Transport Processes I: Heat and Mass Transfer

Prof. V. Kumaran
Department of Chemical Engineering
Indian Institute of Science, Bangalore

Lecture – 02

Transport by convection and diffusion

Welcome to this second half hour of lecture on transport processes, in the previous lecture we had looked at what we are going to be considering in transport processes.

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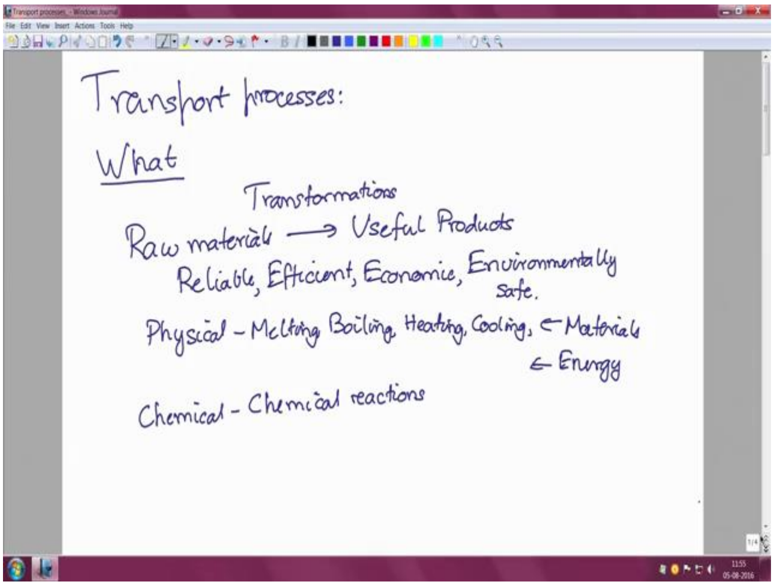

Transport processes:

What

Raw materials $\xrightarrow{\text{Transformations}}$ Useful Products

Reliable, Efficient, Economic, Environmentally safe.

Physical - Melting Boiling, Heating, Cooling, ← Materials
← Energy

Chemical - Chemical reactions

Basically the transformation of materials from raw materials to useful products requires the transfer of materials, energy, whether it involves physical transformations or chemical reactions and doing this in laboratory is not the same as doing it in a reliable, efficient and safe and economic manner, we have taken specific examples.

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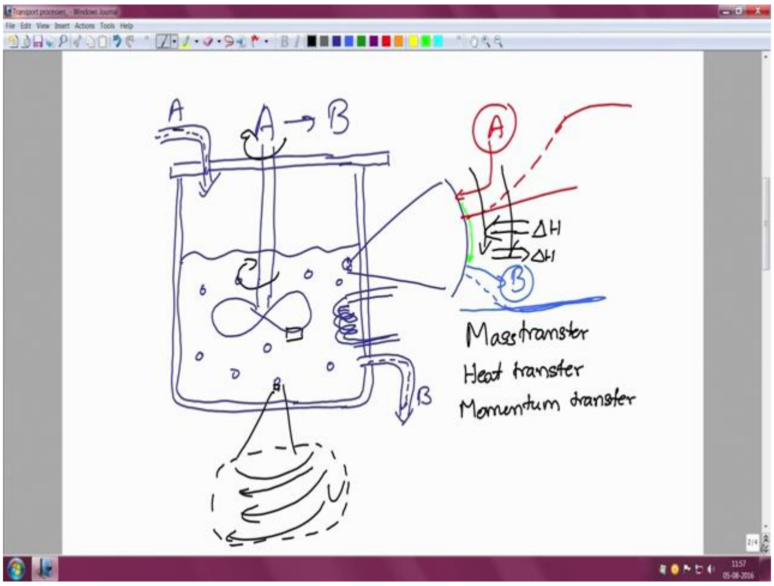

For example, the reactor in which there was a reaction taking place at a catalyst surface and I had explained to you that it is not necessary, is not sufficient to ensure that there is sufficient amount of reactants coming in, you also have to make sure that at the catalyst surface where the reaction is taking place there is sufficient transfer of reactants to the surface at a sufficiently fast rate. Sufficiently fast rate of transfer of products off the surface and the sufficiently fast rate of transfer of energy either to or from the surface of course, these are increased if you use an impeller to mix the fluid in the reactor, but that has its own cost in as far as the power required to drive the reactor.

