Transport Process I: Heat and Mass Transfer
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Lecture – 03
Non-dimensional analysis of beams

I welcome you to this lecture number 3 in our course on Fundamentals of Transport Processes. In the previous introductory lecture, I have tried to explain what it is that we will be doing and why we will be doing it. Now in this lecture, I will try to go through how transport processes are modeled using dimensional analysis. This is the kind of thing that is done in unit operations for example, where we take a microscopic picture of the entire unit operation and try to find correlations between how much material mass, heat has been transported and the conditions that are applied on this system, the temperature differences of the inlet and outlet stream of the concentration differences, the concentrations of the different species in catalyzed reactions and so on.

So, based upon those large scale properties of the system, what is the transport rates that you can expect that is the subject of unit operations and dimensional analysis plays a powerful role in enabling us to model these effectively, as I will show you it reduces significantly; the number of parameters that we have to vary many experimental modeling of the system.

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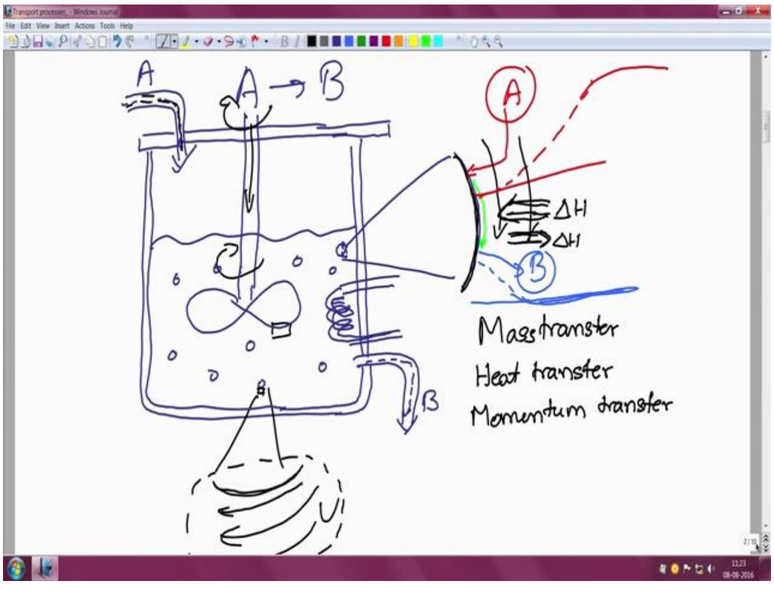

So, last lecture I had taken you through a few different examples and shown you how transport plays an important role in those. The first of these was this reactor system where we have a solid catalyzed reaction and there are particles on which the reaction gets catalyzed and I had tried to explain to you that it is not sufficient to ensure that enough raw materials are put into this reactor, enough products are taken out of the reactor, there is enough heat that is supplied or taken out of the reactor.

It is also necessary at the microscopic scale, at the particle scale to ensure that where the reaction takes place, there is sufficient amount of reactants being sufficient flux of reactants to the surface locally and this flux is determined by a variation concentration close to this surface. Of course, once the reactants reaches the surface, it can react and then the product of course, has to come out of the surface and it is not sufficient if you just ensure the reactant reaches the surface, you have to also ensure that the product comes out of the surface. The conditions have to be such that there is a concentration difference of the product between the surface and the bulk of the fluid.

Similarly in the case of exothermic or endothermic reactions, it is necessary to ensure that there is sufficient flow of heat either to the catalyst surface or from the catalyst surface to ensure that the reaction takes place at these specified rates. If you do not supply heat for an endothermic reaction then of course, the temperature of the catalyst will decrease and the reaction rate will slow down. Alternatively, if you do not take out heat from an exothermic reaction then the temperature will continuously increase and this could lead to hot spots and runaway reactions.

Now, the transport rates are enhanced by mixing in fluids and that mixing basically transfers momentum from this impeller that is being used mixing the fluid into the fluid and that transport of momentum takes place due to shear stresses that are exerted at the surface of the impeller and these are the kinds of processes that we will be studying. Similarly, in the other example that I had taken of the heat transfer in a heat exchanger; it is not sufficient to ensure that there is sufficient amount of heat coming in and going out. You also have to make sure that locally at every point there is sufficient amount of heat being transferred across the surface.

