Transport Processes I: Heat and Mass Transfer

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Lecture – 23
Unidirectional transport: Similarity solution for mass transfer into a falling film

So, welcome to our continuing series of lectures on transport in one dimension. In the last lecture, I had completed for you the problem of transport from a surface. So, we had considered a surface for the heat transfer problem; an infinite surface with an infinite fluid above it and initially the temperature everywhere is a constant, let us say the ambient temperature and at T is equal to 0, you instantaneously increase the temperature of the surface and because of this temperature difference, the heat diffuses upwards and our task was to find out what is the temperature profile and therefore, what is the heat flux.

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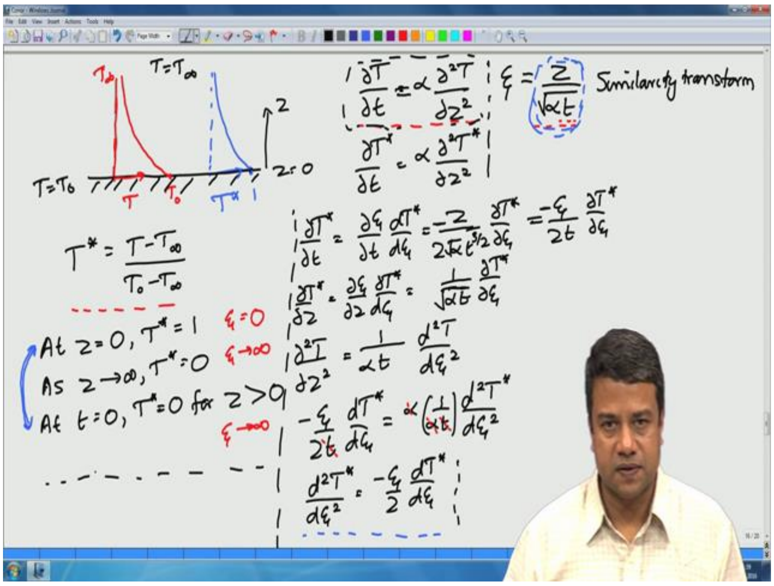

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2}$$

$$\xi = \frac{z}{\sqrt{\alpha t}}$$ Similarity transform

$$\frac{\partial T^*}{\partial t} = \alpha \frac{\partial^2 T^*}{\partial z^2}$$

$$T^* = \frac{T - T_\infty}{T_0 - T_\infty}$$

At $z = 0,\ T^* = 1$ $\quad \xi = 0$

As $z \to \infty,\ T^* = 0$ $\quad \xi \to \infty$

At $t = 0,\ T^* = 0$ for $z > 0$ $\quad \xi \to \infty$

$$\frac{\partial T^*}{\partial t} = \frac{\partial \xi}{\partial t}\frac{dT^*}{d\xi} = \frac{-z}{2\sqrt{\alpha}\,t^{3/2}}\frac{\partial T^*}{\partial \xi} = \frac{-\xi}{2t}\frac{\partial T^*}{\partial \xi}$$

$$\frac{\partial T^*}{\partial z} = \frac{\partial \xi}{\partial z}\frac{\partial T^*}{d\xi} = \frac{1}{\sqrt{\alpha t}}\frac{\partial T^*}{\partial \xi}$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha t}\frac{d^2 T}{d\xi^2}$$

$$-\frac{\xi}{2\cancel{t}}\frac{dT^*}{d\xi} = \cancel{\alpha}\left(\frac{1}{\cancel{\alpha t}}\right)\frac{d^2 T^*}{d\xi^2}$$

$$\frac{d^2 T^*}{d\xi^2} = -\frac{\xi}{2}\frac{dT^*}{d\xi}$$

In the last few lectures, I had derived for you the energy conservation equation as shown here; the diffusion equation. Diffusion equation whether it is of mass momentum or energy looks similar, the time rate of change of temperature is equal to thermal diffusion coefficient times the second derivative of the temperature with respect to the coordinate and in this particular case, this is a partial differential equation. There are variations in both time and in space; however, because the system is infinite, there is no length scale in the problem, there is no time scale either you heat it at time T is equal to 0 and then the temperatures of the bottom surface is a constant for all time. On this basis, there were no dimensional length and time to scale these spatial in the time coordinates and on the basis of dimensional analysis, we could get only one dimensionless parameter.

And therefore, we had used a similarity transform to reduce from two coordinates to one. The similarity variable size was equal to z by root of alpha T and once we did that the equation reduced to just a single equation inside, the boundary conditions could also be expressed explicitly in terms of sight.