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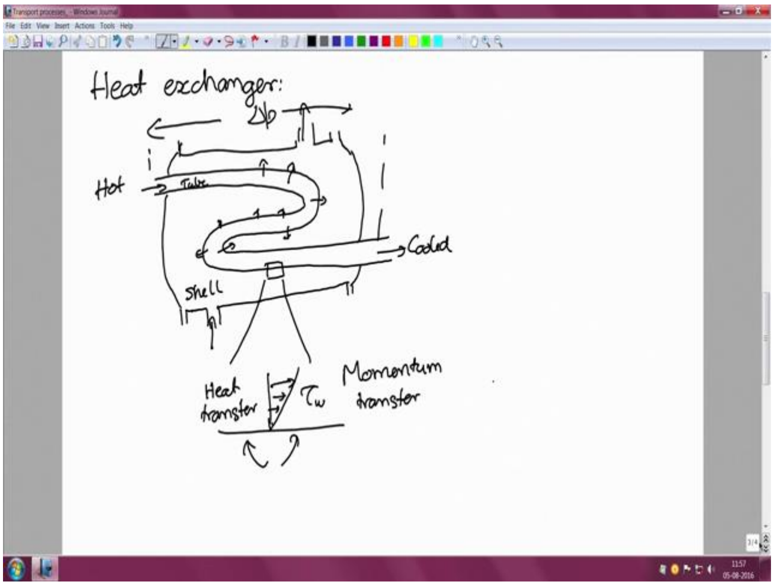

Similarly, in the case of a heat transfer, a heat exchanger when hot fluid is coming in and you want to cool it to a certain temperature, it is not sufficient to just trans pump in the correct flow rate of the fluid, you also have to make sure that there is sufficient surface area of the tube in order to transfer all of the heat across given a temperature difference and you would think that as you increase the length of the tube, there will be an increase in the transfer rates. However, there is an optimization because as you increase the length, the power required, the pressure difference increases and it increases because there is friction at the surface of the tube, due to the shear stress exerted by the wall on the fluid and this has to be compensated for and so the costs increase. So, you end up with an optimization between heat transfer and mass transfer

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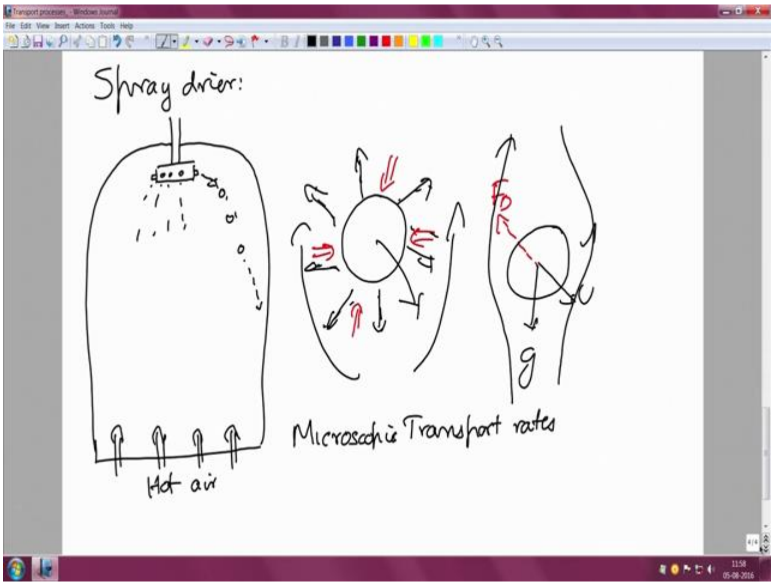

And we looked at the case of a spray drier where there are other compromises to be made, you cannot make the temperature difference to a large, but you still need to transfer a sufficient amount of heat before the droplet which is ejected at the nozzle hits the wall or the bottom of the straight drier.

