Transport Processes I: Heat and Mass Transfer
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Lecture – 12
Dimensional analysis: Natural and forced convection

Welcome to this our 10th lecture on Dimensional Analysis. We were getting to physical understanding of what different dimensionless groups mean and what are the kinds of correlations that result from these different dimensionless groups.

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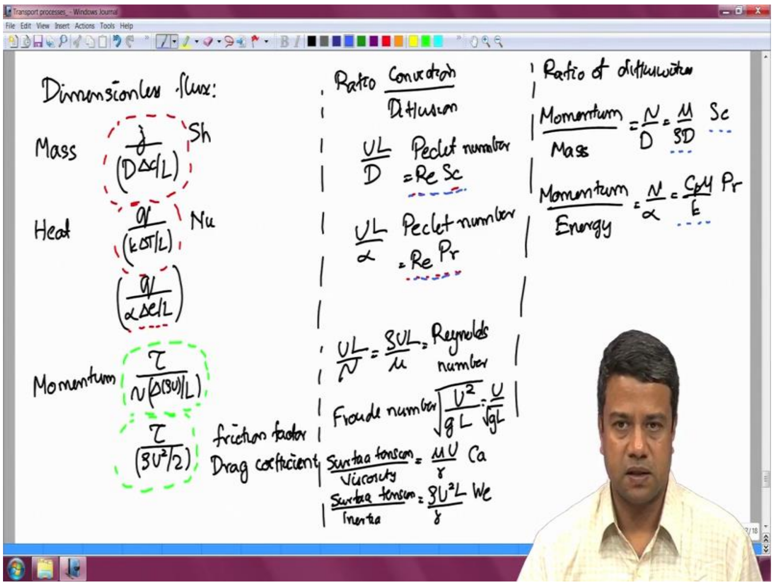

If you recall in the last lecture I had defined dimensionless groups in two broad categories. One is the dimensionless flux, the mass flux scaled by a characteristic right hand side of Fick's law, heat flux scaled by a characteristic right hand side of Fourier's law, momentum flux you would normally expect it to be scaled by a characteristic right hand side of Newton's law.

Traditionally that is not what is done; you scale it by a pressure force or an inertial force. In this case the shear stress is scaled by a kinetic energy density kinetic energy per unit volume. Then there are ratios of convection and diffusion. The Peclet number for mass transfer the velocity times the length scale divided by the mass diffusion coefficient. Heat transfer the velocity times the length scale divided by the heat diffusion coefficient.

And for momentum transfer velocity times length divided by momentum diffusion coefficient.

These are actually fundamental quantities, in correlations these are not written as Peclet number they are often written as products of the Reynolds number and Schmidt number of products with the Reynolds number and the Prandtl number. However, these are the fundamental quantities; ratios of convection and diffusion of mass and momentum and energy.

So, momentum of course it is the Reynolds number. And then of course, you have ratios of diffusivities momentum to mass momentum to energy; these are the Schmidt number in the Prandtl number.

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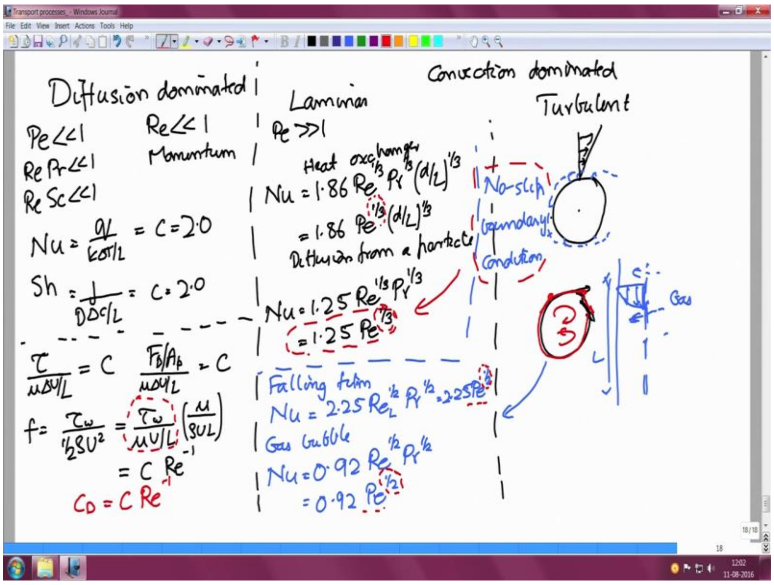

And we were trying to get some broad picture of what kinds of correlations you would expect on physical basis. We have divided the broad parameters regime into diffusion dominated and convection dominated. If diffusion is dominant; diffusion is dominant when the Peclet number is small whether it is for mass or heat transfer or the Reynolds number is small, in that case the fluid velocity field makes no difference to the transport rates it is primarily by diffusion.