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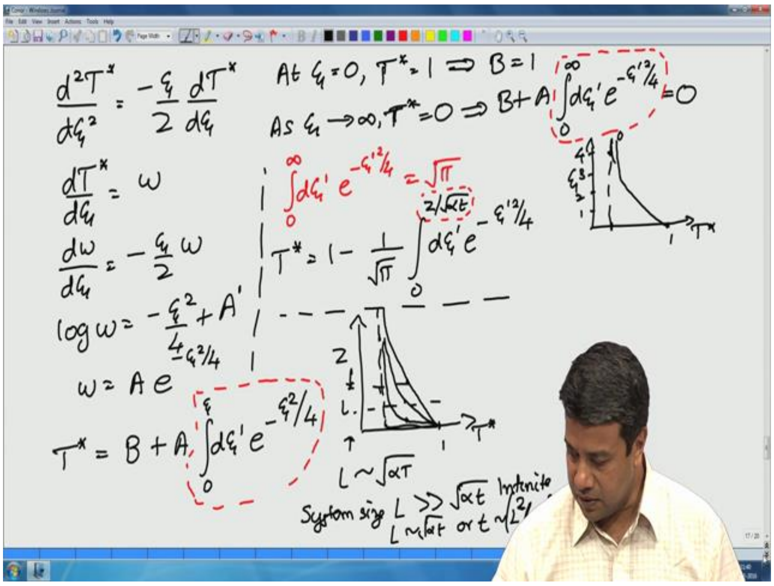

$$\frac{d^2T^*}{d\xi^2} = -\frac{\xi}{2}\frac{dT^*}{d\xi}$$

$$\frac{dT^*}{d\xi} = \omega$$

$$\frac{d\omega}{d\xi} = -\frac{\xi}{2}\omega$$

$$\log\omega = -\frac{\xi^2}{4} + A'$$

$$\omega = A e^{-\xi^2/4}$$

$$T^* = B + A\int_0^{\xi} d\xi' e^{-\xi'^2/4}$$

At $\xi = 0$, $T^* = 1 \Rightarrow B = 1$

As $\xi \to \infty$, $T^* = 0 \Rightarrow B + A\int_0^{\infty} d\xi' e^{-\xi'^2/4} = 0$

$$\int_0^{\infty} d\xi' e^{-\xi'^2/4} = \sqrt{\pi}$$

$$T^* = 1 - \frac{1}{\sqrt{\pi}}\int_0^{z/\sqrt{\alpha t}} d\xi' e^{-\xi'^2/4}$$

$L \sim \sqrt{\alpha T}$

System size $L \gg \sqrt{\alpha t}$ Infinite

$L \sim \sqrt{\alpha t}$ or $t \sim L^2/$

On this basis, we had obtained the solution for the temperature field and I told you that this temperature is only in terms of the similarity variable therefore, if I scale z by root alpha T and I express the temperature field in terms of that parameter, that is independent of individually z and T, it depends only on the combination z by root alpha T (Refer Time: 03:10) transform therefore, a different z at different time if this parameter is the same the temperatures is the same.

This also gives us an estimate for the penetration depth of the temperature field into the fluid as a function of time. The temperature field disturbance due to the bottom surface penetrates a distance comparable to square root of alpha T that distance increases as square root of T.

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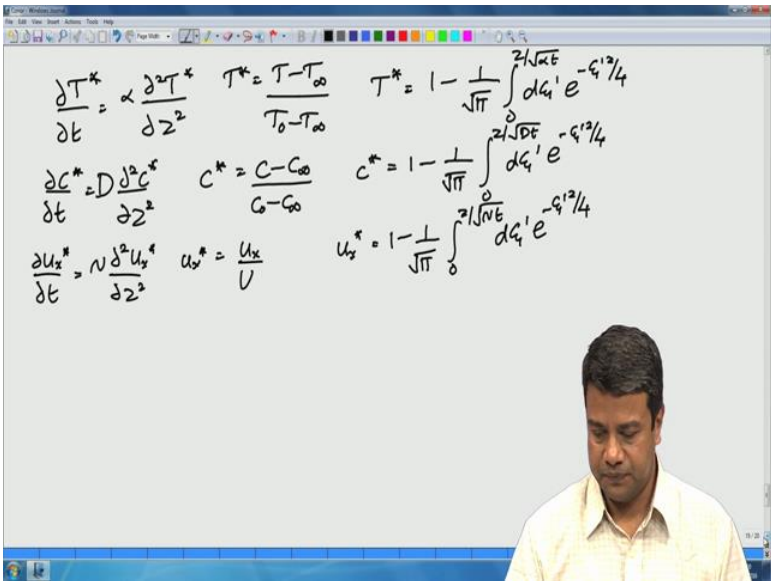

