Transport Process I: Heat and Mass Transfer
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Lecture – 06
Dimensional analysis: Mass transfer from a particle suspended in a fluid

Welcome to this, this is our 4th of our lecture on Dimensional Analysis. I had in the beginning explained to you fundamental dimensions, the derived quantities or could calculate the dimensions of the derived quantities. And we were looking at some examples where we can use dimensional analysis to advantage in reducing the number of parameters in a problem.

If a problem contains n parameters and there are M dimensionless groups amongst all of those parameters, the number of dimension non dimensional quantities is just n minus M and that can greatly simplified problems and many situations. We had looked at the case of partials settling in fluid in which case the drag force non dimensionalized by the viscosity velocity and diameter depends only upon one non dimensional number, the Reynolds number in addition to the aspect ratio for example; the ratio of the diameter of the particle to the size of the system.

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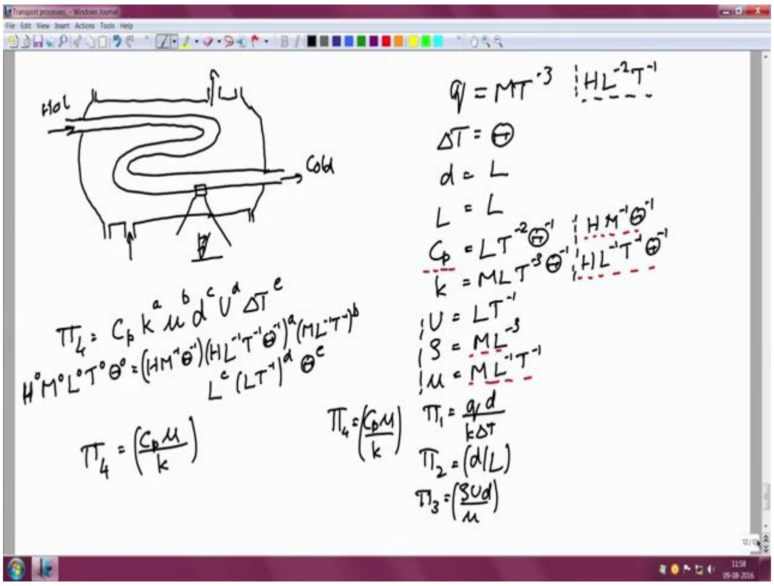

And most recently we were looking at the heat transfer in heat text changer; where we had identified the dimensional groups. The number of dimensional groups is quite large which is 9 in this case. The heat flux is the dependent dimensional quantity, it depends upon the temperature difference it means shell and tube side of course, the average temperature dependence; the length and diameter of the tube, thermal quantities, specifically thermal conductivity, mechanical quantities, and the velocity with density in the viscosity.

And so there are total of 9 and there are 4 dimensions in this problem mass, length, time, and temperature. So, simplistically you would think there should be 5 dimensionless groups. However, in this case we had made a simplification, we had said the there is no interconversion between thermal energy and mechanical energy. And when there is no interconversion between thermal and mechanical energy we can consider the thermal energy to be a surface dimensional in itself. And express all the thermal quantities in terms of this thermal energy.

In this particular case the thermal quantities for the heat flux, the specific heat and the thermal conductivity; the thermal conductivity, the specific heat and the heat flux for thermal quantities. And so those alone we had expressed in terms of the thermal energy, whereas mechanical quantities for express in terms of mass length and time. And on that base we had derived the total of four dimensionless groups. The dependent dimensionless group is a non dimensional heat flux it is called the Nusselt number.