Therefore, if I scale the heat flux and the mass flux by the product of diffusivity times a difference intensity divided by a length scale these should just tend to constant values.

The value of the constant of course you have to solve the problem to figure out what is the value of the constant, but it should be a constant value in the limit where diffusion is dominant.

Similarly, the shear stress scaled by viscous scale, the viscosity times the velocity divided by a landscape you can verify that that has dimensions of stress from Newton's law. That has to tend to a constant whether it is for a pipe flow for the flow past a particle wherever diffusion is dominant or where the Reynolds number is low this ratio has to tend to a constant value. Unfortunately to increase our confusion usually these momentum transport rates are scaled by inertial scales to get the friction factor in the drag coefficient and for that reason these things scale as inverse of Reynolds number in the limitation of number goes to 0.

The second broad category was convection dominated in a laminar flow. In this case the Peclet number is large and therefore the flows connection dominated, and as I said there is convection that is sweeping materials passed the surface and there is diffusion from the surface. When convection dominates the speed at which is being swept back will be much faster than the speed it is diffusing out; which means that the energy or the mass that is coming off the surface will be restricted to thin regions close to the surface. If the mass or energy that is coming off the surface, it will be restricted to thin regions close to the surface. By thin I mean that the characteristic distance is much smaller than the length scale in the problem, the particle diameter, the pipe diameter and so on.

And because it is restricted to thin regions the gradients are much larger there when you would expect, therefore the transport rates are much larger. And because convection is dominant the nature of the flow close to the surface is important. And broadly the kinds of flow can be divided into two broad categories: one is when there is a solid surface at a rigid solid the solid will not deform. And therefore if you are moving in the reference frame of the solid the velocity of the fluid has to come to 0 at the solid surface. So, it is called a no slip boundary condition.

And whenever you have a no slip boundary condition at the surface you have one set of correlations. The exact form of the correlation will change depending upon the configuration. You will have one form for the flow in a heat exchanger and another form for the flow passed a particle, but in all cases a common feature is that the average transport rates increase as the Peclet number to the one third power whenever there is a solid surface.

On the other hand, if you have a gas bubble or a liquid gas surface. At the surface itself the gas can deform, you can have internal circulation within the gas; the gas has very low viscosity. And what that means is that from the continuity of velocity at the surface, the velocity of the fluid at the surface gave the tangential velocity can be non zero. The normal velocity has to be zero so long as there is no penetration of the liquid into the gas. The normal velocity has to be zero, but the tangential velocity is non zero; and in that case you have a different set of correlations.

And of course, once again the form depends upon the exact nature of the flow, but in all cases it goes at Peclet number to the one half power. Why it does that, we will see later in the course. The third broad category is for turbulent flows.

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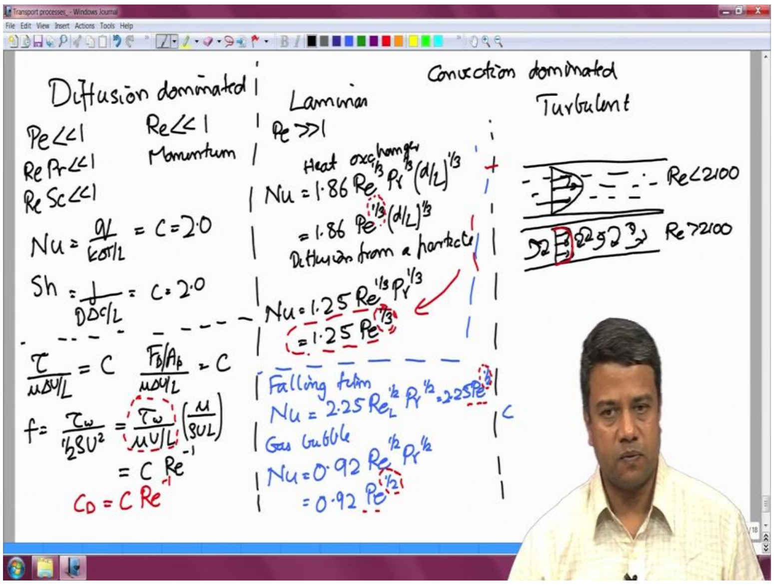

Now, first I should say a word about the distinction between laminar and turbulent flows. Take a flow in a pipe for example, at low Reynolds number you have a parabolic profile, smooth streamlines, and there will the transport across the stream lines has to take place only by molecular diffusion; because there is no fluid velocity perpendicular to the streamlines. This happens when the Reynolds number is less than about 2100. And the Reynolds number increases beyond about 2100; the flow has a remarkably different form.

