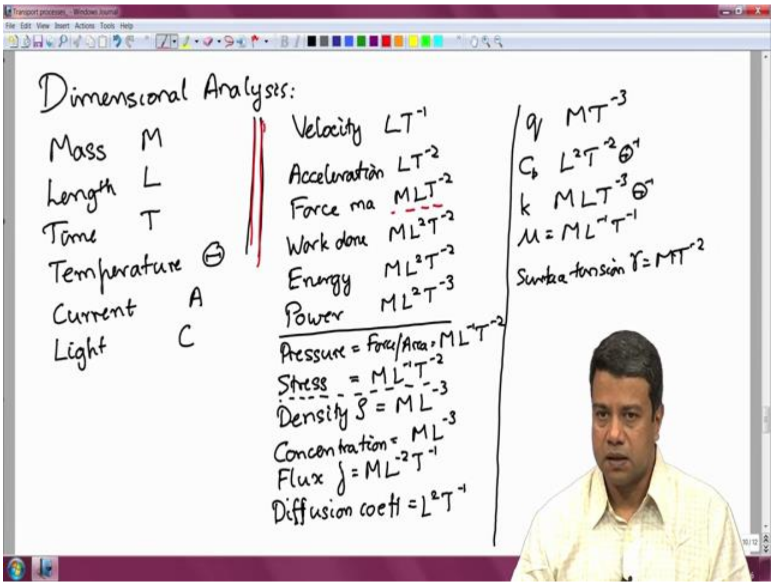
Transport Processes I: Heat and Mass Transfer

Prof. V. Kumaran
Department of Chemical Engineering
Indian Institute of Science, Bangalore

Lecture – 05
Dimensional analysis: Heat transfer in a heat exchanger

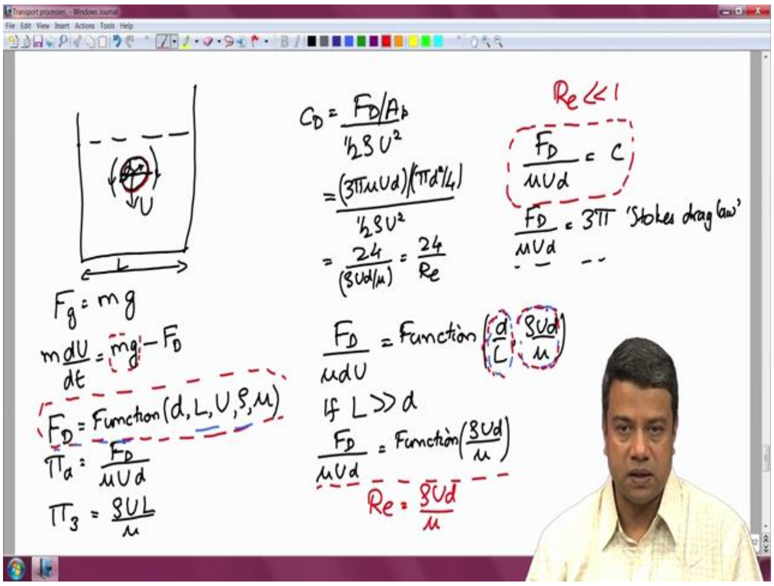
Welcome to this, this is our 5th of our lecture on Fundamentals of Transport Processes. In the first lecture I had given you an introduction to why we study transport processes and what is it that we will be studying in this class. We had started our study transport processes with dimensional analysis in the last two lectures. I had explained to you the distinction between fundamental and derived dimensions in the last two lectures.

(Refer Slide Time: 00:53)



The fundamental dimensions are mass, length, time, temperature, current, and light intensity. And in this course we will primarily be dealing with mass, length, time and temperature. And I had shown you how all the derived quantities can be derived from these fundamental dimensions. And all of the quantities of interest to us in this course the fluxes, mass, heat, and the shear stress are the temperature difference, the concentration difference, the velocity difference. And those coefficients that are in there the diffusion coefficient for mass transfer, viscosity for momentum transfer, thermal conductivity and specific heat for heat transfer.

