Transport Processes I: Heat and Mass Transfer

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Lecture – 04

Dimensional analysis - Force on a particle settling in a fluid

Welcome to this second lecture on dimensional analysis, the 4th lecture of our course. In the previous lecture I had taken you through the fundamental and derived quantities and I have derived for you the dimension of all the derived quantities that we will commonly use during the course; as I said the fundamental quantities are mass, length, time temperature and there are 2 that we do not really use doing most of this course that is current and dimension of light. But from this 4 dimensional quantities: mass, length, time and temperature you can derive a very large number of quantities that we have employed throughout we will be employed throughout this course.

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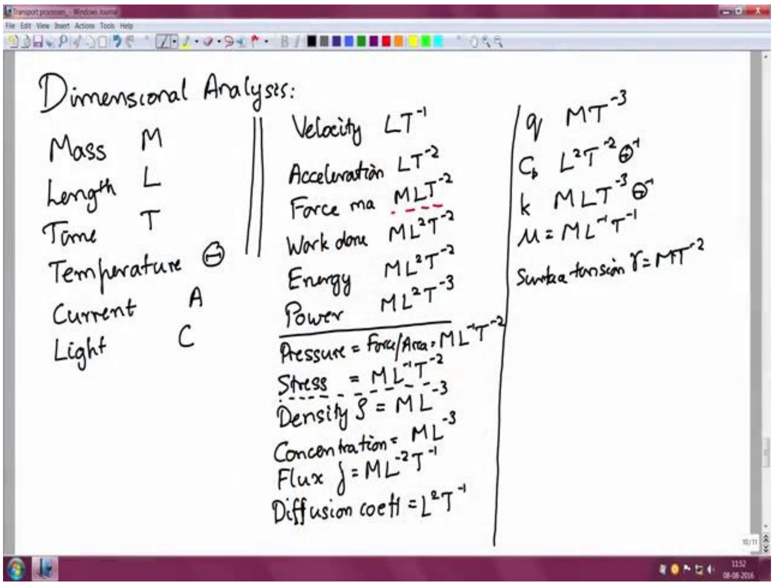

Velocity, acceleration, force, work, energy power, pressure stress density and so on; quantities which are of relevant in mass transfer and heat transfer and the momentum transfer. So, how do we use dimension analysis in order to analyze problems.

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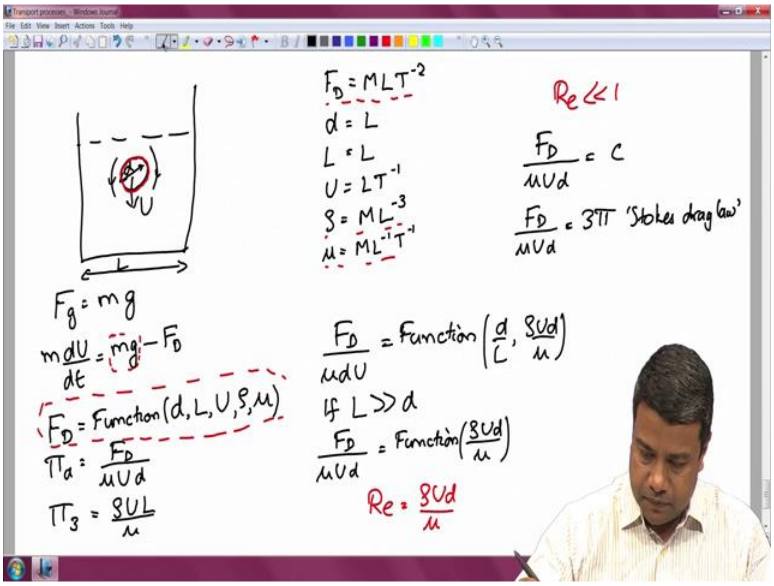

So, we start after with a fundamental with a basic momentum transfer problem, that is that I have this tank of the fluid within which there is a particle that is settling. Now the particle of course, settling because the density of the particle is higher than the density fluid around it and therefore, there is gravitational forces acting downwards. However, there is also another force due to the fluid which tests to the resist the motion of the particle.

