Transport Processes I: Heat and Mass Transfer
Prof. V. Kumaran
Department of Chemical Engineering
Indian Institute of Science, Bangalore

Lecture – 10
Dimensional analysis: Physical interpretation of dimensionless groups

Welcome to this continuing lecture on Dimensional Analysis, where we are now going a little bit deeper and trying to understand the meaning of all these dimensionless groups that arose and we were doing dimensional analysis.

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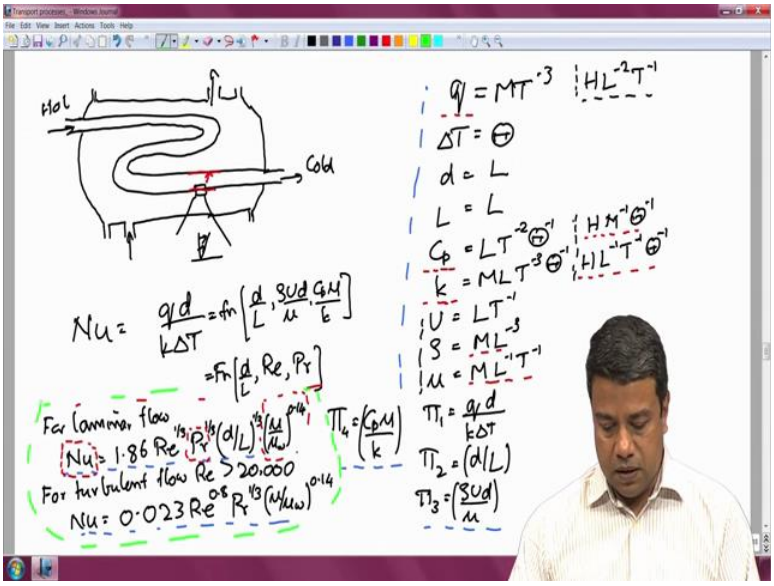

As you recall we did the dimensional analysis for the heat transfer across surface of a heat exchanger, and we got a relationship between these quantities the Nusselt number, the Reynolds number, the Prandtl number and so on.

The considerable reduction from the complexity is the original problem without a doubt, but still we still do not have a good idea what these things called Reynolds number and Prandtl number mean.

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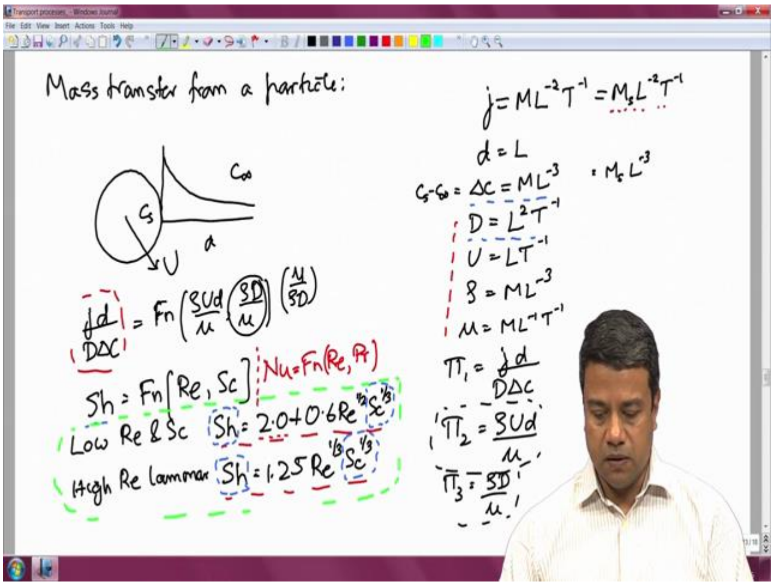

In the mass transfer problem we had mass flux there is Sherwood number dimensionless mass flux a function of Reynolds number and Schmidt number.

