Transport Processes I: Heat and Mass Transfer
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Lecture – 08
Dimensional analysis: Scaling up of an impeller

Welcome to this; this is our sixth of our lecture on dimensional analysis and in the last lecture we were looking at how to estimate the power required by an impeller in reactor. I had actually shown you the importance of reactors in our very first lecture.

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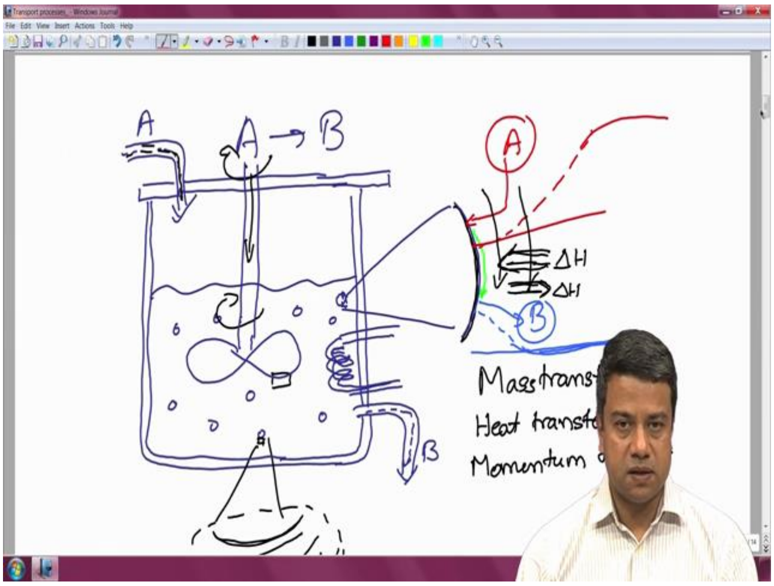

Where I head discuss this process of a chemical reaction and that said that for the chemical reaction to take place fast, you need the reactance on the products to be swept on to the surface of the particles and off the surface of the particles and in order to do that you need to generate the flow and that flow increases the transport rates.

However that flow also costs energy because you have to supply power to this reactor in order to rotate it. So, therefore the flow costs energy and we have to have some idea about what is the utility of this flow as far as increasing the transport reach is concerned and what is the cost with respect to the power requirements. So, that is what we were trying to do at the end of the last lecture; the power of an impeller.

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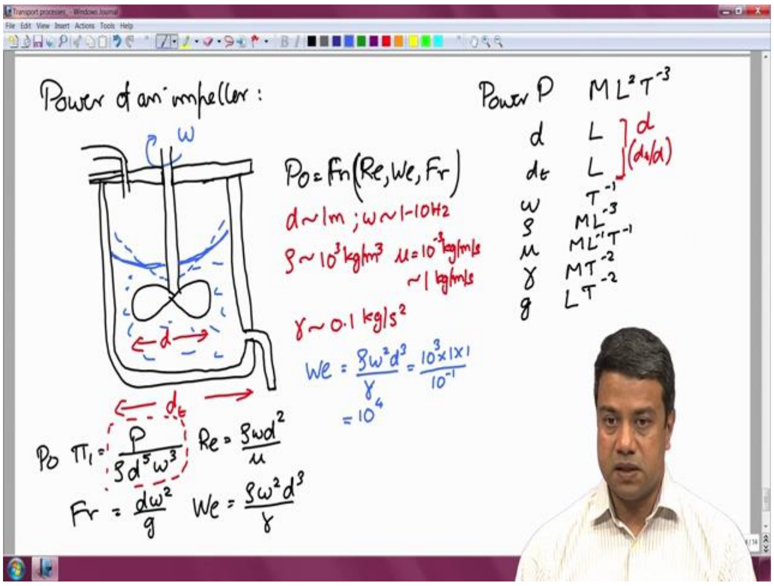

We have listed out all the quantities that are relevant; the power required that is an energy per unit time and energy is force times distance, so we have got the power S ML square T per minus 3; a length scale in this case we have chosen the impeller diameter. There are other length scales that are important, but if in a scale of process; you keep the ratio so of all of these the same then you need consider only 1 of these length scales in the dimensional analysis and it depends of course, on the frequency with which this impeller is rotating because the faster you rotate, you require more energy. Fluid properties density and viscosity and I would also told you that it could well depend upon the surface tension because when you rotate the impeller the surface deforms and the surface tension is a force the tresses the deformation; the deformation is due to the centrifugal forces, the centrifugal forces go simply as rho mega square, with the mass times the angular velocity square.

