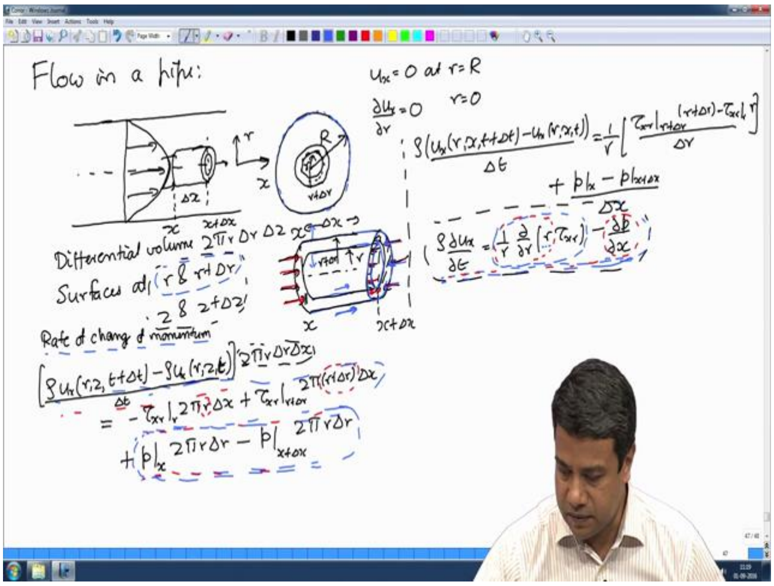
Transport Processes I: Heat and Mass Transfer
Prof. V. Kumaran
Department of Chemical Engineering
Indian Institute of Science, Bangalore

Lecture - 38
Unidirectional transport: Laminar and turbulent flow in a pipe

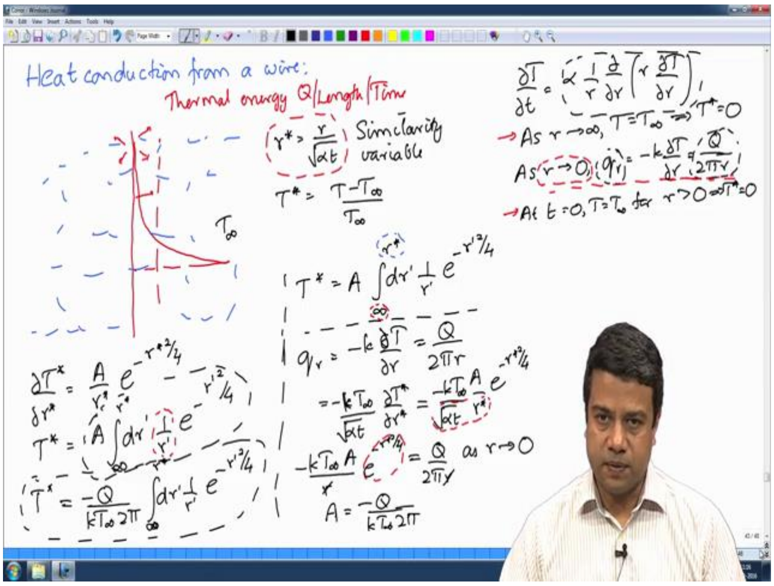
Welcome to our continuing series of lectures on Fundamentals of Transport Processes. We were going through transport in one direction transport of mass momentum or energy in one direction. We looked at mass and heat transfer from a flux surface using a Cartesian coordinate system; that was the simplest coordinate system that we can consider because the three axes are straight independent of position and the plane is constant coordinate or perpendicular to each other.

(Refer Slide Time: 00:53)



We had also progressed to a cylindrical coordinate system, where the surfaces of constant coordinate were cylindrical surfaces. So, this coordinate system was defined by one coordinate was the distance from the axis of the cylinder the radial distance from the axis. And the other was the distance along the axis the axial distance. And we had solved a heat transfer problem in that cylindrical coordinate system using separation of variables. We had also solved a heat transfer problem using the similarity solution if you recall we had solved the problem of heat conduction from a wire using a similarity solution that had the distinguishing feature that the temperature there was going to the temperature in that case is going to infinity at the wire itself, but despite that we were able to get an analytical solution using the similarity transform in this case of a heat conduction from a wire.