As the particle is settling downwards, the fluid there is fluid flow around the particle and that fluid flow causes friction within the particle and due to the fluid there is stress that is exerted at every location on the particle surface here force for unit area stress and as we know that stress comes from Newton law of viscosity. And this stress this upward force exerted on the particle, a forces the first two gravity and therefore, if I were to write the first balance equation for this particle, it would be of the form m d U by d t is equal to this gravitational force minus this drag force.

Our task now is to find out what is the form of this drag force, in particular what are we have to start off we are trying to find out, what are the quantities the dimensional quantities on which this drag force could depend? As I said this drag force could be a function of first thing is it has to depend upon the diameter of the particle, it should depend upon the diameter of the particle and it could also very well depend in some way on the dimension of this tank, it could dependence on some way on the dimensional of this tank. And then it has to dependent upon the fluid properties because this drag force is exerted due to the fluid flow around the particles therefore, the drag force has to depended upon the fluid properties.

First thing it will of course depend upon the velocity; you would accept that as the particle moves faster there is going to be a grater drag that is to resist it is flow. So, it should depend upon the velocity, but it should also depend upon fluid properties; what are the fluid properties mechanical properties of the fluid are 2 in number, that is one is the density and the other is the viscosity; after all it is the viscosity that causes the resistance to flow the frictional force is due to the fluid viscosity. So, it could depend upon the density and the viscosity as well; while numerating these dimensional groups one has to be careful, the particle density for example, does not affect the drag force.

The drag force due to the fluid flow around the particle and for the fluid flow around the particle on the only relevant quantity is going to be the fluid density. Similarly the acceleration due to gravity does not affect the drag force, drag is a separate force in itself that is a separate force in a itself, that is what we call a body force; force that is exerted on the body it is propositional to the volume of the material or the mass of the material. Whereas, the drag force is due to a surface force, due to force exerted on the surface of the particle due to the motion of the fluid. So, therefore, the acceleration due to gravity for example, cannot affect this drag force and by a last this is the total dimensional quantities that can affect the fluid drag.

Now how do we find the form of the drag force? So, in this problem now there are how many dimensional groups there is the of course, the dependent dimensional group the drag force and then there are the independent dimensional groups, I have 1, 2, 3, 4, 5 of them; the dimensional of the particle, the dimension of the tank, the velocity, the density and the viscosity. So, let us list out all of these and look out what are the dimensions in these. The drag force is a cause of force, it has dimensions of mass length T to the minus 2 mass times acceleration that is the drag force.

The diameter of the particle and the length of the dimension of the tank both have dimensions of length. The velocity of courses distance moved by unit time, the density is mass per unit volume mass per volume and we had seen what is the dimension of the viscosity mass length inverse T inverse. We got that by from the Newton's law of viscosity, stress is equal to viscosity times velocity it reference by time and that is how we got this dimension. So, I have 1, 2, 3, 4, 5, 6 dimensional variables and how many dimensions are there between all of these it is the total of 3 dimension: mass, length and time and therefore, I should be able to get total of 3 dimensionless groups from these from 6 dimensional variables, and the 3 dimensions, there are total of 3 dimensionless groups.

One of them is an independent dimensionless group I am sorry, 2 of one of them is a dependent dimensional group because it is involves the dependent variable the drag force. Drag force is dependent variable it depends on the other quantities, therefore one of them is a dependent dimensionless group and the two have to be independent dimensionless groups which just contain the properties of the fluid, the particle diameter the velocity and the length the tank. So, how do we get the first dependent dimensionless group? So, for that I have to scale the force which has the mass dimensional unit with either the mass, the density or the force which has mass dimension unit with either the density or the viscosity, combine with the velocity and the length scale, I can choose either of this both are valid choices.

I will come back and tell you in which cases we should choose the density and in which cases we choose the viscosity, but at this stage both of these are valid choices, In this case I will choose the viscosity as the independent variable. So, I will construct dimensionless group, I will call it as the dependent dimensionless groups containing the force non dimensionalized by suitable powers of the viscosity, the length scale there is particle diameter and the velocity; to some powers we do not yet know what powers these are, we will find out right dimensional analysis.