So, that is already incorporated in the density and angular velocity; what is not incorporated at two other effects; one is the gravitational force which tends to bring the surface back to a flat state and the surface tension force which tends to reduce surface area both of these act to make the surface flat where is a centrifugal force is studying to destruct the surface because we have a force acting outwards. So, based upon all of these we had identified minimal set of dimension less parameters, the dependent dimension less parameter was the power number. We have chosen to scale it by the density and because of that we (Refer Time: 03:34) this power number as P by rho d to the fifth power and omega cube and that is function of the Reynolds number.

In this particular case, the Reynolds number is based upon the frequency of rotation Reynolds number the Froude number which gives you some idea about the centrifugal forces to the gravitational forces because acceleration due to gravity in the denominator and then you have a Weber number which is the ratio of inertial to surface tension forces, so these are the dimensionless parameters. Now if you want to analyse the system, we should have some idea of what the values of these dimensionless parameters are.

If you take a typical configuration, the diameters about to meter or so, the frequencies of Theo-graph 1 to 10 hertz; we want to 10 revolutions per second. Liquids normally we will have density of Theo-graph 10 power 3 kilograms per meter cubed, gasses will have density of Theo-graph 1 kilo gram per meter cubed, so this factor of 10 power 3 difference between the two. In this case, we considering a liquid system the density of Theo-graph 10 power 3 kilo transfer per meter cubed; viscosity of water is 10 power minus 3 kilogram per meter per second, sometimes we have very viscous fluids in these tanks and there the viscosity could increases much as 1 kilogram per meter per second that happens for very viscous fluids like polymer solutions are melts.

In those cases typically the frequencies are also much smaller; the rotation rates are also much smaller. Surface tension of fluid like water is of Theo-graph 0.1, so for this set of parameters can we estimate what are these dimensionless numbers. First Weber number which is equal to rho omega square d cube by gamma, rho is of their of 10 power 3, omega square if you take 1 revolution per second resist 1, if you take 10 revolutions per second it will become 100 and d cube; I said impeller typical diameter of 1 meter. So, this is also 1 and gamma is 10 power minus 1. So, you can see that even at rotation rates of 1 hertz, 1 revolution per second; this Weber number is coming to the order of 10 power 4, for number of fluids like this.

What that implies is that the inertial forces are much larger than the surface tension forces. So, most likely in a configuration like this; of this size surface tension is not important and therefore, the Weber number is no longer a parameter. This simple dimension analysis was able to tell you that in this particular case most likely the Weber number is not a parameter, surface tension is not important simply because the inertial scales of much much larger than this surface tension scale, so that is one simplification.

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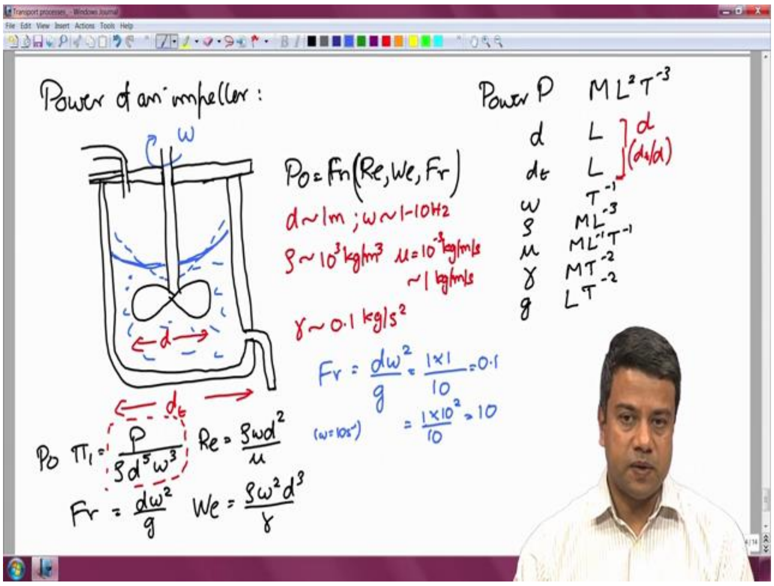

Now how what gravitational effects, the Froude number that I had, was equal to d omega square by g; g is 10 meters per second square; so with the nominator I have 10. The diameter I have taken as 1, if the revolution is 1 hertz; I will just get 1 which is gives me a value of 0.1. On the other hand if I take omega is equal to 10 hertz, I will get 1 into 10 square divided by 10 which is about 10. So, for the range of parameters that we are interested in the Froude number varies approximately between 0.1 and 10. So, it is most likely and important parameter because both the gravitational as well as the centrifugal effects are roughly comparable and because of that the Froude number is relevant parameter in this problem what that also tells you is that most likely the surface is going to be distorted because both of these forces are comparable to each other.