(Refer Slide Time: 02:08)



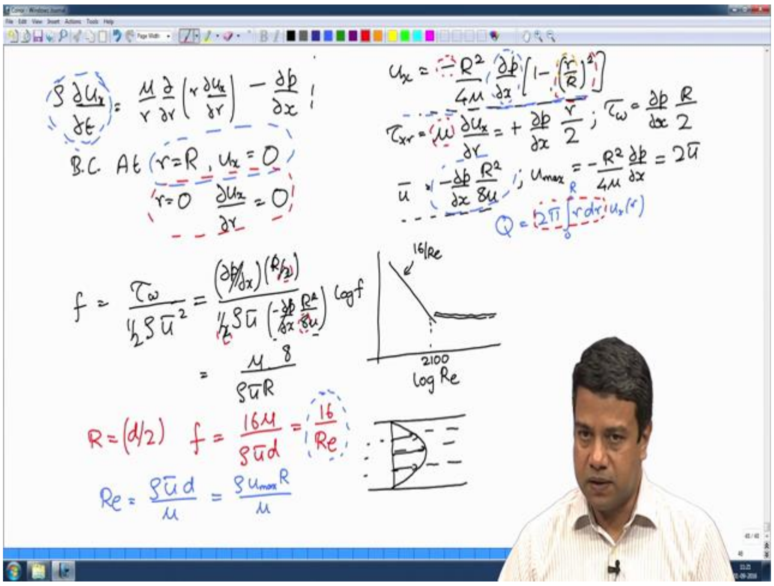
And then we had gone on to momentum transfer problems. As I said in momentum transfer there is one additional factor which is the pressure gradient, which is not in general present in heat and mass transfer problems. And last lecture we had looked at the flow in a pipe a cylindrical coordinate system in which the surfaces are surfaces of constant distance from the axis of the pipe. And in this case we were considering fully developed flow the assumption being that there is no variation of the velocity along the stream wise or the axial direction. The velocity is the same at every downstream location that is a fully developed flow. And return a momentum balance equation for that particular situation where there could be variations in time, but there was no variation in the axial direction for the x momentum the axial momentum; the momentum along the axis of the pipe.

And we are chosen surfaces at z and z plus delta z written down the balance and finally we had got this balance equation. The new feature here was the pressure, the pressure between the ends of the pipe was different that is what is driving the flow; the pressure difference between the two ends of the pipe. Therefore, whenever we consider a surface and I consider any surface there is an additional pressure that is exerted on that surface. Pressure is always compressive it acts along the inward unit normal to the surface.

So, for the differential volume that we are considering we are considering the momentum balance in the x direction in the stream wise direction. The pressure is in that direction only at the two annular faces at x and x plus delta x. Therefore, that pressure has to come in at these two annular surfaces in addition to the shear stress, the momentum transfer across the cylindrical surfaces. As I had explained in the last lecture the shear stresses are parallel to the surface, the pressure is perpendicular to the surface and compressor.

So, in this x momentum balance equation the pressure at the surfaces in x and x plus delta x does contribute to the net force on this volume. The shear stress at the surfaces at r and r plus delta r does contribute to the net force. The pressure at these surfaces does not, because the pressure at these surfaces is in the radial direction. So, on that basis of momentum balance we had got this momentum conservation equation for the unsteady flow. And if you recall in the last lecture we had solved it for the special case of a steady flow.

(Refer Slide Time: 05:04)