That involves heat transfer to evaporate the water that is inside the droplet which could be as large as 80 percent mass transfer of the vapor from the droplet to the air and of course, the forces exerted on the droplet will determine how long it takes to travel from the nozzle to the wall of the container and all of these things have to be optimized in order to get the desired rate of transfer of get the desired product quality because if you do not evaporate all of the water then the product quality the droplets will stick and the product quality will degrade.

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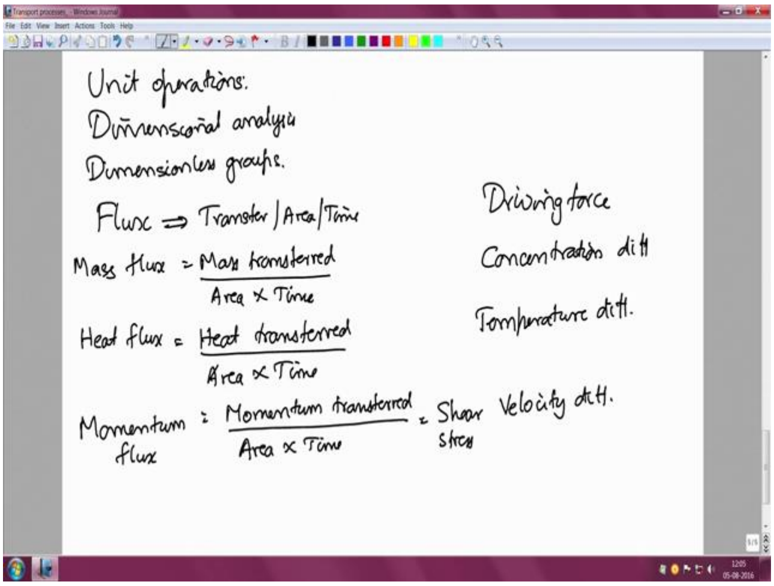

Unit operations:

Dimensional analysis

Dimensionless groups.

Flux ⇒ Transfer / Area / Time

Driving force

$$\text{Mass flux} = \frac{\text{Mass transferred}}{\text{Area} \times \text{Time}}$$

Concentration diff

$$\text{Heat flux} = \frac{\text{Heat transferred}}{\text{Area} \times \text{Time}}$$

Temperature diff.

$$\text{Momentum flux} = \frac{\text{Momentum transferred}}{\text{Area} \times \text{Time}} = \text{Shear stress}$$

Velocity diff.

Currently how are these things done? So, for example, we typically in unit operations, we use dimensional analysis and dimensionless groups why do we do this? So, if I wanted to design a heat exchanger the length of the tube and the size of the shell will of course, depend upon how much temperature difference is there between the shell sale in the tube site, how much the temperature of the hot fluid has to be cooled in addition to material properties, it depends upon the specific heat of the material the conductivity of the material and therefore, you have a relatively large number of parameters on which it depends.

Of course, momentum transfer is also important because you need to determine, what will be the flow rate for a given pressure drop and that will depend upon the velocity the viscosity and the density of the fluid. So, basically you have a large number of parameters which determine ultimately how this transfer takes place and if you just wanted to work in terms of those parameters then you would have to design the specific heat exchanger for each kind of fluid with the each specific conductivity specific heat viscosity density and so on.