If the Froude number is small, the destruction will be less because gravitational effects are larger compared to centrifugal effects and vice versa and the Froude number becomes larger to destructions going to be more. So, Froude number is a relevant parameter and what about the Reynolds number.

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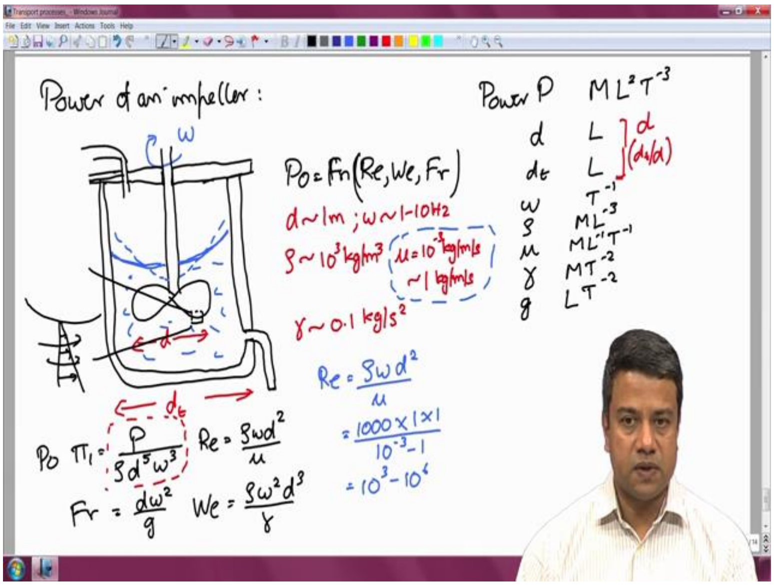

Density is 1000 and angular velocity; we said it could be 1 to 10 hertz. So, let us take a value of about 1, second inverse and diameter is also 1 divided by viscosity it can vary between 10 power minus 3 to 1. I said the viscosity if our things like waters 10 power minus 3 even us for very viscous flow be of Theo-graph 1. So, this is coming out to be a large number if this is 10 power 3 to 10 power 6 for systems like this.

Reynolds number is large, so you would expect inertial effects to be dominant and therefore, that actually proves that the initial choice that we are made to scale it by density rather than viscosity is the correct one because in this case inertial effects are dominant and therefore, I should choose the density to scale by the power number. If I had instead chose in the viscosity for scaling the power in the power number and I came to this stage in found the Reynolds number was large, I should go back in change because the dominant effects in this case are inertia.

You would simplistically think that because the Reynolds number is large, I can neglect viscous effects but as I have quotient many times during this lectures when the Reynolds number is basically a ratio of inertia and viscosity, it is the ratio of convection and diffusion of movement. When the Reynolds number is small; that means, viscosity is large therefore, diffusion is large therefore, we can neglect convection all together and the Reynolds number is large on the other hand you might simplistically think that you can neglect viscosity all together; however, if you look close to the surface; there is fluid passing across the surface and from the no slip condition, the velocity profile has to increase linearly with distance from the surface. Even though convection can transport momentum along the fluid flow convection does not take place perpendicular to the surface and you need to transport momentum perpendicular to the surface in order to exact to force on the surface.

That force can be exhausted only by diffusion; so you will convection is large compare to diffusion. You cannot just neglect diffusion, you still have to retain the viscosity as a relevant parameter and that is something to be kept in mind when discussing all forms of diffusion, if diffusion is large compared to convection you can in general neglect convection, if convection is large compared to diffusion; you cannot in general neglect diffusion because at the surface ultimately transport as to take place due to diffusion, whether it is mass heat or momentum transport. So, the Reynolds number is still a relevant parameter. So, from the discussion we have had so far; we said that the Weber number is no longer a parameter, the Froude number is a parameter and we have to retain the Reynolds number. Now how can we use this for some design application, so let us say that I have this large tank which I want to design but before that I want to make a smaller tank and then find out what is the power a finite plant if we will.