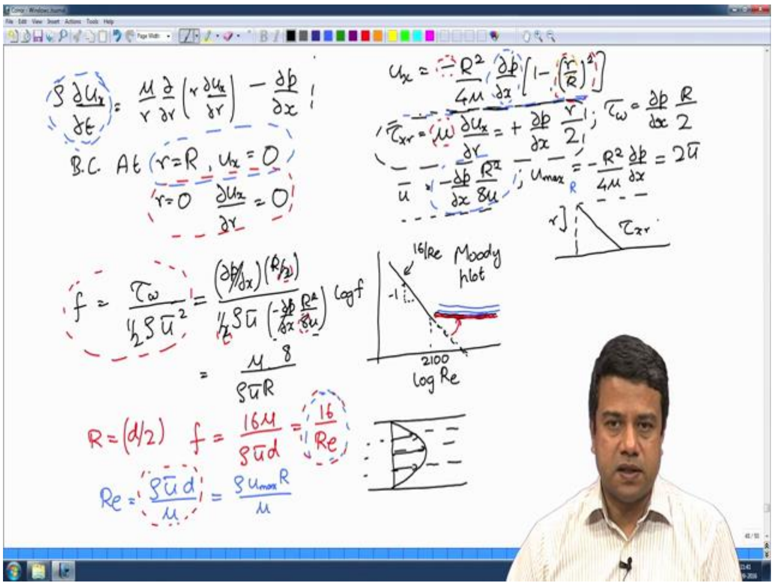


So, it considered that there is no time variation we considered a steady fully developed flow, and we solve for the velocity profile of that study fully developed flow with the boundary condition that at the surface of this pipe the velocity has to be equal to 0. The no slip condition, basically that at a solid surface the fluid velocity is equal to the velocity of the surface itself. In this particular case the pipe wall is not moving and therefore the velocity is 0 at the surface.

Based upon that we had solved this equation and got an equation for the velocity profile, and from that for the shear stress at the wall all based upon this pressure gradient all based upon this pressure gradient. We had got the maximum velocity the mean velocity in terms of the pressure gradient. If you recall the mean velocity was equal to the integral the flow rate, the integral of the velocity over the cross section. And while doing that calculation one has to be careful, the flow rate itself is equal to 2 pi integral 0 to r r dr times u x at the location r. That is because the area element in a cylindrical coordinate system is 2 pi r times dr. The parameter at any location r is to 2 pi r, and the thickness of that interval is dr. So, the area element is 2 pi r dr the cylindrical coordinate system. And that is one thing that one has to be careful about while doing this calculation.

In any case once you do this calculation you will get the flow rate divided that by the cross sectional area and you get the average velocity. And as I had shown you in the last lecture the average velocity is equal to one half of the maximum velocity for a pipe flow.

(Refer Slide Time: 07:17)

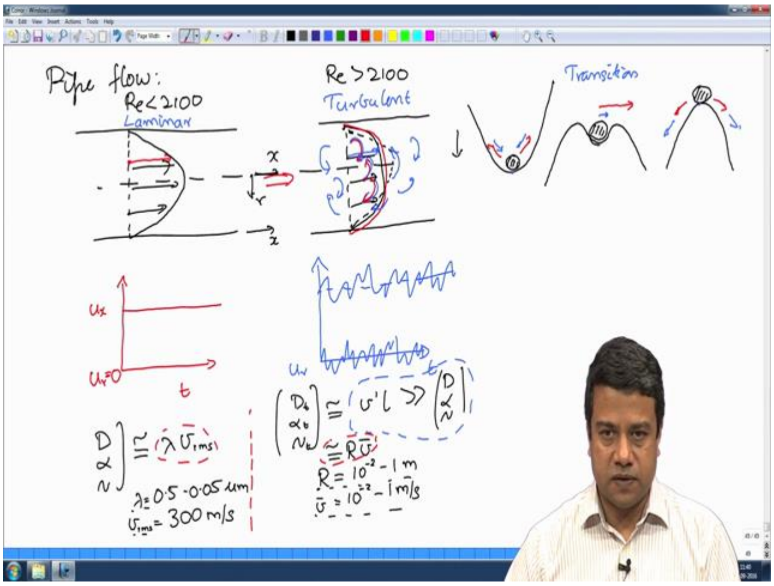


And the friction factor is defined as the wall shear stress scaled by the inertial scales. And from this we had got an expression for the friction factor as a function of Reynolds number as well and we found out that the friction factor is equal to 16 by the Reynolds number, where the Reynolds number could be defined either in terms of the pipe diameter and the average velocity or the pipe radius and the maximum velocity. These two are the same in this case whether you define it in terms of the pipe radius in the mean velocity or the pipe diameter; I am sorry pipe radius and the maximum velocity of the pipe diameter at the mean velocity. This is not true in general for example, in a plane channel this will not be true, but for the particular case of a laminar flow in a pipe this is true.