So, if this is a dimensionless group then it as have dimensions of mass power 0, length power 0 time power 0 should be dimensionless. So, force has dimensionless mass, length T to the minus 2 ok. The viscosity is mass, length inverse T inverse or a length power b and length T inverse power c. So, this combination of quantities has to have a net 0 dimension. So, if I look at the mass dimension all of these then I will get one relation that is 0 is equal to 1 plus a, if I look at the length dimension in each of these I will get 0 is equal to 1 minus a plus b plus c and finally, if I look at the time dimension in each of these, I will get 0 is equal to minus 2, minus a, minus c; now I have 3 equations for 3 coefficients a b and c we can solve these.

The solution quite simply comes of a is equal to minus 1, b is equal to minus 1 and c is equal to minus 1. So, that actually satisfy these 3 relations therefore, my (Refer Time: 12:08) dependent dimensionless group is going to be equal to F D by mu U hence the particle diameter. So, there is the first dimensional group that I had there is dependent dimensionless group, it is a scale force scale for suitable quantity is in such a way that it been dimensionless.

Now I should have 2 other independent dimensionless groups: one of them is quite easy to see since I have 2 lances in the problem the particle size and the length of the tank. The second dimensionless group simply going to be equal 2 d by L and I have to have a third dimensionless groups which does not involve the drag force, but involves on the other quantities and in this dimensionless groups I can of course, set any one quantity to the first power without loss of generality because regardless of which quantity is set to the first power, I will still get a dimensionless group and the dimensionless group to any power is still another dimensionless group. So, it is a valid way to do things.

So, I just choose the density to the first power because we have not yet used it in any our calculation. So, the dimensionless groups will contain the density to the first power, the velocity to the power of a, the particle diameter to the power of b times the viscosity to the power of c. Once again mass power 0, length power 0, T power 0 is equal to the density has dimension of mass L power minus 3, u is L T inverse or a, L power b and viscosity is M L inverse T inverse power c.

Once again if we take the dimension of mass both sides, you get 0 is equal to 1 plus c, we take the dimension of length on both sides you will get 0 is equal to minus 3 plus a plus b minus c (Refer Time: 15:04) and if I take the dimension of time on both sides and I will get 0 is equal to a minus (Refer Time: 15:22) I am sorry minus a minus c. In these are also simple relations they can be solve quite easily, you will find that c is equal to minus 1, a is equal to plus 1 and b equals to plus 1. So therefore, this dimensional group pi 3 rho U L by mu many of you recognize this as the Reynolds number we will come back to the physical interpretation of this dimensionless group. So, now, that we have done this we can write this functional form that I had earlier which involved the total of 6 quantities, I can write it to involve only a total of 3 quantity. So, what I will write is that F D by mu d U is equal to the some function of d by L and rho U d by mu.

So, that has greatly simplified this relationship now. So, you do not have to very 6 quantities independently to find out what is the drag force, usually to very 3 dimensionless groups. Now if the tank is much larger (Refer Time: 17:16) diameter, if the length of the tank, if length is much larger than the particle diameter then you would think that the size of the tank does not really matter here very small particle that settling in a huge tank whether the tank is if we have particle about a millimeter settling in a tank whether it is 1 meter or 10 meters it does not really matter. So, for particles settling in infinite fluid this simply of still further; F D by mu U d is equal to some function of rho U d by mu.

So, we have reduce it even further just one relationship, now what is the physical interpretation of rho U d by mu? This as U is the Reynolds number, it has the fluid density in numerator and it has a fluid viscosity in the denominator. So, it is some ratio of the fluid inertia in the fluid viscosity; if the Reynolds numbers is small then the fluid initial does not really matter. So, therefore, fluid inertia is not an important parameter and the drag force should depend upon the viscous forces along. So, therefore, density should no longer be important if Reynolds number is much smaller than one.