(Refer Slide Time: 01:46)



We had solved the problem in the last lecture; the simple case of the flow of the settling of a particle in a fluid. This is a curve acted upon by a gravitational force, maybe a buoyancy force and therefore the net force will be proportional to the difference in density between the particle and the fluid. That is a body force; it acts at every volume element within the particle.

In addition there is the drag force or surface force, due to the stress exerted by the fluid on the surface of the part. And that surface force acts to retard the motion of the particle. It arises because when the particle moves through the fluid the particle generates fluid flow around it and that fluid flow inevitably results in fluid friction and therefore a dissipation of energy. And this manifests itself as a drag force. This is a surface force, this is basically the stress exerted by the fluid on every surface element of the particle summed up over the entire surface that gives you the total drag force. And as we had seen in the last lectures this drag force could be a function of the particle size, diameter, the velocity of a particle, because the particle does not move there is no drag force; if it is basically a relative velocity between the fluids in the particle.

The density and the viscosity; and based upon dimensional analysis we had obtained dimensionless groups. In this case there were only three dimensionless groups: one of them is the dependent dimensionless group. The drag forces the dependent quantity; it depends upon the velocity, the density viscosity and so on.

So, a dimensionless group which is a non dimensionalized drag force is a dependent quantity and it depends on these independent quantities that are the ratio of the diameter of the particle to the size of the container. And this dimensionless number, the Reynolds number which I had introduced in the last lecture. Reynolds number is the ratio of inertia forces to viscous forces. The inertial forces are basically due to the fluid inertia and they contain the density and the velocity in which the viscous forces are due to the fluid friction, the ratio of these is the Reynolds number.

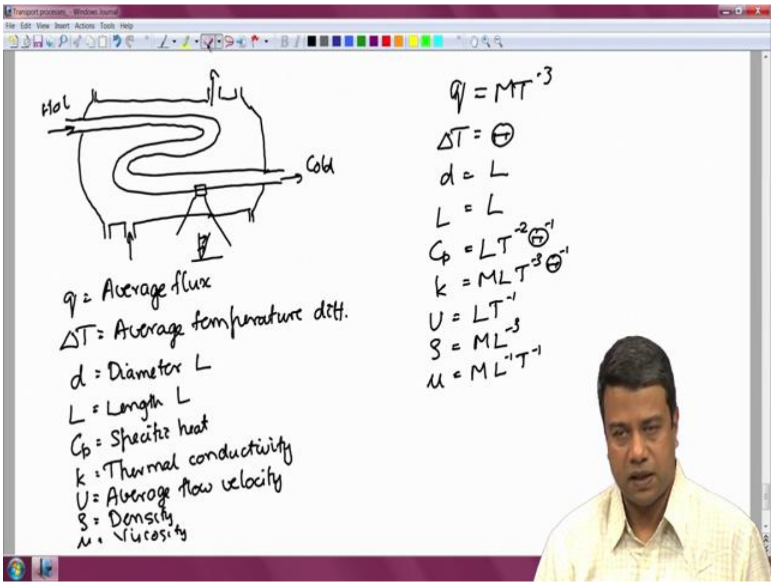
We had gone and interpreted this relation a little bit beyond dimensional analysis. What I had told you was that in this kind of a relationship if the Reynolds number is small that is particle inertia is much smaller than flow inertia then the drag force should not depend upon the particle inertia; I am sorry if the fluid inertia is much smaller than the fluid viscosity then the drag force should not depend upon fluid inertia, it should depend upon the viscosity along.

In that case the regime is when the Reynolds number is much smaller than 1 when the fluid inertia is much smaller than the fluid viscosity. In that case the density of the fluid should no longer be a relevant parameter, and therefore we had still further simplified this, the scaled drag force has to be just a constant. That constant of course we do not know from dimensional analysis. We need to do an exact calculation and you will find that that constant is equal to 3 pi in this case; it is called the Stokes Drag law.