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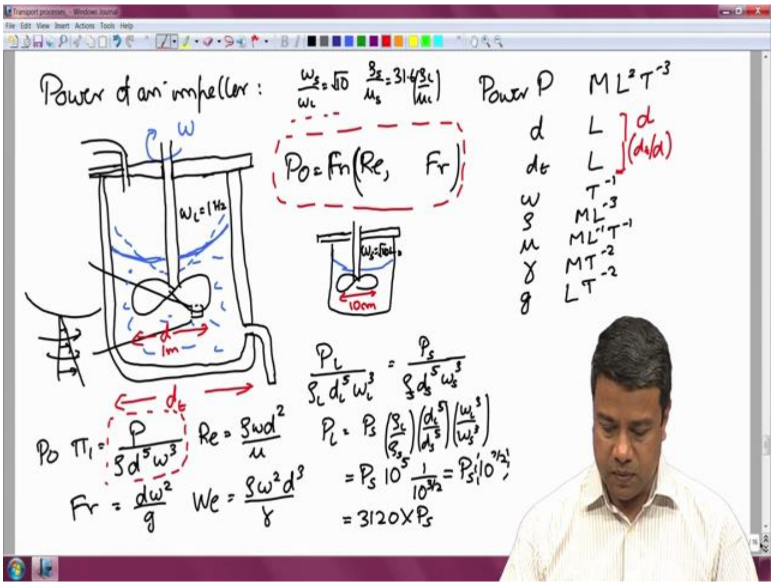

And if you recall when we looked at the transport we had a power number which was a function of other dimensionless groups, the Reynolds number, and the Froude number and so o. And I was trying to give you some idea of what exactly these dimensionless groups mean.

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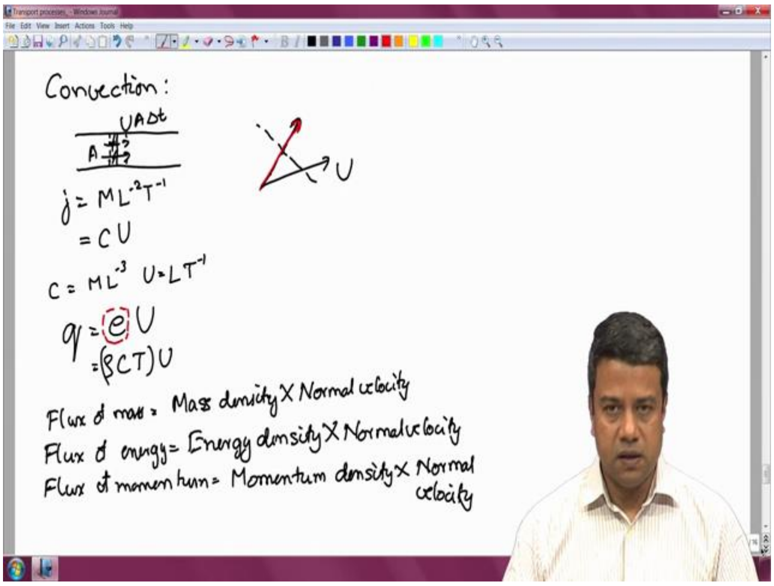

Two broad mechanisms of transport that we will be considering in this course; the first one is convection. Convection is transport because material is carried along with the fluid flow, because energy is carried along with the fluid flow, momentum is carried along with the fluid flow. The flux, the convective flux of material; material transported across a surface per unit area per unit time that is just equal to the density of the quantity mass momentum or energy with density of that quantity times the velocity perpendicular to the surface.

So, the mass density times the velocity there is a concentration times velocity. The energy density times the velocity and the momentum density times the velocity; that is convective transport.

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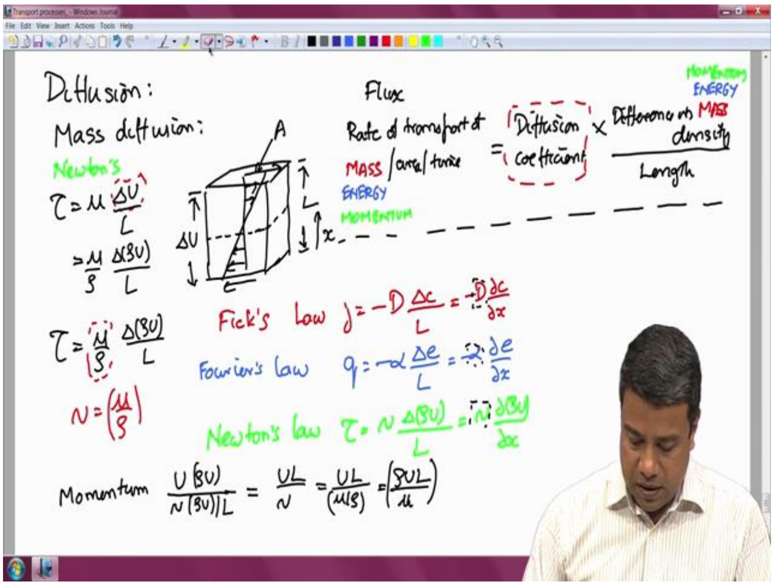

Diffusion transport on the other hand is basically a transport due to molecular velocity fluctuations which takes place even when there is no fluid flow across the surface. You know there is no fluid flow across the surface, material, energy, momentum, will still be transported across the surface because the fluctuating velocity of the molecules which carry along with that with material.