Density is no longer parameter because it represent fluid inertia therefore, the result that I will get use that in this limit F D by mu U d should just be equal to a constant. If the fluid inertia no longer important, the drag force divided by the viscosity the velocity in the drag of it is just equal to constant. So, we could even get form of the drag force just base upon dimensional analysis, if fluid inertia not important of course, beyond this you cannot actually determine value of the constant, if we actually what it to determine value of the constant you have to actually solve the problem find out what is the fluid flow around this particle. Due to that fluid flow what is the stress that is exerted on this particle, that stress some upon the entire surface will give you the net force that is exerted and that is an elaborate calculation which will be a little bit advances to this course, but at the end of it you find that the result is that F D by mu U d is just equal to 3 pi, you can do that calculation analytically and this is what is call Stokes drag law.

Stokes drag law for this drag force exerted on particle in the limit of where the fluid viscosity is much larger than the fluid inertia and this kind of Stokes drag law actually gives you the correlation for the drag coefficient, which I was talking about in the last lecture. I told you that we express dimensional groups in terms of ratio of dimensionless numbers which is the Nusselt number or the Sherwood number and so on. For momentum transfer the appropriate group or what are call the drag coefficients and the in this case drag coefficient define as (Refer Time: 21:12) for the flow on the particle the drag coefficient is define as the drag force divided by the projected area by half rho U square.

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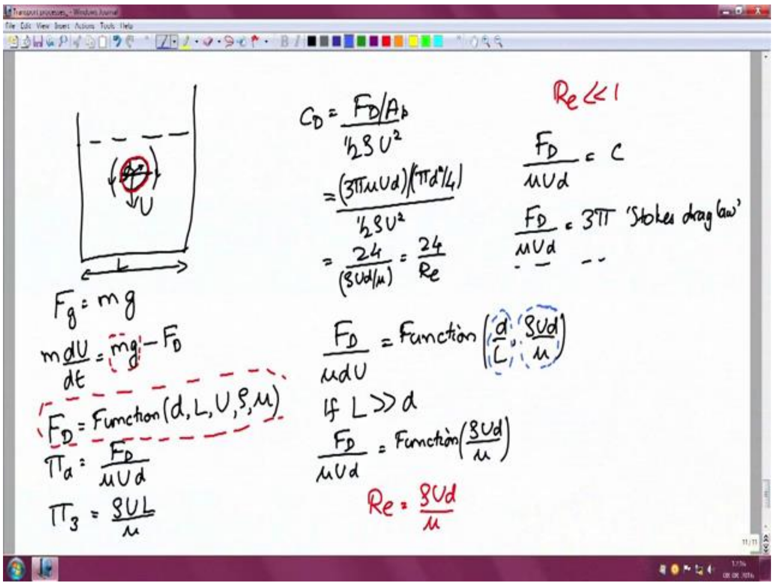

Where the projected area is the area of projection, area projected by particle to the fluid, so we know that the projected area the drag force is 3 pi mu U d from the stokes drag law, the area of projection is pi r square which is pi d square by 4 divided by half rho U square. And if you use this we simplify this will find that is just equal to 24 by rho U d write mu is equal to 24 by Reynolds number.

So, there is a first correlation that we got for the drag coefficient on, it is spheres settling in a fluid when it satisfies it is stokes drag law. So, these are often plotted in drag law glass the drag coefficient verses the Reynolds numbers and the this drag coefficient is valid as I said law Reynolds numbers, has increase the Reynolds number the flow became more complicated and the drag coefficient tense to some constant value, but this is in limit of low Reynolds number. Reynolds number increases this F D by mu U d that is depend upon the Reynolds number, that Reynolds number is depended little complicated and I will discuss it when we actually do momentum transfer round this week.

So, this is a simplest example, we have done dimensional analysis but a little bit more. We said that one this number is small we can neglect this as a parametric dependence because the tank volume is much small than much larger than that particle size and then we can look at different limits whether Reynolds number is large or small. When the Reynolds number is small I said that we can neglect density all together and we get back the Stokes drag law and the Reynolds number is large can we neglect viscosity all together, simplistically you might thing at let us just neglect viscosity and solve the rest of the problem. However, as I said the momentum transfer is due to the stresses exerted by the fluid on the particle surface.