When fluid inertia is much larger than fluid viscosity the situation is a little more complicated. You might simplistically think that fluid viscosity can be neglected all together. However, in that case there is effectively no friction at the surface, and therefore there will be no drag force. Fluid inertia is the equivalent of convection of momentum. Fluid viscosity is equivalent of diffusion of momentum. And as I told you in the last lecture even though convection and transport momentum within the fluid ultimately at surfaces there is no convection perpendicular to the surface. And therefore, ultimately the momentum has to be transported due to diffusion.

So, when the convection is much faster than the diffusion it is not always possible to neglect diffusion. That is a point that we will come to a little later, it is not always possible to neglect diffusion. You may need to consider diffusion within ten regions close to the surface. So, that is the summary of the dimensional analysis that we had done in the previous lecture.

(Refer Slide Time: 07:05)



Now, let us turn our attention to the subject of the present lecture which is a correlation based on dimensional analysis for the heat transfer in a heat exchanger. Rather than talk about the total transport heat; the total transport of heat per unit time across the surface. It is conventional to talk about the average flux. This is basically the heat transported per unit time across the wall divided by the area of the wall, so it is called an average flux. And that average flux is of course dependent upon the average temperature difference between the inside and the outside.

In addition to the average temperature dependence it also depends upon other dimensional quantities. Of course, it will depend upon the diameter of the tube with dimension L, the total length of the tube with the dimensional length. It depends upon the thermal properties of the fluid, because the thermal properties do affect the rate of transfer of energy across the surface. Thermal properties in this case are the specific heat, and the thermal conductivity. And of course the rate of heat transfer will also depend at which the fluid is bringing in heat into the tube due to convection, because it is the heat that is convicted along the flow that is being transported across the surface. And that rate at which heat is being gotten due to convection will depend upon the flow velocity. And this will also depend upon the mechanical properties of the fluid, the density, and the viscosity.

So we now have a total of nine quantities here, the dependent quantity is the average flux and that depends upon all of these other quantities the temperature difference, the diameter length, specific heat, the thermal conductivity, the velocity, density, and viscosity. And of course, now the program is clear we have to write down the dimensions of each of these and then based on dimensional analyses see how many non dimensional groups are there, and then try to derive expressions for these non dimensional groups.

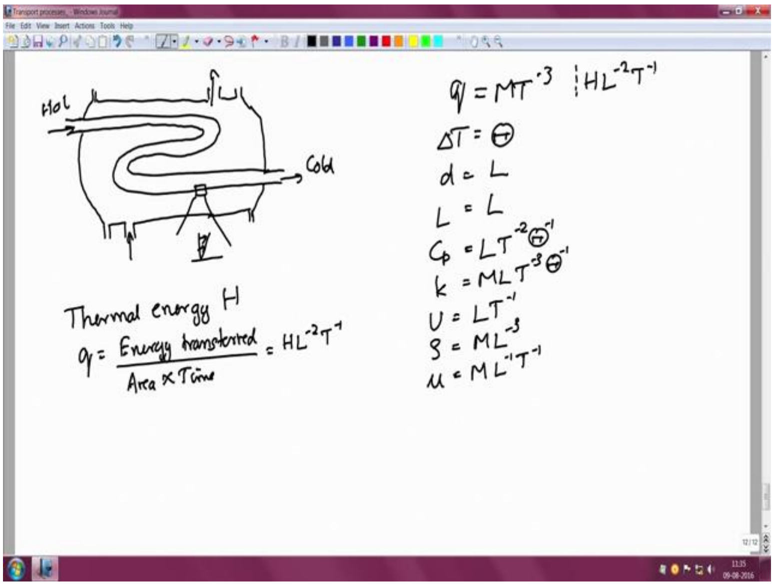
The heat flux, we had derived equations the dimensions of each of these in the previous lecture if you recall. So the heat flux, as dimensions of MT power minus 3; the delta T the temperature difference has dimensions of temperature itself, the diameter has dimensions of length, the length of the pipe has dimensions of length. The specific heat if you recall if you can derive that; the specific heat has dimensions of length T power minus 2 theta inverse. The thermal conductivity had dimensions of MLT power minus 3 theta inverse. The average velocity had dimensions of length per unit time. The density was a mass per unit length. And the viscosity if you recall, we had call it over here and ML inverse T inverse.

