Transport Process I: Heat and Mass Transfer

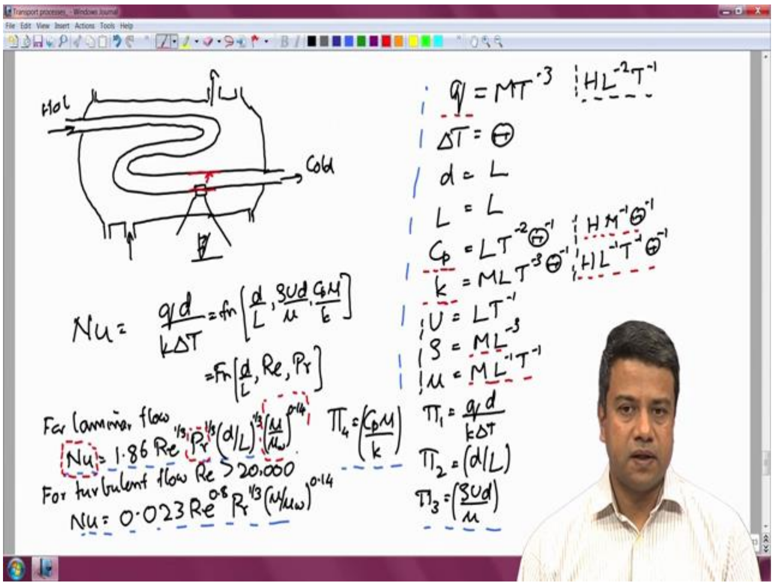
Prof. V. Kumaran
Department of Chemical Engineering
Indian Institute of Science, Bangalore

Lecture – 07
Dimensional analysis: Power of an impeller

Welcome to this; this is our course on transport phenomena and after a couple of introductory lectures, I have given you about four lectures now far; far each on dimensional analysis, but there is still a lot more to go, a lot more to understand.

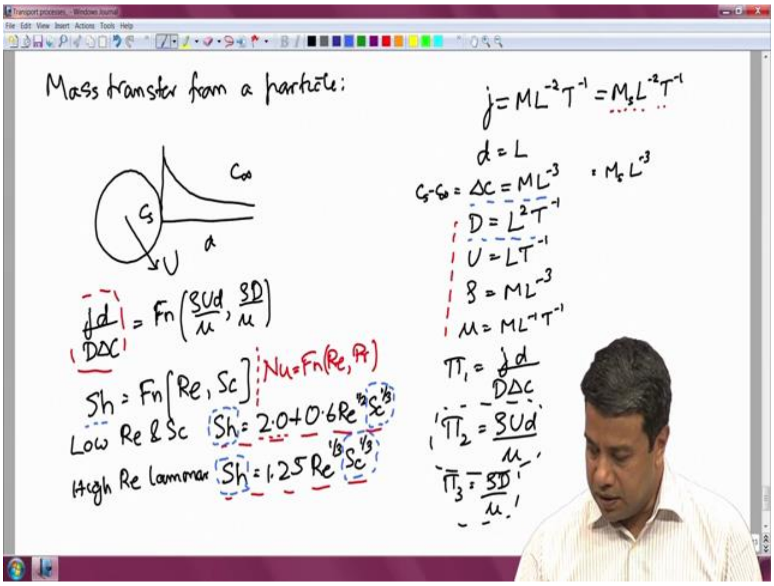
If you recall in the last lecture, we had actually looked at how dimensional analysis can help us in design. We had done two specific examples in the last lecture, one was the flow through a heat exchanger; where we wanted to predict the average heat flux as a function of the average temperature differences the shell side and the heat side and the tube side.

(Refer Slide Time: 01:07)



And this of course, depends as you can see on a large number of parameters; it depends upon a large number of parameters, the dependent variable the heat flux depends upon eight other parameters in the calculation that we did, but we were still able to reduce it down to just four parameters using dimensional analysis and a bit more, the physical insight that there is no inter conversion of energy between mechanical and heat energy and therefore, I could have considered heat energy as a dimension in itself. Based upon this we found that the Nusselt number, the non dimensional average flux depends upon 3 other quantities, the ratio of diameter to length of the pipe, the Reynolds number and the Prandtl number where we have defined these dimensionless groups, the Reynolds number is rho u d by mu and the Prandtl number is c p mu by k.