$$\frac{\partial T^*}{\partial t} = \alpha \frac{\partial^2 T^*}{\partial z^2} \qquad T^* = \frac{T-T_\infty}{T_0-T_\infty} \qquad T^* = 1 - \frac{1}{\sqrt{\pi}}\int_0^{z/\sqrt{\alpha t}} d\zeta'\, e^{-\zeta'^2/4}$$

$$\frac{\partial c^*}{\partial t} = D \frac{\partial^2 c^*}{\partial z^2} \qquad c^* = \frac{c-c_\infty}{c_0-c_\infty} \qquad c^* = 1 - \frac{1}{\sqrt{\pi}}\int_0^{z/\sqrt{Dt}} d\zeta'\, e^{-\zeta'^2/4}$$

$$\frac{\partial u_x^*}{\partial t} = \nu \frac{\partial^2 u_x^*}{\partial z^2} \qquad u_x^* = \frac{u_x}{U} \qquad u_x^* = 1 - \frac{1}{\sqrt{\pi}}\int_0^{z/\sqrt{\nu t}} d\zeta'\, e^{-\zeta'^2/4}$$

So, the infinite fluid approximation can be used only when this distance is much smaller than the system size and we had also got expression for the other problems; the mass transfer and the momentum transfer problems. Equations are exactly the same; except that the variables are changed, temperature to concentration or velocity and thermal diffusivity to mass diffusivity or kinematic viscosity; therefore, you can straightaway get these expressions.

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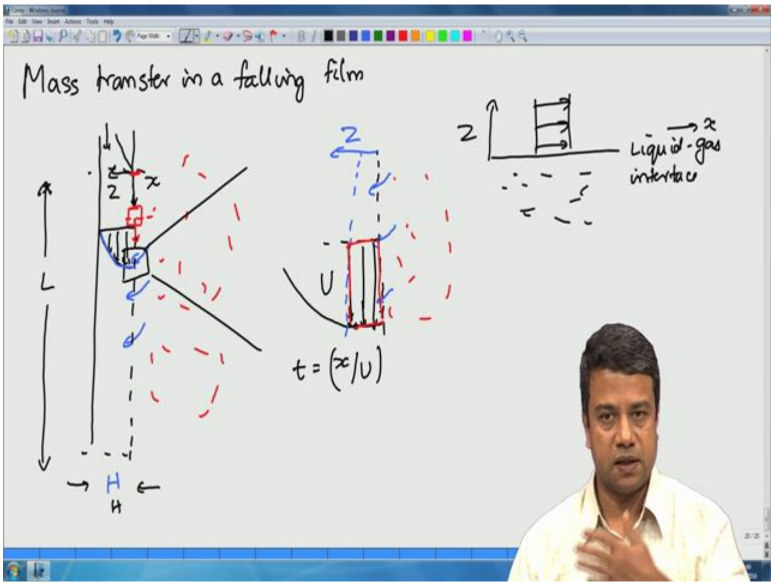

Mass transfer in a falling film

And I had said earlier that I would solve a problem for you to show where this can be applicable and that was the mass transfer in a falling film. Typically what happens is that you have contacting equipment through which fluid flows in. The fluid has some thickness; let us call this thickness as H and the length go which the film is in contact that let us call that as L. So, this is going to be the stream wise coordinate; I will call that as x and this is going to be the cross stream coordinate, I will call that as z. This fluid film is moving in contact with a gas outside which has some molecules that you want to transfer into the liquid film because this is typically used in pollution control equipment where you want to absorb poisons or toxic gases from an exhaust gas stream so that they not affect the environment.

So, you have this gas going up and you have this liquid film coming down and these toxic molecules are transferred from the gas into the liquid. If you recall, I had told you that the diffusion coefficients in gases are typically of the order of 10 power minus 5 meter square per second, it is comparable to the product of the root mean square fluctuating velocity at the mean free path. In liquids they are much smaller of the order of 10 power minus 9. So, the resistance to diffusion in the liquid side is higher than the resistance to diffusion in the gases side and therefore, we can consider the gas concentration to be more or less uniform; no variation in the concentration of the gas phase; however, the mass from the surface will diffuse into the fluid; into the liquid.