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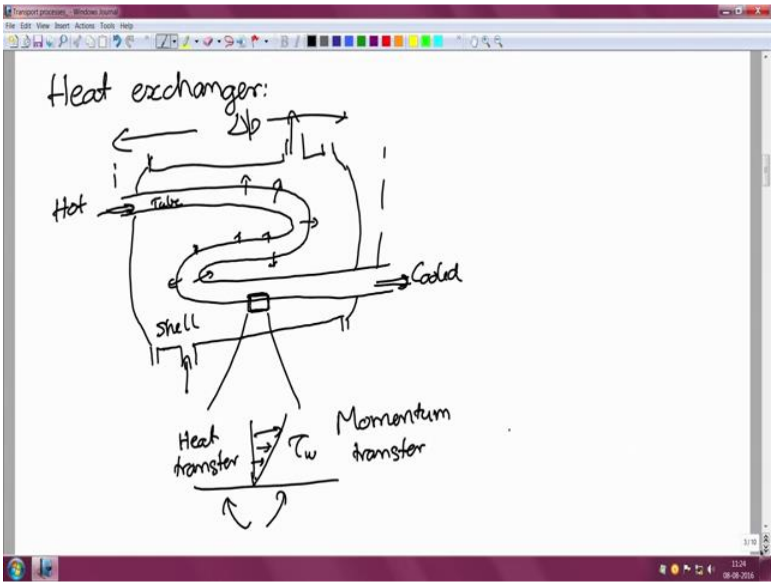

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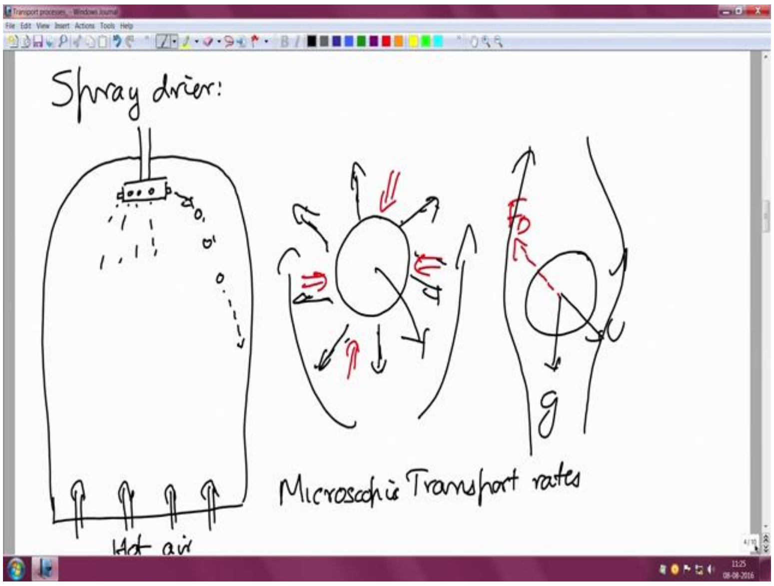

So that cumulatively when you add up all the heat that is transferred, it is sufficient to reduce the temperature of this heated fluid to the level that you have specified and we have also taken an example of a cumulative spray drier which includes both mass transfer and heat transfer. In this case droplets there are ejected from a surface have to cool and dry before they heat the wall of the spray drier.

These droplets are usually contain lot of moisture; roughly 80 percent in many cases, but relatively small in size about 100 microns and as they move through the fluid, there has to be heat transfer through the droplet to evaporate all the moisture that is inside. This heat transfer evaporates the water and that water has to be transferred out of the droplet. Both of these take place through a combination of convection and diffusion, these heat these spray driers are quite large in size they could be a meter in diameter, maybe a few meters in height. They have to be large enough that the entire drawing process takes place before the droplet hits the wall and for that what is important is what happens at the particle scale, at the droplet scale at 100 microns scale. That is where we have to focus our attention look at the heat transfer, the mass transfer and to find out how long it will take for the droplet to hit the wall, we need to know what is the velocity with which the droplet moves as it moves through the fluid and that is the problem of momentum transfer.