There will be a net transport of material only if there is a difference in the concentration or temperature across the surface, if there is no difference then what goes up will be equal to what comes down due to molecular fluctuations, there will be no net transport. And there is a difference, for example with the concentration below is higher than the concentration above, then the molecules from below will carry a higher concentration along with them molecules may above will carry a lower concentration, therefore there will be a net transport of mass across the surface.

Similarly, if there is a temperature difference the energy of molecules going up will be higher than the energy of molecules going down, therefore there will be a net transport of energy. The same things calls momentum; if there is a net flow tangential to the surface molecules going above will have a different momentum than those going down. So, diffusive transport basically depends upon the difference in concentration across the surface or more particularly the gradient in the concentration variation in concentration with respect to length across the surface.

And the rate of transport of all of these due to molecular fluctuations can be written in this column form. For any quantity the rate of transport of that quantity per unit area per unit time is equal to a diffusion coefficient times the gradient in the density of that quantity; density of that quantity that quantity per unit volume. The gradient of that gives you one more inverse length scale. And if you put those two together, this diffusion coefficient for any quantity as to have dimensions of length square per unit time.

So, we had expressed the Fourier's law, Fick's law and Newton's law in terms of this. And what we had got was diffusion coefficients for mass momentum and energy. I have inter changed these two here my apologies this should be Fick's law and this one should be Fourier's law. And these things here are the diffusion coefficients. Now when we do dimensional analysis if you want to find out what is the dominant mechanism of transport we have to find out the ratio of convection and diffusion.

For example, for momentum the ratio of convection and diffusion I told you that convection of momentum we will go as the velocity times the momentum that is been carried along with the flow. So, this will go as the velocity times the momentum which has being carried along with the flow which is just the momentum density itself. So, that is the convective effective, the momentum density times the velocity. Diffusion, it goes as the kinematic viscosity times the momentum density divided by a length; so nu into rho U by a length. So, the ratio of this will just be equal to UL by nu the ratio of this will be just equal to UL by nu the ratio of convection and diffusion. Ultimately, it will not depend upon the quantity that you considering because there is both and convection as well as diffusion the density comes in both.

Now, the kinematic viscosity is equal to the ratio of viscosity intensity, so if I write this way I get rho UL by mu. As you will recall this is our familiar Reynolds number. So, the Reynolds number actually is just the ratio of momentum convection divided by momentum diffusion.

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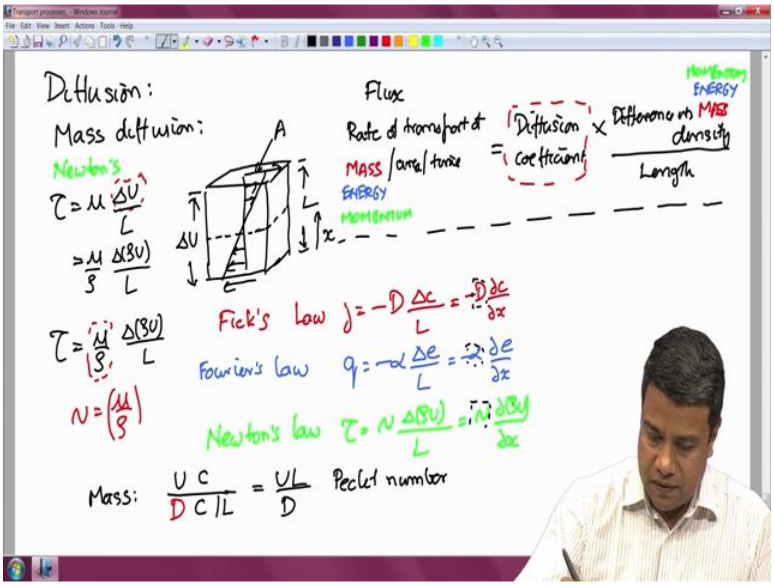

Now if I wanted to do this for mass or heat all I need to do is to change for mass transfer for example. I just need to change this to a concentration; instead of the momentum density I will have the mass density. So, this is concentration and then instead of the momentum diffusivity I will have the mass diffusivity. Therefore, this is just equal UL by D. This goes by the name of Peclet number from mass diffusion.

