Transport Processes I: Heat and Mass Transfer
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Lecture – 13
Continuum description of fluids

Welcome to this, this is our 13th lecture in the course on fundamentals of transport processes. In the previous 6 hours of lectures, I have tried to give you some idea of what is it that we are trying to study by going through the process of using dimensional analysis to derive correlations and then trying to give you a physical understanding of what those correlations mean in terms of the ratios of convection and diffusion.

If you recall in the last lecture I had said that convection is the process by which the fluid carries along with it, whatever mass is dissolved in it, whatever thermal energy it has or whatever momentum it has with it and the flux due to convection across the surface it is just given by the product of in the case of mass flux, the concentration times the velocity. In the case of heat flux, the energy density times the velocity.

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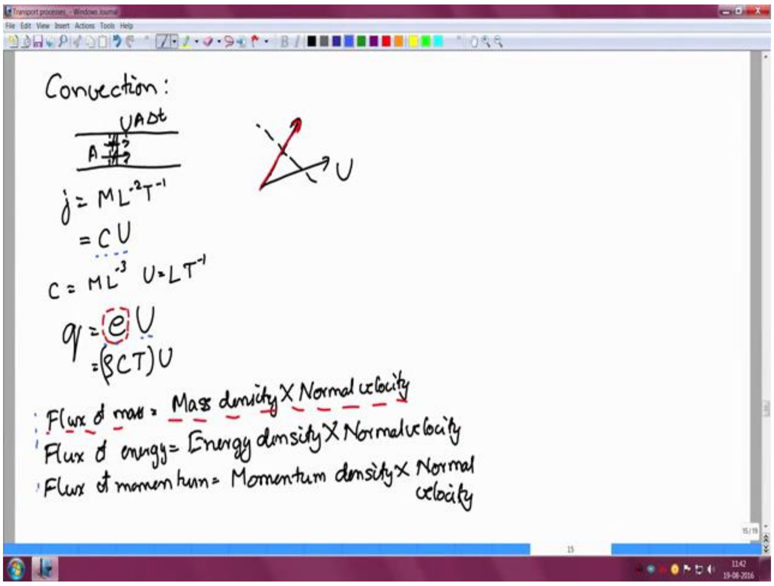

Convection:

$j = ML^{-2}T^{-1}$

$= CU$

$C = ML^{-3}$ $U = LT^{-1}$

$q = \boxed{e}\,U$

$= (\rho C T)U$

Flux of mass = Mass density X Normal velocity

Flux of energy = Energy density X Normal velocity

Flux of momentum = Momentum density X Normal velocity

So, you can have a unified description of the fluxes of mass momentum and energy.

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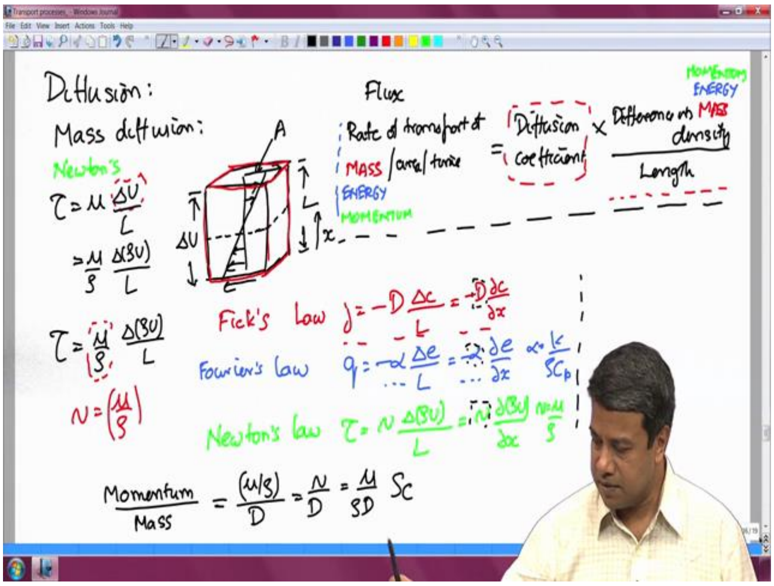

Diffusion:

Mass diffusion:

Newton's

$\tau = \mu \frac{\Delta u}{L}$

$= \frac{\mu}{\rho} \frac{\Delta(\rho u)}{L}$

$\tau = \frac{\mu}{\rho} \frac{\Delta(\rho u)}{L}$

$\nu = \left(\frac{\mu}{\rho}\right)$

Flux

Rate of transport of MASS/area/time ENERGY MOMENTUM = Diffusion coefficient × Difference in MASS ENERGY MOMENTUM density / Length