The velocity is more plug like with a smaller curvature at the center and larger gradients close to the walls to the gradients close to the walls are larger. And you have turbulent eddies of different sizes circulating throughout the flow. The streamlines are not straight and smooth, but rather you have these turbulent eddies. And this takes place when the Reynolds number is greater than 2100.

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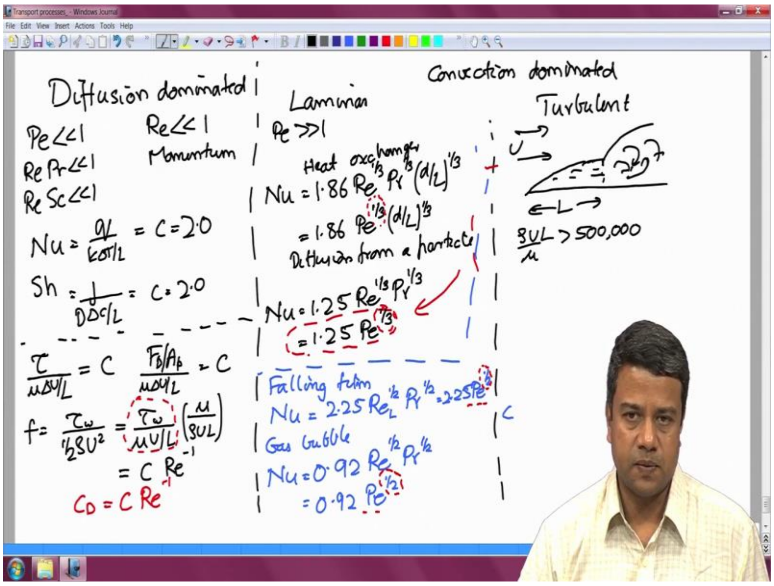

A similar thing happens for the flow past a flat plate for example; if you have the flow past a flat plate initially the flow will be nice and laminar at some point you will have smooth streamlines, some point you will get a transition to turbulence. This happens at the point where if this is the length along the plate into the free stream velocity. This happens when the Reynolds numbers rho UL by mu becomes greater than 500,000. This transition happens the flow past a flat plate it also happens in the flow past particle.

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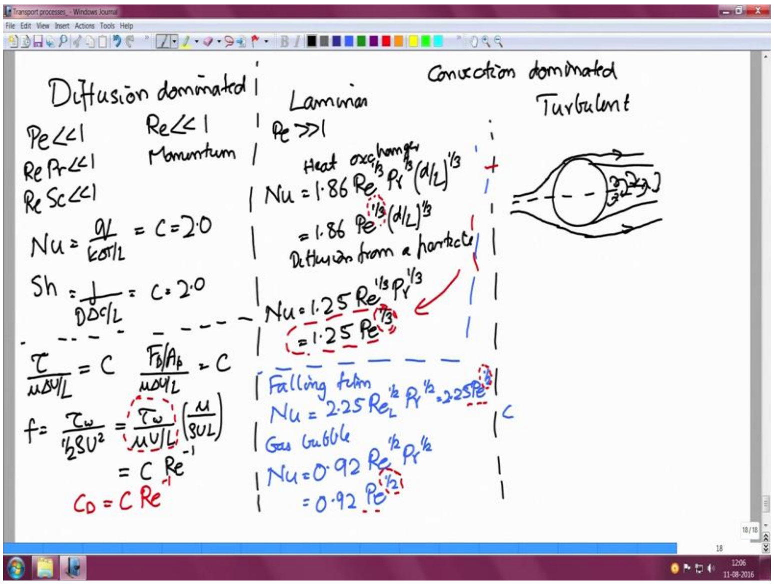

If you had the flow passed a spherical particle for example, initially the flow at low Reynolds numbers will be nice and laminar, smooth streamlines. As the Reynolds number keeps increasing at some point you will have the flow is separating out from the surface of the particle, it no longer is symmetric between the front and the rear. And then you have this wake region at the rear where you have circulation.