So, there are 9 quantities. And there are how many dimensions? 4 dimensions, because you have mass, length, time and temperature. So, total of nine quantities 4 dimensions, so you should have 5 dimensionless groups which determine this average heat flux. One is the dependent dimensionless groups and the others are all independent dimensions groups.

However, we can do something more than just dimensional analysis. If we recognize that there are two different kinds of energy that we are talking about here. One of them is the thermal energy due to the temperature that is being transported across the surface, and the other is the mechanical energy due to the kinetic energy of the fluid itself. Now, in the event that there is no inter conversion between thermal energy and mechanical energy we can consider thermal energy could be different dimensions, because it is not in the converted to mechanical energy.

So, as far as the transport is concerned the thermal energy is in itself conserved quantity. If you make this distinction between thermal energy and mechanical energy we get one more dimensionless dimensional in this problem and that is a thermal energy. And all the thermal quantities can be expressed in terms of thermal energy, while all the mechanical quantities can be expressed in terms of the mechanical energy.

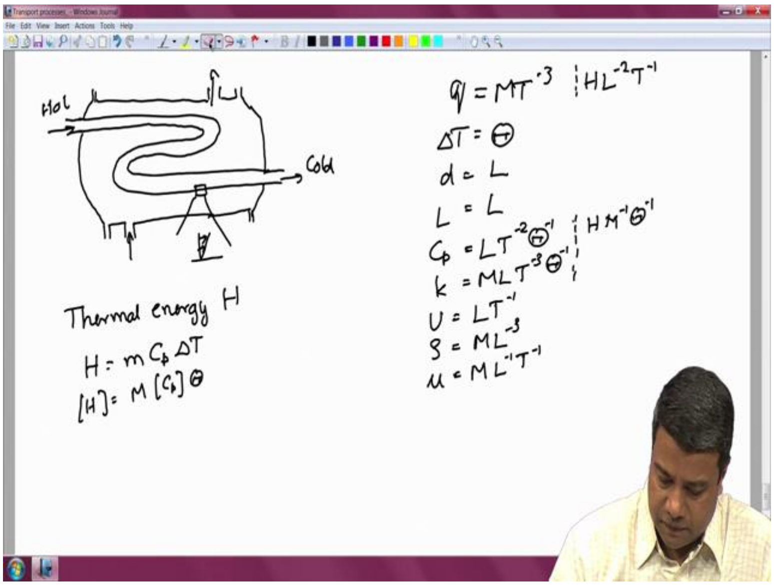
(Refer Slide Time: 13:38)



So, let us write down another dimension for thermal energy. As H, then the thermal quantities the heat flux the specific heat and the thermal conductivity have now got to be expressed in terms of this thermal energy H. So, this we are going one step beyond dimensional analysis and identifying another independent dimension.

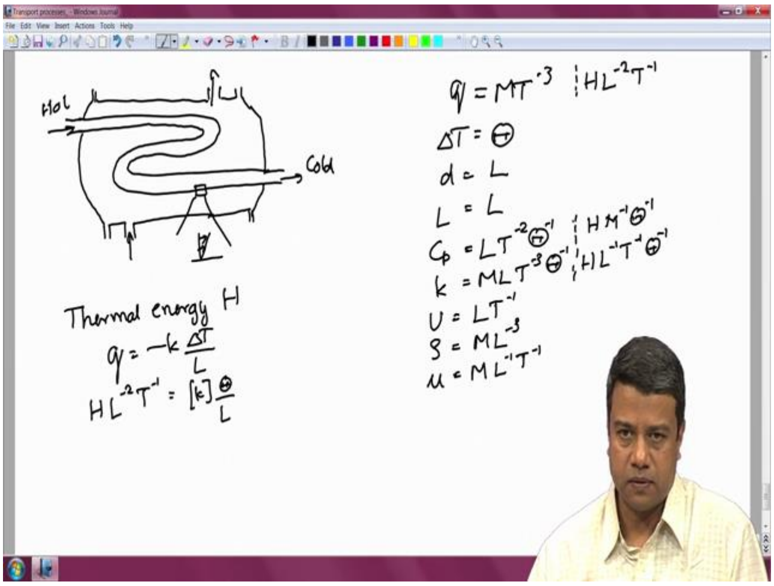
So, what is the heat flux in terms of the thermal energy? The heat flux is equal to energy transfer per unit area and unit time. So therefore, this will have dimensions of H alpha minus 2 T inverse. How do we get the dimension of the specific heat?