Fick's Law $j = -D \frac{\Delta c}{L} = -D \frac{\partial c}{\partial x}$

Fourier's law $q = -\alpha \frac{\Delta e}{L} = -\alpha \frac{\partial e}{\partial x}$ $\alpha = \frac{k}{\rho C_p}$

Newton's law $\tau = \nu \frac{\Delta(\rho u)}{L} = \nu \frac{\partial(\rho u)}{\partial x}$ $\nu = \frac{\mu}{\rho}$

$\frac{\text{Momentum}}{\text{Mass}} = \frac{(\mu/\rho)}{D} = \frac{\nu}{D} = \frac{\mu}{\rho D}$ Sc

Similarly, I had also described for you the process of diffusion; Diffusion takes place whenever there is a gradient, a variation in the concentration the energy or the moment and the flux which is the rate of transport per unit area, per unit time give the flux. It can be written as a diffusion coefficient times, the difference in the density of the whatever is diffusing; the density of mass, the density of momentum or the density of energy divided by a length the cross stream or the length across which there is a variation.

So, therefore, these diffusion laws can be written in a unified framework. Fick's law for diffusion; Fourier's law for heat conduction; Newton's law for viscosity; the flux is equal to diffusion coefficient times a difference in density divided by a length or diffusion coefficient times a gradient in the density, it change in density divided by length whether the density is that of mass momentum or energy and from these we get the diffusion coefficients. The mass diffusion coefficient for mass, the thermal diffusion coefficient for energy which is written as k by rho C p, the momentum diffusion coefficient for momentum which is the ratio of the dynamic viscosity and the density.

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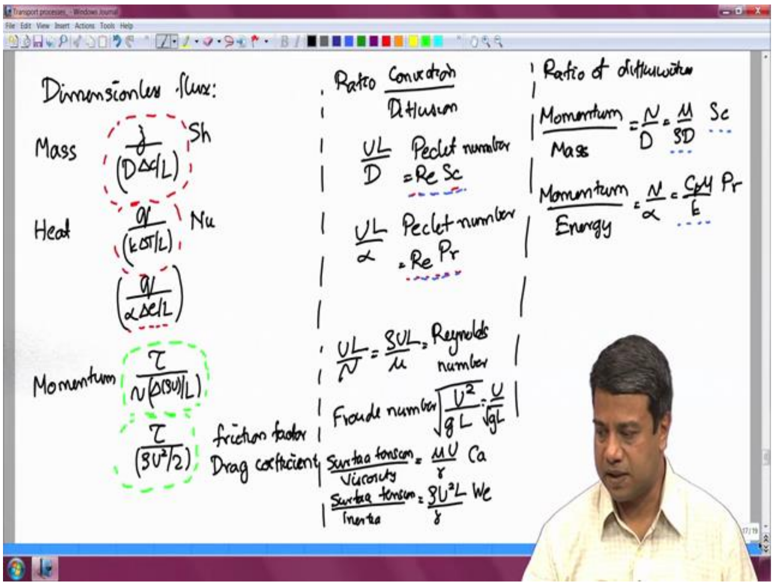

Dimensionless flux:

Mass $\frac{j}{(D\Delta c/L)}$ Sh

Heat $\frac{q}{(k\Delta T/L)}$ Nu

$\left(\frac{q}{\alpha\Delta e/L}\right)$

Momentum $\frac{\tau}{\nu(\Delta(\rho U)/L)}$

$\frac{\tau}{(\rho U^2/2)}$ friction factor Drag coefficient

Ratio $\frac{\text{Convection}}{\text{Diffusion}}$

$\frac{UL}{D}$ Peclet number $= Re\,Sc$

$\frac{UL}{\alpha}$ Peclet number $= Re\,Pr$

$\frac{UL}{\nu} = \frac{\rho UL}{\mu} =$ Reynolds number

Froude number $\sqrt{\frac{U^2}{gL}} = \frac{U}{\sqrt{gL}}$

$\frac{\text{Surface tension}}{\text{Viscosity}} = \frac{\mu U}{\gamma}$ Ca

$\frac{\text{Surface tension}}{\text{Inertia}} = \frac{\rho U^2 L}{\gamma}$ We

Ratio of diffusivities

$\frac{\text{Momentum}}{\text{Mass}} = \frac{\nu}{D} = \frac{\mu}{\rho D}$ Sc

$\frac{\text{Momentum}}{\text{Energy}} = \frac{\nu}{\alpha} = \frac{C_p\mu}{k}$ Pr

So, these are the three diffusion coefficients, all three of them have dimensions of length square per unit time. Dimensionless numbers can be broadly written as dimensionless fluxes, mass momentum and heat fluxes, the mass flux is the Sherwood number, the heat flux is the nusselt number. Momentum flux is non-dimensionalized differently, conventionally in comparison to mass and heat flux whereas, mass and heat flux are non-dimensionalized by the diffusive non dimensionalization, the ratio of the flux and the right side of the constitutive relations D delta C by L or k delta T by L. The momentum flux is traditionally scaled by the kinetic energy density; it is scaled by inertial scale rather than by viscous scale whereas mass and heat are scaled by the diffusive scale that is the way that the moment of flux is traditionally defined.

