Fundamentals of Transport Processes II
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Lecture - 6
Curvilinear Co-ordinates

Welcome to this lecture number 6 in our course on fundamentals of transport processes. We are going through right now, some preliminary material on calculus of vectors. So, that our future development becomes easier.

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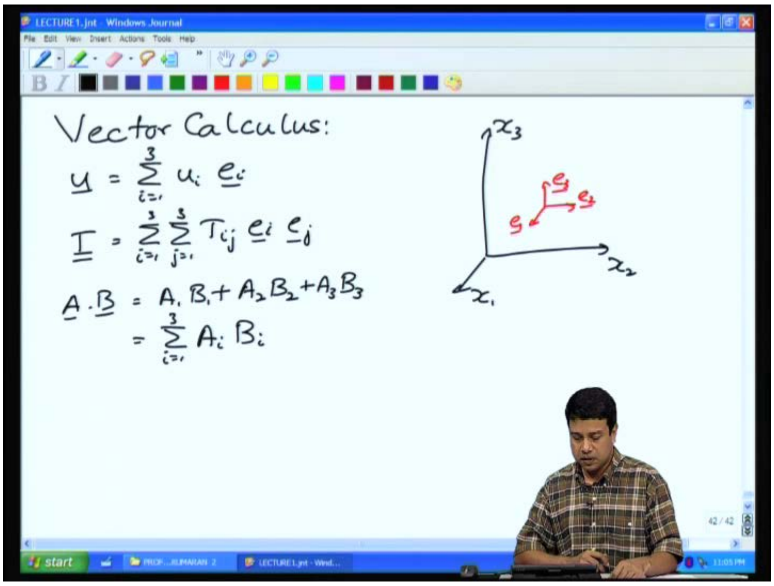

So, to review briefly, what we had done in the previous lecture, previous few lectures, we will consider vectors and tensors as objects in themselves, with some one or more directions associated with them in space. For example, the velocity vector typically it is written as summation i is equal to 1 to 3 u i times e i. If you recall we using a cartesian coordinate system in which the three coordinates are x 1, x 2, x 3 and we have unit vectors e 1, e 2, e 3. So, this is a vector it has three components, but we will consider the vector as an object in itself.

Similarly, I had defined for you the stress tensor equal to 1 to 3 T i j e i e j. So, there is a stress tensor it has two directions associated with it, direction of the force acting at the surface. The direction of the unit normal to the surface and the dot product of two vectors is written as A dot B is equal to A 1 B 1 plus A 2 B 2 plus A 3 B 3. And I had written this for you as summation i is equal to 1 to 3 of A i B i dot product there are no unit vectors because once, I take the dot product of two vectors it becomes a scalar.

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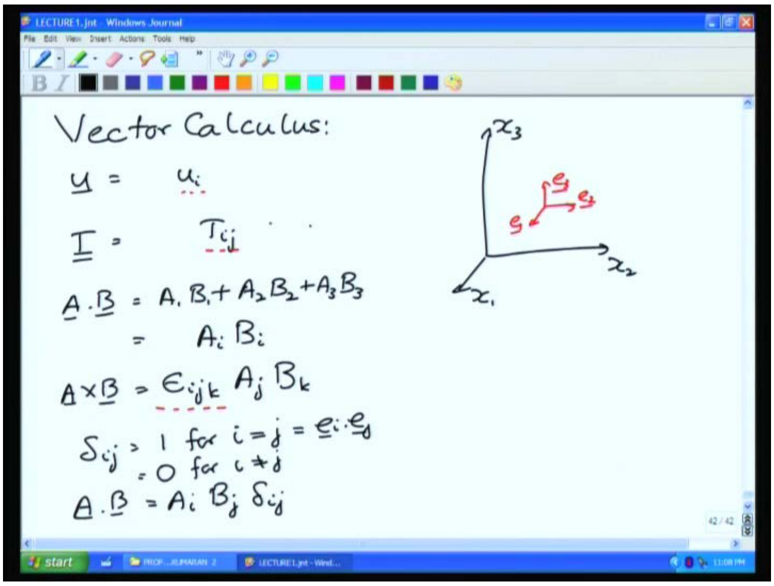

And we had made a notational simplification, we got loss of generality one can always remove the summations and the unit vectors in all of these. Whenever there is an index that is not repeated, it implies that there is a summation and a unit vector. When an index is repeated two times there is a summation, but there is no unit vector. So, there is no direction associated, with that index that is repeated two times because it has already become a dot product.

Cross products we had written in a manner similar to dot products A cross B is equal to epsilon i j k A j B k where epsilon i j k was the anti symmetric tensor is equal to 1. If i j k is 1 2 3, 3 1 2 or 2 3 1 it is minus 1, if it is the other way 1 3 2, 3 2 1 or 2 1 3 and its equal to 0, if any two indices are repeating. So, this anti symmetric tensor third order, it has three indices that is one of the special tensors.

The other one that we had seen earlier was the isotropic tensor delta i j is equal to 1 for i is equal to j i is equal to 0 for i is not equal to j. So, delta 1 1 is 1 delta 1 2 is 0 and so on. Delta i j is also the dot product of the unit vectors e i dot e j. So, this can also be written as e i dot e j. So, if i and j are the same then this dot product is equal to 1, if i and j are different it is equal to 0. The dot product can be written equivalently as A i B j delta i j. So, that is another way of writing dot products and then we had gone on just to briefly review the rules that we will use here, one free index means there is a unit vector and the summation. So, it represents one direction.

So, the order of a tensor is the number of unrepeated indices that are there, when an index is repeated it is a dot product. So, there is no unit vector associated with that index and in general, the order of the tensor of all terms in an equation have to be the same because one cannot equate a scalar to a vector or a vector to a tensor, a second order tensor you do not have the same number of components.

So, the order of all terms has to be the same and the when there is one cross product between two real vectors, what one gets is a pseudo vector something that changes sign, when the coordinate system changes from right to left handed coordinate system. So, if term in equation is a pseudo vector then all other terms should also be pseudo vectors. A couple of other things before we go on to the next, subject the first thing is that.

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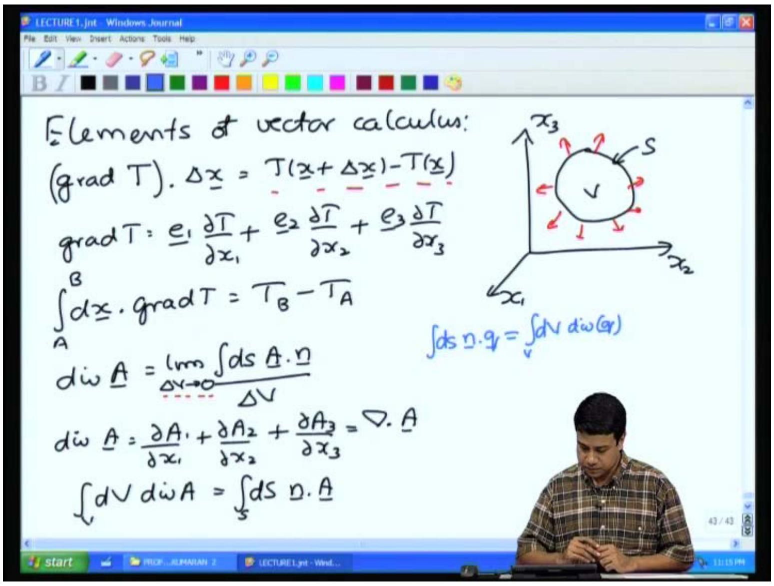

So, the first thing is that we are also defined for you the elements of vector calculus, vector calculus the gradient of phi of a scalar function phi. Let us just assume that this is a temperature field the gradient of a temperature field is defined such that, this thing dotted with delta x is equal to T at x plus x minus T at x. Physically, what this represents if I am sitting in a room with some temperature variations, I am sitting at some location x that travel a small distance to some other location x plus delta x, the difference in temperature between these two locations T at x plus delta x minus T at x is related is equal to distance travelled dotted with the gradient of T.