So this was the log, it is called the Moody plot log of friction factor versus log of Reynolds number and our log to log graph the slope is minus 1 if f is equal to 16 by Re log f versus log Re will have a slope of minus 1. And at Reynolds number of about 2100 a transition takes place. You will no longer this law is no longer satisfied. And there is a transition to some other velocity profile in which the friction factor is much higher than what it would have been for a laminar flow. For a laminar flow if I had extended this line of slope minus 1 the friction factor that ever have got is much lower than the friction factor that is actually observed in experiments.

And the friction factor in experiments is dependent in general on the roughness at the wall of the pipe. You have a lower value for a smooth wall and as the wall roughness increases you get a higher and higher value depending upon the roughness. Those are characteristics of what is called a turbulent flow. So, in this lecture I will try to give you some physical insight into what happens in turbulence. It will not be as quantitative as we were able to do for a laminar flow, because turbulence is a complicated phenomenon not completely understood yet.

(Refer Slide Time: 10:03)



At low Reynolds number as I just shown you up to a Reynolds number of about 2100 you have a parabolic velocity profile. And at a Reynolds number of about 2100 there undergoes a transition to different kind of profile; that is flattered at the center and steeper at the walls in comparison to a parabolic profile. A parabolic profile will look something like. This profile is flattered at the center and steeper at the wall. So, this is what is called a Turbulent flow, and this is what is called a Laminar flows. And turbulent flows typically for Reynolds number greater than 2100.

However, if you do take care to maintain to reduce the level of fluctuations in the flow you can maintain the flow in a laminar state even at a higher Reynolds number. So, even when the Reynolds number exceeds 2100 the laminar velocity profile is still a valid solution of the momentum balance equation. Even when the velocity exceeds 2100 the lamina profile is still a valid solution of the momentum balance equation. The problem is that it is not a stable solution. And any small disturbance in the flow was spontaneously make it go to a turbulent state.

If you want what to describe it simplistically, one could say in one dimension for example the transition to turbulence looks something like this. If I have an object at the minimum of a potential well say and there is a force downwards, if I do displace this object a little bit it is an e at an equilibrium state at the minimum point, it is at an equilibrium state all forces are balanced. If I do displace it a little bit the force tends to return it back to its original location. So, that is a stable steady state.

On the other hand, if I had an object on a hill rather than a valley; this location here is a steady state when the object is kept at the maximum of that hill this location is a steady state the forces are balanced in this configuration. So, that is a steady solution. The problem is that if I display it a little bit in either direction the forces tend to take it further away from the steady state. So, in that sense this is an unstable steady state. And any small disturbance will tend to make the system go away from that steady state.

So, the laminar flow is the steady state. I have shown it schematically only in one dimension similar the flow is more complicated the velocity is defined at each point within the flow and so on. But simplistically this is the picture, and like a configuration like this is called configuration of neutral stability because if you displace it a little bit it does not come back it does not go away. So, that is the transition; so this is the transition simplistically put.

This picture I should warn you does not apply for the laminar to turbulent transition in a pipe flow. It applies for other kinds of stability problems, you often see this kind of a picture drawn it applies for other kinds of stability problems it turns out that the landscape is much more complicated. For the laminar to turbulent transition the landscape is actually much more complicated. What it actually looks like is something like this, or it actually looks like is something like this. What that implies is that it is stable for infinitesimal displacements where it displaced only by a small amount it was still be stable, if I displaced by a large amount it will go to some other state. That is the situation for pipe flows.

So, if you maintain the disturbance level below certain value it will come back to the laminar state, but however when it goes beyond a certain value it will go to a turbulent state. Therefore, one could in principle maintain the flow in the laminar state by completely damping out disturbances, but in practical applications there are always disturbances in nature and those disturbances will spontaneously make the system go to a turbulent state. And in most cases that transition takes place at a Reynolds number of about 2100. For reasons I should not that are not yet completely understood, but once the disturbance takes it to a turbulent state you get to something that is a universal velocity profile, average velocity profile, independent of the disturbance that took you to the state.