There are ratios of convection and diffusion, the Peclet number the for mass and heat transfer in the Reynolds number and then there are ratios of diffusivities, momentum by mass is the Schmidt number, momentum by heat is the Prandtl number. So, these are the broad categories of dimensionless numbers.

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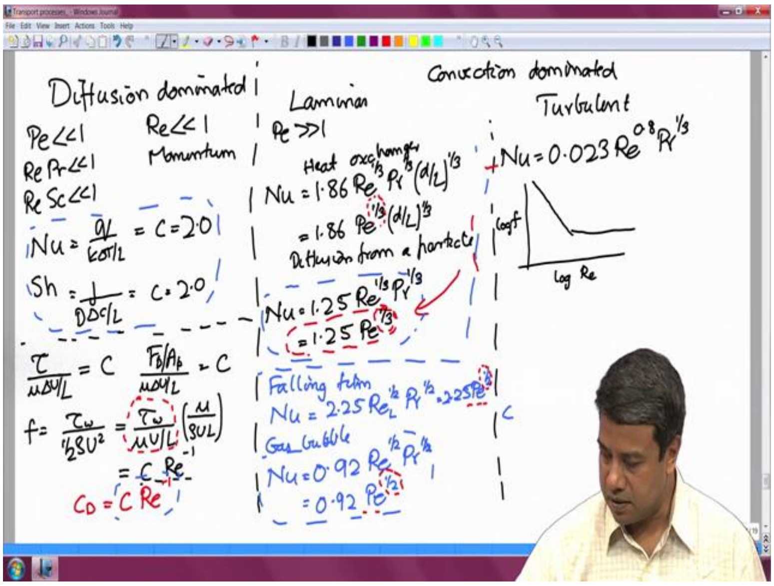

Diffusion dominated

$Pe \ll 1$ $Re \ll 1$

$Re\,Pr \ll 1$ Momentum

$Re\,Sc \ll 1$

$Nu = \frac{q/L}{k\Delta T/L} = C = 2.0$

$Sh = \frac{j}{D\Delta c/L} = C = 2.0$

$\frac{\tau}{\mu\Delta U/L} = C$ $\frac{F_D/A_p}{\mu\Delta U/L} = C$

$f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{\tau_w}{\mu U/L}\left(\frac{\mu}{\rho U L}\right) = C\,Re^{-1}$

$C_D = C\,Re^{-1}$

Laminar

$Pe \gg 1$

Heat exchanger

$Nu = 1.86\,Re^{1/3}\,Pr^{1/3}(d/L)^{1/3}$

$= 1.86\,Pe^{1/3}(d/L)^{1/3}$

Diffusion from a particle

$Nu = 1.25\,Re^{1/3}\,Pr^{1/3}$

$= 1.25\,Pe^{1/3}$

Falling film

$Nu = 2.25\,Re_L^{1/2}\,Pr^{1/2} = 2.25\,Pe^{1/2}$

Gas bubble

$Nu = 0.92\,Re^{1/2}\,Pr^{1/2}$

$= 0.92\,Pe^{1/2}$

Convection dominated

Turbulent

$Nu = 0.023\,Re^{0.8}\,Pr^{1/3}$

All the correlations that we have derived can be written in terms of these dimensionless numbers. If you recall we derived the correlation for the heat transfer in which the dependent dimensionless number, the dimensionless flux as been written as the function of the Reynolds number and the Prandtl number. Similar is the case for other such problems, we saw for a falling film problem we have the Reynolds number and the Prandtl number, to a different power the nusselt number is equal the Reynolds number times a central number to a different part.

Similarly, in diffusion dominated regime, the Peclet numbers are smaller, the Reynolds numbers are small; in that case inertia should not matter and therefore, the nusselt in the Schmidt numbers tend to a constant. Then there is a third regime of a turbulent flow; where the dimensionless numbers are a lot more complicated.