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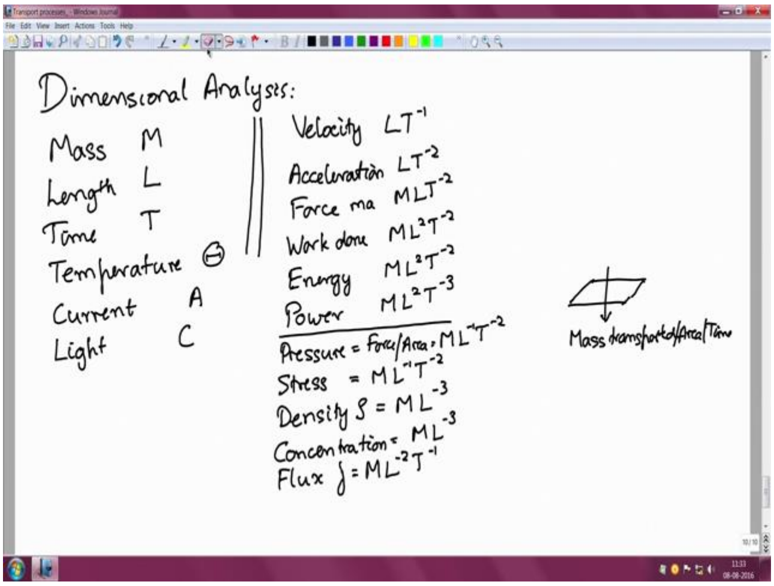

So now we will start looking at dimensional analysis; how are these processes carried out in unit operations. There are fundamental dimensions and there are derived quantities, the fundamental dimensions in all of these cases by convention these are assumed to be mass with dimension M, length dimension L, time dimension T. So, these are the 3 mechanical quantities and then we have the thermal quantity temperature, the dimension I will call by theta big theta to distinguish it from time which has dimensions T.

Then there are 2 others current amperes and then there is a unit of light energy which is usually is written as chandelier. Sometimes mole is also considered a dimension and one mole is just 6.023 into 10 power 23 molecules. So, it is just a number you could include as one of the fundamental dimensions, but in this particular case there is no particular need.

Now of these fundamental quantities, we will be concerned with the first 4 in this course. Sometimes we will also deal with current, but for most of the lectures; we will be dealing only with the mechanical quantities and the thermal quantities.

Now, these are considered to be the fundamental dimensions of course, there is some subjectivity in how you choose, what is fundamental and what is not; however, for the present course or for the present purposes, these are considered by universal consensus as the fundamental dimensions. These you can derive the dimensions of any other quantity, all quantities that are known to us can be written in terms of these fundamental dimensions and that is what makes it so powerful. There is only these sects there are no others in addition everything else can be derived.

The velocity for example, is the distance traveled per unit time. So, it has dimensions of length times T inverse. The acceleration is the rate of change of velocity, the velocity change in velocity per unit time. So, therefore, the acceleration is the velocity per unit times, so this is L; T to the minus 2. Force is mass times acceleration; Newton's third law is equal to mass times acceleration; therefore this will be M L T power minus 2. Work done is equal to force moved times distance therefore, the work done is the force times distance that is M L square T power minus 2 and similarly energy also has the same dimensions as work. Power is the energy input per unit time, so it will have dimensions of energy divided by time, so that is the dimensions of power. So, these are the dimensions of some common mechanical quantities.

Now, we will come to dimensions of quantities that we will use in this course. Pressure is force per area. So, force has dimensions of M L T to the minus 2 and the area is length square; length to the second power. Therefore, the force per unit area will have dimensions of M L inverse; T power minus 2, so as the pressure force exerted per unit area of a surface. The stress also has dimensions of force per unit area except that commonly pressure acts perpendicular to the surface and stress acts parallel to the surface. So, that is the only distinction, but stress as well has dimensions of M L inverse T power minus 2.

Now we often deal with the fluid density, the mass per unit volume of the fluid. So, the density is an important quantity; it is written by the simple rho, density is mass per unit volume; volume has dimensions of length to the minus 3 power, so this is M L power minus 3. Concentration of a solute in a solvent also has dimensions of mass per unit volume; mass of solute per unit volume of fluid. So, concentration will have the same dimensions as total densities, it also has dimensions of M L power minus 3. The difference is that the densities refers to the total mass per unit volume whereas, concentration refers to the mass of a particular solute per unit volume.