So, the point I am making is that this distinction between laminar and turbulent flows takes place abruptly at a particular value of the Reynolds number. Beyond that there are no smooth streamlines, but rather there are large crustal velocity fluctuations. And in that case you have transport not just due to molecular diffusion, but also due to the fluid velocity fluctuations. The fluid eddies that are present in a turbulent flow carry with them mass momentum or energy.

And this significantly increases for transport trades; it was significantly increases the transport rates. And for that reason you get different correlations in the turbulent flow in comparison to a laminar flow.

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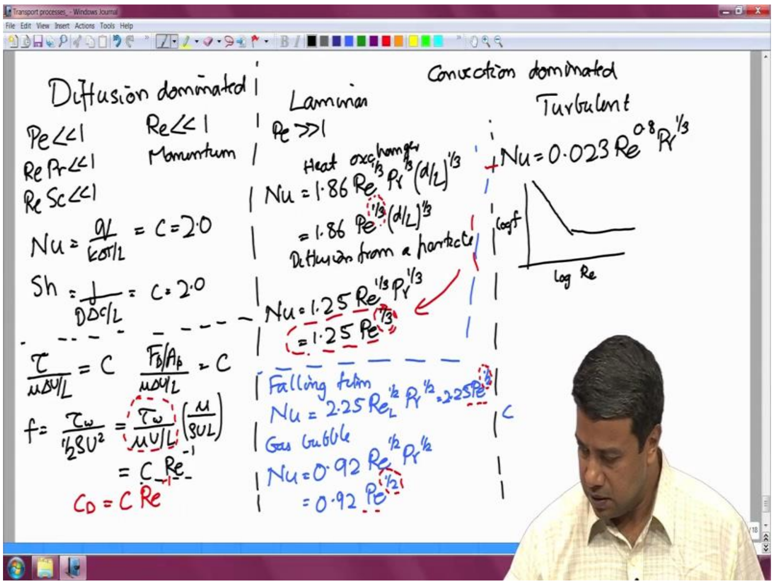

So, in a heat exchanger for example, the correlations are turbulent flow we had seen it earlier Nusselt number is equal to 0.023 R e power 0.8 Pr power one third. You have similar correlations for the flow past a particle and so on. And similarly the friction factor versus Reynolds number: a plot log of friction factor versus log of Reynolds number. For a laminar flow this goes has one over R e because it has to scale as the inverse of R e over here. For a turbulent flow at some point it undergoes a transition to some other value that value does depend upon the exact configuration.

So, the correlations in a turbulent flow are different from those in a laminar flow. And the transport rates are significantly higher because transport takes place due to both fluid velocity fluctuations primarily and not so much due to molecular velocity fluctuations And this process is called the process of dispersion rather than diffusion, it is not really molecular diffusion it is dispersion of mass and energy due to the fluid velocity fluctuations.

The final topic that we need to discuss correlations for is what is called as natural convection.

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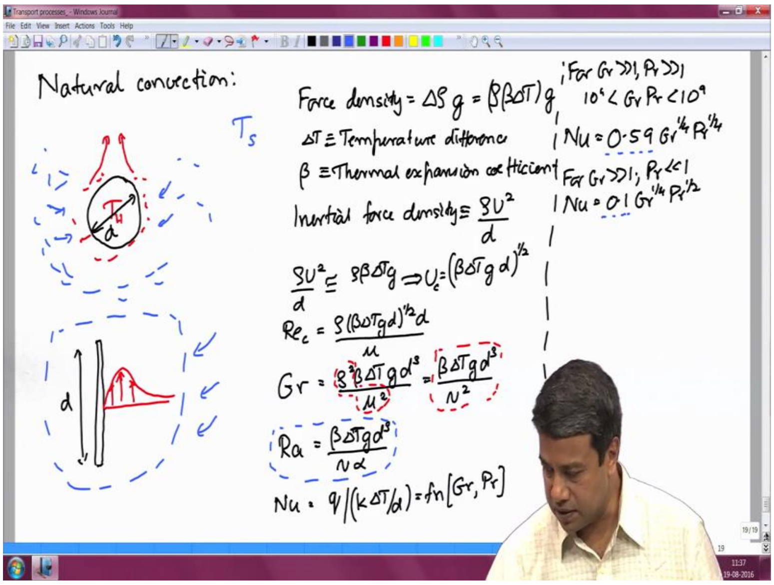