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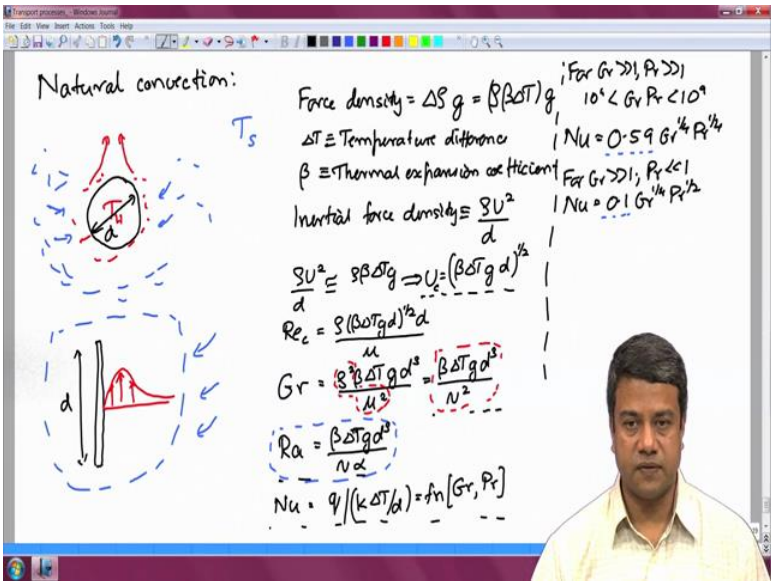

Natural convection:

Force density $= \Delta\rho g = \rho(\beta\Delta T)g$

$\Delta T \equiv$ Temperature difference

$\beta \equiv$ Thermal expansion coefficient

Inertial force density $\equiv \frac{\rho U^2}{d}$

$$\frac{\rho U^2}{d} \cong \rho\beta\Delta T g \Rightarrow U_c = (\beta\Delta T g d)^{1/2}$$

$$Re_c = \frac{\rho(\beta\Delta T g d)^{1/2} d}{\mu}$$

$$Gr = \frac{\rho^2\beta\Delta T g d^3}{\mu^2} = \boxed{\frac{\beta\Delta T g d^3}{\nu^2}}$$

$$\boxed{Ra = \frac{\beta\Delta T g d^3}{\nu\alpha}}$$

$$Nu = q/(k\Delta T/d) = fn[Gr, Pr]$$

For $Gr \gg 1$, $Pr \gg 1$

$10^4 < Gr\,Pr < 10^9$

$Nu = 0.59\, Gr^{1/4} Pr^{1/4}$

For $Gr \gg 1$; $Pr \ll 1$

$Nu = 0.1\, Gr^{1/4} Pr^{1/2}$

We also talked about natural connection; in this case the velocity is determined from the force density exerted due to the temperature difference due to a heated object and therefore, the dimensionless fluxes defined the same way as we had earlier.

However in our expressions for the ratio of convection and diffusion, the convection velocity is not specified, it is not you do not have food that is being forced passed the object to a specific velocity. The velocity itself has generated due to the temperature difference and therefore, you need to calculate the velocity scale based upon a balance between the buoyancy force density due to the temperature difference and the inertial force density and then use this velocity in order to define dimensionless numbers and we saw the dimensionless numbers that I may emerge the Grashof number of the Rayleigh number, these are Reynolds numbers and the Grashof number is the square of a Reynolds number based upon the conduction velocity, based upon a balance between the force density and fluid inertia and the Rayleigh number instead of the new square in the denominator is you substitute that with the product of the kinematic viscosity and the thermal diffusivity and we had seen once again there are certain correlations that arise in this case.

The program for the neck rest of this course is going to be able to solve the entire problem analytically in order to have physical understanding of how these correlations arise? What is the difference whether the Reynolds number is large or small or the Peclet number is large or small and why do different regimes as well as different conditions at the surface of objects, why do these gives rise to different kinds of correlations?

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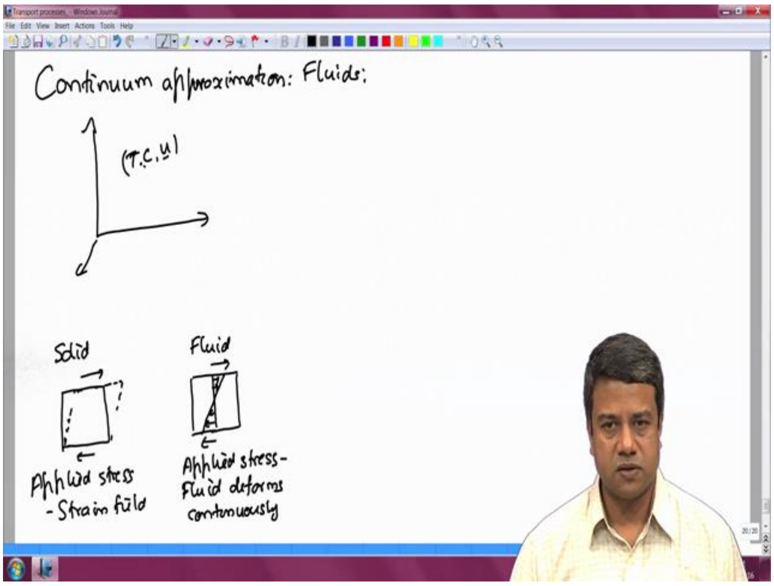