Dimensional analysis enables you to make the number of parameters much smaller using dimensionless groups because we know that if you express everything, it also dimensionless groups, you can reduce the number of parameters and in similarly in this case of the heat flux and the mass flux and so on, all of these depend upon the properties of the fluid the air around the specific heat the conductivity the humidity and so on, they depend upon the latent heat of what is to be evaporated and so you end up with a large number of parameters and the way to reduce this number of parameters is to use dimensional analysis it is a very powerful technique which has been used extensively, but it has its limitations. So, first I will try to explain what is the usefulness of dimensionless groups? And then I will go to the limitations and at the end I will try to explain what is it that we will be doing more in this course than just dimensional analysis.

The dimensionless groups for example, for heat transfer are usually defined as dimensionless fluxes a flux is basically the transfer per unit area per unit time across the surface. So, if I have a particle surface the flux of mass to the surface or from the surface would be the mass transferred per unit area to time.

Similarly, the flux of heat in the case of heat transfer for example, is equal to heat transferred per unit area into time and similarly in the case of momentum transfer actually the flux is just the shear stress is the shear stress and the transfer of each of these fluxes depends upon what are called driving forces. So, let us just get rid of them. So, there is not going to be no transfer of mass to the surface unless there is a concentration difference between the bulks of the fluid in the surface. So, driving force for mass flux is the concentration difference.

Similarly, there is going to be no flux of heat from the fluid to the surface unless the bulk temperature is higher than the surface temperature heat always moves from regions of high temperature to low temperature. So, transfer necessarily has to involve a temperature difference and similarly the momentum flux has to necessarily involve a difference in velocity I will make these a little more precise a little later it is not exactly the difference, but rather the concentration, but for now let us just consider these as the concentration difference the temperature difference in the velocity difference.

We know of course, that the fluxes change when you impose a higher velocity of fluid for example, if the flow around the particle is faster the fluxes change; however, that is what is taken into account by the correlations. So, for example, the Nusselt number can be written for the heat flux as the heat flux divided by.

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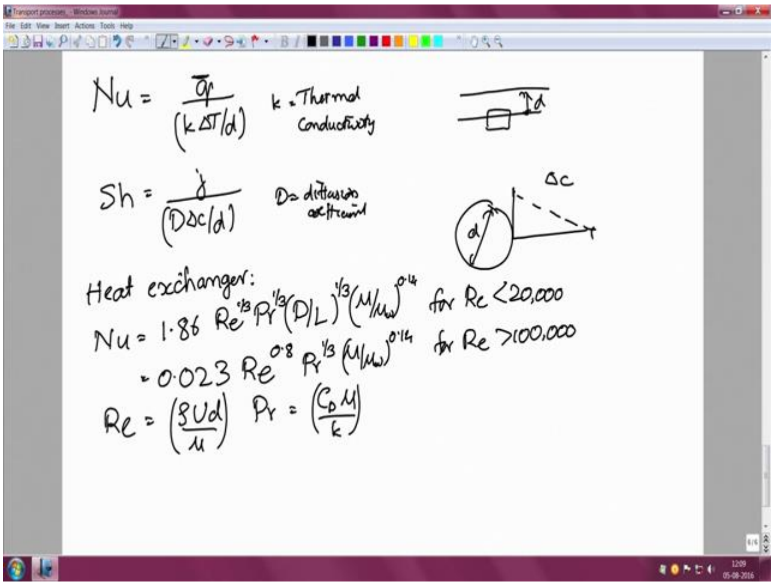

$$Nu = \frac{\bar{q}}{(k \Delta T/d)}$$ k = Thermal Conductivity

$$Sh = \frac{j}{(D \Delta c/d)}$$ D = diffusion coefficient

Heat exchanger:

$$Nu = 1.86\, Re^{1/3} Pr^{1/3} (D/L)^{1/3} (\mu/\mu_w)^{0.14} \text{ for } Re < 20{,}000$$

$$= 0.023\, Re^{0.8} Pr^{1/3} (\mu/\mu_w)^{0.14} \text{ for } Re > 100{,}000$$

$$Re = \left(\frac{\rho U d}{\mu}\right) \quad Pr = \left(\frac{C_p \mu}{k}\right)$$