(Refer Slide Time: 02:15)



Similarly, for the problem that I have talked about in the first lecture of the mass transfer from a spherical particle, we had once again written down expression for the average flux. The flux depending upon the concentration difference between the surface of the particle and the ambient curve which is dry in case of water evaporation and of course, once again this average flux depended upon six other parameters, but we had reduced it to just two; based upon dimensional analysis and a little bit more and those dimensionless groups at the Sherwood number which is a dimensionless average mass flux as a function of the Reynolds number and the Schmidt number.

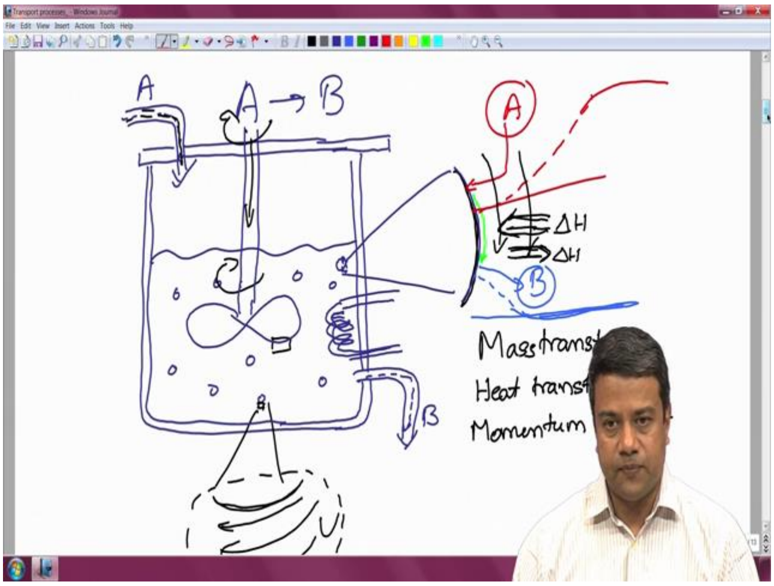
Now, I should note here that there are analogies between heat and mass transfer. The same problem if you recall when we talked about the drying of a droplet in a spray drier, we had said that the latent heat required has to go in and the moisture has to come out. Moisture coming out is a mass transfer problem, the latent heat going in is a heat transfer problem which where the heat flux is driven by the difference between the droplet temperature and that of the outside air and if were to write a correlation for that; I would get it to be of exactly the same form.

Dimensionless flux in that case is the Nusselt number; the dimensionless heat flux analogous to the dimensionless mass flux which is the Sherwood number is equal to some function of the Reynolds number and the other parameter that we had in the case of a heat transfer problem was the Prandtl number and if I wrote the correlation in this way, I could just substitute for the Nusselt number instead of the Sherwood number in the left side and the Prandtl number instead of the Schmidt number on the right side and I would actually get the exact same correlations.

Similarly, in the case of heat transfer for example, in this case if I for the heat transfer problem, I had got the Nusselt number as a function of the Reynolds number and the Prandtl number. If on the other hand, I have a mass transfer problem where there was some material that was either coming of the surface and getting absorbed into the fluid or the other way around if it was getting absorbed by the surface from the fluid or reacting or something like that, in the same configuration if I wrote down the correlation for this. It turns out I would get the same correlation except that instead of the dimensionless heat flux, I would have to substitute the dimensionless mass flux which is the Sherwood number and instead of the Prandtl number, I would have to substitute the Schmidt number that requires a little deeper understanding of the transfer processes which we will come to in a little bit, but these analogies are important because what you get from a calculation for heat transfer can easily be used for mass transfer as well and I will tell you a little later why that is so.

Before we proceed onto that into a fundamental understanding, I would like to spend some time with a momentum transfer problem. If you recall the problem that we had started off with was a reactor.