So, how do we treat the problem? The first thing we do is a continuum, that is we assume that the temperature, the concentration of the velocity fields or continuum fields that is if you are analyzing some object in some coordinate system, we assume that this temperature is continuously varying as a function of position. If the temperature, the concentration or the velocity these are continuously varying functions of position, it is they are defined at each and every point within the flow. So, that is a continuum approximation for the velocity, the concentration or the temperature fields.

These continuum fields are defined within fluids; in this course we will be dealing primarily with fluids. The difference between solids and fluids is of course, that for a solid if you apply some kind of a stress, it will deform for some extent and then it will stop, if you apply under continuous application of a stress provided that stress is small enough that it does not break the solid, it is going to deform to some extent and then stop. So, therefore, in a solid in applied stress generates a strange field whereas, to a fluid if you apply a stress, there is a continuous velocity as long as stress is applied the fluid will continue to flow, it will not stop as long as the stress is applied because if you remove the stress then it will come to a rest. So, fluid deformed continuously under applied stress. So, this is a solid.

At the microscopic level and the molecular picture the difference arises because in solids molecules have fixed equilibrium positions, we have a fixed crystal lattice or at least crystalline solids will have a fixed crystal lattice within which all the molecules are sitting within the potential well that is created by their neighbors.

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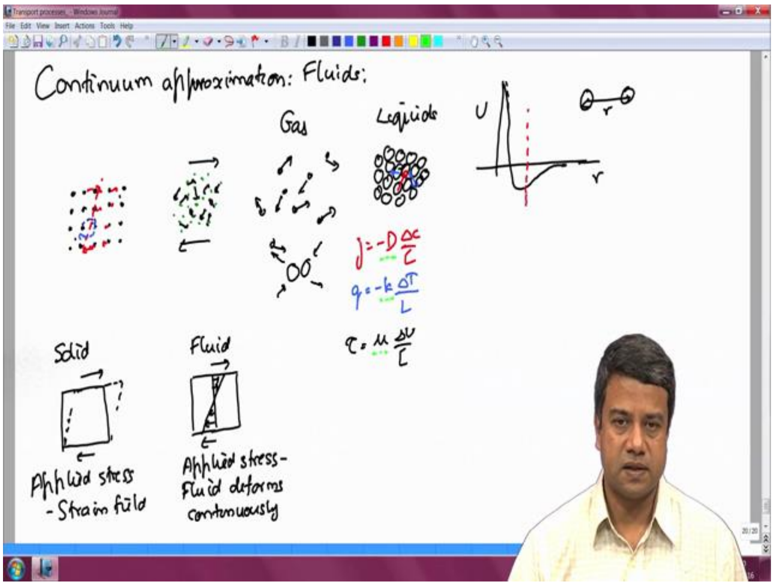

Continuum approximation: Fluids:

$J = -D \frac{\Delta c}{L}$

$q = -k \frac{\Delta T}{L}$

$\tau = \mu \frac{\Delta v}{L}$

So, there is a fixed lattice and therefore, if you deform this lattice a little bit, if you apply a stress all the molecules will shift a little bit from their equilibrium positions and that shift of the molecules exerts a restoring force because molecules would prefer to come back to their equilibrium locations. So, that shift of the molecules from their equilibrium locations see there is a restoring force and that causes a stress.

So, this happens because the molecules have defined locations within a solid. There is of course, diffusion in solids it takes place by two means, one is the interstitial diffusion due to molecules which are imperfectly located within the interstitial space of the solids and the other is due to vacancy migrations, certain locations may not be filled with the molecule. So, here we can see within a solid at some location.

You could have another salt molecules move in and consequently the vacancy actually migrates the other way you know solid. So, you do have processes of diffusion in solids as well, you try the interstitial of vacancy diffusion, but these are extremely slow processes these processes takes place over extremely long time scales, over many years and therefore, diffusion over reasonable time scales required for material transformations does not really take place in solids.

In fluids on the other hand the molecules do not have equilibrium positions and therefore, there are free to move about in any direction and therefore, in a fluid if you apply a stress the molecules will move continuously in response to that stress, they do not have equilibrium positions and because of that, the molecules to move in such a way as to accommodate that stress and thereby generate a fluid flow. So, the fluid flow is continuous in liquids and gases in all fluids and that is a fundamental difference.