The stresses exerted tangential to the surface entirely discuss and therefore, even when inertia is large you may not able to neglect stresses exerted on the surface we will look that in a little bit more detail a little later, so this simplest example. Let us take the next more complicate example and that is the heat exchange of (Refer Time: 24:56), I will just introduce the configuration the present class and then we will continue with this in the next lecture.

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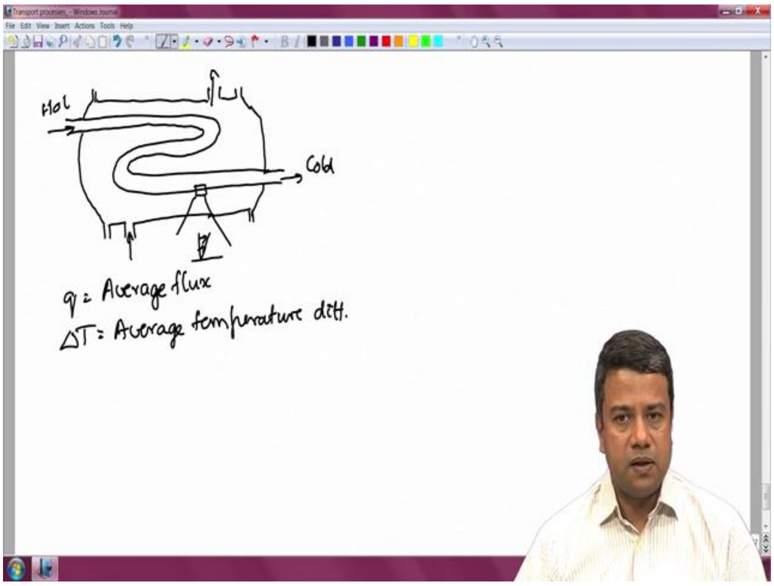

So, we have a shell and tube heat exchanger; in this case we are considered the there is a hot fluid coming in cold fluid going out, this is being cool by coolant that is coming in through the shell side. Now in this case what we like to know actually what is the total heat that is the transferred across the tube wall, but this heat there is transfer of course, depends on length of the heat exchanger the diameter and so on. So, what is normally is to define and average flux q, average flux across the tube wall that is the amount of heat that is the transfer per unit area per unit time.

So, where as the fundamental quantity here is the heat exchange itself the rate of heat exchange, the heat that is transferred per unit time across the wall, we express this in term of the average flux which is the total heat transferred divided by the area. Obviously, there is a not convenient way to do it because has you keep increasing the area simplistically you would think that the flux has to increase and that is going to a function of the average temperature the difference. It is going the function of the average temperature difference between the shell side and the tube side; there is more than that depends upon other quantities, it depends upon the thermal properties of fluid, the thermal conductivity the specific heat.

In addition the heat that has been transferred is being is being swept in to the tube due to the fluid flow itself, and so if it look at a differential volume close to the surface, that is fluid that is flowing across the surface carrying the heat with it as it flows and that has to diffused through the surface to the other side. So, in addition to the thermal properties the mechanical properties in the fluid will also be important, the density the viscosity and so on. In addition these properties are also important for determining, what is the pressure difference between the hands of the pipe, which is also an important design consideration.

So, on the basis dimensional analysis can get some idea of what are the dimensionless groups rather than the dimensional parameter that effect the transfer process and once we know that are dimensionless groups, we can then try to get the relationship between these dimensionless groups, so that we will continue in the next lecture.

Next lecture I will give you dimensional analysis of the heat transfer in heat exchanger and then will go back and look at the other problem that we had looked at the in introductory lectures and that was the mass transfer from the spherical particles. And then I will give you some physical inside into the dimensionless numbers that arrives in this kind of transport process. So, we continue in this in the next lecture and I will see you then.