(Refer Slide Time: 15:04)



The thermal energy is equal to m C p delta T. Therefore, the dimension of specific heat has to be the dimension of thermal energy times mass times T. Therefore, the specific heat will have dimensions of H M inverse theta inverse. And finally, the thermal conductivity it has to be obtained from some relation which involves a thermal conductivity in which we know all other quantities. In this case it is the considered in relation, the Fourier's law for heat conduction.

(Refer Slide Time: 16:00)



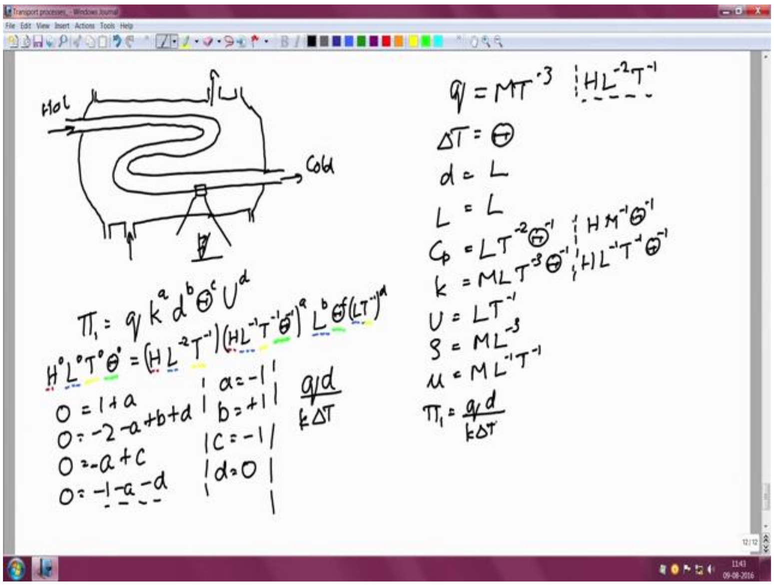
Q is equal to minus k delta T by L.

Now, q has dimensions of H alpha minus 2 T inverse is equal to the dimension of k times theta divided by L. And therefore, you can see that the dimension of the thermal conductivity has to be H L inverse T inverse theta inverse. All the others are of course, either length dimension or mechanical quantity, so they do not involve thermal energy.

So, now in this modified scheme that we have there are now 5 dimensions, and therefore there should be only 4 dimensionless groups. One of them is the dependent dimensionless group which basically involves the average heat flux. So, the dependent dimensionless group involves the average heat flux, it involves the dimension of thermal energy and that dimension of thermal energy can be canceled only against one of the other thermal energy dimensions of this problem. Those are either the specific heat or the thermal conductivity.

You can see that the specific heat involves the master machine as well. So, if you use them this specific heat you will have to bring in one of the mechanical quantities using the thermal conductivity is simpler because it does not involve the mass dimension.

(Refer Slide Time: 17:49)



So, my dependent dimensionless group has to involve the gamma flux times the specific heat; I am sorry the thermal conductivity to some power a. And now you can see that there is a length and time scale involved here. The length scale is of course equal to the diameter power b. There is a scale of temperature involved, so I like to have theta power c. And then there has to be some time scale in the problem, you have to choose the quantity which has dimensions of time. We will just choose the velocity since the first dimensions of time U power d.

