Transport Processes I: Heat and Mass Transfer
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Lecture – 11
Dimensional analysis: Correlations for dimensionless groups

Welcome to this is our lecture number 11 in our course on transport processes, so we are well on our way to analyzing transport processes in a more quantitative manner. I had in the previous two lectures solved a couple of practical problems for you; how does one go from the complicated system that we have to a smaller set of parameters that describe the entire system. We had done that for the heat exchanger problem if you recall.

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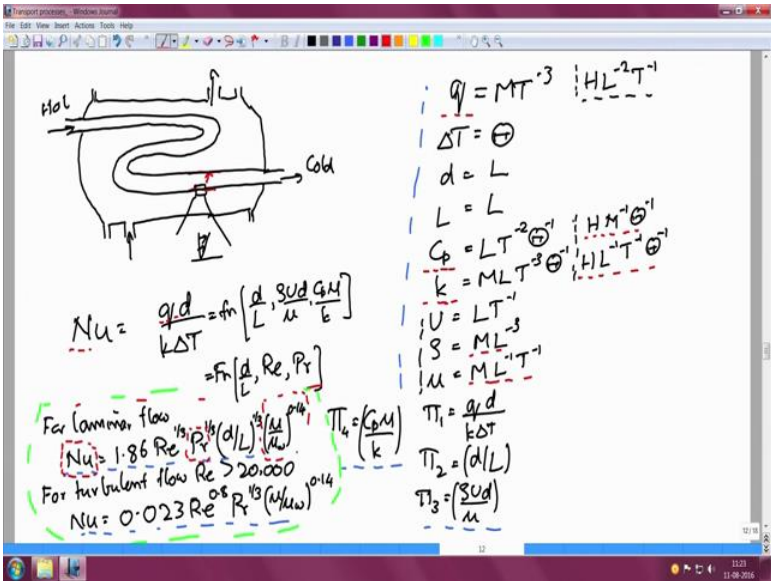

Where we had defined the dimensionless Nusselt number which is basically a heat flux scaled suitably so that it becomes dimensionless. We found that it depends only upon two other dimensionless numbers called the Reynolds number and the Prandtl number. And for a given configuration if you knew what the heat flux were for one particular set of parameters you could find the heat flux for other parameters using relations of the kind shown at the bottom left of your screen. For a laminar flow you have one set of relations for turbulent flow you have another set of relations. In both of these cases you can see that it depends on Reynolds number to the one-third power, Prandtl number to the one-third power and some parametric dependence on the dimensions of the problem.

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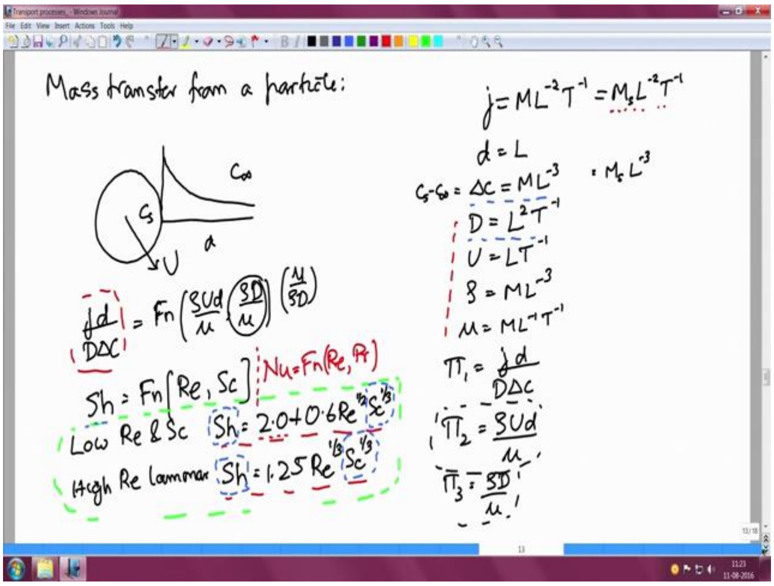

We had done that for mass transfer for particle. We had got a similar functional form for the Sherwood number. The non dimensional flux the Sherwood number as a function of two other parameters. The Reynolds number and another number called the Schmidt number. And so the original problem had a total of 7 parameters we reduced it to just 3. And I told you the kinds of relations that would result for laminar flows for low Reynolds number for high Reynolds number and so on.

