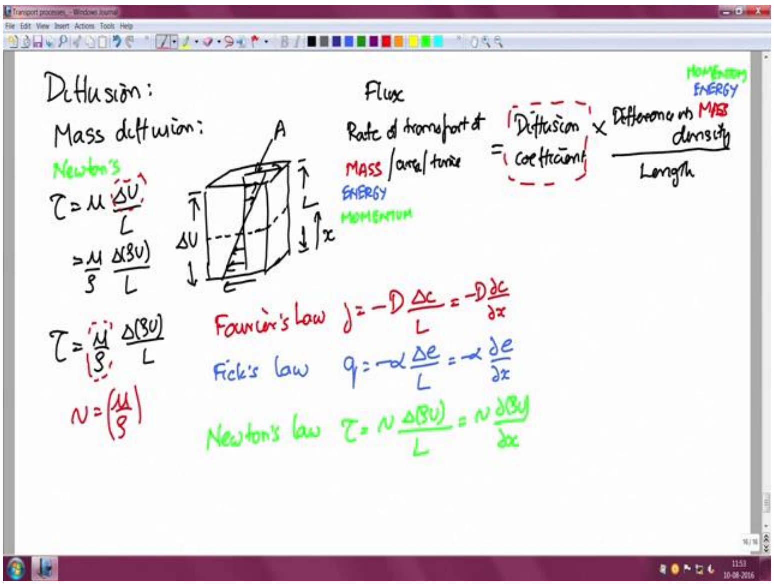

So, therefore, when I expressed in terms of the momentum density, I will get the constitutive relation of the form tau is equal to mu by rho, delta of the momentum density divided by L and this once again has to be a diffusion coefficient, it has to have dimensions of length square per time. This is what is called the kinematic viscosity mue is equal to mue by rho, ratio of the viscosity and the mass density what is a diffusivity for momentum. So, try to explain to how look an express the constitutive relations this for example, is Newton's law. So, the Fourier's law of heat conduction can be expressed as j is equal to minus D delta C by L.

Rate of transport of mass per unit area per unit time is equal to diffusion coefficient times thus difference in the mass density divided by the distance. The Fick's law for diffusion can be expressed as q is equal to minus k delta e by L, q is energy transported per unit area per unit time, delta e: e is the energy density and similarly Newton's law for viscosity, my apology is which should not be k which should be the diffusivity alpha and Newton's law for viscosity can be written as tau is equal to nu delta of the momentum density divided by L and express this way all of these D, alpha and nu are all diffusion coefficients. They have dimensions of length square per unit time.

So, these basically give you the relative rates of transfer of mass, momentum and energy; very often these are written in differential form. So, this kind of a relationship that we have here is valid only when you have a linear gradient across two surfaces in the fluid, you can also applied this locally at each location was in the fluid in which case you reduce the differential form minus D partial c by partial x. Where x the direction perpendicular to the surface; x is the direction perpendicular surface across which the transfer to taken place, these differential relations or obtained in the limit us L goes to 0, in the limit is L goes to 0 delta C by L will be d c by d x and it is important to note that you have to take the derivative perpendicular to the surface across which transfer to taken place. In this particular case the x coordinate will be in this direction.

The other point to note in both Fourier's law and Fick's law there is a negative of sign, there is because mass transported from higher concentration to lower concentration. Energy is transported from higher temperature to lower temperature or from higher energy density to lower energy density. And Newton's law we will write it here with the positive sign, I will explain a little later how that positive sin comes about.

So, these are the fundamentals of diffusion any quantity, rate of transport of that quantity per unit area per unit time that is the flux of that quantity is equal to a diffusion coefficient, times the difference in the density of that quantity. (Refer Time: 27:08) mass momentum or energy, the density of that quantity divided by a length perpendicular to the surface or the gradient perpendicular to the surface and written this way all the diffusion coefficients have dimensions of length square per unit time. How can we use this to frame our dimension less groups, I will come to that in the next lecture.

So, we have look at how we can interpret dimensionless groups in terms of ratios of diffusivities and ratios of convection and diffusion and that will give you a much better idea of what these dimensionless groups actually mean, will start that in the next lecture we will see you then.