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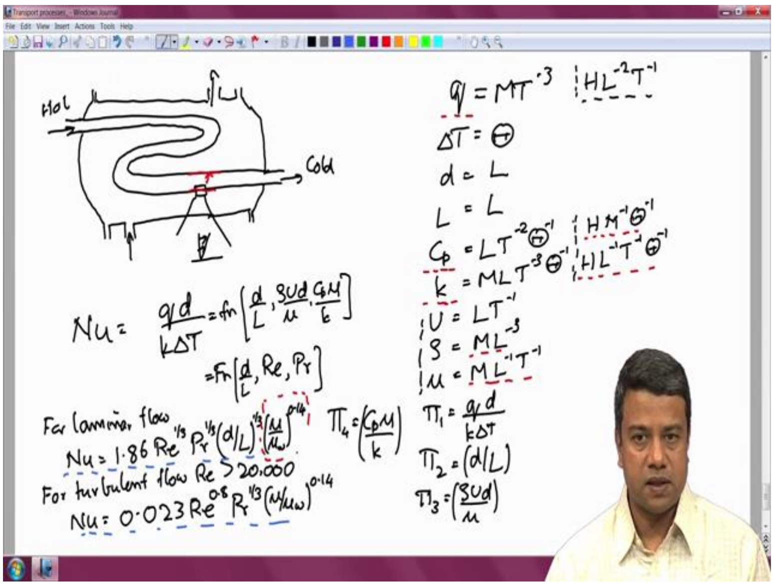

The Nusselt number is a non dimensional heat flux which is q d by k delta T; that is the dependent what you want to measure or what you want to estimate is the heat flux in the non dimensionalize that is the dependent quantity the non dimensional heat flux. And this is some function of d by L rho u d by mu C p mu by 3. The first one we know it just an aspect ratio. The second one is for what we have seen r in the earlier lecture the Reynolds number, ratio of fluid inertia and fluid viscosity. The third one is you can see it contains both thermal and mechanical quantities. It is called the Prandtl number; C p mu by k. I will show you later that it is a ratio of diffusivities or momentum in for energy.

So, we have got now the dependent quantity in an non dimensional form the Nusselt number in terms of the ratio of diameter and length the Reynolds number and the Prandtl number you will doubtless agree that this is a significant simplication over the original problem where I had the dependent quantity depending up on total of eight other quantities and that really is the power of dimensional analysis. Which as simple dimensional analysis I could have reduced it to five, but since I recognised the thermal energy is different and minus 2 reduce it to just four.

Beyond this point of course you cannot proceed by just dimensional analysis you have to either do experiments or you have to do derive formations analytical you not to get the relationship between these two. And these correlations are known for laminar flow the correlation is Nusselt number is 1.86 Re power 1 by 3 Pr power 1 by 3 d by L power 1 by 2 mu by mu w power 0.14 what often added over here. This is something that I do go through in this lecture. In this case mu by mu w takes it to the count the factor to viscosity of the fluid changes with temperature.

Now since heat is been conducted from the bulk of the fluid to the wall and then to the shall side; the temperature in the bulk is often different from the temperature at the wall, which higher if heat is conducted out and its lower if heat is conducted in. This affects the viscosity of the fluid and therefore the mechanical properties. And this factor mu by mu w for 0.14 is added in order to take into account this variation in viscosity. This particular factor we should not concern (Refer Time: 07:28) of this course.

You have another correlations for turbulent floor in the Reynolds numbers is greater than about 20000 where the Nusselt number is equal to 0.023 Re or 0.8 Pr for 1 by 3 into mu by mu w, we have different correlation in the turbulent. Now you can see that for laminar flow as the length of the pipe increases the Nusselt number actually decreases. So, the marginal utility of a long pipe decrease as the length keeps on increase. And during this course we shall see how to derive this relationship. We shall see how to derive this relationship based upon the microscopic details of how energy is transferred at microscopic level. We shall not go in to turbulent flows, but I will try to motivate this kind of a relationship for you in the later lecture where we consider turbulent course.

So, there is a considerable reduction but we still need to do some more work to get these functional forms. Why does it have to go as Reynolds numbers to the one Prandtl number to the one third or d by L to the one third why this dependent to the this particular manner. That is something that we shell discuss in this course. So, far in dimensional analysis we consider the whole system as a unit and we try to right down average correlations for this entire system. In the rest of this course we shell deal with the local transparently based upon to local terms.