The gradient of T is a quantity, which is defined uniquely a vector quantity defined uniquely at each position in space and I have derived the equation for grad T, which is in cartesian coordinates e 1 partial t by partial x 1 plus T 2 partial T by partial x 2 plus e 3 partial T by partial x 3. So, this is one type of derivative acting on this scalar field, the inverse of this derivative is the integral and that integral relation basically, relates the integral of grad T dotted with distance travelled longer path to the difference in the temperature between the two end parts.

So, the integral equivalent of this is that if I have two points here A and B, if I take a path between these two points then integral along that path. The vector distance, the vector displacement dotted with grad T between these two end points is equal to the difference in temperature between those two end points. Corollaries, the difference in temperature is only a function of the n points. Therefore, along any path I should get the same integral of d x dot grad T and if I go around and come back to the same location, this has to be equal to the 0. So, those that was one element of vector calculus and that was the gradient.

The second is the divergence, the divergence of a vector A is defined as if I want to know what is the divergence at some location x, at some location x. I want to know what is the divergence of A I construct a small volume delta v with a surface S I construct a small little volume delta v around this point with a surface S, that surface has the unit normal, outward unit normal at various points along that surface.

So, what I do is I take integral over that surface of a vector dotted with the unit normal at that location on the surface at each location of the surface I take A. And dotted with a unit normal and divided by the volume delta v in the limit as delta v goes to 0. That is the definition of the divergence, we had derived this for a cubic differential volume and for that we got divergence of A is equal to partial A 1 by partial x 1 plus partial A 2 by partial x 2 plus partial A 3 by partial x 3.

This can also be written as del dotted with A, where the del operator of course, is e 1 d by d x 1 plus e 2 d by d x 2 plus e 3 d by d x 3. So, there is a definition of divergence, once again it is a derivative, it contains partial derivatives of components of a with respect to the different coordinates. The integral equivalent of this is for any macroscopic large differential volume, you can show that integral over the volume d v of divergence of A is equal to integral over the surface, surrounding that volume of n dot A is called the divergence theorem, the greens theorem.

And for this rather than taking a differential volume for which in the limit as this volume goes to 0, rather than taking differential volume in the limit as the volume goes to 0, we actually take a large volume macroscopic volume v with a surface S and with unit normal defined at each point on the surface. And if I take integral d s of n dot a over this surface, this is equal to integral over the volume of divergence of A. So, what this means is that the divergence of a integration over the entire volume depends only on the values of A on the surface. And we will see some physical examples of where this might be applicable we will see at a little later also, but briefly if you.

For example, A over a heat flux, if A over a heat flux, then you know that integral ds n dot q, q is the energy transported per unit area per unit time, q dot n is the energy transported perpendicular to the surface, integrating that over the entire surface that is equal to the total energy that comes out of this surface, and that is equal to integral over the volume d v of divergence of q. So, only if there is a net amount of energy coming out of this surface with the divergence of q integrated over the volume be non zero.

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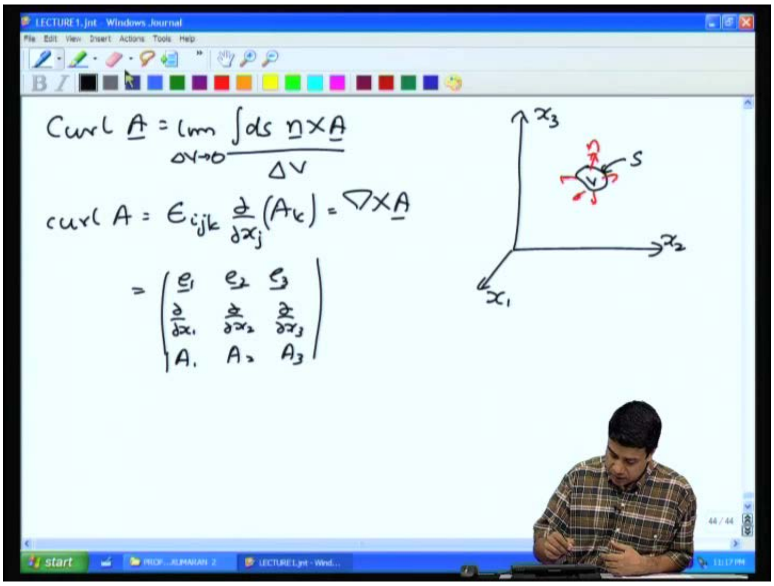

The third element that we had considered in the last lecture was curl of a vector. Once, again you have our coordinate system x 1, x 2, x 3 and we construct once again a small volume v over the surface S, there is a unit normal at each point on the surface. And the curl of A is defined as integral over the surface of now, the unit normal crossed with A, the cross product of the unit normal length A divided by the volume, and limit as delta v goes to 0.

So, that is the third class of derivatives of vectors, note that all of these are defined independent of the coordinate systems, they are not derivatives of components, their derivates are integrals of the vectors themselves. So, I got an expression for curl of A in the last lecture is equal to epsilon i j k partial by partial x j of A k and it is also equal to I can write it in matrix form e 1, e 2, e 3 partial by partial x 1, A 1, A 2, A 3 also del cross A. So, this is the definition of curl the gradient operator cross product of the gradient operator acting on this vector A.

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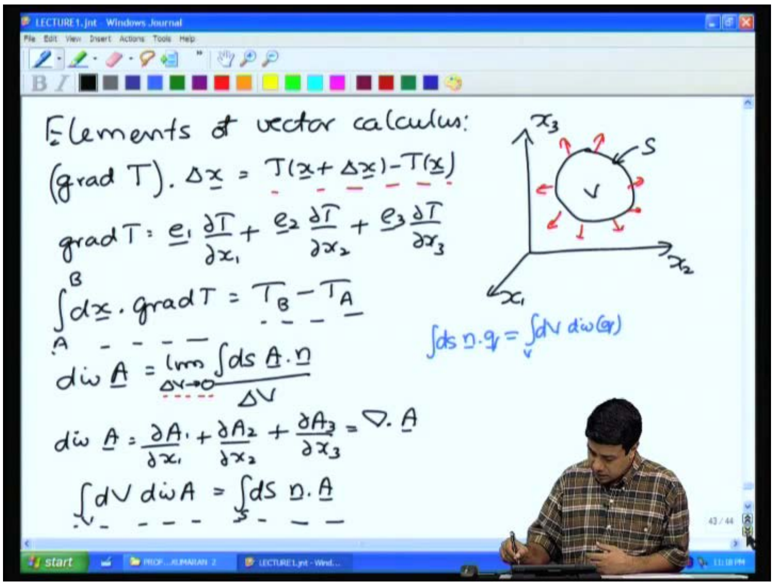

The equivalent integral theorem for this is, as I said for the gradient the integral theorem relates the line integral to the difference in a function between its end points, line integral to the difference in the function between its end parts. The divergence theorem relates the volume integral to a surface integral. So, that is the divergence theorem.