Where k is the thermal conductivity T is the temperature and d is let us say the diameter of the pipe in the case of the shell and tube heat exchanger. This is a dimensionless group because we know that from furious law of conduction the heat flux is equal to the thermal conductivity times the temperature difference divided by the distance. So, this is a dimensionless number. Similarly the Sherwood number can be written as the mass flux divided by d delta c by d for this is for the heat exchanger problem where d is the diameter of the pipe this could be for the spherical particle problem where d is the diameter of the particle. So, if I have delta c the concentration of the surface is higher than the concentration of the bulk this difference is delta c and here d is the diffusion coefficient and k is the thermal conductivity.

Now, these dimensionless numbers can be used to compress the dependences on those large numbers of dimensional parameters that I had. So, for example, for a heat exchanger problem you have correlations of the kind Nusselt number is equal to 1.86 Reynolds number power one-third, Prandtl numbers power one-third D by L power one-third and for Re less than 20000.

These compress the dependencies into just a small number of parameters in the scale the Nusselt number the Reynolds number, Prandtl number where the Reynolds number is defined as rho U d by mu where d is the diameter mu viscosity and rho is the density Prandtl number is to find us C p mu by k where mu is the viscosity case the thermal conductivity and c is the specific heat. So, rather than having to very a large number of parameters you can just vary these dimensionless numbers alone and then you will get an expression for the Nusselt number where the Nusselt number is now defined in the basis the average flux ok here the Nusselt number is defined in terms of the average flux which is an average over the entire heat exchanger.

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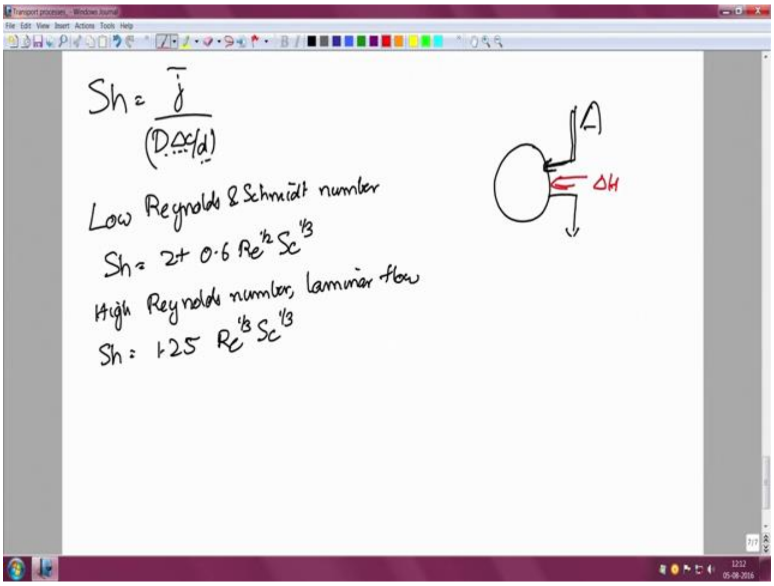

$$Sh = \frac{\bar{j}}{(D\Delta c/d)}$$

Low Reynolds & Schmidt number

$$Sh = 2 + 0.6\, Re^{1/2} Sc^{1/3}$$

High Reynolds number, laminar flow

$$Sh = 1.25\, Re^{1/3} Sc^{1/3}$$

These dimensionless variables are certainly reduce the number of parameters that we have. For the mass transfer around a spherical particle for example, where I have some component that is coming in, I can define the Sherwood number which is equal to the mass flux mass transferred per unit area per unit time divided by d delta c by the particle diameter where d is the diffusion coefficient this number is dimensionless because we know that from Fick’s law of diffusion the flux is equal to d delta c by the diameter; however, this thing is once again it is written for the average flux.