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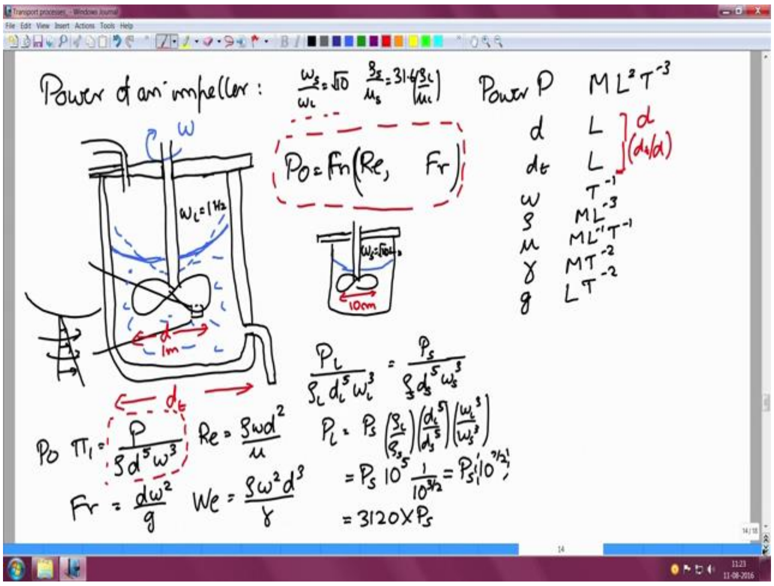

And I had also solved for you problem of power of an impeller required for stirring the fluid and showed you how you can reduce the total number of parameters which is eight down to just three or four of them based upon dimensional analysis. And a further reduction can be effected based upon the dimensions of the dimensionless parameters that arise from our physical understanding of what these dimensionless parameters mean in terms of ratios of different kind of forces.

For example, Reynolds number is the ratio of inertial and viscous forces. If Reynolds number is small you can neglect inertial. Similarly, the Weber number is the ratio of the inertial forces and the surface tension forces; if that is large you can neglect surface tension and so forth, and we used it for a concrete scaling problem.

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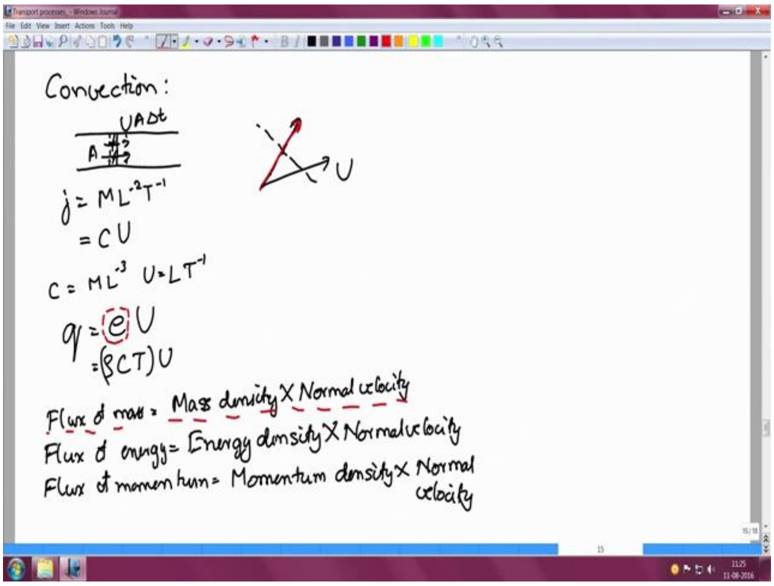

Now, we have taken a deeper look at what these parameters mean as a ratio of convection and diffusion. Convection is transport along with the mean fluid velocity and the convective flux transport of any quantity per unit area per unit time is equal to the density of that quantity times the flow velocity perpendicular to the surface.