(Refer Slide Time: 06:08)

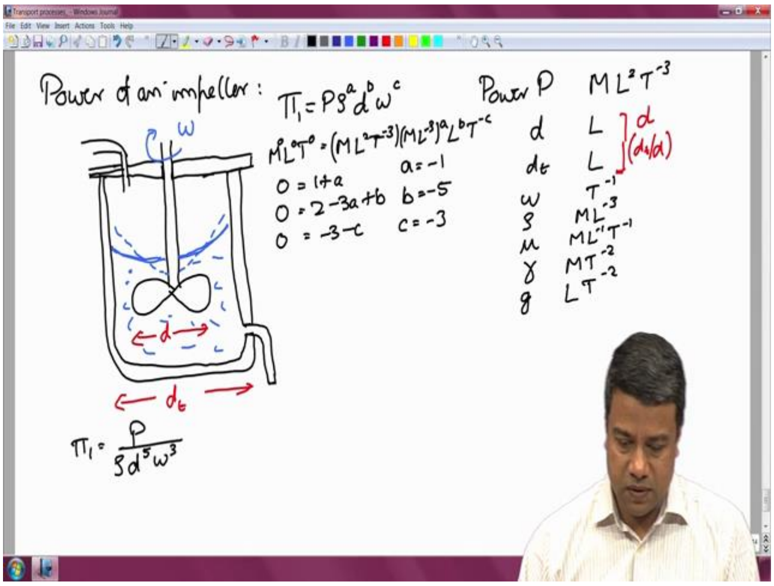


And we looked at the mass transfer from the surface, the heat transfer to the surface and so on when we consider a single particle and we saw how the dependent variable can be described in terms of dimensionless groups. The dimensionless flux has a function of other parameters the Reynolds number and Prandtl number or the Schmidt number.

Now, what about the power required for stirring this reactor, the power is required for stirring the impellors, so that you increase the transport rates, but as I said you cannot keep increasing the transport rates in these cases just because you keep increasing the speed because at some point as you increase the speed, the power increases and one has to know how the power increases with the frequency or the angular velocity with which speed this impellor is being stirred because that will basically design determine how much energy you have to spend in order to increase the transport rates at the microscopic scale.

So, therefore we will look at the case of the power required by an impellor in a rector and trying to see what are the dimensionless groups that arise in this problem.

(Refer Slide Time: 07:45)



So, have a reactor here with some particular dimensions some shape and so on and I have space for the inlet and space for the outlet must draw probably. Let us assume that he impellor has some diameter d, the tank has some diameter of the tank and it is rotating with some angular velocity omega. Omega can either be expressed in the number of revolutions per second or it can be expressed in radiance per second, there is only a factor of 2 pi between these two. For the present, I will assume that omega is expressed in terms of the revolutions per second.

So, now we would like to know how much power is required for rotating this; in a tank that is filled with some fluid. Assuming that we know the properties of the fluid, the density the viscosity and so on, so in this case the dependent variable is the power p and it has dimensions of rate of change of energy. If you recall energy is force into distance, force is mass into acceleration. So, therefore, energy has dimensions of mass n square T the minus 2; rate of change of energy, the rate of input of energy has dimensions of M L square T to minus 3 and this will of course, depend upon the diameter of the impellor with dimensions L, we will depend upon the dimensions of the tank that also has dimensions L.

It has to depend upon the frequency; frequency is number of revolutions per second or per minute, the number of revolutions per second. So, it is a number per unit time angular velocity is also a number per unit time, if you recall the linear velocity is length per time and the angular velocity is the linear velocity divided by the diameter, so it has just dimensions of T inverse.

Now it depends also on the fluid properties, the density and the viscosity. It does depend upon also the gravitational acceleration and the surface tension, the reason is because as this impellor is being rotated; you will find the due to centrifugal forces, this interface shift will actually deform; at low rotations it looks something like that, at very high rotations it looks something like this and that deformed shape depends upon of course, the rotation speed of the centrifugal force. It also depends upon the gravitational force because the gravitational force is turning to bring it back to a flat surface. It could also depend upon surface tension because surface tension tends to reduce surface area. So, therefore, the surface tension is also a relevant parameter that is a fluid property and then I have the gravitational force which is of course, the body force it acts everywhere equally.