And you have dimensionless correlations of the kind for example, for low Reynolds number, the Sherwood number is equal to 2 plus and for High, the Sherwood number is of the form 1.25 Re power one-third e r power one third I am sorry I should express it in terms of the (Refer Time: 15:36) number that is what is appropriate for mass transfer.

And identical correlations are available between the Nusselt number the Reynolds number the Prandtl number and you can use these. So, at the microscopic level if you know that the average flux is given by this kind of a correlation then knowing the difference in concentration the particle diameter the diffusivity and all of the properties the density the viscosity the velocity the diffusivity and so on, you can determine what is going to be the average flux on this surface, you can determine what is going to be the average flux that is exerted on the surface.

And from that knowing, the surface area and the number of particles you know how much of a is reaching the surface same thing holds you know how much will be is leaving the surface and from that you can determine whether the conditions that you have applied the concentration difference the velocity and all of the temperature difference are sufficient for achieving the objectives of your process.

Now, these are of course, empirical correlation they are done with a large number of experimental observations, the question is how do we get these at the from a microscopic understanding of the system; similarly in the case of momentum transfer.

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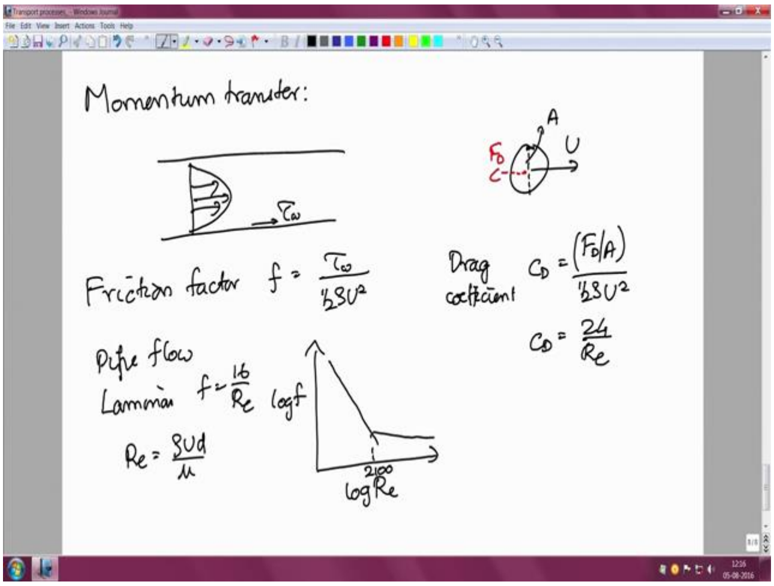

Momentum transfer:

Friction factor $f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$

Drag coefficient $C_D = \frac{(F_D/A)}{\frac{1}{2}\rho U^2}$

$C_D = \frac{24}{Re}$

Pipe flow

Laminar $f = \frac{16}{Re}$

$Re = \frac{\rho U d}{\mu}$

If for example, for the shell and tube heat exchanger that I had and generating the certain flow within the fluid that flow exerts a certain wall shear stress on the wall of the pipe. If I know the shear stress exerting, it exerted each point on the wall of the pipe from that I can find out what is the total force because the force is just the shear stress times the area and therefore, I know what is the pressure difference between the inlet and the outlet for these also you have correlations they are defined slightly differently for flow in a pipe for example, we can define the friction factor – f is defined as tau w by half rho u square, you can easily verify that half rho u square has the same dimensions as a shear stress and on the other hand for a particle moving with a certain velocity u in the fluid this is usually defined in terms of the drag coefficient.

If there is a certain listed drag force that is exerted by the fluid on the particle you can define the drag coefficient as the average force per unit area of cross section of the particle where the area of cross section is the area cross section which is across which the fluid is flowing. So, this is the area this once again is defined by half rho u square because the ratio of drag force and area also has the dimensions of stress.