So if I expand out this region; what I have is a liquid film in contact with the gas on the side and within this there is a diffusion of the material in and because diffusion on the liquid side is slow, it is four orders of magnitude slower than that in the gas even for small molecules. Therefore, this will penetrate only to some finite depth within the liquid film and if this depth to which it penetrates is much smaller than the thickness here H; if this depth is much smaller than this thickness H, I can consider the diffusion to be within an infinite film; infinite in this z direction, the direction of diffusion.

The fluid is flowing, it has a certain non zero velocity; if you plot the velocity profile in detail as I have done here, for a liquid gas system as I said the shear stress in the liquid has to be 0 at the surface that is because the liquid viscosity is much higher than the gas viscosity, if the strain rates are comparable then the shear stress in the liquid has to be much smaller than the shear stress in the gas.

Therefore for fluid flows for liquid interfaces with gases, the boundary condition that is applied is that the shear stress is equal to 0. In this particular case, the boundary condition that could be applied is that tau x z is equal to mu; d u x by d z equal to 0. What that implies is that the velocity gradient is 0 at the surface, so if I look at a small region; very close to the surface, if the gradient is 0, the velocity is approximately constant so that of course, when you go closer to this; the solid surface here the velocity has to go to 0 at the solid surface; however, so long as I am very close to the liquid gas interface, the velocity gradient is 0 therefore, the velocity can be considered approximately a constant.

So, my purpose in going through this boundary condition for the velocity was to just indicate that at this liquid gas surface, the velocity close to the interface is approximately a constant the maximum velocity, it goes to 0 at the solid surface of course but if you are close to the liquid gas interface, the velocity is equal to some velocity. Now this is not a time dependent problem, this is a steady state problem.

Basically what it says is that as the liquid is flowing downwards, the thickness of the film is invariant in time and the diffusion fluxes are invariant in time, but they could depend upon the stream wise distance x in this case; the concentration and fluxes will in general depend upon the stream wise location; however, if I consider this parcel of fluid, the partial of fluid over here as it translates downwards if I consider this parcel of fluid as it is going downwards at a constant velocity, along the x direction. The time that has elapsed since it has started to come in contact with the gas, it is time that has elapsed since it has started to come in contact with the gas that time at a given location x; the time that has elapsed too since it has started to come in contact with the gas, it is just going to be equal to x by u because it was moving with roughly a constant velocity close to the surface, the velocity is a constant.

That constant velocity is u therefore, the time since it has come into contact with the gas is going to be equal to x by u. So, rather than considering a fluid volume element that is moving downward with a velocity u, again that is costs a certain distance x; I can as well consider a fluid volume element which the time elapsed since it has come into contact with the gas is going to be equal to x by u. So, in that sense if I now consider this problem, you turn it around if you will; this is the liquid gas interface and I have gas over here and I am moving in a reference frame that is moving with this parcel of fluid, then this is like a problem in an infinite; let me plot it let me draw the gas the other way.

So this is the gas; I have a liquid in the upper half, so this is the coordinates z and have liquid in the upper half that liquid is of course, moving with a velocity in the x direction that liquid is moving with the velocity in the x direction close to the surface that velocity is a constant; however, if I move in a reference frame moving with this velocity, the liquid is stationary; if I move in a reference frame moving with this velocity that liquid is essentially stationary.

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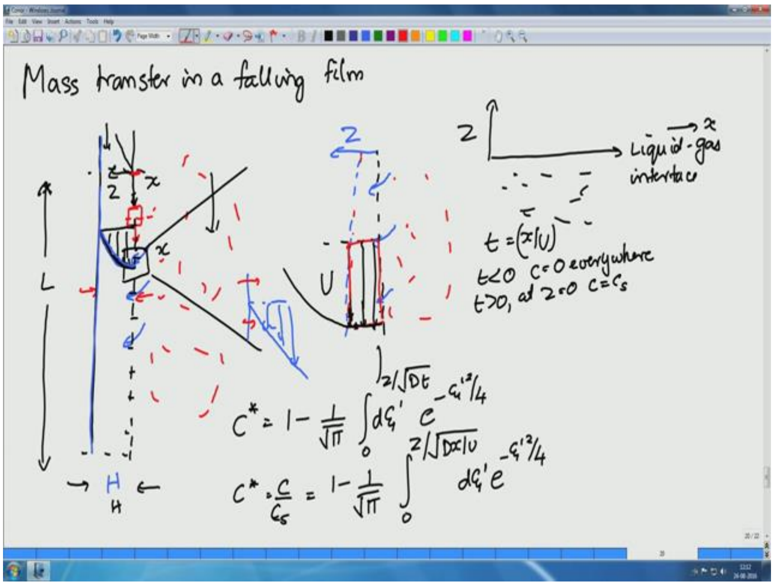