This is a process that significantly enhances heat transfer from objects primarily used for heat transfer; from objects which are immersed in a fluid, it could be either just air or water. The object itself is at a higher temperature which was significantly higher than the surroundings, whereas the surrounding temperature is much colder. And because of that what happens is that the air close to the object is heated up and because it is hotter it has a lower density. Whereas, the air that is further away from the object it is actually colder, and therefore it has a higher density.

And therefore, this hot air actually rises because it is has a lower density it is lighter close to the object therefore it rises, and as it rises the cold air comes in. And in this way the heat is swept away by the hot air in a region above the object which is called the thermal plume above the object and the cold air continuously comes in. So, in a sense the temperature difference itself is creating a flow which is convicting heat away. And this mechanism of heat transfer could result in significantly higher transport rates in comparison to just thermal conduction.

In fact, this is the primary mechanism of cooling in air of those hot objects or hot fluids. Typically used either as an object of this kind with certain diameter or just a flat plate give it is heated a fin if you will which has significantly higher temperature, and therefore you generate a velocity field close to the object which carries away the heat and then you have fluid coming in from far from the object. And in order to model this now there is no imposed velocity here in contrast to the forced convection problems that we had considered so far. The velocity is generated automatically due to the temperature difference and the consequent density difference. Therefore, we need away of estimating the velocity itself before we can go ahead and write down dimensionless numbers to describe the natural convection process.

Now, the velocity is of course created by the buoyancy forces, it is a buoyancy force density. And therefore, the velocity will be determined from the buoyancy force density balanced by some other terms in the momentum conservation equation. Usually, these free convection natural convection problems the length and the velocity scales are sufficiently large; that for these kinds of problems the inertial terms in the momentum conservation equation are higher than the viscous terms. When we do the analysis of natural convection problems, we will come back and see how to scale all of these parameters. But for the moment we will proceed with the assumption that the buoyancy forces are balanced by fluid inertia.

So, what is the buoyancy force density due to the temperature difference? The force density is of course the force per unit volume which can be written as the difference in density between the hot and cold parts times the gravitational acceleration. So, mass times acceleration is a force, the difference in density times the acceleration is a force density.

Now, the change in density due to temperature can be written as rho beta delta T times g; where delta T is the temperature difference, and beta is what is called as the thermal expansion coefficient. This thermal expansion coefficient is the fractional change in density due to a unit change in temperature; it tells you how much the density changes. So, rho beta times delta T is the change in density due to a change in temperature delta T. And so this force density is the driving force for the natural convection, and this has of course to be balanced by the inertial force density if we assume that fluid inertia is large compared to fluid viscosity for these kinds of problems.

Now, what is the inertial force density? The inertial force density; the kinetic energy density can be written as half rho v square where rho is the density and u is the velocity, it is kinetic energy per unit volume. To get the force density I have to divide the kinetic energy density by a characteristic length scale in the problem. That length scale could be for example the size of this heated object or the length of this heated plate. Therefore, the inertial force density I will just get from the kinetic energy density divided by the characteristic length. So, this will scale as rho u square divided by d; where d is the length of the plate or the characteristic dimension of the object. And by balancing this inertial force density with the force density due to the temperature difference the thermal for the buoyancy force density we can estimate what is going to be the velocity that is generated due to the force exerted by buoyancy due to the temperature difference.

So therefore, I will get rho u square by d is rho beta delta T g; which gives me a velocity scale u is equal to beta delta T g times d to the half power. Now the beta delta T is a dimensionless it is a fractional change in volume due to with change in temperature delta T. So, this is the characteristic velocity scale due to convection, the characteristic velocity scale that is generated by the buoyancy forces.

Now, one can define dimensionless numbers based upon this catalytic velocity scale. The Reynolds number based upon this characteristic velocity scale will be defined as rho times the velocity divided by multiplied by the length divided by a viscosity. The density times the velocity times the characteristic length scale divided by the viscosity so that is a Reynolds number.