From these kinds of non dimensionalization you can then get correlations for how this friction factor depends upon other dimensionless numbers for example, for a pipe flow. So, a laminar flow the friction factor is equal to 16 by the Reynolds number where the Reynolds number is equal to and you have certain diagrams which give you the friction factors function of Reynolds number for turbulent flow as well. Typically these are plotted these plots are what are called moody plots we are plotted on a log scale, this is log of the Reynolds number and you have the slope of minus one 16 by Re and at some point there is a transition, it is a Reynolds also above twenty one hundred this transition to a turbulent flow.

Similarly, for the drag coefficient on a sphere the correlation is for the laminar flow this is 24 by Re and then once again once there is boundary layer separation in. So, on across the surface there will be a different correlation. So, these correlations can be used to predict what is going to be the velocity given a certain force or what is going to be the flow rate given a certain pressure drop.

However these are all macroscopic relations they are at the macroscopic level. In this course our attempt is going to be to derive these relations from a microscopic description from knowing what is happening in each volume element around the particle.

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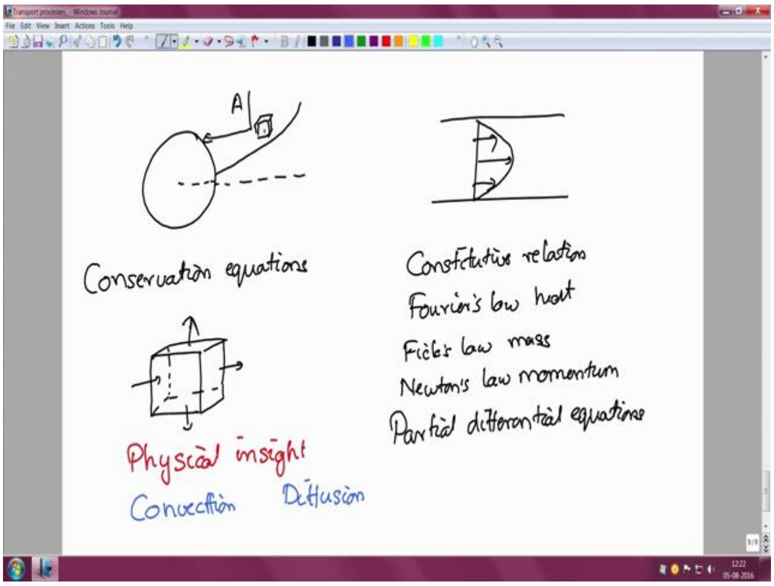

For example, if I take the mass transfer around a particle, you know that at the particle surface itself if you have transferred to the surface the concentration the bulk is higher than the concentration of that surface, but; however, the concentration is a smoothly varying function it varies continuously as you approach the particle. So, our attempt will be to first be able to predict what is the entire concentration field around the entire particle how does it very with position, at each location around the particle and once I have that solution for how it varies at each location around the particle I can then find out what is the flux at the surface of the particle and from that find out the average flux.

This is a microscopic description where I am trying to describe what is happening in each differential volume element around the particle of course, the fluid around the particle can be separated into a large number of differential volumes and I need to know how the concentration varies at each location around the particle and then use that in order to find out what is the concentration difference and therefore, what is the flux at the surface. Similarly for the pipe flow what I will try to do is to try to get the entire velocity as a function of distance from the wall, once I have that solution for the velocity everywhere, I can then find out what is the shear stress at the surface from Newton’s laws, from Newton’s law of viscosity and from that I will know what is the wall shear stress and therefore, what is the pressure difference that has to be applied. So, this involves doing writing down equations which are applicable each differential volume at each location.

These equations have two parts one is what are called conservation equations, conservation equations are applicable for all for mass heat and momentum transfer mass is always conserved unless here close to the speed of sound. Similarly energy is conserved and momentum is also conserved. So, if I have some small differential volume that I had written here.