Mass transfer in a falling film

$t = (x/U)$

$t<0$ $C=0$ everywhere

$t>0$, at $z=0$ $C=C_s$

$$C^* = 1 - \frac{1}{\sqrt{\pi}} \int_0^{z/\sqrt{Dt}} d\xi' \, e^{-\xi'^2/4}$$

$$C^* = \frac{C}{C_s} = 1 - \frac{1}{\sqrt{\pi}} \int_0^{z/\sqrt{Dx/U}} d\xi' \, e^{-\xi'^2/4}$$

However the time that it has been in contact with the gas T is equal to x by u at the location x; at the downstream location x, the time that has elapsed since it has come into contact is equal to an x and x by u. So, this resembles a problem where for T less than 0, the concentration of the gas is equal to 0 everywhere C equal to 0 everywhere and for T greater than 0 at z is equal to 0 C is equal to C S, where C S is the saturation concentration of the gas in the liquid.

When we have an interface between liquid and a gas, the assumption is that instantaneously thermodynamic equilibrium is attained between the concentration of the gas and the concentration of the liquid at the interface; that means; of the concentration of the species being transferred is equal to the saturation concentration at the surface itself. So, at T is equal to 0 instantaneously the concentration at the surface has increased to C S. This problem is exactly the same problem that we have solved for C T less than 0, the concentration is equal to C infinity everywhere, for T greater than 0 the concentration is equal to this C naught at the surface and then we have seen how the concentration involves in time.

The only difference is that in this particular case time is equal to x by u, in doing this transformation there are a couple of assumptions here. The first is of course, that the velocity is a constant, so the velocity is a constant so I can just write down the time as x by u at the close to the surface. Therefore, the penetration depth has to be much smaller than the length scale over which this velocity is (Refer Time: 15:34).

Similarly we have made the assumption an infinite fluid, in an infinite fluid the length scale over which the fluid that the gas has penetrated is much smaller than the thickness of the film. So, those are the two assumptions; there is actually a third assumption and that is there is no variation in the x direction of the concentration, that is not strictly true and I will come back to that when we do boundary layer theory; however, this is valid so long as the length scale for variation in the stream wise direction is much longer than the penetration depth of the gas.

If the length scale for variation in the stream wise direction which is the length L itself there is much larger than the penetration depth, you can assume that over distance is comparable to the penetration depth there is no variation in the concentration; in the stream wise direction and then the system exactly resembles the system of the transport into an infinite fluid that we have already solved for, we know what is the solution C star is equal to 1 minus 1 over root pi integral 0, 2 z by root of Dt; e power minus xi prime square by 4 times d xi prime.

Only difference now is that, the time is the time elapsed since the liquid has come into contact with the gas. Therefore, this time is effectively x by u; C star in this case; C is equal to 0 as at T less than 0, so C infinity is 0; C naught is CS. So, C star is equal to C by C s. So, this 1 minus 1 by root pi integral 0 to; now here instead of time I will substitute x by u, so that is the solution for this particular problem; this physical problem. This gives me the concentration as function of both x as well as z; in this case of course, I have two coordinates x and z. So, I could not have simply used similarity solution in this case because I have another length scale, but; however, since I have made the analogy of time with x by u; I get this exact result.

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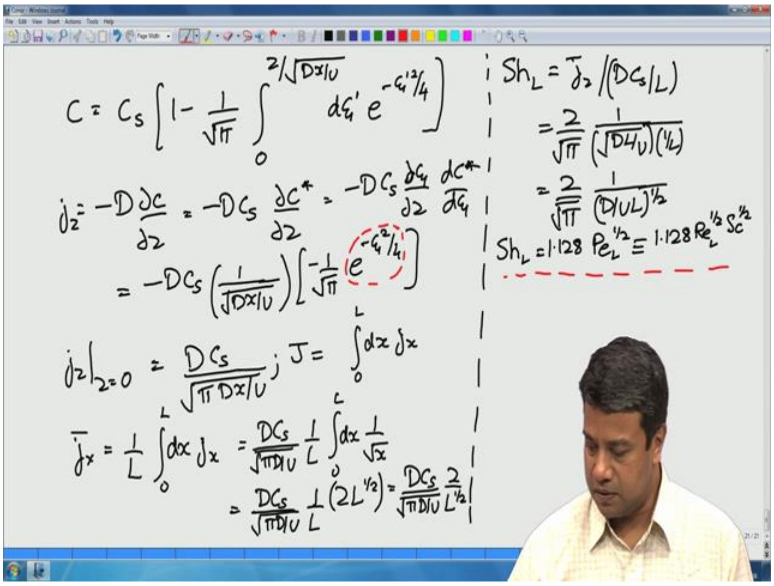