Now, let us come to fundamental quantities that we deal with in mass transport. The flux of a solute, the flux is defined as the mass that is being transported across the surface; it has the symbol j, mass transported across the surface per unit area; per unit time. Flux has dimensions of mass transported across the surface of the solute per unit area; per unit time. So mass has dimensions of M; area is length square, so I will get L to the minus 2 and T to the minus 1. So, that is flux of a solute across the surface.

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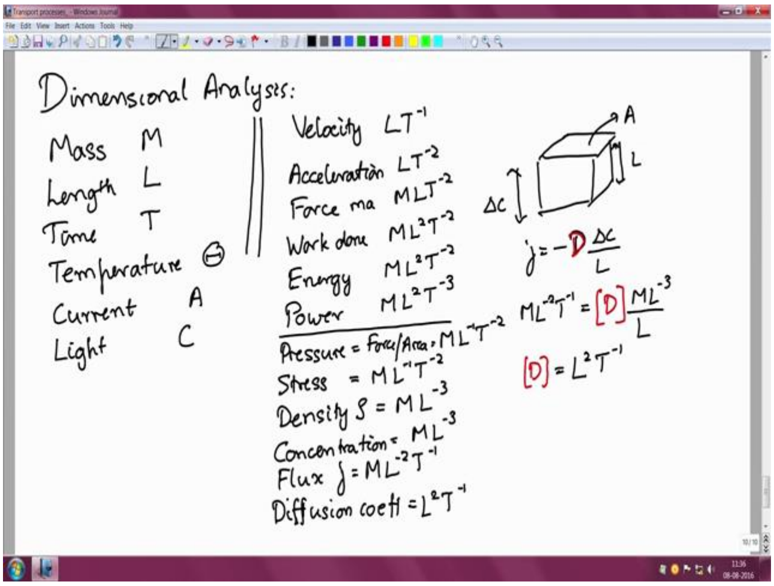

Now in mass transfer; the other quantity that is of importance is the diffusion coefficient. How do we get the diffusion coefficient? The dimensions of the diffusion coefficient; for that we have to use some relationship in which we know the dimensions of all quantities except for the diffusion coefficient; in this particular case for mass transfer that relationship is the fixed law of diffusion, which states that if I have some volume of fluid of area A; across which area A and length L, across which I apply there is a difference in concentration in delta C across which there is a difference in concentration in delta C, then the flux that results will be equal to minus the diffusion coefficient times delta C by L, the flux is the mass transported per unit are per unit time.

So, in this we know the dimensions of everything apart from this diffusion coefficient D. So, if you look at the dimension for the left hand side; it is mass length to the minus 2; T inverse that is equal to the dimension of this diffusion coefficient D, times; delta C is at concentration difference. So, it has dimensions of mass length to the minus 3; it has dimensions of mass length to the minus 3 because it is a concentration difference, the difference in concentration between 2 locations, that has to be divided by the distance between those 2 locations and from this you can see quite easily that the dimension of diffusion coefficient, the mass dimensional cancel out on both sides and this will turn out to be L square T inverse. So, the dimension of the diffusion coefficient is length square times T inverse.

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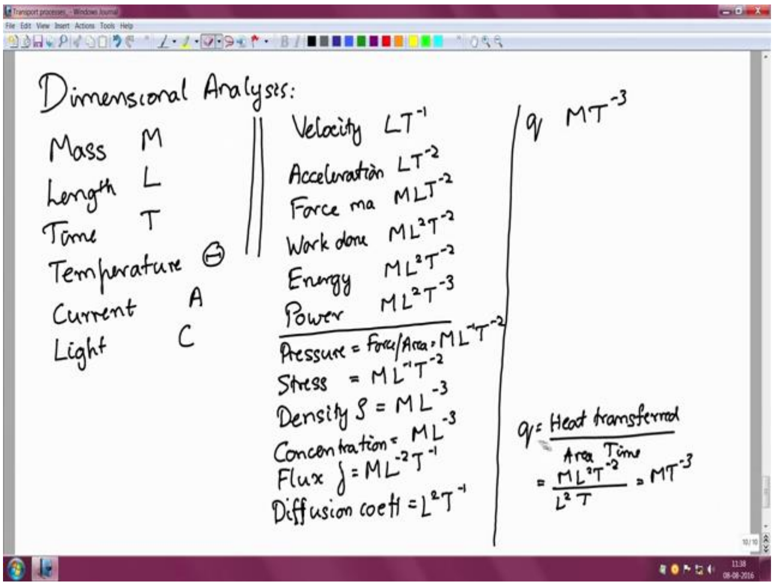