The reason that most of these transfer processes are carried out in fluids is for those reason, because of the molecular motion of the molecules in a fluid, these transport rates are much higher. There are molecular motions in solids as well, in all cases the root mean square velocity is given by root of 3 k t by m the thermal energy; however, in the case of solids this is about and this motion takes place about defined or specified locations on a lattice and therefore, when you apply stress on a solid you will deform to some extent and then it will stop when the restoring forces become equal to the applied stress.

In a fluid on the other hand, molecules can freely move about anywhere within the system because they already have thermal fluctuating velocities and because the molecules in this fluid are agitated and they do not have equilibrium positions, any applied stress is going to generate a flow which will accommodate the applied stress and if the stress is continuous the flow will be continuous.

We will deal exclusively with fluids in this entire course. Fluids of course, a compressed of liquids and gases; the distinction between liquids and gases at the molecular levels is clear, but at the continuum level we are not distinct; at the molecular level in a gas, the separation between molecules is much larger than the molecular diameter, the molecules are once again in a random state of fluctuations due to the thermal energy, but the distance between molecules is much larger than the molecular diameter, typically about 10 to 100 things.

And for this reason the interaction between molecules in gas can be considered as a series of instantaneous collisions between pairs of molecules, followed by the range of interaction is much smaller than the distance between collector interactions therefore, these can be considered as a series of instantaneous collisions between molecules in the gas, for most of the time they are each molecule is out of range of the interaction with any other molecule, but there are brief occasions on which an interaction takes place and this scatters the velocities of the molecules in between such collisions the molecules follow a straight line path or a ballistic path in case there is an applied force.

In liquids on the other hand; the molecules are all located in at distances which are comparable to the molecular diameter, effectively in liquids in case you draw the interaction potential between molecules, interaction potential between as a distance of distance between two molecules, we call this is as r to distinguish it from the molecular diameter.

For a very short distances there is a repulsive part and then at long distances this an attraction again then as you go further away the force comes down to 0, this is a typical potential versus distance curve for molecules. In the case of liquid molecules, each molecule is close to the potential minimum of the neighboring molecules, the distance is comparable to the molecular dimensions. So, that is the fundamental difference, if a molecule has to move some distance within a liquid, the other molecules have to move out of the way. So, the interaction is continuous and it is multi body, simultaneous interaction between many molecules.

At the molecular level this is the difference. At the macroscopic level the difference is not actually that much. At the macroscopic level the fundamental laws the constitutive relations whether it is Fick’s law or Fourier’s law of heat conduction or it is Newton’s law of viscosity, these laws are applicable both to liquids as well as gases. So, laws are identical therefore, as far as calculations are concerned we can use the same equations whether it is a liquid or gas.

The only difference is that between liquids and gases these coefficients change. The diffusion coefficient, the thermal conduction coefficient and the viscosity coefficient these coefficients are different depending upon where is the liquid or gas; however, the constitutive relations themselves are identical, independent of whether we are dealing with a liquid or a gas, so long as it is a fluid. So, we can deal with both of these on equal footing and during the course itself we shall really make no distinction about whether the system we are considering is actually a liquid or a gas. So, we will deal exclusively with the fluids in this course and the discussion itself does not distinguish between liquids and gases, the only difference arises when we have to write down the diffusion coefficients, these turn out to be the numerical values of these turn out to be a very different depending upon whether it is a liquid or a gas and we will look a little later at how do we derive these constitutive relations and the diffusion coefficients.

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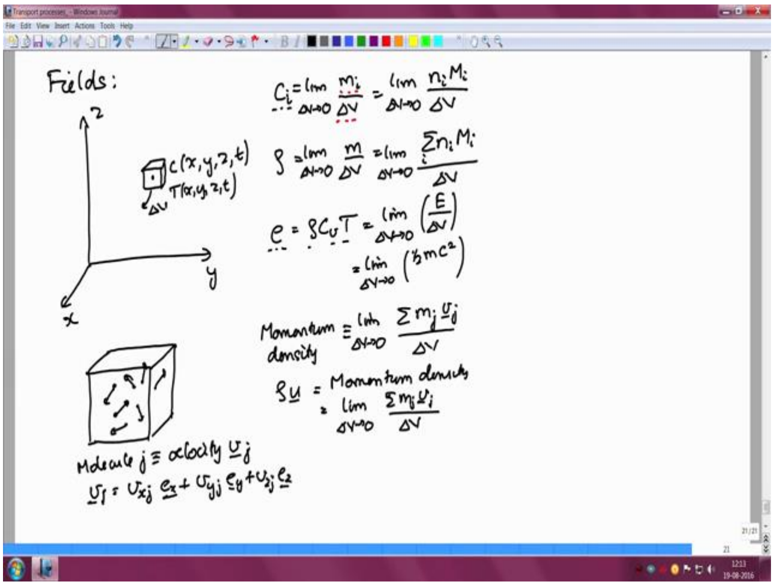