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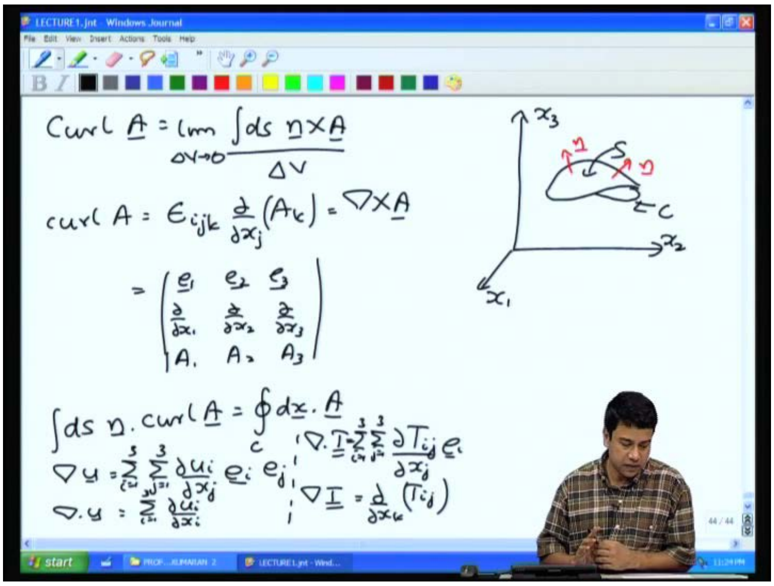

The integral theorem for the curl relates the surface integral to a line integral. So, if you have some surface like this with some perimeter c. So, this is the surface S this is the perimeter of the surface c and this surface of course, has its own unit normal and the integral theorem for the curl states, that integral d s of n dot curl A is equal to integral over the perimeter c over the closed perimeter c of d x dotted with A. So, this is the integral theorem for curl, it is also called the stocks theorem.

This relates the surface integral over an open surface to the integral over the perimeter, the contour that is the perimeter of this surface has some physical implications. Integral over the perimeter is the same for all surfaces that are bound by the same perimeter. So, you can have different surfaces, which have exactly the same perimeter this integral is the same for all of those surfaces. And for a closed surface it has no perimeter. So, the integral of n dot curl a has to be equal to 0, over a closed surface.

Now, you have taught in mathematics courses that these the gradient acts on the scalar that divergence acts on a vector, the curl acts on the vector it does not have to be. The divergence could act on a vector, a tensor the gradient could act on a scalar, a vector, a tensor and so on. The only distinction between these is whether, there is a dot product or not. So, I could take the gradient of a vector if I write this as partial u i by partial x j.

Now, let me just leave some space for myself, if I write this as partial u i by partial x j. Now, there is no repeated index i appears only once, j appears only once. That means, there is one unit vector associated with i and one summation associated with i, there is one unit vector associated with j and one summation associated with j. So, this is a second order tensor, the gradient of the velocity is a second order tensor. It has two directions one is the direction of the velocity i, in this case is the direction of the velocity the direction in which the fluid is moving, j is the direction of the gradient, the direction in which you are moving to find out what is the variation in velocity.

So, this give me a second order tensor. On the other hand if I have del dot u I will write this as partial u i by partial x i del dot u is partial u i by partial x i. In this case i is repeated so, there is no unit vector and there is only one summation. So, this is now a scalar it is the divergence I could also write the divergence of the second order tensor. For example, the stress tensor that I have del dot tin this case there is repeated index, involving this gradient operator and one of the two directions of T, it is not clear from this notation what the direction should be, but let us just define this for ambiguity without ambiguity. T has two components i j and there is a dot product.

So, that means that the dot product can be with respect to either i or with respect to j. So, let me just write this without ambiguity as T partial by partial x j, this is the divergence of the second order stress tensor, it has one unrepeated index that is j that is i and therefore, there is one unit vector and a summation, i is equal to 1, 2, 3. There is one repeated index j and there is no summation over that I am saying, there is no unit vector for that, but there is still a summation for j.

So, this represents the divergence of the stress tensor, one dot product a gradient and a second order tensor because of one dot product, it reduces the order by 2 and I get back a vector, when I take a divergence you take a divergence of velocity vector you get a scalar. If you take the divergence of a second order tensor, you get a first order tensor or a vector. You could as well take the gradient of the stress this is also possible.

So, this is now a third order tensor because this is partial by partial x k of d i j. So, I am taking the derivative with respect to index k of T, which has indices i and j three unrepeated indices. So, you take a gradient of a second order tensor, you get a third order tensor. So, taking the gradient increases the order of tensor by one, divergence decreases it by one and you can do it for any scalar vector tensors. Of course, divergence and curl can be taken only for vectors and tensors gradient can be taken for all scalar, vector and tensor.

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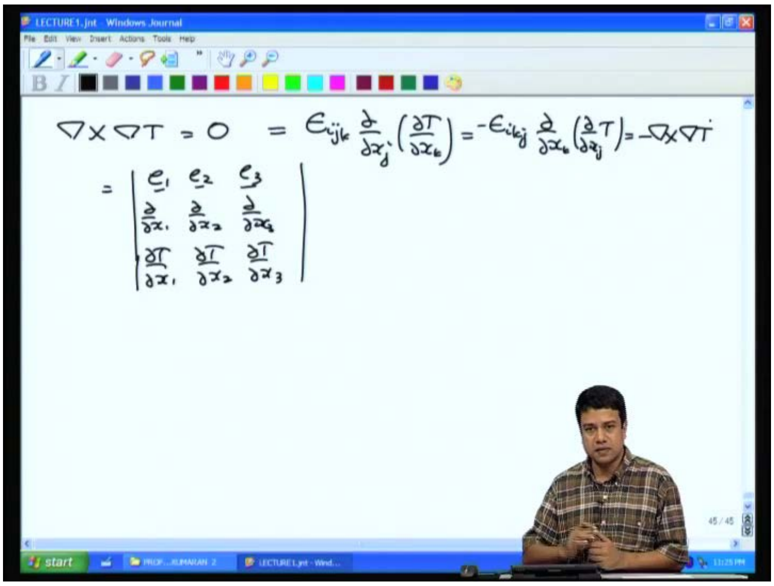

A couple of other things the first is that if I take the curl of the gradient of something, this has to be equal to 0 it is because this is if I write it on in in long hand notation, if this is the matrix e 1, e 2, e 3 d by d x 1, you can evaluate this determinant into integral c is equal to 0. So, the curl of the gradient of anything is equal to 0, I could have told you this quiet easily because this is equal to epsilon i j k partial by partial x j of partial T by partial x k.

Now, in this one if I interchange two indices I will get this is equal to minus epsilon i k j partial by partial x k, partial by partial x j of T. Let us interchange two indices and I get minus epsilon i j k because I interchange two indices, I get the negative sign. So, this is equal to minus curl of grad T, this is also equal to minus curl of grad T, if a number is equal to the negative of itself it can only be 0. So, the curl of the gradient of anything is equal to 0. The converse is also true, the converse is also true.