The fundamental relation for convection; the flux across the surface is equal to the density of that quantity the mass per unit volume energy per unit volume momentum per unit volume times the velocity perpendicular to that surface. So therefore, these convective fluxes always scale as quantity times the velocity.

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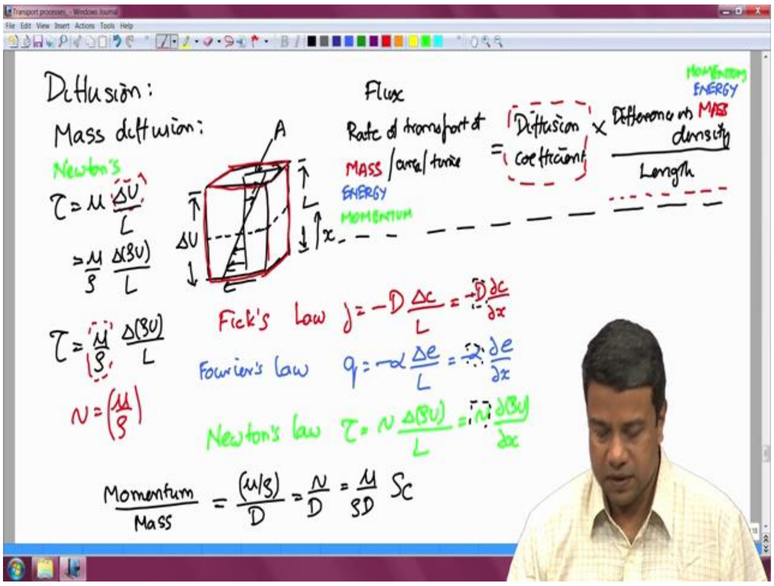

This is the crux of the matter, convection and diffusion. Diffusive flux I had told you that it is equal to can be written as rate of transport of anything per unit area per unit time; this can be written as a diffusion coefficient times the difference in the density of that quantity divided by a length. Diffusion, typically takes place across a particular area of fluid across a certain length. And the diffusion flux of any quantity can be written as a diffusion coefficient times the difference in that quantity divided by the length across which should differs.

So, this gives you the flux due to diffusion. Diffusion at it is origin is of molecular phenomenon, it takes place due to the fluctuating velocity of the molecules. Therefore, in order for mass to diffuse you require that there should be a difference in concentration, whereas convection takes place due to mean fluid velocity. So, convection it takes place even when there is the concentrations are constant, whereas for diffusion you need a driving force that driving force is the difference in the density of that quantity divided by the length and express this way all diffusion coefficients for all quantities will have dimensions of length square per unit time.

So, Fick's law for diffusion you have the diffusivity here the molecular diffusivity for mass. Fourier's law; if I write it in terms of the difference in the energy with density instead of the difference in temperature I do get a diffusion coefficient here, that is a thermal diffusion coefficient. As I said the thermal diffusion coefficient is basically equal to alpha is equal to k by rho c p, if I write the driving force of the difference in the energy density instead of in the difference in temperature.

Similarly, Newton's laws viscosity if I write the driving force as the difference in the momentum density, the momentum density is mass density times velocity. So, if I write the equation in terms of the momentum density then the diffusion coefficient is the kinematic viscosity which is the viscosity divided by the density. So, these are three diffusion coefficients for mass, momentum, and the energy.