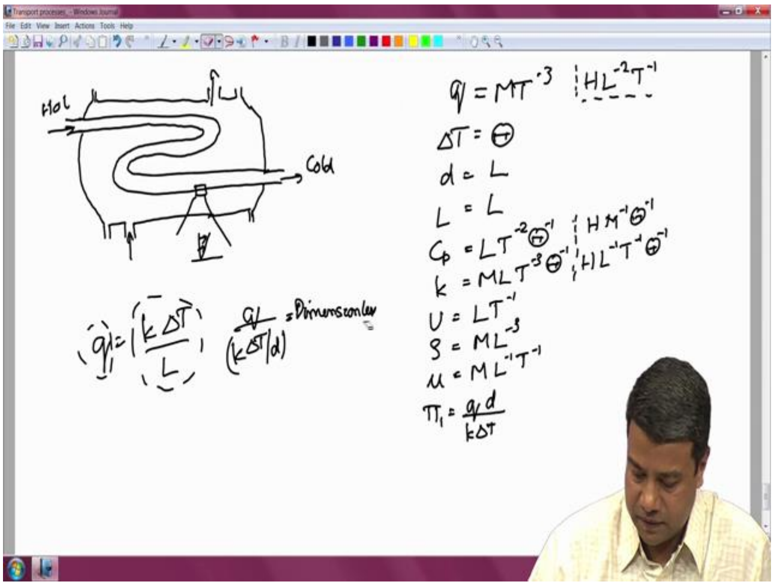
So, that will be the dimensionless loop that we assemble. And this dimensionless group has the dimensions H L T and theta. So, 4 dimension, 4 unknowns: a b c and d and we can obtain each one of those explicitly. Therefore, H power 0, L power 0, T power 0, theta power 0 is equal to H alpha minus 2 T inverse into the thermal conductivity power a H L inverse T inverse theta inverse of a into L power b theta power c and L T inverse power d.

So, with this I can get equations for a b c and d. The first one if I take equal dimensions of H, what I will get is that 0 is equal to 1 plus a. If I take equal dimensions of L, I will get 0 is equal to minus 2 minus a plus b plus d. If I take equal dimensions of theta I will get 0 is equal to a plus c. And finally if I take equal dimensions of time you will get 0 is equal to minus 1 minus a plus d, I am sorry minus d.

Now, these can be easily solved. If the first equation gives me that a is equal to minus 1 straight away. This equation here since this is 1 plus a plus b equals 0 this gives me that d is equal to 0; this equation here will give me that c is equal to plus 1, I am sorry minus 1. It is I absorb the mistake here in the equation for theta which should be minus a plus c is equal to 0 c, should be minus a plus c is equal to 0 which means that a is equal to c and so c is equal to minus 1. And from this equation you should end up with d is equal to; I am sorry d equals b is equals plus 1.

So therefore, this dimensionless group that I have a sample this way will be equal to q T by k delta T. That is the dependent dimensionless group. Therefore, my first dimensionless group pi 1 is equal to q d by k delta T. When you give a physical insight here you could have got this without going through the details of the dimensional analysis. Just by realizing that at the local scale, at every volume element within the fluid Fourier's law of heat conduction applies for the conduction of energy.

(Refer Slide Time: 23:24)



And Fourier’s law of heat conductions is that q is equal to k delta T by L. Therefore if I assemble a dimensionless group this applies at every local position where delta T is the difference in temperature locally. This does not apply for the unit as a whole; it does not apply for the unit as a whole, because convection is a very important part of heat transfer within this unit as a whole.