The same thing I could do for thermal diffusion; in that case the concentration has to be change to the energy density, we have to change for the thermal diffusion, I have to change the concentration to the energy density, and I have to change the diffusivity from mass diffusivity to the thermal diffusivity.

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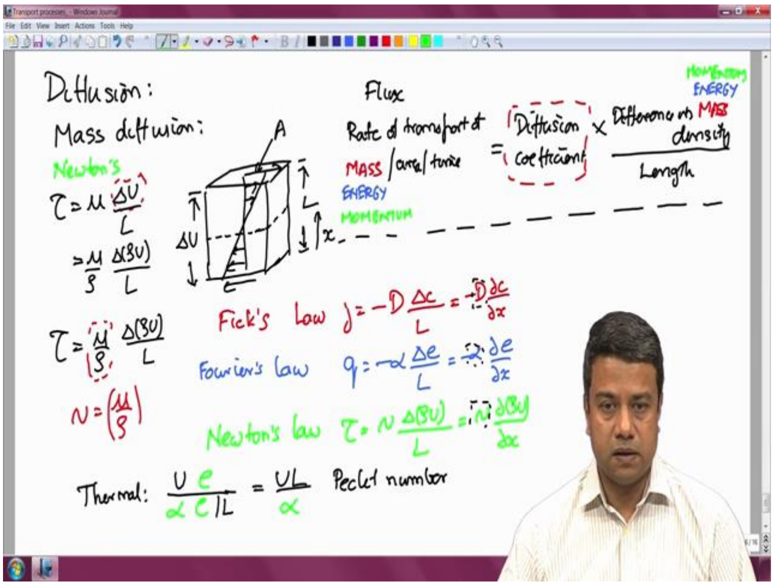

And therefore, finally I will just get instead of UL by D I will just get UL by alpha. That is the Peclet number for thermal diffusion. So, these are the dimensionless numbers that are ratios of convection and diffusion. You could also have dimensionless numbers which are ratios of two diffusivities.

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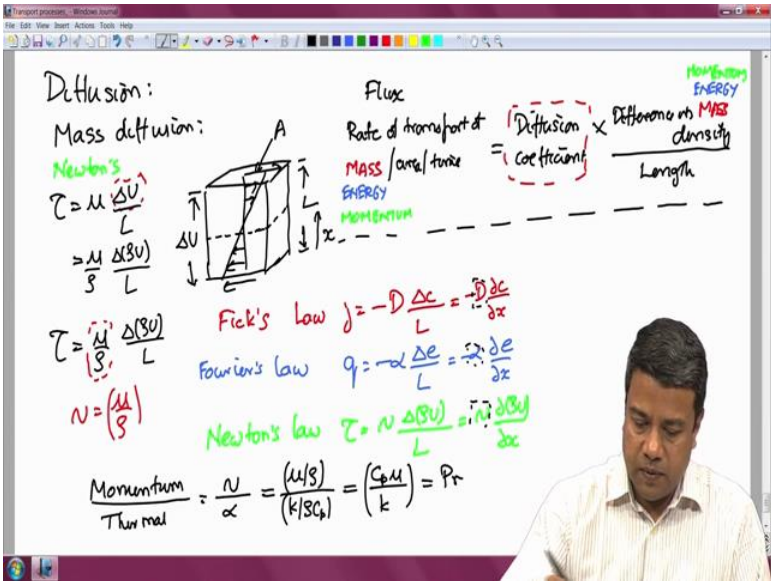

How would that look? If I take the ratio of momentum by thermal diffusivities, the momentum diffusivity was the kinematic viscosity nu; the thermal diffusivity was the thermal diffusivity alpha. I can express that back in terms of the original quantities; this is mu by rho if you recall the thermal the momentum diffusivity is the kinematic viscosity, ratio of the viscosity and the density. The thermal diffusivity was k by rho c p, when I had expressed in terms of the energy density thermal diffusivity of k by rho c p. And this gives me c p mu by k. If you recall when we did the heat transfer problem this was the Peclet number.