However, traditionally it is not written this way it has written as the square of this, the ratio of the inertia and viscosity which is the Reynolds number. Traditionally what is defined as the Grashof number is the square of this; rho square beta delta T g d cubed by mu square it is called the Grashof number. You can also be written as beta delta T g d q divided by the kinematic viscosity square, because this combination it is just the inverse square of the kinetic viscosity I had shown you earlier that the kinematic viscosity is equal to the dynamic viscosity mu divided by the density that is also the momentum diffusivity.

So, this is how the Grashof number is defined. And this basically done is the ratio of inertia and viscosity based upon the characteristic velocity generated by the difference in density, so this is the physical meaning of the Grashof number. This written this is the square of the Reynolds number by convention rather than the Reynolds number itself.

Other dimensionless number that is often used is rather than writing the denominator in this case as the square of the kinematic viscosity I can also write it as the product of the kinematic viscosity times thermal diffusivity, and that is what is called the Rayleigh number. In the denominator instead of having the square of the kinematic viscosity is write it as a prognostic kinematic viscosity and the thermal diffusivity. Of course, you get the Rayleigh number from the Grashof number by just multiplying it by the Prandtl number; so I just simple way to think about it. You just multiply the Grashof number by the Prandtl number. Recall the Prandtl number is the ratio of momentum diffusivity by thermal diffusivity.

So, if I multiply the Grashof number by the ratio of momentum diffusivity divided by thermal diffusivity I will get this Rayleigh number. So, in these natural convection problems there is of course the Nusselt number, the dimensionless heat flux scaled suitably by the diffusion scales the thermal conduction scales. In addition so this is the dependent variable, is the Nusselt number is the thermal heat flux divided by k delta T by d; scaled by the diffusion scales that is the Nusselt number. That now has to be a function of either the Grashof number or the Rayleigh number.

It is traditionally assumed to be a function of the Grashof number as well as the Prandtl number. So therefore, the thermal heat flux it scaled thermal heat flux the dependent variable is written as functions of the Grashof number and the Prandtl number. So, in this natural convection problem we have reduced it to the relation between the Nusselt number and just two other numbers; the Grashof and the Prandtl number.

Beyond this of course one cannot proceed just based upon dimensional analysis alone, you need to have some functional dependency. And there are correlations for this natural connection as well. For example, for Grashof number and Prandtl number large, typically the range is given as 10 power 4 less than Grashof Prandtl less than 10 power 9. The Nusselt number correlation of the form Grashof number power one fourth Prandtl number power one forth.

This correlation is for this problem of the heat transfer from a vertical plate, for the heat transfer from objects of other shapes this coefficient will change. However, the dependence on the Grashof and the Prandtl numbers will not change even if the shape of the object changes; though the numerical coefficient in this correlation will change.

Similarly, in the limit where the Grashof number is much larger than 1 and the Prandtl number is small; the correlation has another form here would 0.1 Grashof power one fourth Prandtl power half. That is once again for a vertical heated plate. Once again this numerical coefficient will change it depending upon the geometry that is of the system. However, the dependence on the Grashof and Prandtl number will not change.

How do we get these correlations? As well as of all those other correlations that we have seen in forced convection problems, the heat exchanger problem, the flow from, the mass transfer from a particle, the momentum transfer correlations, the drag coefficient, the friction factor and so on. How do we derive these? That will be the subject of this course.

So, in the next lecture I will start looking at how to derive these kinds of correlations from microscopic description, which takes into account the balance between convection and diffusion at each location within the flow. Based upon that we reconstruct what is the temperature field or the concentration field everywhere. And from that we find out what is the flux from the surface. And from that get back and determine what that value of the correlation should be.

Seems rather roundabout way to go about getting this correlation, but once we have done the problem in this way we know what is the value of the temperature at every location within the system or the concentration based upon the velocity field. So, this is a much more detailed description of the entire system it does not restricted to just hanging out the flux on the surface, but rather we are solving for the temperature and velocity field at each location.

How do we go about doing that? I will start the next lecture talking about how we go about deriving these equations for heat mass and momentum class were based upon balances. And once we have done that I will give you a brief introduction to the process of diffusion. How does diffusion take place? Why do diffusivities have the values that they have? And how can one understand the molecular origins of diffusion? That will be the program for the next module. And once we finish that we will actually solve problems and actually start deriving correlations of this kind.

So, that is the program for the next few lectures. I will start by giving an introduction of the methods that we use for delivering these relations. And then talk about diffusion, then we will actually start for the problems. So, we will start that in the next lecture. I will see you then.