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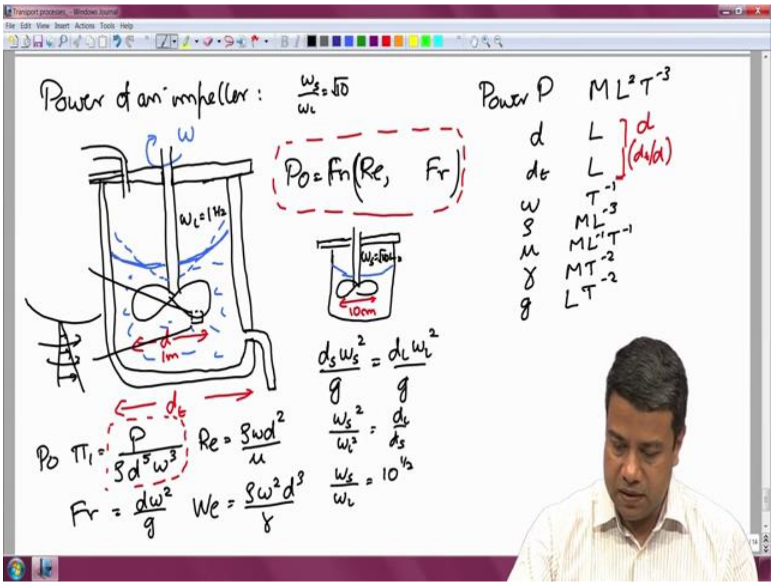

Make a smaller tank which is about one-tenth the size of the large tank and then drawn experiments in the smaller tank and then use that in order to be able to predict what is the power in the larger tank. So, the diameter is one-tenth, so the volume is actually 1 by 1000, so in terms of volume this is the rather large scalar. Now what I need to do between the smaller tank, so this is diameter say 1 meter and this diameter I will make it 10 times smaller. We have got 10 centimetres and if I run experiments in this and I find out what is the power required in this smaller tank can I then find out what is the power that will be required in the larger tank that is the problem with scalar.

So, how do I do that first; thing between the larger tank and the smaller tank it is essential to keep all the dimension less groups the same because this is a relation only between dimension less groups. So, if I keep the Reynolds number and the Froude number the same between; the smaller tank and the large tank then I know that the power number will be the same, provided the ratios of all the length dimensions of the same that is for make proper scale down replica of the large tank in this smaller tank and I keep all of the dimension less groups the same then I know that the power number will be the same between this large and small tank. So, I have to keep the dimension less groups the same; that means that the Froude number has to be the same.

For the small tank d small omegas; omegas smalls square by g is equal to d large omega large square by g of course, we are on earth. So, we cannot change the acceleration due to gravity that has to be the same between the two; however, we can change the frequency in such a way that the Froude number is exactly the same between the large and the small tank. So, what that you replay is that omega small square by omega large square is equal to d large by d small; d large by d small is a factor of 10. Therefore, omega small square I am sorry divided by omega large as to be equal to 10 to the half the square root of 10.

So, if for example, I want to run this large tank at a frequency of 1 hertz, I should be running the small tank at a frequency that is root 10 hertz. Therefore, omega small by omega large is equal to square root of 10; that is equal to about 3.016 roughly. So, in that way I can make sure that the Froude number is the same between the two systems.

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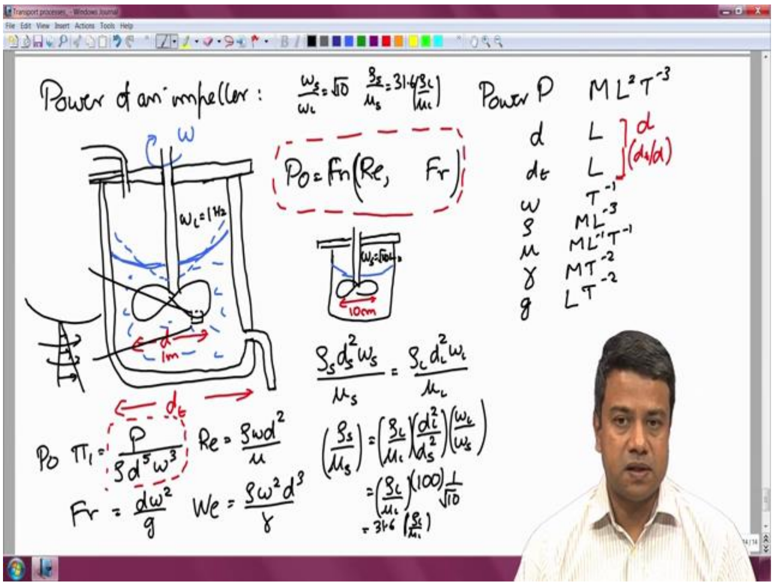