So, the Peclet number is the ratio of momentum diffusivity and the thermal diffusivity. You can take the ratio of momentum diffusivity and mass diffusivity.

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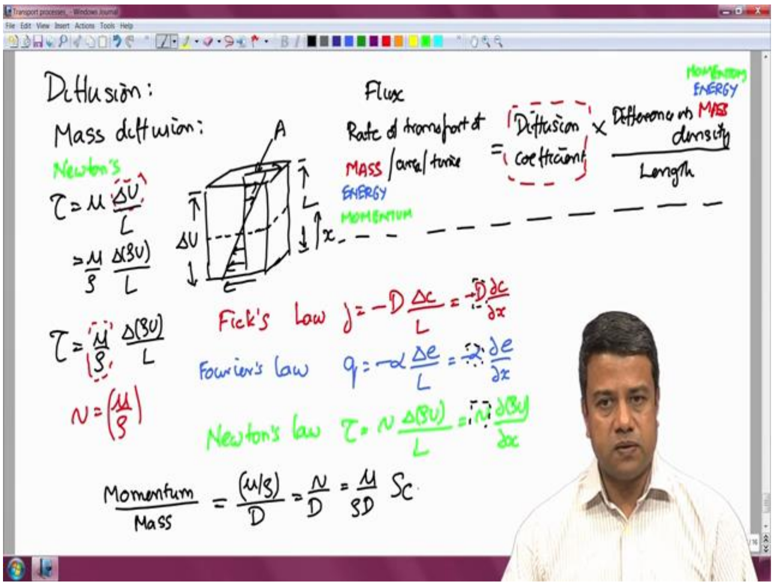

Since, all diffusion coefficients are the same dimensions you can take ratios of each of them against the other and all of those will end up being dimensionless groups. So, this turn out to be mu by rho divided by the mass diffusion coefficient. Basically that is equal to nu by the mass diffusion coefficient the kinematic viscosity, so you get mu by rho d.

And if you recall when we did the mass transfer problem that was what was call the Schmidt numbers mu by rho D (Refer Time: 13:01) in the terms of mu by rho D use the inverse instead of rho D by mu and that another being Schmidt number. So, once we understood these fundamentals of convection and diffusion all of those dimensionless groups that we had can be classified into a few fundamental forms.

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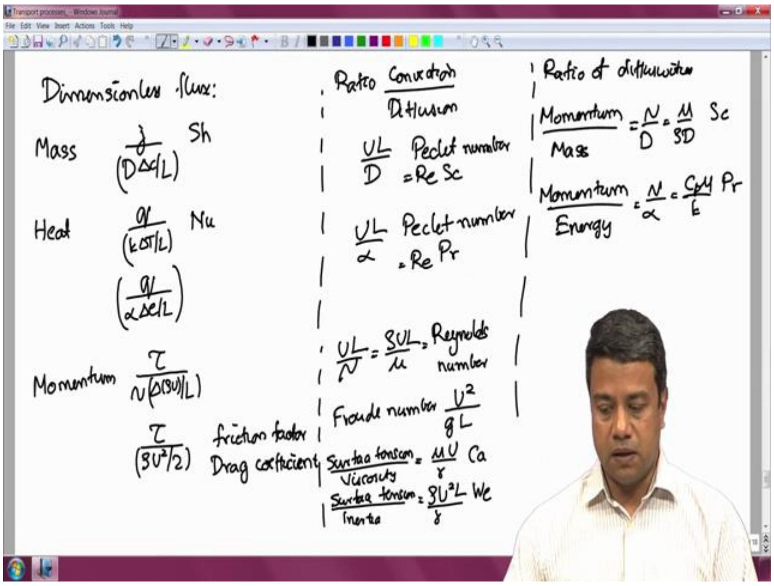

One is of course, the dimensionless fluxes. As I said these dimensionless fluxes emerge from a relation of this kind; rate of transport of material is equal to diffusion coefficient times the change in density divided by a length. So, dimensionless fluxes in this case for mass transfer we will just turn out to be j by D delta c by L, because I know that D times delta c divided by distance as to have the same dimensions as j. So, j by D delta c by L will be a dimensionless group; so that is the Sherwood number. For heat transfer this will be q by k delta T by L and this is the Nusselt number, because from Fourier's law of heat conduction q by k delta T by L has to have the same dimension, has to be dimensionless.