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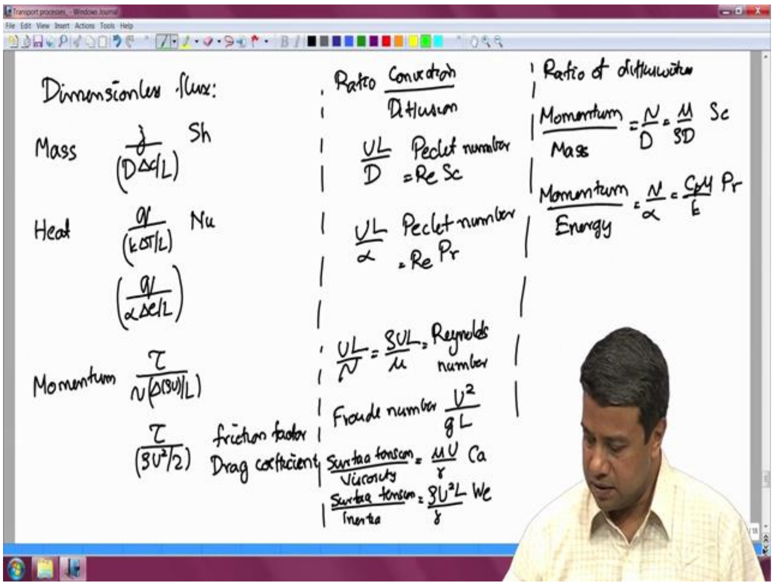

And now all dimensionless parameters as I said can be expressed in terms of ratios of these. First, I have the dimensionless fluxes, the dimensionless mass flux; mass flux is mass per unit area per unit time. The simple way to non dimensionalize it is to just realize that the Fourier's law, I am sorry the Fick's law for mass diffusion states that the flux is equal to minus D times the difference in concentration by the length. This is valid locally at every point. However, if you look at the global mass transport from a droplet for example transport takes place due to diffusion as well as convection. So, for the entire system a relation of this kind will not be valid because you have to take into account convective effects as well.

However, this Fick's law enables me to define dimensionless quantities because I know that in this Fick's law the dimensions of all quantities on both sides are equal. So, I could get a dimensionless flux by this scaling the mass flux with D times the difference in the concentration difference divided by a length scale; that is the dimensionless flux for mass the Sherwood number. Dimensionless flux for heat use Fourier's law you know that term on both sides scale the same thing. So, I can define the dimensionless flux of heat as a Nusselt number q by k delta T by L, if expressed in terms of the temperature difference what comes in is the thermal conductivity.

On the other hand if I express in terms of the difference in energy density what comes in is the mass diffusion coefficient. So, that is the simple way the dimensionless numbers. For momentum transport, if I were to scale it by the diffusion coefficients times the difference in momentum density divided by length what you will get is a number that looks something like this. As I have told you in the last class this is not the way it is usually done, what is usually done is to scale it by rho U square by 2. And as I will just show you shortly this non dimensionalization differs by a factor of Reynolds number from the viscous non dimensionalization.

So, these are the dimensionless fluxes the dependent quantities, they will depend on some independent dimensionless parameters. These are of two kinds: one is ratio of convection and diffusion and the other is the ratio of two different kinds of diffusion. Ratio of convection and diffusion Peclet numbers for mass transfer UL by D it is the convective transport is equal to U times c U times delta c if it was there is a concentration variation. Diffusive transport is D delta c by L and you divide the two you will get UL by D the Peclet number for mass transfer. For heat transfer you just replace the diffusivities.

Convection remains the same it is all proportional to the velocity, replace the mass diffusivity by the heat diffusivity and you will get UL by alpha. And as I showed you this is a Peclet number for mass transfer for heat transfer and it can also be written as the product of the Reynolds number and the Prandtl number. The Peclet number for mass transfer is the probability of the Reynolds number and the Schmidt number. For momentum transfer you have UL by kinematic viscosity. And if you substitute for the kinematic viscosity you have the Reynolds number which is the ratio of convection and diffusion of momentum or it can also be interpreted as the ratio of inertia and viscosity.