So, the density mass per unit volume, the viscosity we had derived the dimension earlier; it is the stress divided by the strain rate. So, it is mass L inverse, T inverse; the surface tension is a mass per I am sorry a force per unit length, force is mass times acceleration; acceleration is L T to the minus 2. So, force by distance will be give me just M T to the minus 2 and g is a gravitational acceleration, the gravitational acceleration is length times T to the minus 2.

So, now these are the different parameters that are of importance; the power, the dimensions, the angular velocity, the density, the viscosity, the gravitational acceleration and the surface tension. So, of course, these I can sort of combine, so I can just consider one dimension; the diameter of the impellor and try to dimensionless group as the tank dimension divided by the impellor diameter. So, long as I keep the ratio of all of these length scales are constant, this will be a constant and in that case I have only one relevant length scale that is the diameter of the impellor.

So, I have 1, 2, 3, 4, 5, 6 independent dimensional groups; the diameter, frequency, density, viscosity, the surface tension and acceleration due to gravity and I have one dependent it is power number, so 7; so therefore and there are 3 dimensions. So, I should be able to get four dimensionless groups, out of which one is a dependent dimensionless group involves the power and the other 3 are independent dimensionless groups, so what does the dependent dimensionless group. So the power has a mass dimension in it and in order to scale that mass dimension, I have to use either the density, the viscosity or the surface tension.

As I said the choice of which one to use depends upon which is the dominant effect in these systems. Normally these impellors are made in large tanks with diameters of the order of a meter or so and the density of the fluid is of the order of 1000 meters per second. Therefore, you might expect as the first case that inertia is important compared to viscosity and surface tension and therefore, I should scale the power by the density rather than the viscosity of surface tension.

We can always go back and check which is the dominant effect and then scale it appropriately. For the present, I will assume that the density is the dominant effect, so therefore, I have to define my dependent dimensionless group as power density power that is the mass dimension and then I need a length dimension and the time dimension.

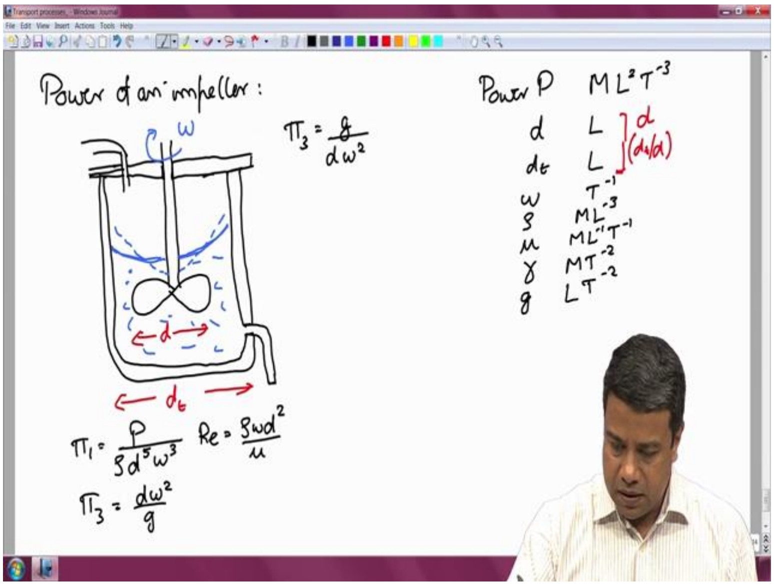
For the length dimension I can just use the diameter of the impellor and a time dimension as you can see the angular velocity has dimensions of T inverse. So, I can use omega power c and then we go through the usual M L power 0, T power 0 is equal to the power what is the relation; density power a, density is mass per unit volume, length power b the diameter of power b, angular velocity has dimensions of T inverse, so that is power c; so mistake in the power that I have used it should be minus 3.