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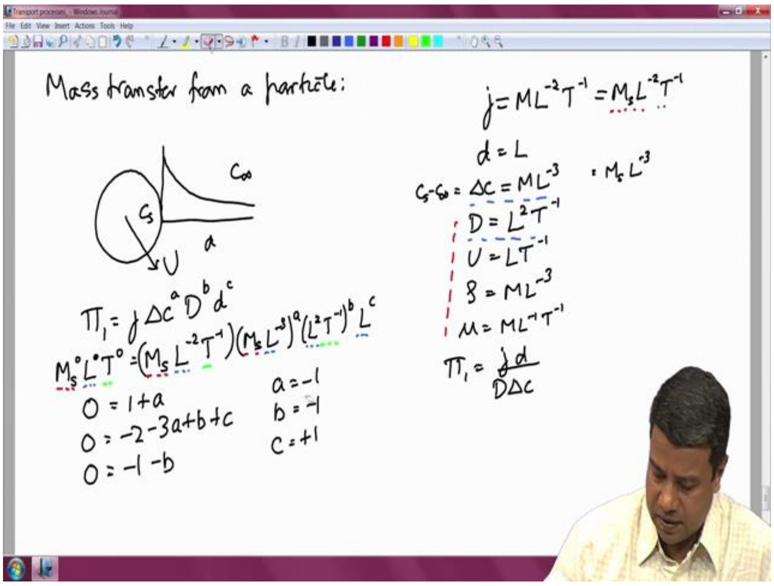

So let us take one more example; the mass transfer from a particle. So, we have let us consider this spherical particle for simplicity. There is some species with concentration c s on the surface and will consider the concentration at far away from this surface there to be c infinity. And that is this concentration difference it is of a plot as a functional locality distance from the surface plot the concentration. Concentration of the surfaces larger in an decrease you go away from the surface or particle. This difference in concentration results in a flux across the surface. And therefore for example, in the problem of the straight liner the drop let dries because of the moisture of the surface as a higher concentration then the moisture in the bulk, and that cause is transfer of moisture because in the droplet dries as in time progresses.

Now, of course if it was just a simple droplet that was drying then the rate of drying would depend only upon how fast the moisture defuses from the surface taken by. But as in the straight drawer problem the particle is also moving with some velocity used and the faculty is particles in getting spelt trough the fluid can distract the variation of concentration around part. To the fact that fluid is getting swept around the particle will distract the concentration variation around the particle and could see difficulty and enhance the little derivative diffusion. And that is what to try to analysis with it.

So, for this mass transfer problem the dependent variable is of course the mass flux j. The mass flux is the rate of transfer of mass per unit area per unit time; so this is equal to rate of transfer of mass per unit area per unit time. This of course will firstly will depend in general on the diameter of the particle dimensions of length, it will depend upon the concentration difference between the surface and the bulk that is c s minus c infinity; if this is positive than there will be drying of droplet, if is negative there will moisture getting absorbed to the (Refer Time: 12:33).

In this particular case it should be noted that what I mean by concentration is the concentration of water in the gas face on the surface of the droplet. So, it is not the concentration within the liquid we assume that the liquid at the surface is in equilibrium with the gas face at the pressure which responds to the saturation pressure, it is what you mean here is the concentration in the gas fields at the surface, the equilibrium concentration which is given by thermo diameter relations. So, this has dimensions of mass per unit volume; concentration as dimensions of mass equilibrium problem. What can also affect this is the diffusion coefficient. We if you recall we derived it from fix law of diffusion that j is equal to D delta C by L, these are dimensions of M L power minus 2 T power 1 is the dimensions of equations delta c has dimensions of mass L power minus 3 divided by (Refer Time: 13:50).