For this differential volume in the case of mass for example, what comes in minus what goes out has to be the net rate of accumulation within this volume that holds for mass it holds for heat and it also momentum as well what comes in minus what goes out has to be the net rate of accumulation.

In order to apply these conservation equations I need to know what is the rate at which mass is coming in and going out across the surface and that rate at which mass is coming in or going out across the surface that is the surface flux, flux of mass per unit volume per unit time. That is related to the concentration difference across the surface or rather the rate of change of concentration with position across the surface in other words the gradient of the concentration and that relation is what is called the constitutive relation.

This tells you, given a certain variation of concentration temperature or velocity across the surface, what is the flux of mass momentum or energy, these are constitutive relations and these are given by what are well known laws. So, for example, the 4 years law for heat conduction for heat, Fick’s law for mass diffusion and of course, you have Newton’s law of viscosity for momentum transfer and these depend upon the variations the derivatives of the concentration, momentum and temperature fields. These 2 together will give us a set of equations which apply at each differential volume surrounding these particles and then of course, using that we have to solve those differential equations at each location you have to get the entire temperature concentration of momentum field.

These differential equations they turn out to be what are called partial differential equations these are called partial differential equations because they vary both in spaces, in position as well as in time. So, there are derivatives with respect to all 3 coordinates in general as well as delta with respect to time again once you post it into the terms of partial differential equations you have to solve them. Now the point is that that solution procedure is not straightforward given an equation you cannot always find a simple solution procedure for partial differential equations. In order to solve these equations you also have to have importantly some physical insight into the processes that are taking place in the system and the important processes in all of these are one of two types - one is convection and the other is diffusion.

And this balance between convection and diffusion is what will determine what the rate at which transport is expects. So, for example, in the problem on this reactor design for the catalyst surface as the fluid is being mixed around that fluid is carrying with it all of the reactants and products the reactants are coming in to the reactor through convection it is being carried along with the fluid flow similarly the products are flowing out due to convection, convection is the transport of materials heat or momentum due to the flow of the fluid.

They have been carried out due to the fluid flow; however, at the surface itself the fluid cannot flow perpendicular to the surface the fluid has to flow along the surface and at the surface ultimately transfer perpendicular to the surface has to happen due to diffusion and that is why diffusion is so important in transport processes, no matter how fast the convection is ultimately at the surface transport has to take place due to diffusion. Similar is the case of heat transfer in the case of a heat exchangers heat is being conducted in along with the fluid at the inlet the cold fluid is being connected out; however, the transport across the surface has to take this due to diffusion because there is no heat transfer across the surface.

And similar is the case of whom the droplets heat transfer across the surface ultimately whereas, heat mass or momentum has to take place due to diffusion and the interplay between these two that will ultimately determine how efficient the process is.

Diffusion the transport of material due to the random molecular fluctuating velocities of the molecules in the fluid, so, first in this course, I will first go through dimensional analysis to show you how non dimensionalization greatly simplifies problems, but more importantly I will try to give you some physical insight into these dimensionless numbers. I just returned these dimensionless numbers for you just on the basis of non dimensionalization making sure that the dimensions of all the terms add up to 0; however, each of these dimensionless numbers has its only physical insight what is that that; I will try to explain to you in the next series of lectures on dimensional analysis and after that we will look at diffusion some further detail, how do we estimate the rate at which things diffuse in liquids and gases and after that we will go on to the main topic of the course that is to be able to predict all of these temperature momentum and concentration fields writing down the differential equations that I talked about.

Writing down the differential equations for each little differential volume there and then how do you integrate it to get the entire field and from that how can you get back the kinds of correlations that I have written for you here for pipe flow and for flow around a particle as well as the correlations for the Sherwood number and the Nusselt number. So, the Nusselt number correlations that are shown here, the Sherwood number correlations that are shown here, how can we derive these from microscopic description which takes into account convection in diffusion. There is going to be the drought of length of this course.

We will start off on dimensional analysis in the next lecture and I will see you then, good bye.