Now, let us go through some fundamental quantities in heat transfer. The properties of the fluid in heat transfer are the specific heat at the thermal conductivity and the flux is the heat flux. So, the heat flux q is defined as q is equal to heat transferred per unit area; per unit time, just as j is mass transferred to unit area per unit time, q will be the heat transferred per unit area per unit time. The heat energy has dimensions of M L square T to the minus 2 that is the dimension of energy; area is length square and time is T.

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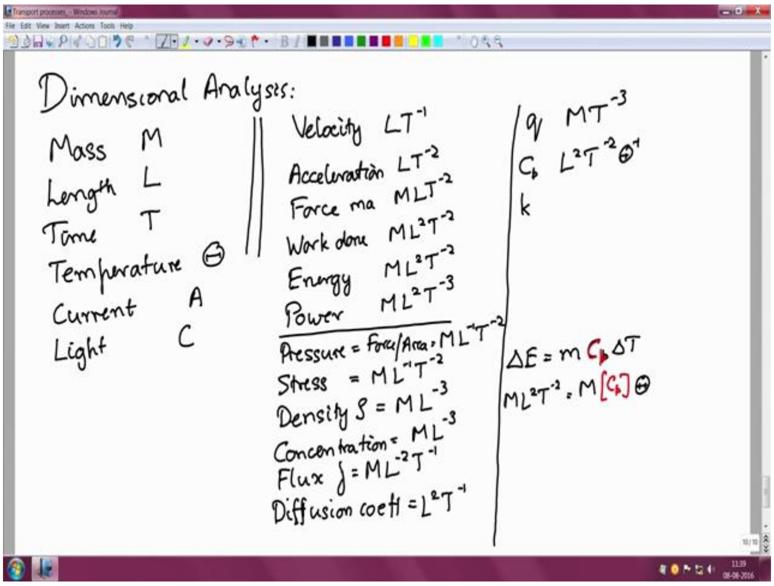

So, therefore the dimension of the heat flux will be equal to M T to the minus 3 and then we have two thermal properties, the specific heat Cp and the thermal conductivity k. The dimension of the specific heat, how do we get that? We know that; the change in energy of a fluid is related to the specific heat by mass M Cp times delta T; where M is the mass of the system of the fluid and delta T is the temperature difference and in this we know the dimensions of everything except for this specific heat Cp.

So, on the left hand side I have energy; M L square T power minus 2 is equal to the mass M, the dimension of the specific heat; times temperature that is theta and from this you can easily see that the dimension of specific heat is L square T power minus 2 theta inverse. Now, next is the thermal conductivity and for the thermal conductivity once again we need a relationship in which we know all the dimensions of all quantities apart from the thermal conductivity.

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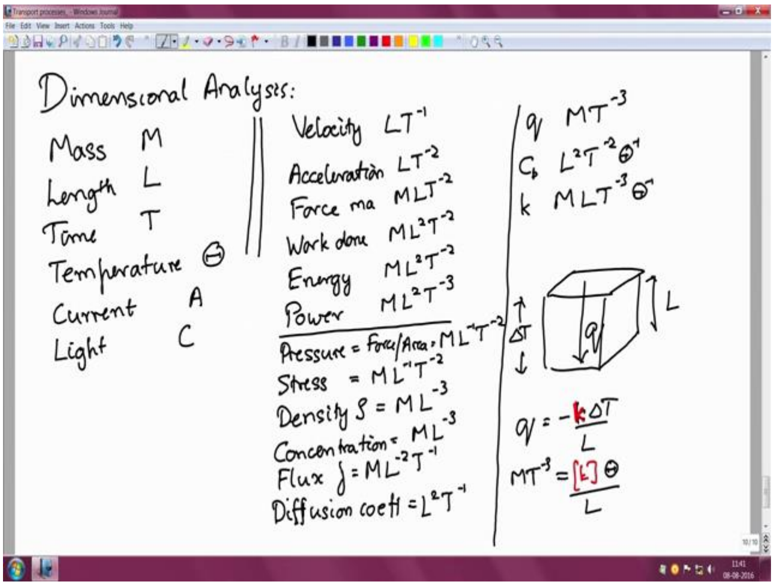

The relation in this case is of course, is Fourier's law of heat conduction; Fourier's law of heat conduction states that if I have some volume of fluid of length L across which I have applied the temperature difference delta T, then the heat flux that is going through this q is equal to minus k delta T by L; negative sign because heat is transferred from regions of higher temperature to regions of lower temperature.