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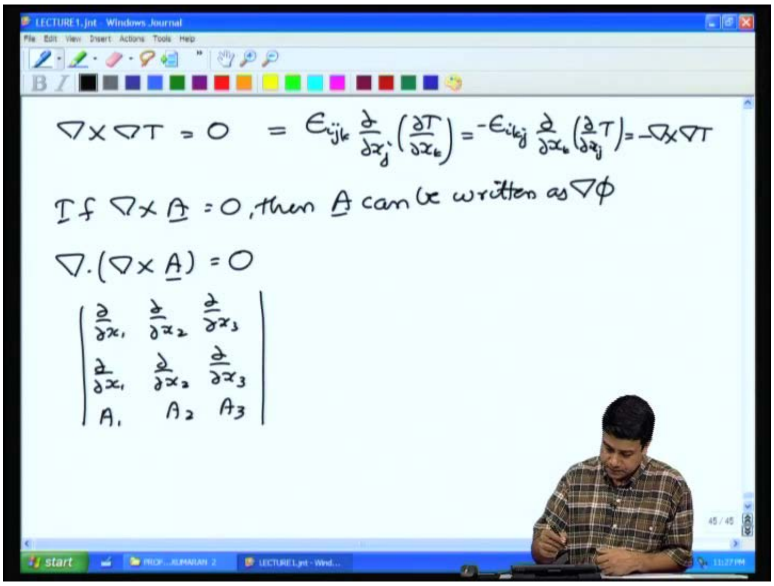

If del cross a is equal to 0 then A can be written as the gradient of some scalar function, if the curl of A is equal to 0 then A can be written as the gradient of some scalar function. The other thing, if A is a vector then I also have the divergence of del cross A is equal to 0. So, this as you recall is a triple product the divergence of the curl of A is also equal to 0, this you can easily see is equal to d by d x 1 d by d x 2 d by d x 3, two rows of this matrix are identical. Therefore, the determinant has to be equal to 0.

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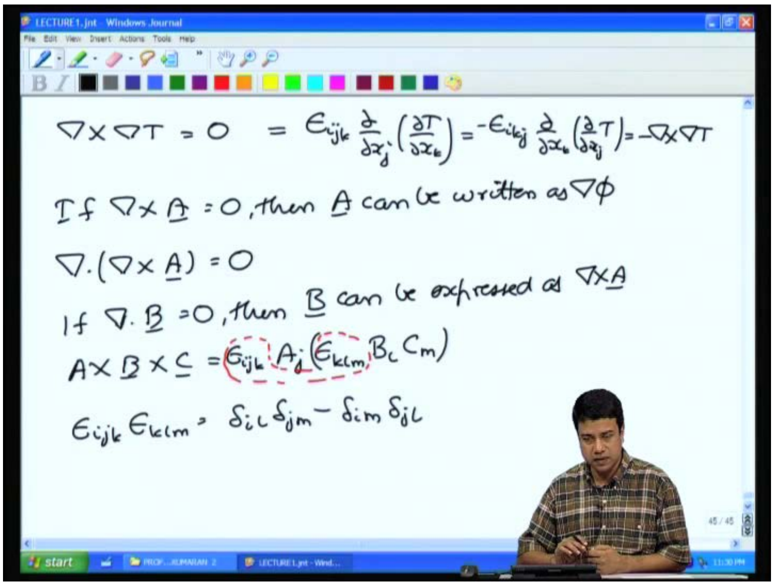

And corollary of this is that if del dot any vector B is equal to 0, then B can be expressed as del cross some vector A. If del dot B is equal to 0 then B can be expressed as some vector as the curl of some vector A. Finally, often you will come A cross, A double cross product A cross B, cross C. If there is one cross product it becomes a pseudo vector because the direction changes sign.

When you go from a right to a left handed coordinate system, if there are two cross products both of those change sign and so, you get back a real vector. If I have to write this in long hand notation I would write B cross C as epsilon k l m B l C m. So, this is B cross C it is a vector, which has direction k and I am taking a crossed with this vector. So, I take A cross with this vector I get epsilon i j k, A j times epsilon k l m, B l and C m.

Now, this involves the product, this involves product of two epsilons, this involves product of two epsilons. And there is a short hand there is an identity, which relates this to the delta functions epsilon i j k epsilon k l m is equal to delta i l delta j m minus delta i m delta j l. So, this identity you can derive for yourself verify that if any two of epsilon i j k and epsilon k l m are equal then it should be 0.

You can verify that the right hand is 0, where deltas are the identity tensors and if all three are different you can verify you actually get this result for epsilon i j k times epsilon k l m. So, these are the kinds of identities that we will use as we progress in the course. So, everything I have done for you so, far is for a cartesian coordinate system. How does all this carry over to a curvilinear co-ordinate system? If you recall when we did fundamentals of transport processes one, we actually did shell balances in those coordinate systems, where the surfaces of the control volume were chosen to be surfaces of constant coordinate.

And when we looked at the fluxes going through those volumes took the difference to find out, what rate of change of the temperature or concentration within that volume was. And once, we got the equations then we identified the divergences the gradients etcetera, within those equations. There is a another way to do it, and that is to look at the transformation of unit vectors themselves. So, I will just briefly go through that as to how you transform unit vectors, in order to get relations between unit vectors.

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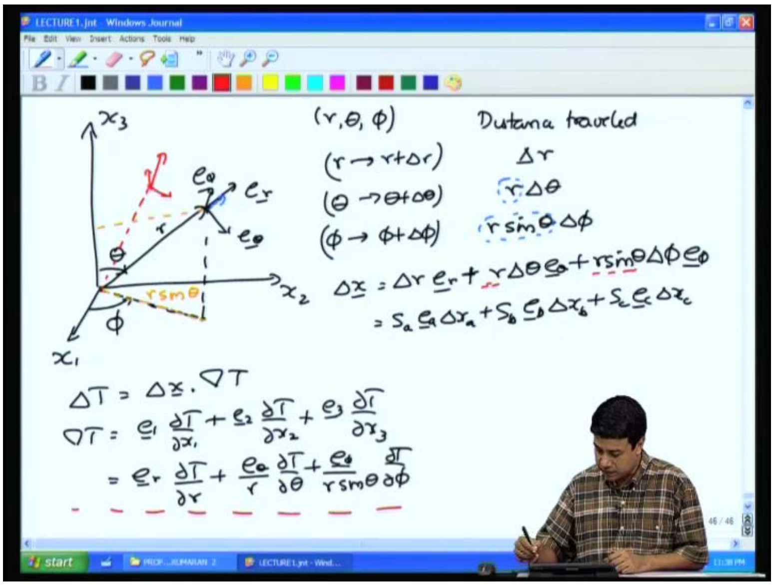

So, let us take the spherical coordinate system for as an example x 1, x 2, x 3 and this spherical coordinate system, the three coordinates are one is r the distance from the origin. So, r theta and phi, r is the distance from the origin, theta is the angle that this makes with the x 3 coordinate, theta is the angle it is called the azimuthal angle. The angle that the radius vector makes with the x 3 coordinate.

And phi is the angle in the x-y plane made by the projection, in the projection of the radius vector on to the x-y plane. So, those are the three angles r theta I am sorry three coordinates r theta and phi note that r is a distance theta, and phi are angles. So, they are dimensional less. Now, at each point you have, a unit vector in the r direction e r unit vector in the theta direction, the unit vector in the theta direction goes in the direction of increasing theta, theta increases downwards.

So, the unit vector in the theta direction will go in the direction of increasing theta, and the unit vector at the phi direction will go in the direction of increasing phi. And obviously, these unit vectors now depend up on position. So, if I go to some other location here the unit vectors are in different directions, the unit vectors are in different directions. So, the unit vectors do depend up on location. Now, if I go a small distance delta r in the, if I change r coordinate from r to delta r, the distance travelled if I go from r to r plus delta r. The distance travelled is delta r itself because if I go a small distance in the r direction along that along the r direction. If I go a small distance, the distance travelled is delta r itself. On the other hand if I sit at this point and then move a small change theta to theta plus delta theta, the distance travelled is not delta theta because the distance travelled is actually, it has to have dimensions of length.