Now, how about the Reynolds number; rho small, d small, omega small square by nu small is equal to rho large d large omega large square by nu large therefore, straight in terms the viscosity. Now, therefore rho small by nu small is equal to rho large mu large into d large by d small into omega large square by omega small square. Now I know that d large by d small is a factor of 10 and omega small by omega large is factor of root 10 therefore, omega small by omega large is omega large by omega small. So, equal to 1 by root 10, and this set of came out of the Froude number itself because we knew that I am sorry the Reynolds number is not correct it correct and d small square d large square. So, it is d omega square rather than d square omega I am sorry d square omega and not d omega square the Reynolds number.

So, this d large square by d small square; d large by d small is factor of 100; omega large by omega small is a factor of 1 over root 10 and therefore, the factor that I get is approximately 10 power 3 halves, so about 31.6. Therefore, I require for the scale up for the Reynolds number to be the constant between the two systems, I require that rho small by mu small is equal to 31.6 times rho large by mu large. What this is telling me is that to keep the Reynolds number a constant, I cannot use the same fluid; I have to use a different fluid whose ratio of density to viscosities 31.6 times, the ratio of density to viscosity of the large tank, so if the fluid in the large tank.

Let us look at what this actually means that means that for the fluid in the small tank the small model system that I have constructed.

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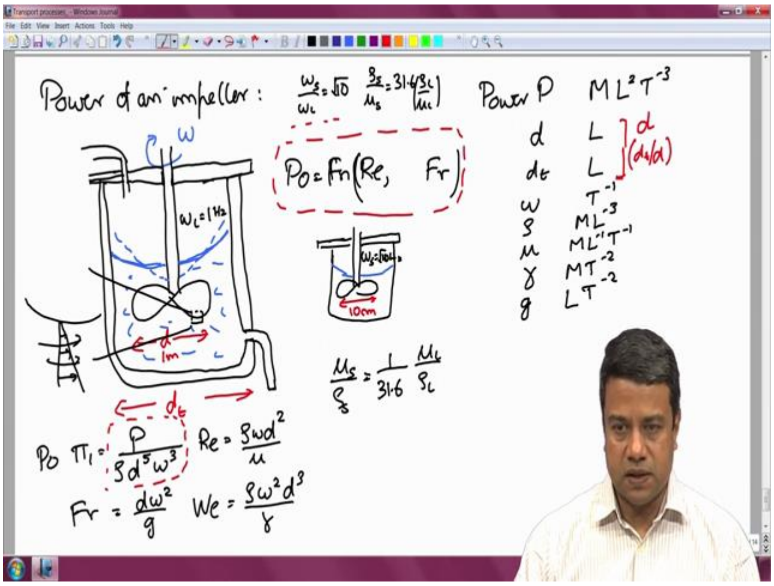

Mu small by rho small will be equal to 1 by 31.6 mu large by rho large. So, I am using the particular viscosity in the large tank and you need to use a fluid with a much smaller viscosity in the small tank and because of that I cannot use a same fluid, density is a fluids or approximately of the order of 1000 kilograms per meter cube, there is very little variation in that. Now the viscosity is can vary by a lot, so if I am have to use a large viscosity fluid in the large tank, I have to suitably scale down the viscosity of the fluid that I am using in the small tank.

If I can find that fluid then I can do that experiment in this small tank and then I will get a certain power required to rotate at this particular frequency in the small tank, let a frequency that is 3.16 times the frequency that was required for the large tank and using the fluid whose viscosity is 1 by 31.6 times the viscosity of the velocity and so once I do that then I note that the power number is the same between the large tank and the small tank.