Now, the turbulent velocity profile is not a steady velocity profile. The laminar velocity profile is a steady velocity profile. So, if I look at some particular point and I ask, what is the fluid mean velocity? That fluid mean velocity is only in the x direction; so I have the x direction here and I have the r direction here. That fluid mean velocity is only in the x direction and it is independent of time. So, for the laminar flow if I plot the velocity at any location u x is functions of time at one particular location say it has a steady value. And u y and u z; I am sorry this is a radial coordinate system and u r the velocity perpendicular to the axial coordinate this will be equal to 0.

On the other hand, in a turbulent flow there is an average velocity at any location. However, if you take the instantaneous velocity it is fluctuating about this average value and it is never equal to a constant. Similarly, in the cross stream directions as well if I plot the velocity at any location these velocities are also fluctuating, on average they are 0 but they are fluctuating about 0 and as a function of time right you do get a fluctuation in the velocity profile.

Now, this fluctuation the velocity profile is because of the presence of what are called turbulent eddies. These are velocity fluctuations within the fluid basically an eddy is a parcel of fluid that is in correlated motion. These eddies have different sizes all the way from the large scale; large scale is comparable to the type pipe diameter all the way to very small eddies. And the transport of mass momentum and energy takes place due to these turbulent eddies rather than due to the diffusion due to the molecular velocity fluctuations.

Primarily, the transport takes place due to turbulence eddies; the transport of mass momentum and energy due to the turbulent velocity fluctuations and not due to the molecular velocity fluctuations. For this reason the rate of transport in a turbulent flow is much higher than it is in a laminar flow. If you recall when we did the kinetic theory for gases I had shown you that all of these diffusion coefficients; the diffusion mass diffusivity, the thermal diffusivity and the momentum diffusivity all of these scale as the mean free path times the root means square fluctuating velocity of the molecules. We all have dimensions of length square per time. This is due to molecular diffusion. You had estimated these diffusion coefficients. The mean free path is of the order of 0.5 to 0.05 micrometers.

So, proximately 10 power minus 7 to 10 power minus 8 meters is the mean free path. And the v rms is of the order of 300 meters per second in gases. In liquids the diffusivity depends upon the mechanism of transport. In a turbulent flow at the simplest level one could also define the turbulent diffusivities; one could also define at the simplest level turbulent diffusivities. These will scale as the fluctuating turbulent velocity, look at the turbulent fluctuations the velocity of the turbulent fluctuations times a length scale for the turbulence. And that length scale for the turbulence is what is called the eddy size can be eddy length scale.

So, you have a length scale for eddies a distance over which there is correlated motion. This correlated motion is what is transferring mass momentum or energy. Therefore, you have a length scale for the turbulent eddies and you have a velocity for this turbulent eddies. And at the simplistic level this product is what would give you the turbulent diffusivity. Now, in most cases the largest scales of turbulence, the largest eddies that I told you or comparable to the system size; the largest eddies are comparable to the system size. And the largest velocity fluctuations are comparable to the mean velocity.

The velocity fluctuations in the stream wise direction in the turbulent flow are typically about 40 percent of the mean velocity. So, this will typically scale us in large scale times the mean velocity. Slightly smaller a factor of 10 or so smaller because the largest eddies are not exactly of the same velocity as the fluid mean velocity, but it will scale similarly. And you can see that in this case for typical application R would be about a centimeter to a meter. And the fluid mean velocities would once again be similar in magnitude that they would be of the order of 10 power minus 2 to about 1 meter per second.

So, the mean velocity is much smaller than the rms velocity. The average eddy size is much larger than the turbulent than the molecular mean free path. However, this product is usually much larger than the molecular diffusion coefficients. In this product you can see the product will vary from about 10 power minus 4 to about 1, whereas this the mall is the molecular diffusion coefficients are 10 power minus 5 in gases and even less in liquids. So, for this reason the turbulent diffusion coefficients are much larger than the molecular diffusion coefficient. I should caution that this picture is not exact. When we calculated the molecular diffusion coefficients I told you the basic assumption then we did the gradient expansion for the concentration about the location z is equal to 0 were expanded in the series that expansion can be truncated at the first term only if the molecular scales are much smaller than the system size.