Since, the distance from the centre the radius vector is r the distance travelled is actually, r times delta theta and if I change phi to phi plus delta phi, the distance travelled is not delta phi. The projection here, the projection on to the x y plane this length, this length is equal to r sign theta because the projection on to the z axis is r cos theta, the projection on to the x y plane is r sine theta. So, the distance travelled is going to be actually r sine theta delta phi. That means, that when I take derivatives with respect to coordinates to define a gradient, I should also use these scale factors which transform the change in coordinate to a distance. That means, I should also use a scale factor, which transforms the change in coordinate to a distance. For example, you know that for the definition of gradient delta T is equal to delta x dot grad T. In a cartesian coordinate system I had written grad T as e 1 partial T by partial x 1 plus e 2, partial T by partial x 2 plus e 3, partial T by partial x 3, but in this coordinate system I have to use scale factors because if I write in terms of r theta, and phi these are not distances.

So, what I have to define this is as e r partial T by partial r plus e theta divided by r partial T by partial theta plus e phi by r sine theta partial T by partial phi because when, I move a small distance delta phi, the distance travelled is actually r sine theta times delta phi. And that infinitesimal displacement delta x has to be written as delta r e r plus r delta theta e theta plus r sine theta delta phi delta phi times, times e phi. So, these things are what are called the scale factors, in the case of r it is just 1, but for theta I have r and for phi I have r sine theta. So, these are what are called as scale factors in any general coordinate systems, where the coordinates themselves may not have dimensions of length, this can in general be written as S a e a delta x a plus S b e b delta x b plus S c e c delta x c. In this case for the cartesian coordinate systems a was equal to one because the pre factor in delta r is equal to ones, b is equal to just r and S c is equal to r sine theta.

When you have these scale factors which are non 0, there is also a variation in the unit vectors with position. And one can derive the derivation the variation in unit vectors with respect to position quiet easily, for the general case I will write down the derivation and then I will apply to the specific case, but the reason I am doing this is because I have defined here the gradient for you. I have defined here the gradient for you it is e r d by d t by d r plus e theta by r d t by d theta plus e phi by r sine theta d t by d phi.

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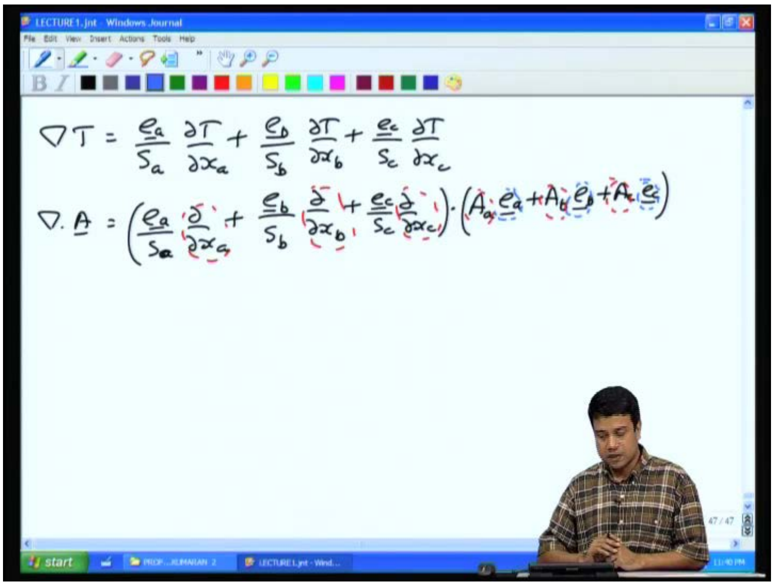

So, I want to go to a general coordinate system, the gradient of T will be equal to 1 by e a by S a partial T by partial x a plus e b by S b partial T by partial x b plus e c by s c partial T by partial x c. Where S a, S b and S c are the scale factors in the a, b and c directions. This is for a general orthogonal coordinate system. However, when it comes to defining divergences it gets a little more complicated, we know that del dot a is equal to partial by partial x a plus e b by S b dotted with. So, when I am taking this divergence I have to take derivatives, I have to take derivates of the components as we did usually in cartesian coordinate system, as well as of the unit vectors here, even you take derivatives of the unit vectors as well.

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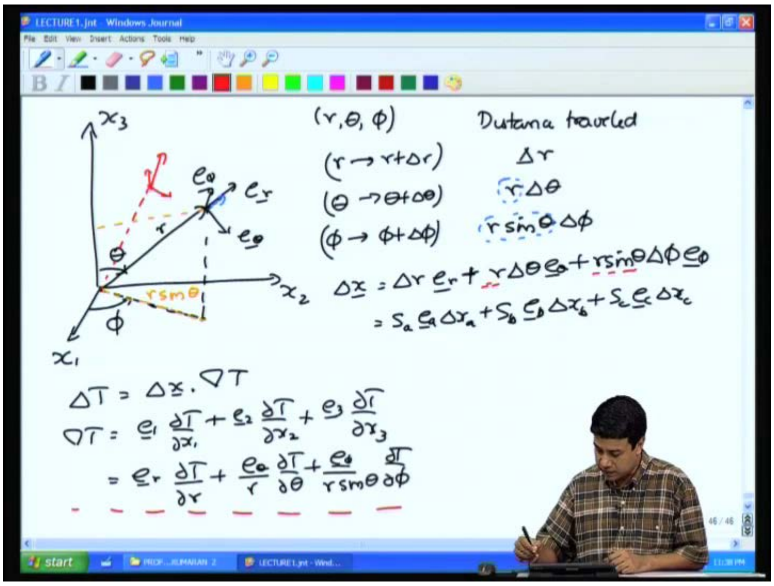

Because as I showed you in the spherical coordinate system, the unit vectors do depend up on position and you have to take derivatives with respect to the unit vector as well.

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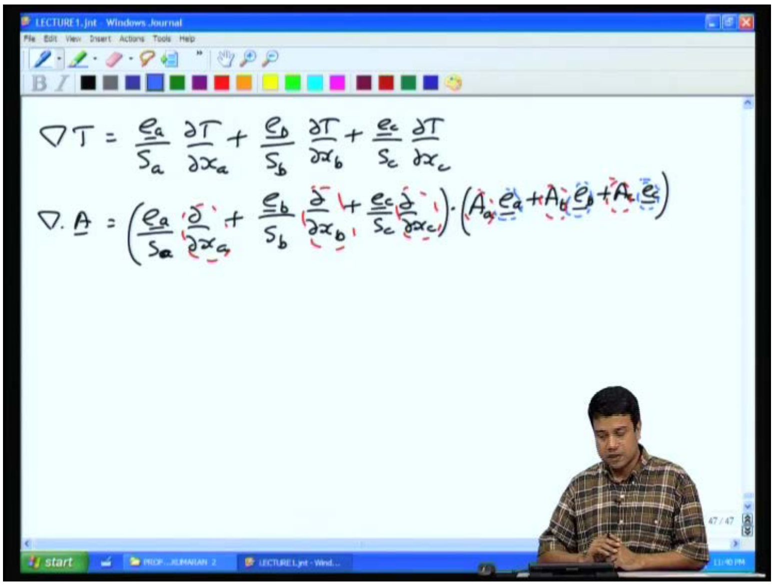

So, for this reason it becomes important to be able to define derivatives of unit vectors, and that is the reason I will spend a little bit of time on that. So, how do we get the derivative of a unit vector with respect to position.