And from that you can see that the mass dimensions cancels out in both sides and you find the dimension of the diffusion coefficient to be L square T inverse, so that is a diffusion coefficient. And of course, you also have the mechanical properties, because that relates to how fast the droplet us moving through the fluid. Therefore, the mechanical properties are also important for the fluid the rate at which the transport will be effected of course by the fluid, you know that when the fluid flows past the surface the rate of transport increases so depends upon the velocity. And the mechanical properties of the fluid in this case it will depend upon the density and the viscosity.

Once again here we can make a simplification. The simplification is that when we talk about mass transfer we are talking only about the mass of the solute, whereas we were talking about mechanical properties the density, the viscosity etcetera we are talking about the mass of the entire fluid. So, there is no interconversion of mass between the mass of the fluid in the mass of the solute we can consider these to be two different things. As far as mass transfer quantities are concerned the dimension should be the mass of the solute, whereas when there mechanical quantities are concern the dimension should be the total mass of the fluid.

So, let us write another dimension for the mass of solute, we will that only for the mass transfer quantities there is the flux and the concentration difference. These alone depend upon the mass of the solute, whereas the mechanical quantities here depend upon the mass of the fluid. So we have seven quantities, originally we just had mass length and time three dimensions and therefore we should have four dimensions less groups, whereas once we in terms of the mass solute we have know four dimensions. And therefore, one can get only three dimensionless groups. So, there is the simplification that we bring that a code when we recognise that there is no inter conversion of mass between solute and fluid.

So, we should have a total of three dimensionless groups here, one is the dependent dimensionless group; dependent dimensionless group in this case as a non dimensional flux. The non dimensional flux as dimension of mass length and time is must solute length in time and that has to be combined with quantities which are related to the mass transfer. So, quantities with to mass transfer are the diffusion coefficient the concentration difference and of course the diameter of the particle.

So, I would write my dependent dimensionless group as equal to the mass flux, the concentration (Refer Time: 17:35) bar a, the diffusion co efficient for bar b and the length scale diameter bar C. And from the dimensions of the mass of solute, the length scale, and the time scale I will find out what are a b and C. Note that I have done the simplification in such a way that this expression does not contain any the total mass of the fluid. So therefore, I have M s power 0 L power 0 T power 0 is equal to M s L power minus 2 T inverse into M s L power minus 3 for a. The diffusion co efficient has lengths square T inverse power b and then I have the length of c. So, with this I can get all the quantities a b and c. So, 0 it adjust the mass solute dimension I will get that 0 is equal to 1 plus a, it take the length dimension; I will get 0 is equal to minus 2 minus 3 a plus b plus C. And finally the time dimensional will give me 0 is equal to minus 1 minus be plus C.

And this you can solve relatively easily. First thing you will get this that a is equal to minus 1 you will get b equals minus 1 and c equals plus 1. We should rewrite that. Take a time dimension I will get 0 equals minus 1 and minus b there is no c in that because this is only two time dimensions there. Therefore, the dimensionless group is equal to j d by D delta C.

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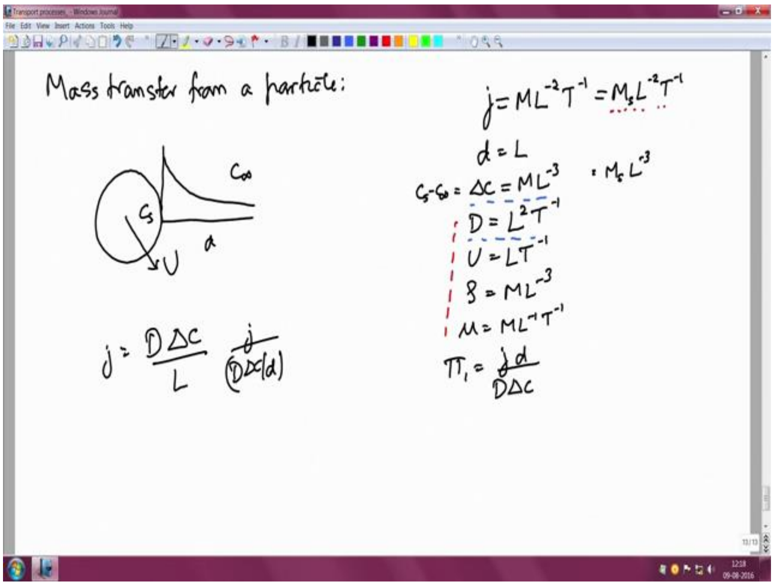