I could also express it in terms of the energy density and the thermal diffusion coefficient. We can easily verify that this can also be written as q by alpha delta t by L for heat transfer I am sorry delta e by L. But diffusion coefficient times the difference in the density of the quantity divided by that the crossing distance of the \ gradient in that direction that is the Nusselt number. And for momentum transfer you would think that this stress could be scaled by the kinematic viscosity, the diffusion coefficient times delta of rho U by L. This is not what is usually used, what is usually used is a friction factor or the drag coefficient and there rather than using the viscous effects what is usually done is to use the inertial effects rho U square by 2.

Because, as I said the transport of momentum across the surface is equal to the velocity times the momentum density the convective transport across the surface is equal to the velocity times the momentum density across that surface. So, the convective transport rates are usually used in correlations. The velocity times the momentum density is used to scale the stress. And these are what are call the friction factor or the drag coefficient. So, that is the dimensionless flux that is the independent variable.

What are the independent dimensionless parameters? There are of two types: one is ratio of convection by diffusion. In the case of mass momentum by mass in the case of mass transfer that turns out to be UL by D that is the Peclet number. In the case of heat transfer that turns to be UL by alpha which is a Peclet number of a heat transfer. And from momentum transfer it is UL by nu which is equal to rho UL by mu is the Reynolds number. So, these are ratio of convection and diffusion. And then of course, you have the ratio of diffusivities: momentum by mass diffusion is equal to nu by D is equal to which is a Schmidt number. And momentum by energy which is equal to nu by alpha is equal to c p mu by k which is the Prandtl number. And these numbers are all inter related.

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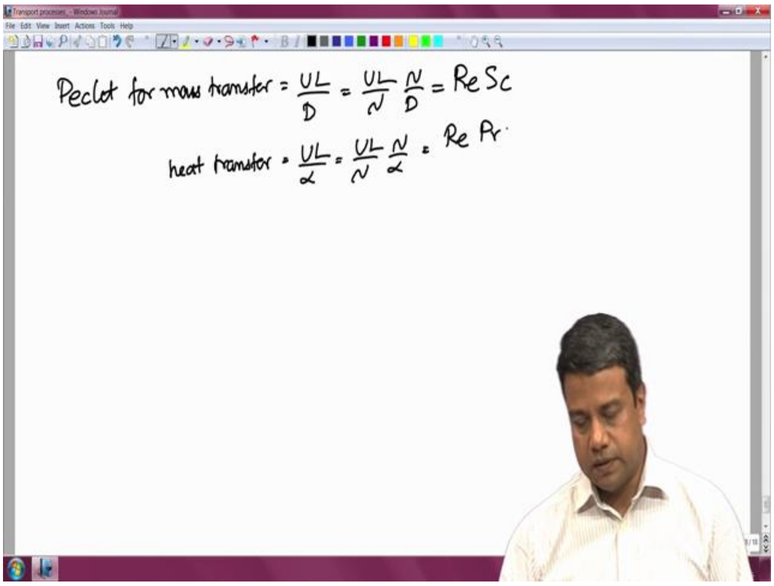

For example, the Peclet number I can write it as; the Peclet number for mass transfer will be equal to UL by D: I can write it as UL by nu into nu by D. You can see the nu and nu cancels out so that is the Peclet number. This UL by nu is the Reynolds number and nu by D is the Schmidt number. Therefore, the Peclet number for mass transport is just equal to the Reynolds number times the Schmidt number. Similarly, the Peclet number for heat transfer is equal to UL by alpha which is equal to UL by nu into nu by alpha is equal to the Reynolds number times the Prandtl number. So, these all are related.

Therefore, I broadly classified the dimensionless numbers that we came across as dimensionless fluxes, ratios of convection and diffusion, and ratios with diffusivities. And you can see that this encompasses all of the dimensionless groups that we had, almost. In the case of the heat exchanger problem all of the groups are included here, the Nusselt number is the dimensionless flux; Reynolds number ratio of inertia and viscosity, ratio of momentum and convection and diffusion; Prandtl number ratio of momentum diffusion and thermal diffusion, in the case of a heat exchanger apart from the ratio of lengths.

In the case of mass transfer from a particle the Sherwood number dimensionless flux the dependent variable, function of Reynolds number, ratio of momentum convection and diffusion; Schmidt number ratio of momentum diffusion and mass diffusion. So, all of these are classified just based upon ratios of convection and diffusion.