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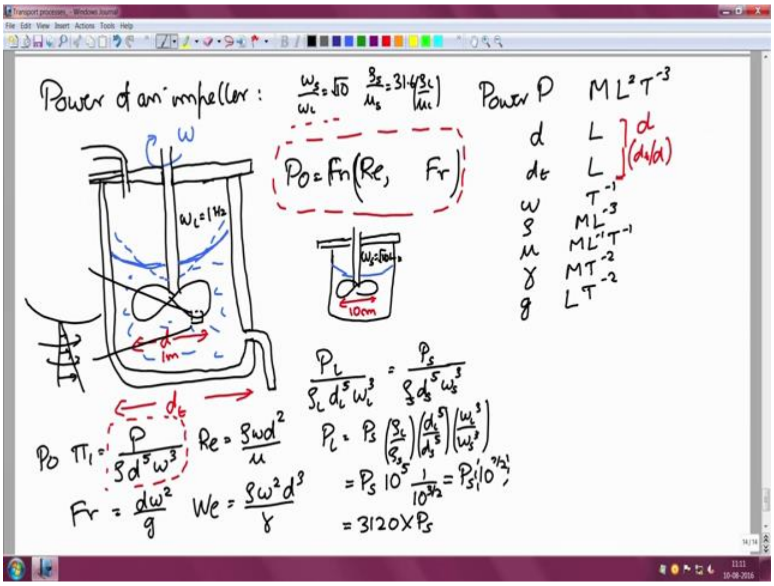

So, therefore, the power in the large tank divided by rho large, d large to the fifth power omega large cubed is equal to power in the small tank by rho small d small to the fifth power omega small cube.

Therefore the power in the large tank is equal to the power in the small tank into rho large by rho small; d large to the fifth power by d small to the fifth power; omega large cubed by omega small cube. So, there is the ratio of the power required then ratio of densities I have told you will be approximately equal to 1 for most fluids, the ratio of density is not failure parameter here will be approximately 1 for all fluids. So, therefore, if you assume that the density does not vary much; d large by d small factor of 10, the larger tank has impeller diameter that is 10 times the smaller tank. So, I get that the factor of 10 power of 5 in this case.

Omega large by omega small is 1 over root 10 note that omega small, the frequency in the smaller tank have to be square root of 10 times that in the larger tank for the Froude number to be a constant. Therefore, omega large by omega small cubed will be 1 over 10 to the 3 halves because omega large by omega small is 1 over square root of 10. So, therefore, this gives me a factor of 10 power 5 10 power minus 3 by 2. So, I will get this power as 10 power 7 by 2; the power required in the large tankers larger by effects (Refer Time: 24:50) 7 by 2 compared to this; this is approximately equal to 3120 times the power in the small tank.

So, this tells you that when you scalar by factor of 10; the power which is going to increase by a 1000 times at least and that is the reason that the power requirements actually as you scale of turnout to be very large, you do gain a lot due to the flow there increasing the transport rates but there is also the significant or requirement when you scale of from their small tank to the large tank.

So, this was an example of momentum transport I derived for you, the dimension less numbers; in this case it was the power number. Usually a momentum transfer you will use either the stress of the pressure difference, in this case if you are considering the system as a whole, you are using the power in the system and that actually scales us the density times the diameter to the fifth power and frequency to the third power and that is the reason power increases substantially you assume make systems larger and larger. That is the reason that reactors are not much larger than on meter in diameter usually because the power requirement increases in dramatically as the diameter increases and as a frequency of rotation increases and the increase in power required does not compensate for what you gain from the increased transport rates.

All of this we have learnt just based upon simple dimensional analysis. What are the advantages you gain when you go to larger system and what are the costs and what is the ratio between these the determines, what kind of a design you will have. If I had to design a tank of a certain dimension and I wanted to make a scale model of a smaller dimension smaller by a factor of 10 in dimension, how do I relate the parameters for the small and large system, indentify the important dimension less groups make sure that those dimension less groups are the same between the large and the small system.

So these are the kinds of things that you can do to use dimensional analysis to advantage in order to do. Next lecture we will start on trying to get slightly more deeper understanding of what these dimension less groups or mean, we have indentified several of them and this is branch of them that keep appearing again and again, the Reynolds number, the Peclet number, the (Refer Time: 27:47) number and so on.

What do all of those mean and how can we identify them without having to go through it list the whole thing, write down different dimensionless parameters and then look it is one of them, how can we get them more simply. So, that is the subject that I will discuss start in the next lecture, what do dimension less groups mean and how can we find relations between these two; dimension less groups. That is something that we will start in the next lecture, I will see you then.