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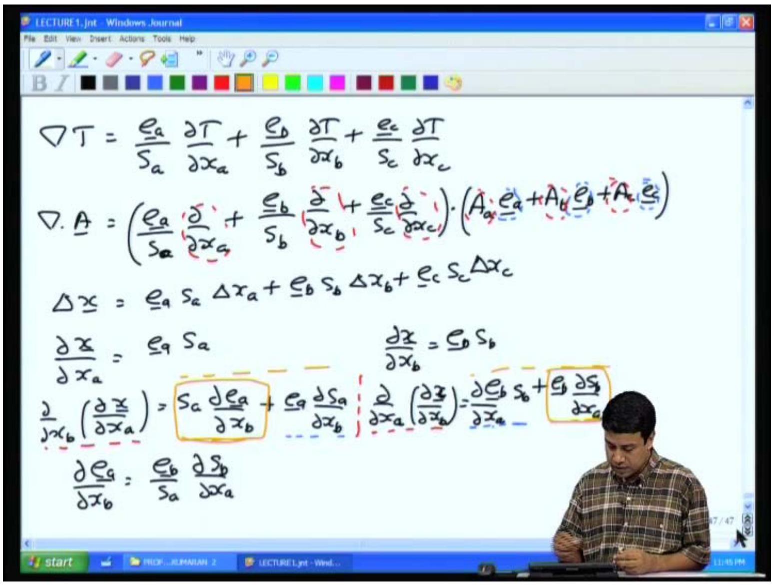

If I take a small displacement delta x, this can be written as e a, S a, delta x a plus e b, S b, delta x b plus e c, S c delta x c. Where the e's are the unit vectors, S are the scale factors and delta x's are the small change in coordinate. If I take the derivative of this with respect to x a for example, I will get e a times S a, if I take the derivative with respect to x b is equal to e b and S b. Now, the derivative with respect to x a that I had taken, I can take one more derivative with respect to x b. If I take partial by partial x b of partial x by partial x a is equal to S a, partial e a by partial x b plus e a partial S a by partial x b.

Now, I can take the derivative of this with respect to a d by d x a of partial x by partial x b. And write it quiet easily once again so, this is equal to partial e b by partial x a, S b plus e b partial S b by partial x a. Now, if we look at this relation of course, when I am taking partial derivatives, the order of the derivative should not matter. So, obviously this one in which I am taking with respect to a and then b should be the same, as this one where I take first with respect to b and then with respect to a both of these are vectors.

In this case the first one is S a partial e a by partial x b plus e a partial s a by partial x b. The second is equal to partial e b by partial x a S b plus e b partial S b by partial x a. The coordinate systems that we are considering are orthogonal. So, e a and e b are perpendicular to each other, you will restrict attention to orthogonal coordinate systems.

Since, e a and e b are perpendicular to each other and these two vectors, these two vectors have to be the same.

Whereas, a and e b are perpendicular to each other, you require that the e a the term proportional to e a here has to be proportional to has to be equal to this term because obviously, e is e a is orthogonal to e b. So, it has no component along e b that means, that e a has to be equal to this term. Similarly, e b times this has to be equal to this. This gives us the relation for the derivatives of unit vectors in terms of the scale factors.

For example, partial e a by partial x b is equal to e b by s a partial s b by partial x a. So, if I know what the scale factors are, then I automatically, know what the derivatives of the unit vectors are. So, this one I got by equating this term with this term because I know that this term cannot be equal to the first term on the right hand side because e a, and e b are perpendicular to each other. So, this gives me a relation for the derivatives of the unit vectors with respect to position. And how do I use that to advantage, in order to derive the divergence.

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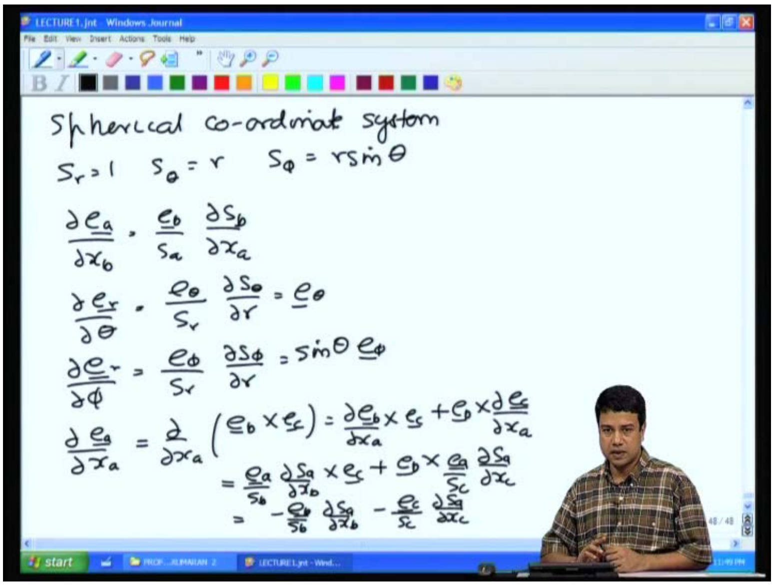

So, this case we the derivative is a unit vectors for a spherical coordinate system. I know that S r is equal to 1 S theta is equal to is equal to r and S phi is equal to r sine theta. I also I know that partial e a by partial x b is equal to e b by S a partial S b by partial x a and from this you can derive the derivatives of the all unit vectors. For example, partial e r by partial theta is equal to e theta by s r partial S theta by partial r.

And partial S theta by partial r is just one because S theta is equal to r and S r is 1, this just becomes equal to e theta. Similarly, partial e r by partial phi is equal to e phi by S r partial S phi by partial r S r is 1 S phi is r sine theta. Therefore, partial S phi by partial r is just sine theta. So, this just gives me sine theta e phi, what about the diagonal terms this just gives you the derivatives of unit vector in one direction, with respect to some other direction.

You can also derive the unit vectors in one direction with respect to that same direction itself, as follows. You know that partial e a by partial x a, if I want to evaluate this one, what I do is to express e a in terms of e b and e c. So, I express d by d x a of e b cross e c. So, this is equal to partial e b by partial x a cross e c plus e b cross partial e c by partial x a. One has to be careful here because the order of the cross product has to remain the same because if you interchange, the order of the cross product it becomes the negative of itself. So, this first term here partial e b by partial x a is equal to e a by S b partial S a by partial x b cross e c plus e b cross e c by S e b cross e a by S c partial S a by partial x c just using the formula that we just derived, and e a cross e c is minus e b e a cross e b is minus e c is minus e b. So, I get minus e b by S b partial S a by partial x b and e b cross e a is minus e c. So, I get minus e c by s c partial s a by partial x c. So, this gives me the derivative with respect to a of the unit vector in the a direction itself. So, in that way you can derive all of the unit vectors good. Now, how do we use this in order to calculate the divergence.