The power number that we had earlier is a ratio of the power and the momentum convection; rho D power four five omega cubed is ratio is the momentum of convection because it contains the density, the momentum is proportion to the density times the velocity. And I scale it by viscous scales it would have been ratio of power and momentum diffusion, but since I have done at this way as it is commonly done in momentum transport, it is common to scale by inertial scales rather than viscous scales, ratio of momentum of power and momentum diffusion.

Function of the Reynolds; number ratio of momentum convection and diffusion; the Froude number, the ratio of momentum transport the acceleration due to gravity, and we also had Weber number which was basically ratio of inertia and surface tension. So, in momentum transport along you have some additional parameters is some. Additional dimensionless groups for momentum transport. In case gravity is important you can have an additional term which goes as the Froude number U square by g L; you can see that this is dimensionless length square per time square in the numerator, length square per times square in the denominator.

So, your gravity is important we will have numbers that represent the ratio of gravity and the inertia and the acceleration due to gravity. In cases were surface tension is important you could have additional numbers; surface tension. You could scale it either by viscosity in which case the dimensionless group will be equal to mu U by gamma, you can verify that this is dimensionless is called a Capillary number.

Surface tension by inertia, we had just got the Weber number in the previous example where we expressed in terms of the frequency in the angular velocity instead if you expressed in terms of a linear velocity you will get rho U square L by gamma. This is called the Weber number. And there are other such numbers dimensionless numbers which represents different effects in the case of momentum transport.

So, in this lecture I have tried to give you fundamental understanding of what all of those dimensionless numbers mean. One category is the dimensionless flux; the flux scaled by the diffusive scales in the case of mass and heat transport it is conventional in momentum transport to scale it by the inertial scales. So, in the case of mass and heat transport scaled by diffusion you get the Sherwood number, Nusselt number momentum transport is scaled by convection and you get the friction factor drag coefficient. These are the independent variables, the average flux that you want to calculate of mass momentum energy. These depend upon ratios of convection and diffusion. The Peclet number for mass and heat transfer, and the Reynolds number for momentum transfer, ratios of convection and diffusion.

And the other independent variables, other ratios of two different kinds of diffusion: momentum to mass is what is call the Schmidt number, momentum to energy is what is call the Prandtl number. And these ratios of convection and diffusion the Peclet numbers can be expressed as the Reynolds number times Schmidt number or the Reynolds number times the Prandtl number. And these encompass all of the correlations that I had got for you earlier.

Now, why do these correlations have these specific forms? Why do the correlations beyond this we cannot do by dimensional analysis, but we can still get some idea of why this correlations have these specific forms. What do the forms of the correlations depend upon? What are the factors that affect? Which correlation is applicable in which limit? Both from mass transfer and heat transfer. For heat transfer I told you that we get different correlations depending upon whether it was laminar flow or turbinate flow.

And then in the mass transfer I told you will get different correlations depending upon whether the Reynolds number, the Schmidt number or small or large. Why is it that you get different correlations in different regimes and what are the factors that determine what correlation you will get in what system. We will try to take a closer look at that in the next lecture.

So, far I have tried to explain things to you in terms of ratios of convection and diffusion, and ratios of diffusion, trying to give you unified frame work for how to understand the transfer of mass momentum and energy as diffusion possesses. The fluxes are related to diffusion coefficient, times the change in the density of that quantity divided by a length of the gradient of that quantity of the gradient of the density of a quantity diffusive flux. Convective flux is just equal to the velocity times the density of that quantity. If you take the ratios of those two you get dimensionless numbers which are ratios of convection and diffusion. You could also get dimensionless numbers that are ratios of two different kinds of diffusion.

Next class, what is it the determines the forms of these correlations, we will take a slightly deeper look at that and try to classify these. Can I get some limited set of correlations which are applicable for a large number of systems and what is it that determines for that sectors. I will do that with respect to mass and heat transfer; in the next I will limit to that. Momentum transfer is slightly different I will come back and explain what are the sophistications that are involved in problems of momentum transfer.

So, we will continue that in the next lecture. I hope I have given you some fundamental understanding of how we are going to approach problems as we progress using just dimensional analysis. Once we complete dimensional analysis I will show you how to solve problems at their local level. And from that reconstruct what the large scale transport rates are. That will be the program of the rest of this course. We will see you in the next lecture.