Now in this particular relation, I know the dimensions of all quantities apart from this thermal conductivity k and therefore, that I can find out. So, in this particular case the dimension of q is M T power minus 3; this is equal to the dimension of k times; the dimension of temperature divided by the dimension of length. Therefore the thermal conductivity, the expression for the thermal conductivity will turn out to be M L T power minus 3 theta inverse. So that gives us the dimension of the specific heat, the heat flux in the thermal conductivity.

And then of course, we have to come to dimensions of quantities of use in momentum transfer. In momentum transfer, the fundamental quantities are the stress and the viscosity; the stress of course, I have already derived for you.

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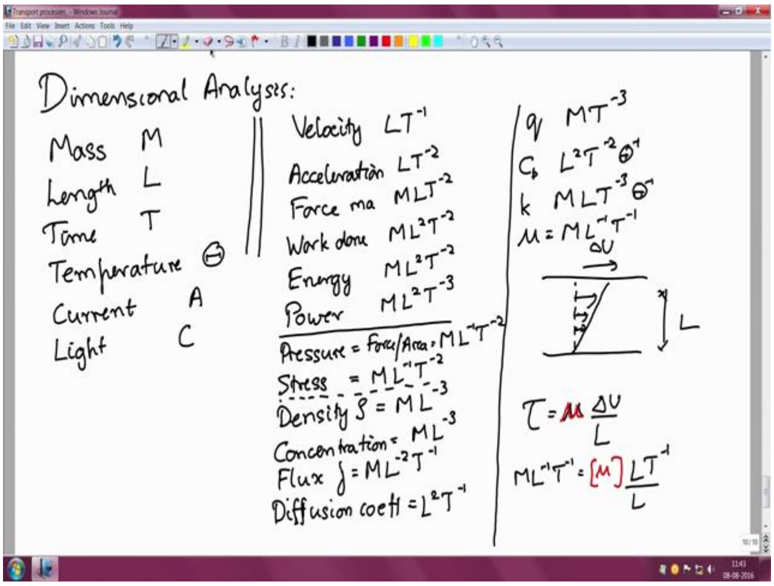

The stress has the same dimension as the pressure, how do I get the viscosity. I need some relationship in which I know the dimensions of all quantities apart from the viscosity and that one is of course, Newton’s law for viscosity; which basically states that the shear stress tau is equal to mu delta U by L. So, if I have some fluid you know gap between two plates and the plates are separated by distance L and if I apply velocity difference delta U between the top and the bottom plates, there will be a flow within this fluid and the shear stress is equal to the viscosity times the velocity difference divided by the length scale; in this case delta U by L is what is called the strain rate of the fluid.

And in this case we know the dimensions of all quantities apart from the viscosity. So, from this I can derive what is the dimension of the viscosity. So, I know that the stress in this case is M L inverse; T inverse is equal to the dimension of the viscosity; times dimensions of velocity L T inverse, divided by the length and from this I can derive what is the dimension of the viscosity mu is equal to M L inverse T inverse, so that is the dimension of viscosity.