And then you have dimensionless numbers which are the ratios of diffusivities, momentum by mass diffusivity which is basically the kinematic viscosity divided by the diffusion coefficient. We have seen that that one was just equal to the Schmidt number that we had in our correlations. Momentum to energy diffusion the ratio of the kinematic viscosity and the thermal diffusivity that is just equal to the Prandtl number. And you can multiply these ratios of diffusivities multiplied by the Reynolds number to get the Peclet numbers for (Refer Time: 11:40).

And I have also listed out a set of other dimensionless parameters for you which are also important. I should remark at this point that the Froude number is written in different ways: one is to write it this way, the other is to write it as the square root of this. So, sometimes you will be square see the Froude number written as U square by square I am sorry U by root of g L that is still the dimensionless number. And then there are dimensionless numbers relating to surface tension and so on.

So, now that we have identified these dimensionless numbers, can we somehow deduce the form of correlations between these dimensionless numbers?

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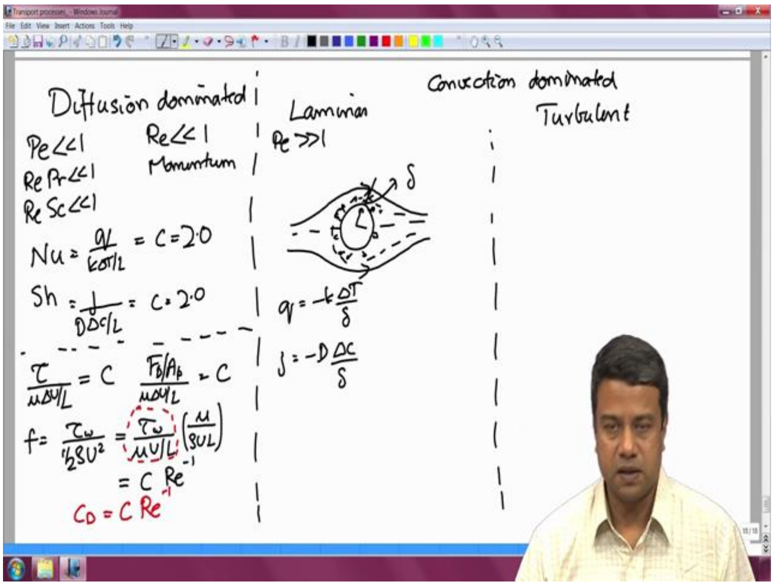

Now all these correlations will depend on the ratios as convection and diffusion. So, you will have different correlations one set of correlations in the regime where it is diffusion dominated. And the other is the regime where it is convection dominated. In the diffusion dominated regime the Peclet numbers are small; I told you the Peclet numbers are ratio of convection and diffusion. So, diffusion is the fastest mechanism of transport then the effect of convection is small. So, this is in the regime where the Peclet number is small that is the Reynolds number times the Prandtl number for heat transfer is small and the Reynolds number times the Schmidt number for mass transfer is small. So, momentum transfer which is the Reynolds number itself.

So, this is the regime when the flow is diffusion dominated, the transport is diffusion dominated. Transport is primarily due to diffusion, therefore convection does not matter. And the fluid velocity field does not matter because diffusion is the fastest mechanism and, therefore whether there is a flow or not is not going to affect the transport rates very much in that case the velocity should not be a relevant parameter the fluid velocity should not be a relevant parameter. And the fluid properties the mechanical properties should not be a relevant parameter, because convection is not important.

And in that case the dimensionless fluxes that is the Nusselt number and the Sherwood number which are fluxes which are non dimensionalized by diffusion. So, q by k delta D by L and j by D delta c by L, both of these should go to constant values. They cannot depend to the Reynolds number because convection is not important, and they cannot depend on the Peclet numbers because convection is not important; Peclet number is going to 0.

And so in this limit correlations will inevitably tell you that the Nusselt number and the Sherwood number go to constant values. And we had actually seen that for the correlation for the mass transfer and the heat transfer from a particle 2 plus 0.6. Therefore, these go to constant values in the limit where the diffusion is dominant. And in fact, in the limit as the Peclet number becomes small all of these will just tend to a constant value which for the case of a spherical particle is just 2. And we will come back and actually calculate this value of 2 for a spherical particle. So, the limit where diffusion is dominant the Nusselt number and the Sherwood number will all go to constant values.