So, now 0 mass dimension; 1 plus a, length dimension 0 is equal to 2 minus 3 a plus b and time dimension 0 is equal to minus 3 minus c. So, this gives me a equals minus 1 c equals minus 3 and if your actually calculate this with c a equals minus 1, you will find that b is equal to minus 5. Therefore, the first dimensionless group the power number is P by rho d power 5 omega cube. You can see that you are scaling it by diameter to the fifth power when density is dominant and therefore, you will expect as the diameter increases this power actually increases a lot.

Similarly, you are scaling it by the frequency to the third power; so as the diameter increases, the frequency increases, the power requirement will increase a lot, I will just do a quick example and show you that. So there is one number; this is a function of 3 other numbers which are all independent parameters.

Now in order to get the three other numbers, we have done some dimension analysis. So, we know the kinds of things that are going to come. Whenever there is a density and a viscosity is going to inevitably end up a Reynolds number coming; except in this case the Reynolds number is not a function of the linear velocity because the velocity linear velocity is not parameter in this problem. It has to be a function of the angular velocity, how do you get a dimension of linear velocity from the angular velocity and the diameter.

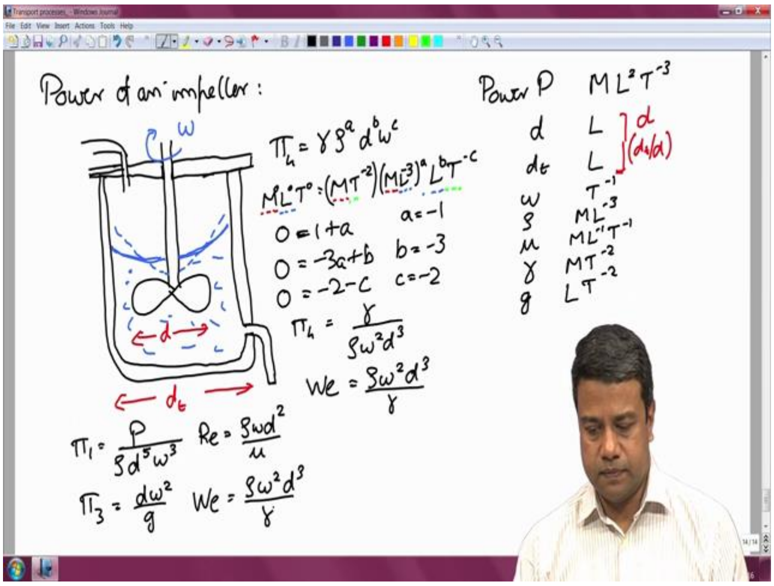
(Refer Slide Time: 19:22)



So, therefore, the Reynolds number that should come out should be of the form rho u d by mu; however, we do not have a linear velocity; we have an angular velocity. Therefore, I can substitute for the linear velocity I can substitute as a diameter times the angular velocity, so that has the same dimension as the linear velocity. So, we will expect this number to come out; the Reynolds number which is equal to rho omega d square by mu, so that is one dimensionless parameter. One dimensionless parameter has to involve the acceleration due to gravity, as you can see the acceleration due to gravity has only the length and time dimensions, so therefore, I can just scale it by the length and time dimensions.

Length dimension in this case is the diameter, time dimension is inverse of the frequency because that has dimensions of time. Therefore, I can get a dimensionless group which is equal to g by the diameter times omega square. It is not usually written this way, the dimensionless group; it is usually written the other way it is written as is equal to d omega square by g; that is also dimensionless and the final dimensionless group has to involve the surface tension, once again that is an independent dimensionless group.