$$C = C_s \left[1 - \frac{1}{\sqrt{\pi}} \int_0^{z/\sqrt{Dx/U}} d\xi' \, e^{-\xi'^2/4}\right]$$

$$j_z = -D\frac{\partial c}{\partial z} = -DC_s \frac{\partial c^*}{\partial z} = -DC_s \frac{\partial \xi}{\partial z}\frac{dc^*}{d\xi}$$

$$= -DC_s\left(\frac{1}{\sqrt{Dx/U}}\right)\left[-\frac{1}{\sqrt{\pi}}\boxed{e^{-\xi^2/4}}\right]$$

$$j_z\Big|_{z=0} = \frac{DC_s}{\sqrt{\pi Dx/U}}; \quad J = \int_0^L dx \, j_x$$

$$\bar{j}_x = \frac{1}{L}\int_0^L dx \, j_x = \frac{DC_s}{\sqrt{\pi D/U}}\frac{1}{L}\int_0^L dx \frac{1}{\sqrt{x}}$$

$$= \frac{DC_s}{\sqrt{\pi D/U}}\frac{1}{L}\left(2L^{1/2}\right) = \frac{DC_s}{\sqrt{\pi D/U}}\frac{2}{L^{1/2}}$$

$$Sh_L = \bar{j}_z / (DC_s/L)$$

$$= \frac{2}{\sqrt{\pi}}\frac{1}{\left(\sqrt{D/U}\right)(1/L)}$$

$$= \frac{2}{\sqrt{\pi}}\frac{1}{(D/UL)^{1/2}}$$

$$Sh_L = 1.128 \, Pe_L^{1/2} \equiv 1.128 \, Re_L^{1/2} Sc^{1/2}$$

So, this is the solutions for the concentration field as a function of x. So, therefore, my solution is C is equal to CS into 1 minus 1 by root pi integral 0 to z by root of Dx by u; d xi prime e power minus xi prime square by 4, so that is my solution for the concentration field. Now what is it that I would like to do in order to design this equipment?

What I need to do is to find out what is the total mass transported, so that if I know that what is the concentration of a particular toxic constituent of this exhaust gas; I can make sure that the concentration at the exit is below the tolerance levels so that it is safe to discharged into the atmosphere. So, therefore I need to know either the total amount of gas of the toxic species that has been transferred or the average flux that has been transferred and that average flux is ultimately going to be given by assault number of share wood number correlation, so we will try to derive that correlation.

In order to find out the average flux; I first have to find out what is the flux at each location x; find out the sum of that over the entire length and divide by the length and that will give me the average flux. In all of this, I will assume that I have a unit length in the direction perpendicular to the board, so that I can do the flux calculation per unit length perpendicular to the surface. So, the flux j equal to minus D d c by d z is equal to minus D CS; the first term integrates 0, the second term is 1 over root pi into d by d z of this will be equal to 1 by root of d x by u into e power minus z square by 4; b x value. So, this is the derivative of an integral; it is just equal to the value of the integrand and I have this additional factor here because let me go through some detail.

The chain rule for differentiation, so this gives me minus D into CS into partial xi by partial z is basically equal to 1 over root of Dx by u and partial C star by partial xi I have a factor of minus 1 over root pi; e power minus xi square by 4, the derivative of T integral. This is the flux in the z direction, the flux at the interface that is z at z is equal to 0; it is just the value of the z is equal to 0. This term here is equal to 1; let us compare e equal to 1; x I equals 0.

So, I just get D CS by root pi D x by u, so that is the flux at z is equal to 0. So, this gives you the flux decreases as x increases 1 over root x. What is the total amount of material transported per unit length perpendicular to the surface, so what I have got is and an expression for the flux; at each value of x, for each value of x downstream I have got a value for the flux. The total amount of material transported is the total amount is the integral of this over the x direction. So, I can do the integral of this over the x direction to get the total amount per unit length in the direction perpendicular to the plane of the board. This will be equal to integral 0 to L; d x times j x and the average flux is just equal to 1 over L integral 0 to L d x; j x.