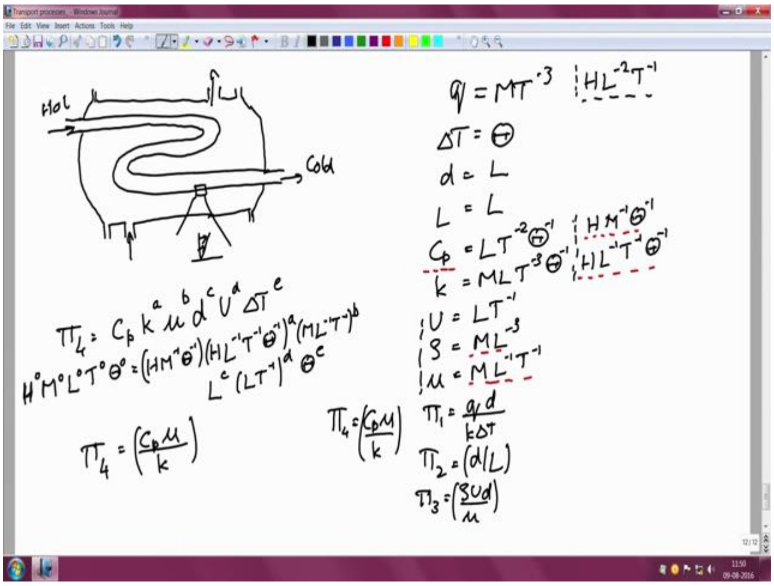
So, this relation Fourier’s law of heat conduction does not apply for the unit as a whole. However, if I scale the right and left sides of this equation I do not get a dimensionless group. Therefore, q by k delta T by a length scale has to be dimensionless. So, for the dependent variables I can use the constitutive relation to guess what is the form of the dimensionless group; so that gives us the first one as I said there are four of them.

The second one is relatively easy to guess. The second one is just equal to d by L, both the diameter and the length of the pipe at the same dimension, same dimensions of length. So, I can just divide 1 by the other and get a dimensionless group. There is a third one which involves the mechanical properties. That is something that we have seen in the last lecture. We showed that rho u d by mu was a dimensionless number. In this problem as well you have u rho d and u. So therefore, I can very easily just try to the third one is rho u d by mu; that is a dimensionless group number three.

There should be four of them. So, the fourth one should contain the specific heat as well, because that is currently not involved in any of the dimensionless groups. The specific heat should also be four dimensionless groups. And the specific heat has relations H m inverse theta inverse, so it contains both mechanical properties and thermal properties, therefore I need to scale it by mechanical quantities as well as thermal quantities I need to scale it by a quantity, an independent quantity which involves thermal energy and an independent quantity which involves mass.

The independent quantity that involves thermal energy is of course, the thermal conductivity. As far as the independent quantity that involves mass is concerned there are two of them; the density and the viscosity. You can without loss of generality choose either one of them. In this case I will choose the viscosity, but you could very well have chosen the density and the density as well.

(Refer Slide Time: 26:40)



Therefore, the dimensionless group that I will ensemble is equal to C p it has to have thermal conductivity unit for the thermal energy dimension the viscosity unit for the mass dimension. And then of course, there is length time and temperature; for the length dimension, the time dimension, in the temperature dimension. The length dimension of course, will be d power c. The time dimension I can get from the velocity u power b. And the temperature dimension I will get from the temperature difference.

So, this involves thermal energy the mass, length, time, and temperature. Therefore, my equation looks a little complicated; H power 0 M power 0 L power 0 T power 0 theta power 0 is equal to into the thermal conductivity that is H L inverse T inverse theta inverse whole power a into M L inverse T inverse power b into the length power c length inverse time power d and theta power e.

So, there are total of five equations that have to be solved. You would think it is a little bit complicated to solve five equations for these five unknowns. It turns out that if you solve these you get a pretty simple relation. The dimensionless group is equal to C p mu by k. It turns out that the powers of the diameter the velocity and the temperature difference are exactly 0. In this case if you solve all five equations, you can do it yourself and verify c is equal to 0 d is equal to 0 and e is equal to 0.

So, we just get the dimensionless group which goes as C p times mu by k so that is the final dimensionless group; you should write that as pi 4, so therefore the fourth dimensionless group that I have. I will show you a little later how this dimensionless group can be inferred without going through this complicated dimensional analysis by trying to give you some more information about what exactly these mean, but for now we have got our four dimensionless groups.

Now of course we have to ensemble relationships between these dimensionless groups. From this point onwards it has to be empirical, I will tell you in the next lecture what are the kinds of relationships that are empirically derived in this case. And after that I will go on to the other problem that we had which was the mass transfer from a particle. So, we will continue with this thermal heat transfer problem in the next lecture. And then we proceed to the mass transfer problem, and see you next time.