(Refer Slide Time: 21:19)

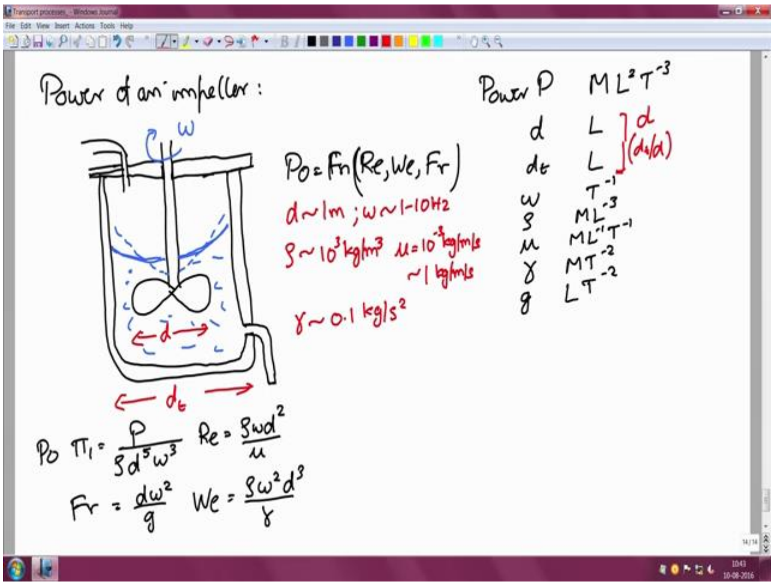


So, therefore I can create a dimensionless group as gamma and then I have to scale it either by the whisker scales or by the inertial scales; if I scale it by. So, gamma has a mass dimension unit, that mass dimension has to be non-dimensionalised either by the viscosity or the density. You have initially postulated that the inertial effects are more important therefore, it should be appropriate to use the density rather than the viscosity.

So, I need a density power a; in order to remove all the mass dimension then I have to have a length dimension and a time dimension. So, the diameter power b and the frequency power c, so that is a dimensionless number is equal to M to the minus 2; M L inverse power a, L power b; T power minus c because angular velocity has dimensions of inverse that. So, therefore I will get for the mass dimension 0; so the mass dimension is equal to 1 plus a; for the length dimension I am sorry the density should be length power minus 3; please correct that, this should be length power minus 3 mass per unit volume, it is for the length dimension I should get alpha minus 3 alpha p.

Therefore 0 is equal to minus 3 a plus b and for the time dimension I get 0 is equal to minus 2 minus c. So, this straightaway gives me a is equal to minus 1, c is equal to minus 2 and b is equal to minus 3. Therefore, the dimensionless group pi 4 will be equal to gamma by rho f square I am sorry I am sorry omega square b cube. It is not once again written usually in this manner, it is usually written the other way round the Weber number is equal to gamma I am sorry the inverse of this rho omega square d cube by gamma r; so our Weber number here .

(Refer Slide Time: 25:10)



So, therefore I have this dimensionless group this power number is equal to some function of the Reynolds number, the Weber number and this is called the Froude number, it is usually written in terms of the linear velocity, but in this case I have written it in terms of the angular velocity. So, I have got this power number as a function of 3 parameters the Reynolds number, the Weber number and the Froude number.

How do we proceed further; usually what we would do is to actually estimate each of these numbers; see what is the magnitude of these numbers and which numbers are relevant and which numbers are not relevant and try to simplify the problem in that manner for that we need some idea about what are the actual dimension of a tank like this

Typically a tank like this would have if it were used in industrial process, it would have diameter of the order of a meter as well. It would have an angular frequency of the order of 1 to 10 hertz. These are usually used for liquids, so the density will be of the order of 10 power 3 kilogram per meter cube. The density of water is 1 gram per cc which should be of the order of 1000 kilograms per meter cube; it is useful to write all of these in dimensional forms in SI units, so that one does not get confused. Similarly the viscosity will be of the order of 10 power minus 3 kg per meter per second; this is the viscosity of water. If you have a very viscous fluid, it could go as large as 1 kg per meter second and the surface tension you know the surface tension of water is of the order of 0.072 kg per second square, so it is the order of 0.1 kg per second square.

So, with these kinds of parameters we need to estimate what are these dimensionless groups and then use that in order to simplify that. So, this topic we will continue in the next lecture, I would show you how we will use these parameters in order to do some dimensional analysis and from that try to get some idea of which are the dominant forces in the system and then try to do a design of this how does one scale up from a small system to a large system, so that is going to be the topic of the next lecture; we will see you then.