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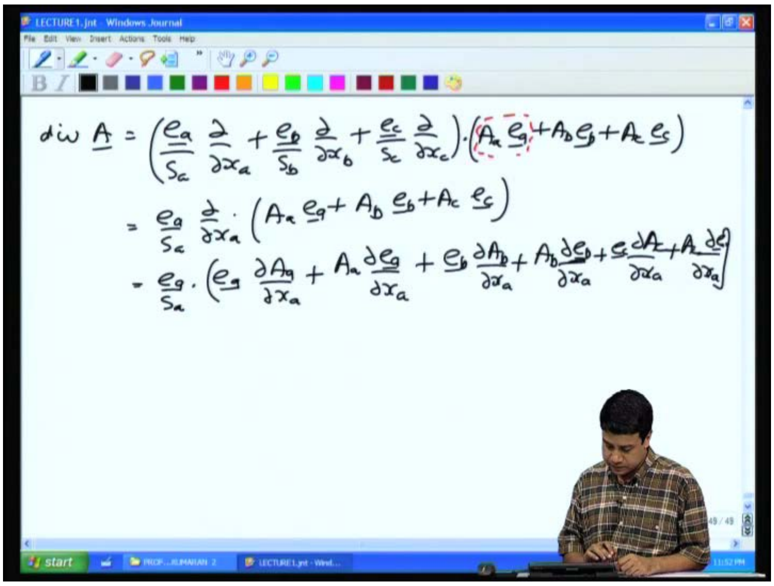

You know that divergence of A is equal to e a by S a partial by partial x a plus e b by S b d by d x b plus e c by s c partial by partial x c dotted with. Let us just take the first derivative alone, in order to illustrate the case, we will just calculate the first derivative alone e a by S a d by d x a of we just take the first term alone. And here I have to take the derivative of both the component and the unit vector. So, I will have e a by S a d by d x a dotted with so, the first term will be the derivative with respect to the component. So, this will be e a by s a dotted with e a partial A a by partial x a plus A a partial e a by partial x a plus e b partial a b by partial x a plus a b partial e b by partial x a plus e c partial a c plus a c partial e c by partial x a. So, I take the derivative of both the component as well as the unit vector point to note here partial e a by partial.

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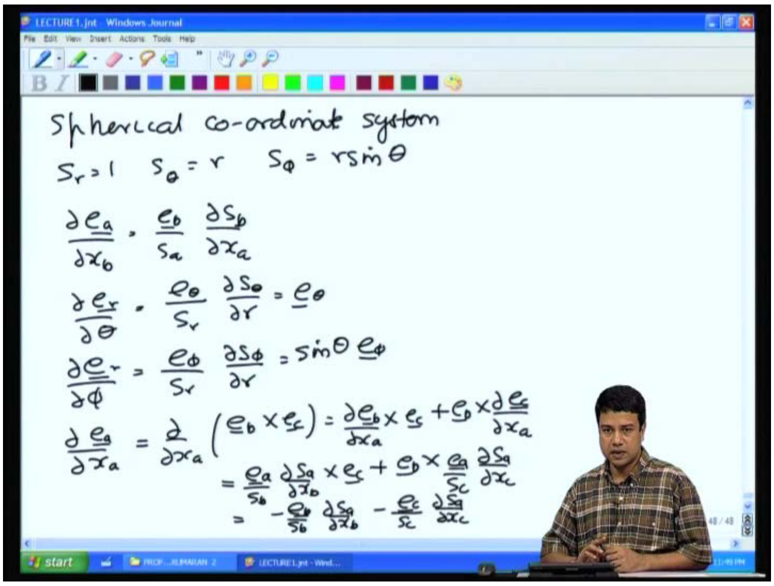

Here I have the derivative of partial e a with respect to x a is equal to minus e b by S b partial S a by x b minus e c by S c partial x S a by partial x c. That means, this derivative is perpendicular to the unit vector itself, it has no components along a it has only components along b and c.

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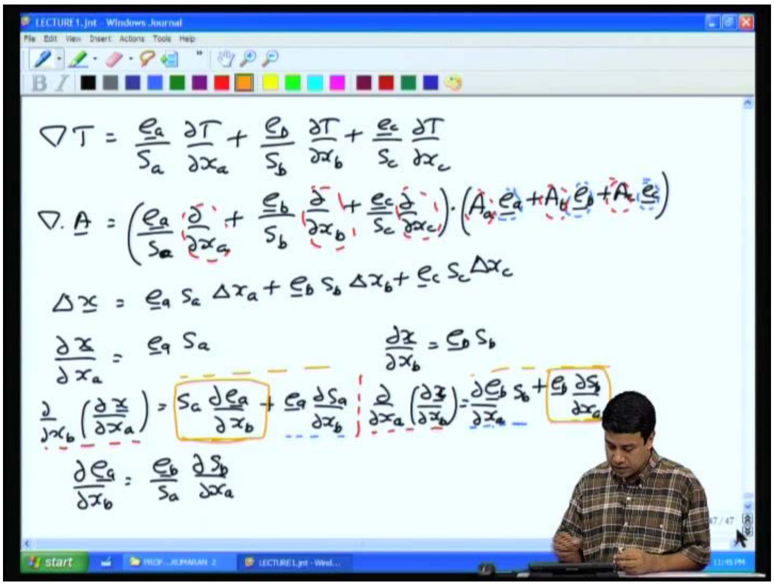

Similarly, partial e a by partial x b is equal to is in the direction of e b that means, the derivative of unit vector with respect to some direction is perpendicular to the unit vector itself, and that I can use to advantage here.

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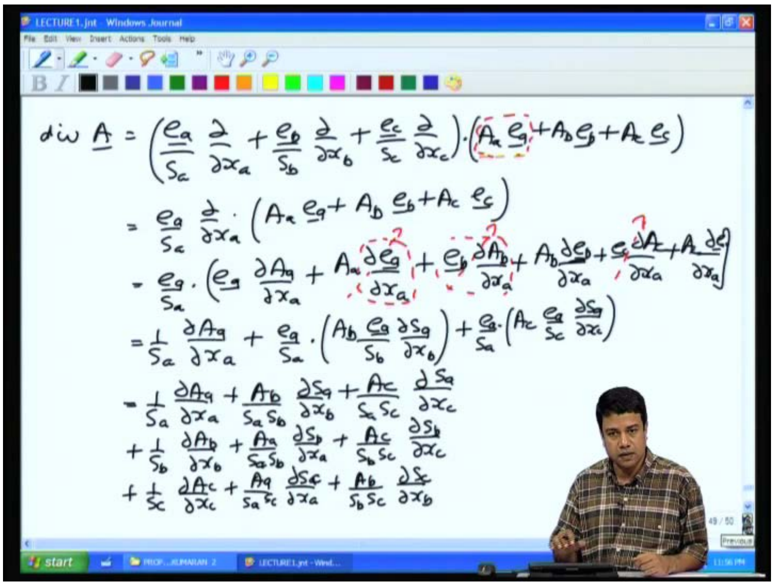

The first term is just e a dotted with e a that gives me 1. So, I just get one by S a partial A a by partial x a. The second is e a dotted with partial e a by partial x a partial e a with respect to x a is perpendicular to e a itself. So, when I take the dot product, this term when I dotted with e a becomes 0. So, I get only due to the variation of component in the direction the second and third terms, I am sorry the third and fourth terms the third term is the component derivative with respect to a times e b, but e b dotted with e a is equal to 0.

So, this term since e b is perpendicular to e a this becomes 0, the next term of course, is not zero because partial e b by partial x a becomes e a by S a dotted with a b partial e b by partial x a is e a by s b partial s a by partial x b. So, that is the expression for partial e b by partial x a. Similarly, in the fifth term e c dot e a is equal to 0, the fifth term e c dot e a is equal to 0. And the sixth term just gives me e a by S a into A c times e a by S c partial S a by partial x c. So, this first term alone if I write it out in expanded form because one over S a partial A a by partial x a plus.