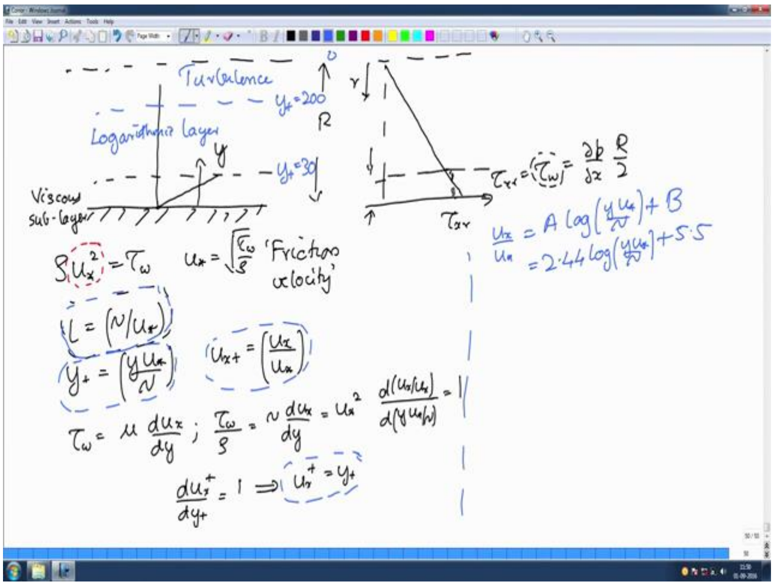
In the case of a turbulent flow the largest eddies are comparable to the system size, and therefore that expansion is not exact you cannot neglect the higher order terms. Strictly speaking because the ratio of the largest eddies size to the system size there is not a small number. But still this serves as a useful analogy to understand why turbulent diffusivities are much larger than molecular diffusivities.

And this also explains why near the center of the pipe the velocity profile is much flatter. The velocity profile near the center of the pipe is much flatter because the turbulent diffusivities are very much faster; momentum is transferred due to the turbulent fluctuations. And these, this is a much faster process. So, that tends to equalize momentum much quicker along the centre of the pipe. So, the centre of the pipe because cross stream fluctuations are equalizing momentum much faster the velocity is approximately a constant its more plug like in comparison to a laminar flow.

Near the wall of the pipe the flow the velocity is much deeper, and it is because of this fast transfer of momentum from the center to the walls momentum transport rates are much higher. Therefore, the shear stress is much higher because the shear stress is just the rate of transport of momentum per unit area per unit time. The shear stresses are much higher and therefore the friction factor is much higher than what you would have for a laminar flow, because these eddies transport momentum much faster across the pipe. And for that reason the friction factor is much higher than what you would expect for a laminar flow.

Similarly the other transport rates; the transport of mass and energy in a turbulent flow are also much higher by orders of magnitude in comparison to a turbulent flow, in comparison to a laminar flow as shown by the orders of magnitude difference in these diffusivities in.

(Refer Slide Time: 25:56)



So, near the center it is almost flat close to the wall; close to the wall if you look close to the wall of the pipe you do have a region where the flow is laminar, there is a region close to the wall of the pipe where the flow is laminar it is called the viscous sub layer.

How do we get the thickness of the viscous sub layer, the Reynolds number is large so as far as the flow near the viscous sub layer is concerned the radius of the pipe is actually not a relevant length scale. The radius of the pipe is not a relevant length scale, because the radius of the pipe is much larger than this viscous sub layer length scale. Across this viscous sub layer the wall shear stress the shear stress is approximately a constant. Now if I were to plot the shear stress across the pipe, we had an expression for the shear stress here this was the expression for the shear stress it is a linear function. At r is equal to 0 the shear stress is equal to 0 and as r increases the shear stress increases. So, shear stress increases linearly as a function of radius, this is if I plot as a function of the radius r if I plot the shear stress tau x the shear stress increases linearly.

So, if I plot as a function of r the shear stress tau x r this increases linearly as a function of r. However, if you are very very close to the wall of the pipe this looks as approximately a constant but the variation is small because the variation is equal to the slope times the length. So, if a very close to the wall of the pipe this variation is small. And within this region we can approximately consider the shear stress is equal to the wall shear stress which we are seen earlier was equal to dp by dx times R, I will see the Constance there; dp by dx times R by 2 at the wall of the pipe.