Fields:

$c(x,y,z,t)$

$T(x,y,z,t)$

$$C_i = \lim_{\Delta V \to 0} \frac{m_i}{\Delta V} = \lim_{\Delta V \to 0} \frac{n_i M_i}{\Delta V}$$

$$\rho = \lim_{\Delta V \to 0} \frac{m}{\Delta V} = \lim_{\Delta V \to 0} \frac{\sum_i n_i M_i}{\Delta V}$$

$$e = \rho C_v T = \lim_{\Delta V \to 0} \left(\frac{E}{\Delta V}\right) = \lim_{\Delta V \to 0} \left(\tfrac{1}{2} m c^2\right)$$

$$\text{Momentum density} \equiv \lim_{\Delta V \to 0} \frac{\sum m_j \underline{v}_j}{\Delta V}$$

$$\rho \underline{u} = \text{Momentum density} = \lim_{\Delta V \to 0} \frac{\sum m_j \underline{v}_j}{\Delta V}$$

Molecule $j \equiv$ velocity $\underline{v}_j$

$$\underline{v}_j = v_{xj} \underline{e}_x + v_{yj} \underline{e}_y + v_{zj} \underline{e}_z$$

So, I said in this course we will be dealing with fields; these are continuously varying position functions of position. The concentration field for example, it is in general a function of all 3 coordinates as well as time is defined at each location within the flow. Similarly the temperature field or the velocity field the only difference is the velocity itself is a vector. So, because of that the velocity field as a vector character, but however it is a field; that means, that the value of this concentration temperature or velocity is defined. Single valued function it has a unique value at each position, within whatever system that we are analyzing.

Now, how do we get the concentration field locally at each point within the flow, what does it mean? So, let us look at the concentration field for simplicity, what it means is that let say I have the concentration of some species within the flow, the concentration of some species within the flow it is written as the total mass of that species, let us say species i, let say you have multi component system with many species, many different types of molecules in there, the concentration of species i is equal to the mass total mass of species i divided by the volume where delta V is this volume in the limit as the volume goes to 0.

So, the idea is that if I want to know the concentration at particular location, I go to that location and consider a small differential volume about that location and find out what is the total mass of all the molecules of species i within that volume, divide that by the volume itself and I get a value for this concentration. If this volume is large then of course, the concentration will depend upon the size of the volume and the location; however, in the limit as the volume goes to 0, as delta V goes to 0, the mass within that volume also goes to 0 in such a way that the ratio of the 2 approach is a constant value that is the value of the concentration at that location.

So, in order to get the concentration of species i, what I need to do is take the limit as delta V goes to 0 of the number of molecules of species i, times the molecular mass of species i get divided by delta V, where capital M i is the molecular mass of species i. So, that is the definition of the concentration. In the limit as if you go to a point and take a small volume around that point can be of arbitrary shape arbitrary size, but as the volume goes to 0 as you shrink it further and further, find out the total mass within that volume divided by the volume itself. The concentration is the ratio of the mass to the total volume.

I can define the density; if the total density is equal to the total mass of everything put together, we divided by the volume in the limit as delta V goes to 0. So, this is going to be equal to summation over all species of number density of each species times the molecular mass of each species, divide it by delta V, it was the total mass per unit volume then the density is of course, the sum of the concentrations of each species within this volume if you had a multi component system; if we have just a single component, the mass and if the concentration and the density will be the same.

Now, for heat transfer we have to define the temperature field. Note that this is a function of position, it could also be a function of time and once again this is defined through the energy density, the energy density at a location is going to be equal to the total energy due to all of the molecules. Nc; divided by the delta V limit as delta V goes to 0.

This is the internal energy if I use specific heat at constant volume, you specific heat at constant pressure that would turn out to be enthalpy of the system. So, what do you do is you take the total mass within the system, the specific heat at constant volume times the temperature divided by delta V there is the energy density within the system and of course, if you had multiple species at equilibrium, the temperatures of all of these have to be equal to the same.

This is actually equal to the limit as delta V goes to 0 of the total internal energy; the total internal energy within this volume divided by delta V. So, we will take the internal energy of each of these molecules and divide by delta V; if you had a monatomic gas for example, the internal energy would be just due to the translational degrees of freedom itself, if we had diatomic molecules you will have a rotational degree of c m s well and for polyatomic molecules you will have vibrational degrees as well, if there are excited vibrated states at room temperature. We take the total energy density within that volume and from that I will find out what the temperature.