Similarly, in the case of momentum transport in the limit where the diffusion is dominant the shear stress when scaled by viscous scales weight mu delta U by L should go to a constant value. If you recall when we did the dimensional analysis of the flow past is sphere.

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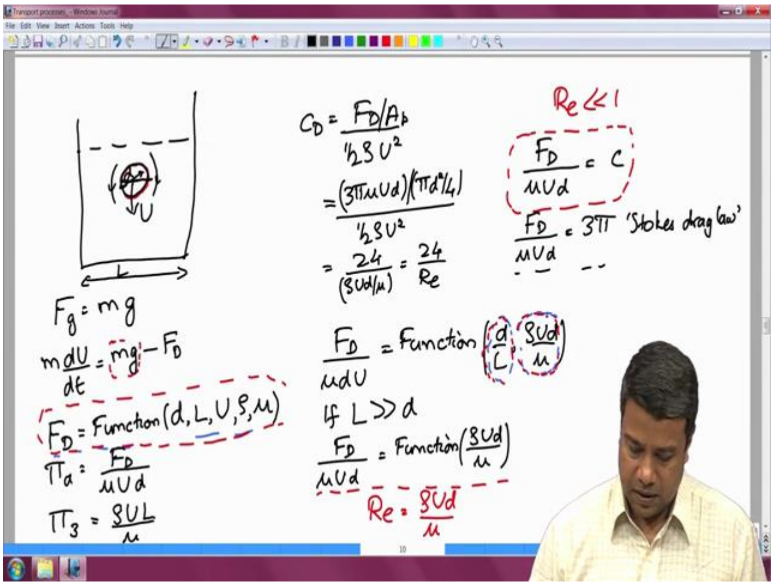

I had got the drag force new type divided by mu D U to these some function of D by L in the Reynolds number. And I said when the Reynolds number is small this goes to a constant value. So, this is stokes drag law that is because it cannot depend upon inertia anymore, and therefore this is the only way you can get a drag force when scale purely by viscous scales.

And the same holds for the flow in a pipe as well, when the viscous effects are dominant the flow is primarily dominant by diffusion and this should go to a constant value. The constant value will depend on configuration whether it is a sphere a spherical particle or whether it is a pipe and so on, but in any case the shear stress divided by mu delta U by L will go to a constant. Similarly, the drag coefficient which is basically the drag force divided by projected area divided by mu delta U by L will go to some constant value. These are different constants of course it will depend upon the configuration.

Now, as I said the drag coefficients are usually defined, the friction factor is usually defined as tau w by half rho U square. Let us traditionally define it scale by the inertial scales. I can write this as tau w by mu cube by L into mu by rho UL. Just rewriting the friction factor in this manner and you can see that this ratio has to go to a constant in the limit when the flow is diffusion dominated. Therefore, this has to be some constant times Reynolds number inverse. That is the reason the friction factor goes as one over Reynolds number in the limit where as diffusion dominated, because historically this friction factors has been scaled by the inertial scales, the pressure scales half rho U square. If I had scaled it by the viscous scales then when viscosity is dominant of course the friction factor the thus the scaled stress will go to a constant.

Same thing applies for the drag force, for the drag force on the particle by L has to be equal to a constant which means that f D by A by half rho U square, I can write it as f t by A by mu U by L into mu by rho UL; it is not quite equal to is proportional to because you have factors here between the area and the. And this thing is tending to a constant in the limit of low Reynolds numbers. And therefore, the drag coefficient c D will also be equal to and this is the inverse of the Reynolds number, so the drag coefficient will also be equal to some constant times the Reynolds number inverse in the limit of low Reynolds number.