So, we can consider the shear stress to be approximately a constant. So, we would write strictly speaking the length scale in terms of the shear stress close to the wall of the pipe, instead the length scale is usually written on the basis of a velocity scale that is derived from this shear stress it is called the friction velocity. And it is defined as rho u star square is equal to tau w; this defines the friction velocity u star. You can easily see that by dimensional analysis the density times the velocity square has the same dimensions as shear stress. Shear stress is a constant in this viscous wall layer, and therefore I can define a velocity scale as u star the friction velocity is equal to root of tau w by rho it is called the Friction velocity, because it is derived from the shear stress at the wall.

Now, in the viscous sub layer the only parameter that is fixed is the shear stress across this viscous sub layer. And since that the flow is viscous in the sub layer the turbulence effects are not important within this region close to the wall. I can define a length scale as nu by u star; where mu is the kinematic viscosity. Because we know that in the center of the flow turbulent diffusion is much faster than molecular diffusion, so the kinetic viscosity is not really important near the center of the flow. Only when you come close to the wall that the flow becomes viscous and the length becomes much smaller than the pipe radius. So, I need Reynolds number I define based upon this length from the wall will be much smaller than the macroscopic Reynolds number.

So, I will come some stage as I approach the wall where viscous effects become important. Within this region what matters is only the wall shear stress and the kinematic viscosity and from that I can get only one length scale. And the distance from the wall is often written by the symbol y scaled by this length scale. It is usually written in wall units y u star by nu; is the relevant length scale as far as the viscous sub layer is concerned. And the velocity u x plus is usually written as u x by u star; the velocity scaled by the friction velocity and the length scale is scaled by this length scale obtained from the friction velocity and the kinematic viscosity. And this comes out of the assumption that the shear stress is approximately a constant across this region so that I can get only one dimensional parameter the shear stress or alternatively the friction velocity.

Now, close to the wall you know that the wall shear stress has got to be equal to mu times d u x by d y; if the flow is purely viscous close to the wall. If the flow is viscous close to the wall then the wall shear stress there is no turbulent fluctuations because they get damped out by the time they come close to the wall. The wall shear stress is equal to just mu times d u x by d y tau w by rho is equal to the kinematic viscosity times d u x by d y close to the wall and this is equal to v star square; where u star as a fiction velocity. I told you the tau w by rho is equal to u star square.

Now, just rewrite this in terms of u by u star and y u star by mu. And you can easily see that d u x plus by d y plus just has to be equal to 1. Because, when I divide this I will get d of u x by u star divided by d of y u star by mu divided throughout by u star square this becomes equal to 1. So, close to the wall I have the velocity profile u x plus is equal to y plus, plus a constant of course what that constant is 0 because at y plus is equal to 0 the velocity has to be equal to 0.

So, expressed in terms of these wall units very close to the wall the equation in the viscous sub layer is just that u x plus is equal to y plus; in the viscous sub layer. Beyond the viscous sub layer there is of course the turbulent flow; there is turbulence here. However, in between the viscous sub layer and the turbulent flow there is what is called a logarithmic layer. And you can derive a velocity profile in this logarithmic layer. I will not go in to the details here because that involves my mass asymptotic analysis, but you can derive a velocity profile. In this case u x by u star is equal to A times log plus B; where A and B are considered to be universal constants, A is about 2.4 close to 2.5 and B is about 5.5 approximately. These constants are determined by actually fitting the velocity profiles to this logarithmic function.

So, expressed in wall units up to about y plus is equal to 30, we have a viscous sub layer; y plus is equal to 30 to about 200. We have a buffer layer, a logarithmic layer should plot it here and beyond that we have the turbulent flow. So, that is the structure of the flow in a turbulent flow the viscous sub layer where u plus is equal to y plus simply because we have scaled in this way, and then a buffer layer where once again the velocity scale is the friction velocity and the length scale is this length scale, again the wall unit length scale.

So, there is a broad structure of a turbulent flow. I will come back and I will device this a little bit in the next lecture, because it probably went a little fast, but once we do that when we are look at other kinds of problems. This is just a steady unidirectional flow you look at more complicated flows as we go along. But this is important because this kind of a turbulent flow structure is something that is used very often in description of flows fast flux surfaces. We continue this in the next lecture. I will see you then.