So, the temperature locally e I can write it as rho C v times T, this is the summation limit as delta V goes to 0, of the total energy internal energy within this volume divided by delta V and as I said this internal energy is in general n C v times, if you had this translational degrees of freedom, it will just be equal to half m into c square for each molecule thank you, there will be the total energy and that has to be equal to equated to rho C v times T to get what is the temperature. So, therefore, from the internal energy equating it to rho C v times T you can find out what is the temperatures. So, this basically defines the temperature field.

The temperature field is defined from the internal energy field or the enthalpy field by take into account the total internal energy of all the molecules within a volume dividing it by the volume itself. In the limit as the volume goes to 0, the number of molecules also goes to 0 therefore, the internal energy also goes to 0, but the ratio of the total energy within that volume divided by the volume itself that will tend towards a constant value and that is the internal energy density and from the internal energy density you write it as the mass density times C v times the temperature, and from that you can define what is the temperature field. This is the fundamental variable in this case is actually the internal energy density of the enthalpy density, depending upon whether you choose working at constant volume or constant pressure.

Now, you can also define the momentum density; the momentum density is in general defined by taking the total momentum within a volume and dividing it by the volume itself. So, I take summation of the mass of each molecule times the vector the vector velocity of each molecule. So, within this differential volume, each molecule has a velocity vector. So, therefore, for molecule i let me call it by different symbol here.

So, each molecule j has a velocity which is equal to vector j, it has 3 components x y z components. So, it is a vector; I can write v j as this written as v x j times e x, plus v y j v y it has 3 components, where e x, e y and e z are the unit vectors, in the x y and z directions. So, therefore, this is the vector, I take the total momentum the mass times the velocity of each molecule and then divide that by the volume and take the limit as delta V goes to 0. So, that is the momentum density and the fluid average velocity is defined as the mass density times the mean velocity of the fluid is equal to the momentum density.

So, that is the definition of the momentum density, and the definition of the mean velocity. So, mean velocity is defined through the momentum density. So, the fundamental quantity here is the momentum density itself, I equate rho times u mass density times the mean velocity to the momentum density and from that I get what is the velocity vector. Where once again the momentum density in order to calculate that you go to a point, construct a small volume around it, look at the total momentum of all the molecules within that volume, divided by the volume itself, in the limit as the volume goes to 0, this momentum density tends to a constant value and that is the mean velocity of the fluid at that location and these fields are specified at every location within the flow and they have a continuous variation with location within the flow.

So, these are the quantities that we will be writing down equations for the fundamental quantities, the densities mass density, the temperature through the energy density and the mean velocity through the momentum density. As far as momentum density is concerned, note that we are adding up the vector velocities. So, if the molecules have random velocity is in such a way that the sum of the vector of all those velocity is equal to 0; that means, of the fluid velocity is equal to 0 or the fluid is at rest.

So, fluid being at rest implies that the average velocity is zero does not mean that the molecular velocity is zero; even when a fluid is at rest all the molecules have their own random fluctuating velocities, the point is that the sum of all of those velocities the vector velocity is equal to zero. Similarly if a fluid is moving, the sum of all those velocities is equal to the mean velocity of the fluid, it is not the molecular velocity. The molecular velocity is actually much faster, the fluid molecules typically move at a velocity comparable to the speed of sound and whereas, the mean velocity of fluids in most common applications it is actually much less, it is much less than the speed of sound.

Therefore the fluid velocity is the vector sum for all the molecular velocities is usually much smaller than the molecular velocity itself. So, even in the fluid is at rest it does not mean to the molecules are stationary. The molecules are in the state of constant motion and it is this motion that gives rise to diffusive transport. Similarly the thermal energy of the molecules in the fluid is nonzero even when the fluid is at rest, and that is the reason why the molecules have a nonzero temperature.

The thermal energy is given by the velocity fluctuations about the mean velocity; is the mean velocity enters only to the momentum density; the thermal energy contains only the fluctuating velocities, which is the difference between the molecular velocity and the mean velocity. So, there is a distinction between the thermal energy and the kinetic energy of the fluid. Kinetic energy of the fluid as defined the mechanical kinetic energy is based upon the fluid mean velocity whereas, the temperature the thermal energy is based upon the fluctuating velocities of the molecules that is the distinction.

So, what do these fields actually mean and how do we go about writing down equations for these fields? I will try to give you a little more insight into that when we continue this discussion in the next lecture. So, I will see you then.