Friction factor for a pipe for example goes as 16 by R e. Drag coefficient for a sphere goes as 24 by re. Those constant values do depend upon the configuration, but the proportional p dependence on the Reynolds number inverse does not depend upon the configuration because in this limit inertia is no longer important and therefore if you scale the drag force by the viscous scales you should get something that goes to a constant. So, that is in the diffusion dominated regime.

Nusselt number Sherwood number will all tend towards a constant because convection is not important, whereas the friction factor and the drag coefficient go as Reynolds number inverse because I have scaled these by inertial scales or pressure scales. The connection dominated regime you have to two broad subcategories: one is for laminar flows and the others for turbulent flows. These are the two broad categories in convection dominated regimes.

Now, take the simple case of the flow past a sphere for example, and if you have some fluid going past this. If it were diffusion dominated of course the fluid velocity field will not matter, you can just diffusion that would transport material of this particle. Take the droplet of that was drying in the spray dryer for example, diffusion would be the down.

When convection is dominant there is material that is diffusing out of the surface at the same time that material is also being swept by the fluid flow past that surface. So, there are two effects that are happening. What does it mean when you say that convection is dominant? That means, that the Peclet numbers are much larger than 1. Convective forces are much faster than diffusive effects in this regime. However, as I had emphasized in the past when convection is dominant, convection can only transport material along the direction of the fluid velocity. There is no convection perpendicular to the droplet the convection is sweeping the moisture past the droplet.

Ultimately, the surface diffusion has to become important. And when diffusion becomes important if you are in the limit of high Peclet numbers then the material that diffuses out; will not diffuse out very far before it is swept backwards by the flow in the material that diffuses out it is not going to diffuse very far before it is swept by the flow, because the Peclet numbers are large. And therefore, this diffusion will be restricted to thin regions near the surface. This diffusion will be restricted to thin regions near the surface whether it is mass or heat and the Peclet numbers are large. And this is something that we will analyze later it is called Boundary Layer Theory.

But because it is restricted to thin regions for these thin regions if I write the equation for the heat flux or the mass flux simplistically, this length scale that I would normally use would just be the particle diameter D. You would think that that is the only length scale in the problem, and therefore this is how these would scale. That is not true as I said this mass flux or heat flux before it penetrates very far into the fluid it gets swept backwards.

So, therefore the flux is the concentration difference is across a small region close to the surface. Therefore, I have to take the length scale of that small region close to the surface over which the material gets swept before it diffuses. And that thickness can be much smaller than the thick the diameter of the particle when the Peclet number is large. So, what I need to scale it is not by this diameter itself, but by the thickness of this region; in the limit of high Peclet number. This region can be much smaller than the particle diameter which means the fluxes can be much larger.

One of the things we will do in this course is to find out how to estimate the thickness of that region. And because of this you do have a dependence on the Peclet number for the coefficients.

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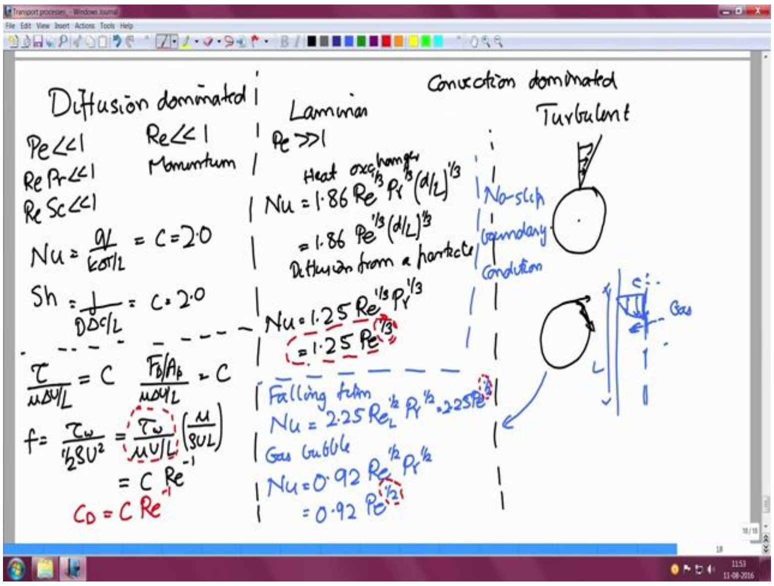