So, I have here D CS by root of pi; d by u, 1 over L integral 0 to L of d x times 1 over root of x and if I do this integral I will get this as equal to D CS by root of pi; d pi u 1 over x power half, the integral of that is two times x power half. So, I will get 1 over L into 2 L to the half which is equal to D CS by root of pi D by u into 2 pi L to the half. So, that is the average flux.

The Sherwood number in this case; Sherwood number based upon the length is this average flux divided by D CS by L, I said that when you define non dimensional fluxes we scale it by the terms in the constitutive relation. A diffusion coefficient concentration and a length scale, in this case the length scale is the total length L. So, therefore, this is the Sherwood number and you can see easily that this equal to 2 by square root of pi. The D CS will cancel out and I will get 1 by root of DL by u into a factor of 1 over L equal to 2 pi root pi into 1 by D by u L or this D by u L is the inverse of the Peclet number, the ratio of mask convection and diffusion. So, therefore, the Sherwood number based upon the length, it is equal to the Peclet number to the half once again based upon the length, times the constant which is 2 by square root of pi and 2 by square root of pi is approximately 1.128.

I told you that the Peclet number can we also expressed as Reynolds number times the Schmidt number. So, therefore, this can also be written as 1.128, the Reynolds number per half, the Schmidt number per half Peclet number, number is convection by mass diffusion, it can be written as convection by momentum diffusion Reynolds number times momentum diffusion by mass diffusion Schmidt number and you will recognize this these are the kinds of correlations that we have got that we had discussed earlier.

So, we have got a correlation now for the mass transfer the dependent dimensionless flux, the show number as a function of the Reynolds number in the Schmidt number, it gives us a half power and this we have managed to get analytically; the entire correlation using a simple model we basically use the analogy between the transport of mass in the following film as a function of downstream location with the transport into a static film as a function of time and of course, there were different assumptions here. One of the assumptions was that as time progresses, the thickness of this penetration depth of mass is small compared to the total thickness of the film that was one assumption, the second assumption was that close to the surface; the velocity can be considered approximately a constant.

With both of these assumptions we went and got the correlation, we can go back and check the validity of these two assumptions. So, what is the penetration depth in this case? The penetration depth in this case is approximately square root of Dt.

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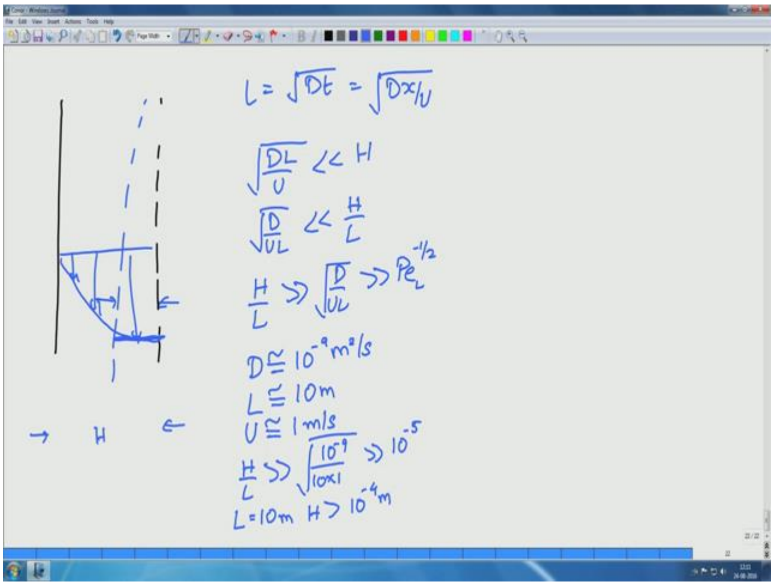

$$L = \sqrt{Dt} = \sqrt{Dx/U}$$

$$\sqrt{\frac{DL}{U}} << H$$

$$\sqrt{\frac{D}{UL}} << \frac{H}{L}$$

$$\frac{H}{L} >> \sqrt{\frac{D}{UL}} >> Pe_L^{-1/2}$$

$$D \cong 10^{-9}\, m^2/s$$

$$L \cong 10\, m$$

$$U \cong 1\, m/s$$

$$\frac{H}{L} >> \sqrt{\frac{10^{-9}}{10 \times 1}} >> 10^{-5}$$

$$L = 10\, m \quad H > 10^{-4}\, m$$

So, let me just draw the following film here for reference and I have a certain concentration profile of the gas, what is this penetration depth. I told you that the total thickness was equal to H, the penetration depth L is equal to square root of Dt is equal to square root of Dx by u.