And this also I could very well have known that from just fixed law of diffusion sates that the mass flux D delta C by L, this is of course applicable only local every point within the floor. However, the dimensions on the left and the right are the same. This low of diffusion is not valid for the entire system, because mass transport on the larger scale is affected by conventions as well as diffusion. But however, the dimensions on the left and right are the same, so if I could just scale j by something that was D delta C by d I know that would be dimensions there is a first dimensionless number.

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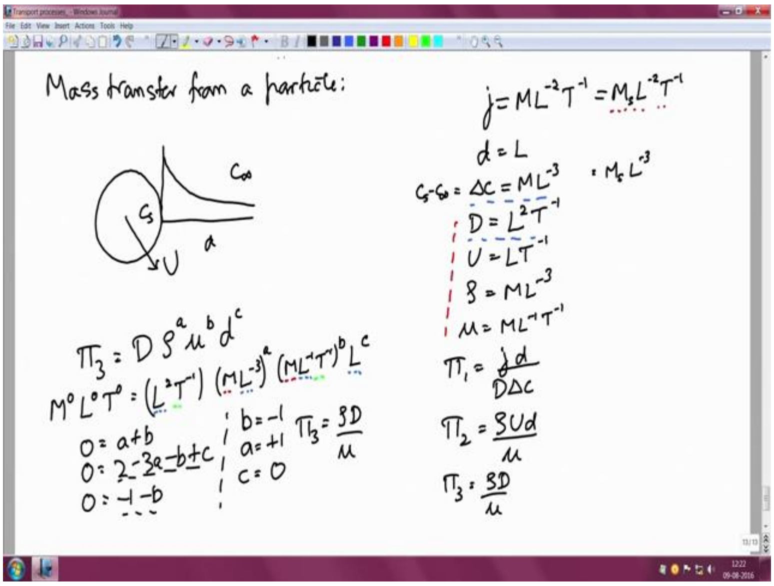

The second one we already know we already know one other dimensionless number which is rho u d by mu, The Reynolds number in alls purely mechanical quantities, there is number two and there should be one more. The third one should involve both thermal and mechanical quantities; I am sorry both mass transfers as well as mechanical quantities. Quantities relate to both mass diffusion as well as with respect to (Refer Time: 22:14). You can assemble that by considering the diffusion coefficient along with mechanical properties.

So, if I consider a dimensionless group pie 3, diffusion co efficient it has dimensional of lengths square per unit time. So, I need to non dimensionaliz that with two other mechanical quantities, in this particular case I could take them as the density power a and the viscosity power b. There is a simplest consideration, because both density and viscosity have dimensions of mass. Therefore, I would take it as density power a times viscosity power b. And from that I could get a dimensionless group mass length and time you should cancel out.

In general, I would have to have lengths scale as well, but as you will see this co efficient turns out be 0; I will tell you why it is. So, a b and c are calculated from the dimensions of length mass and time. Therefore, once again M power 0 L power 0 T power 0 is equal to L square T inverse power a M L power minus 3 b c. So, if I take the mass dimension I will get 0 is equal to a plus b, if I take the length dimension I will get 0 is equal to 2 minus 3 a minus b plus c, and if I take the time dimension I find that 0 equals minus 1 minus b. Therefore, from this I will get b equals minus 1, from this relation and since a plus b has to be equal to 0 I will get a equals plus 1. And, it turns out that c is just equals to 0 if you work it out b is equal to minus 1 and c is equal to c is equal to 0 therefore, the third dimensionless group is equal to rho D by mu.