If you write the Nusselt number for laminar flow for example, for the heat transfer in a pipe if you recall we can wrote this 1.86 R e power one-third P r power one-third b by L power one-third or high Peclet numbers laminar flow. I could as well have written this as 1.86 the Peclet number power one-third, because the Peclet number is just the Reynolds number times the Prandtl number. The Reynolds number is convection to momentum diffusion, Prandtl number is momentum diffusion to thermal diffusion. Therefore, the product of those two becomes the Peclet number.

Similarly, this was for a heat exchanger. For the diffusion from a particle the Nusselt number was equal to 1.25 R e power one-third P r power one-third. I could also write this as 1.25 times the Peclet number power one-three. So, it depends on the Peclet number it depends upon the ratio of convection and diffusion, this one-third power why the one-third power will come back to a little later, but the other thing that it depends upon is the details of the fluid velocity field.

For example if I had a solid particle, I would require that the velocity of the fluid at the surface of the particle has got to be equal to the velocity of the particle itself. If I am sitting in a reference frame on this particle then the fluid velocity field has to decrease to zero at the particle, there is a no slip boundary condition for the particle. In that case you get this relationship; this is with the no slip boundary condition.

I could have for example a gas bubble; a gas bubble does not afford resistance to flow. So, at the bubble surface itself the surface can have a tangential velocity. It cannot have a velocity perpendicular to the surface because there can be no penetration, if you are moving in the reference frame of the bubble itself, but it can have a tangential velocity. And the details of the velocity very close to the surface are important; because I had said that the concentration or the temperature difference is restricted to a thin region near the surface and the details of the velocity in those thin regions is important for determining the transport rates.

So, what I have given you here are correlations which work when you have a particle, so called no slip boundary condition. These are valid when you have a no slip boundary condition at the surface. On the other hand, if you have a slip boundary condition at the surface the correlations are different. For a following film for example, even if I have a gas film that is falling with a certain velocity field and there is a mass diffusion happening into the surface. So, you have a liquid film there is a gas over here and from the gas you have mass diffusion into the surface. The difference between this and the heat exchanger is that at the surface where transport is taking place the fluid velocity is non zero; at the surface where the transport is taking place the fluid velocity is non zero in that case the correlation will change, the correlation that we will get will be is equal to about 0.91 R e power half, this R e is based upon the length of the film.

Similarly for a gas bubble, the correlation is of the form; I am sorry this should be 2.25. The Nusselt number will be equal to 0.92 R e power half P r power half. In both of these cases they depend only upon the Peclet number. So, you can see this boundary condition difference makes a big difference, this exponent changes depending upon what the boundary condition is. If you have a rigid solid surface with no slip boundary condition at the surface you will get one-third power for the scaling. The coefficient of course depends upon the specific geometry, but you will always get a one-third power. When you have a surface with the zero stress condition where those the fluid can slip along the surface, the tangential velocity is non zero along the surface, you always end up with the Peclet number to the half power scale. One of the things that we will do in this course is to try to examine why these kinds of power loss emerge.

Next class I will continue with these non dimensional correlations; we look at turbulent, flows how turbulent flows are different from laminar flows, what the correlations are.

And one final topic that we will have to go through before progressing to actual transport phenomena is the phenomenon of natural convection. A heated object in air will generate heat because of the increase in temperature that heat is convicted along with the fluid because close to the object the air is hotter it is lighter and it tends to rise.

And for those kinds of natural convection problems as well there are correlations. And you look at those correlations before we go on to that the core of the subject how do we analyze these systems at a microscopic level. So, that is the broad plan for this course. I will see you in the next lecture.