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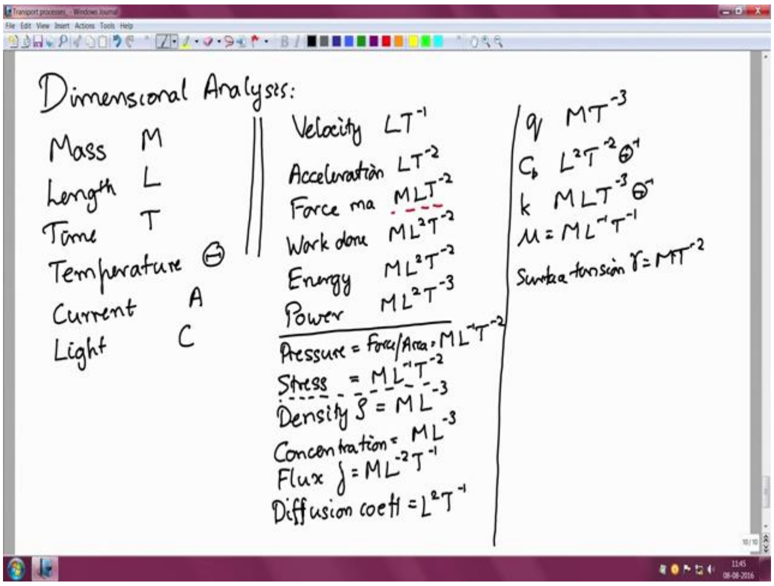

So, these are the dimensions of some common quantities that we will use during the course. There are others for example, surface tension for example gamma; surface tension is the force per unit length or energy per unit area. So, therefore, force per unit length will basically be force M L T inverse T to the minus 2 divided by length. So, this is just equal to M T to the minus 2 and these are the dimensions of the common quantities that we will use during this lecture. So, dimensional analysis is simple there are fundamental quantities, there are derived quantities. Fundamental quantities, mass, length, time, temperature, current of course, and light these are 2 things that we will not use very often in this course, but with just mass, length, time and temperature; you can see the multiplicity of quantities that can be described. All the way from velocity, acceleration force, work you know pressure stress flux and so on.

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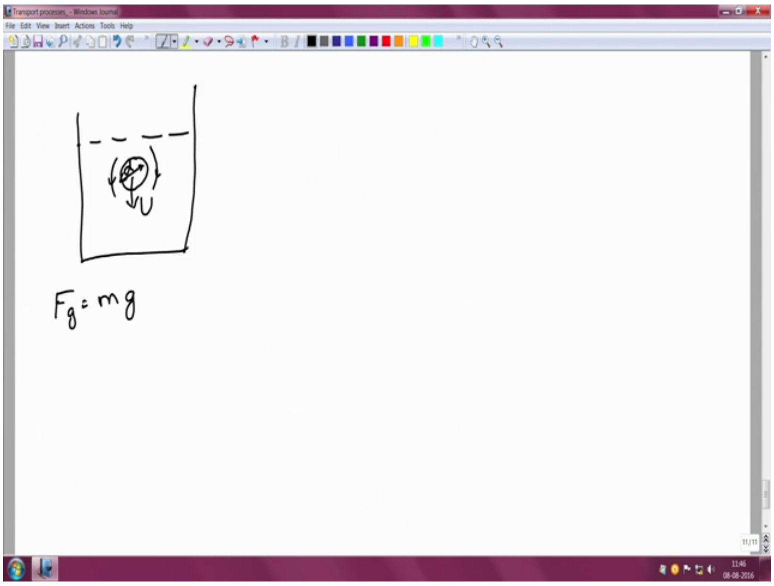

So with these fundamental quantities, we can now go ahead and try to look at problems. So, the first problem that I will consider in the next lecture is the problem of a particle in a fluid that is settling under the action of gravity. Let us assume for simplicity that the particle is spherical particle with diameter D and it is moving with velocity U. Of course, on this particle it is settling because of the density difference between the particle and the external fluid. So, the force on the particle due to gravity is just equal to mass times the acceleration due to gravity.

However, because the particle is translating through the fluid this does generate a fluid flow around the particle. This just generate a fluid flow around the particle; that fluid flow results in certain frictional losses because whenever fluids flow more accurately whenever there is a strain rate in the fluid, there is deformation of the fluid, there is a friction due to the fluid viscosity and that results in a force which opposes the motion of the particle and our task will be to find out based upon dimensional analysis what is the force that is exerted on the particle due to the fluid flow around it.

So, this one we will continue in the next lecture, I will start off with dimensional analysis for this simple problem and then I will try to progress into more complex problems; such as the heat exchanger problem that we saw in the first two lectures. The problem of the transfer from a spherical particle that we saw in the first two lectures, we will progress onto those which are little more complex problems. So, I will see you in the next lecture.