Now I could very well have scaled the diffusion co efficient why the particle diameter in the velocity as well in which case I would have recorded different dimensionless group, but in this case what I try to do is make sure that there is only one quantity which depends upon the particle velocity; and this other one is just a material parameter does not depend upon the particle diameter or the particle velocity is just a material parameter.

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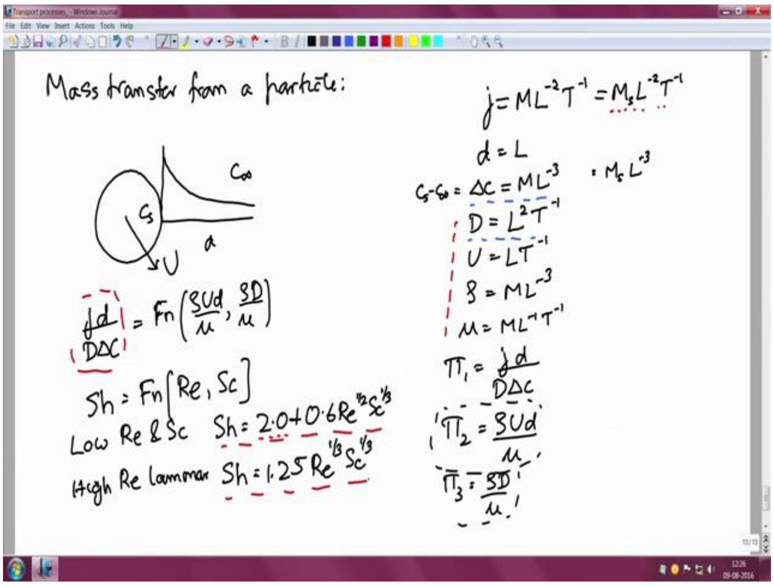

These also have names: the first one, j d by D delta C it has to be some function of and rho D by mu. It is convectional to write this third dimensionless group pie 3 not in this fashion, but rather as its inverse; pie 3 is usually defined as mu by rho D, there is convicted and this is what is called the Schmidt number. Therefore, what I have on the left hand side this one the dependent quantity the non dimensional flux it is called the Sherwood number is equal to some function of this first one is the Reynolds number and the second one is Schmidt number. So, in this problem whereas I had total of seven quantities initially I have reduce it to just one dependent and two independent quantities. Beyond these of course you cannot progress by this dimensional analysis you have to go co relations.

And co relations for example typically for low Reynolds number and Schmidt number the Sherwood number is written as 2.0 plus 0.6 hourly per half S c power one-third. So, in the limit as the Reynolds number goes to 0, this Sherwood number just goes to a constant value. Why is that we will see as we go through the course. At high Reynolds number; Reynolds number for laminar flow Sherwood number is written as 1.25 Re power 1 by 3 S c power 1 by 3. So, these are the kinds of correlation that you have between these non dimensional quantities. And in this class we will see how to derive these kinds of co relations. Sherwood number correlations kind Lawrence.

In the next lecture I will try to demystify these dimensionless numbers are little bit. We manage to get dimensionless numbers in terms Reynolds number, the Schmidt number, the Prandtl number and so on. What to these physically mean? So, I try to a give you some physical inside what these dimensionless numbers mean and why the relations that we have our: of this particular form. For mass and heat transfer as also the track co efficient of movement of transfer, so I will give you some insight of that. But before that I will do one problem which involves the power on a impeller to illustrate our scale of Visteon, how these dimensionless group can assist us in scaling them from small systems and large systems, what are the kinds of things that I have to be maintain constantly when we things the kind of a system. Obviously, you have to scale at economical process, but it is not straight forward to do it if you have you something that works in the laboratory it is not clear that it will work in industry.

So, what are considerations there; you are scale up considerations I will talk about first then I will come back and try to give you some physical idea what these dimensionless group to be. We see you in next lecture.