So for this approximation to be valid, the penetration depth even at the bottom of the film has to be much smaller than the thickness H. The penetration depth even when I go to x is equal to L that penetration depth has to be much smaller than the thickness. So, what I require is that square root of DL by u has to be much smaller than H or alternatively if I were to scale both sides by the length itself then I required that H by L has to be as to be much smaller than H by l. So, the thickness has to be sufficiently large such that H by L is much greater than root of D by u L.

So, therefore, this approximation that it is effectively transferred into an infinite film is valid only when H by L is much greater than root of D by u L, which is effectively the Peclet number to the minus half based upon the length the total length of the contact. How severe is this restriction is this, so let us look at that I told you that in gases for transport of small molecules and liquids, the diffusion coefficient is approximately 10 power minus 9 meter square per second, the diffusion coefficient is approximately 10 power minus 9 meter square per second, a typical length of the contactor; even if you take a very large one, it will not be more than about 10 meters.

If you take a very large contactor, it is not going to be large of the order of 10 meters then not more than that and if you look at the velocity of the liquid falling down the film; a maximum velocity is approximately 1 meter per second. So, based upon this H by L has to be much greater than root of 10 power minus 9 x 10 into 1; it has to be much greater than 10 power minus 5. So, even if I have a length of 10 meters, H has to be greater than. So, for L is equal to 10 meters H has to be greater than 10 power minus 4 meters. So, even if I had a contactor which was very long, the film thickness that I require is actually very small, this gives me 0.1 millimeter; one-tenth of a millimeter.

This is actually an underestimate, but this is approximately the penetration depth with an equipment of this kind a millimeter or so our approximation is valid so long as the film thickness is larger than a millimeter, now that is not a very restrictive assumption. Similarly you require that the length scale for the variation of the velocity has to be much larger than the penetration depth, only then can approximate the velocity as a constant within this region.

In this particular case the length scale for the variation of the velocity is also the thickness H because the velocity varies or a thickness H. So, long as the penetration depth is much smaller than the thickness of the film and I say showed you just now the restriction is not a very severe one, I can assume that the system is in at infinite fluid and that the velocity close to the surface is a constant, therefore, if I move in a reference frame; moving with that velocity, I can consider it to be a stationary system in which you have penetration into the buffer.

So, based upon the simple analogy between time variations in an infinite fluid that is stationary and spatial variations in a film that is moving with time; we were able to get our first correlation for the mass transfer into an infinite film. The Sherwood number as a function of the Nusselt number.

One caution I should make at this point, this kind of analysis will not be valid unless the velocity is close to a constant near the surface. If you had what right trying to do mass transfer at the other end, let us say that I was trying to do the mass transfer from the solid film into the liquid. If I look close to the surface, the velocity actually varies linearly with position close to the surface and if I had transport happening at this side at the solid liquid interface rather than the gas liquid interface, this kind of an analogy will not work.

For doing this kind of a problem, we actually have to go back and solve the similarity solution; it is a little bit more sophisticated than what I have done in this particular example. We will come and see that later when we do boundary layer analysis, in that case the correlation is actually different.

Instead of Peclet number to the half power, what you will get is Peclet number to the one third power. So, this is only when there is a 0 shear stress boundary condition at the surface, if there was a no slip boundary condition the correlation would be different. This is consistent with what I had told you earlier that these correlations depend upon the boundary conditions for the velocity close to the surface. In this particular case since we are able to assume a constant velocity, we managed to get the correlation.

The same may not be applicable in other cases as well. So, I have got for you the first correlation over here; for this Sherwood numbers for a following film based upon an analogy with a simple problem where we have an infinite fluid and from similarity transform, we were able to get the concentration field as a function with just one similarity variable. Just by dimensional analysis, penetration depth goes a square root of D times D; where D is the diffusion coefficient there is a mass momentum or energy.

Next lecture we will look at another such simple problem, which is the decay of a pulse input. That is applicable in cases for example, where you have tresses diffusing in gas, let us say you had smoke being ejected into the air then a diffused in time, you get a different kind of solution and I will discuss that in the next lecture, we will see you then.