This e a dotted with e a is equal to 1. So, I get a b by s a s b partial s a by partial x b plus A c by S a S c partial x a. I am sorry partial by partial x c that was only for this first term here. And now, I need to add up for the second and third term, I will just write it down, I would not go into the details it is just an it is just algebra of the form 1 by S b partial A b by partial x b plus A a by S a S b partial S b by partial x a plus A a plus A c. A c by S b S c partial S b by partial x c plus 1 by s c partial A c by partial x c plus partial S c by partial x a plus A b by S b S c partial S c by partial x b. So, that is the final expression that you get after incorporating all the derivatives. I can write this, I can just collect terms here I can just collects terms in this equation.

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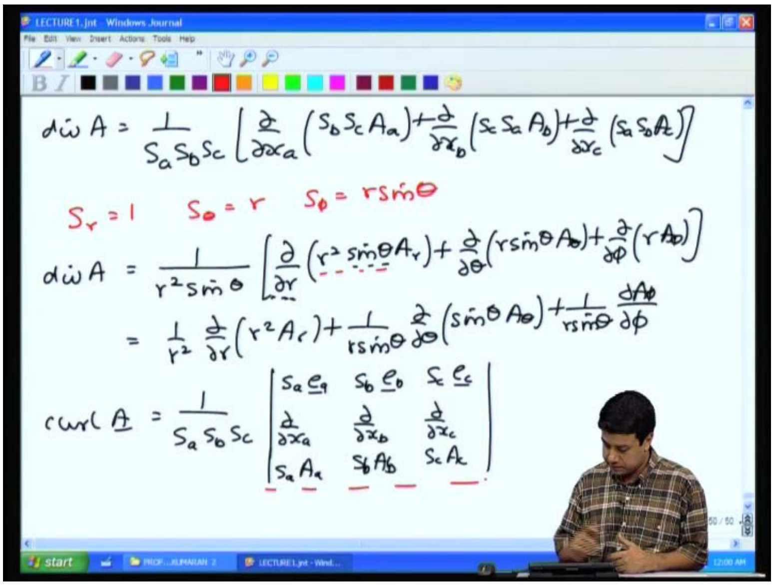

You can write it compactly as divergence of A is equal to 1 by S a S b S c into partial by partial x a of S b S c A a plus partial by partial x b of S c S a A b plus partial by partial x c of S a S b A c. This is the divergence in terms of the scale factors, what is it for a cartesian coordinate system, where I have S r is equal to 1 S theta is equal to r and S phi is equal to r sine theta. This is the divergence of A is equal to 1 over S a S b S c that is r square sine theta of A is r. So, d by d r of S b times S c r square sine theta. I am sorry S c S a r plus d by d theta because this is S theta times S phi that is r into r sine theta.

Second one is S phi into S r, this becomes r sine theta A theta plus d by d phi of r times A phi because d by d phi of S r into S phi. So, this becomes r times a phi. So, this in the first one I can cancel out r and I will get in the first one. Since, I am taking the derivative with respect to theta here, I am sorry derivative with respect to r of sine theta I can cancel out sine theta, and I will get 1 by r square d by d r of r square A r. The second one I can cancel out one r. So, I will get 1 by r sine theta d by d theta of sine theta A theta plus 1 by r sine theta partial A phi by partial phi.

Go back to the lectures, were we calculated shell balances in a spherical coordinate system, and you will find that the divergence of q that I calculated del dot q the expression was exactly the same in the spherical coordinate system, we got the same thing by taking into account the variation of unit vector with respect to position. Similarly, one can get the simplified expression for the curl, I would not go through the details here, it is just algebra of the same kind that we done before. The curl of A can be calculated as 1 by S a S b S c times the determinant of S a e a S b e b S c e c d by d x a d by d x b d by d x c times S a A a S b A b S c A c. Good exercise to undertake, calculate the value of curl in this coordinate system. I mean the spherical coordinate system use this formula here to calculate the value of the curl.

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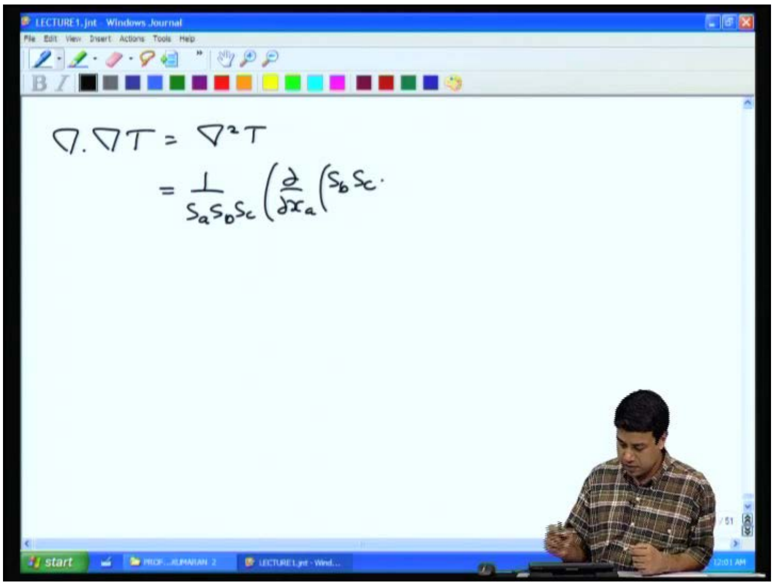

And the other thing that we saw in the course on fundamentals of transport processes was the Laplace del dot grad of T just l square T. So, this I can write simply as 1 by S a S b S c into partial by partial x a of S b S c.

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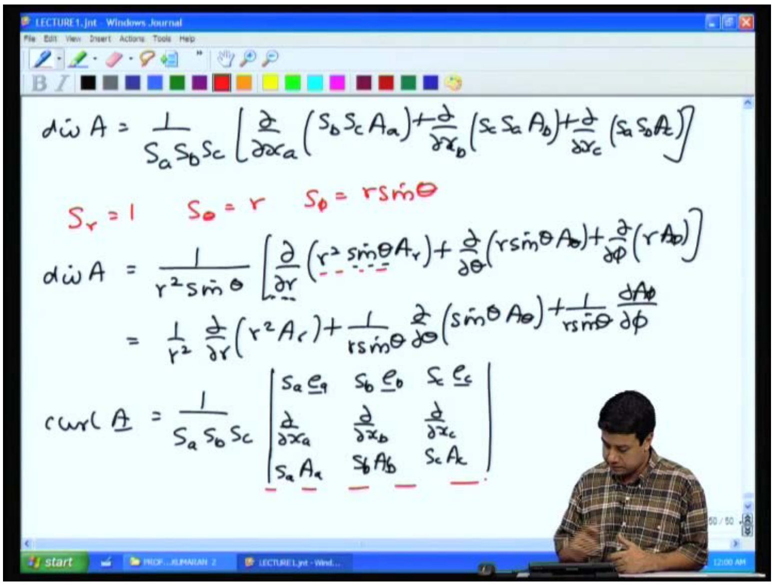

Into A a here the component A a is equal to the gradient of T in that direction, which is 1 over S a partial T by partial x a plus d by d x b of S a S c 1 by S b partial d by partial x b plus d by d x c of S a S b by S c partial d by partial x c.

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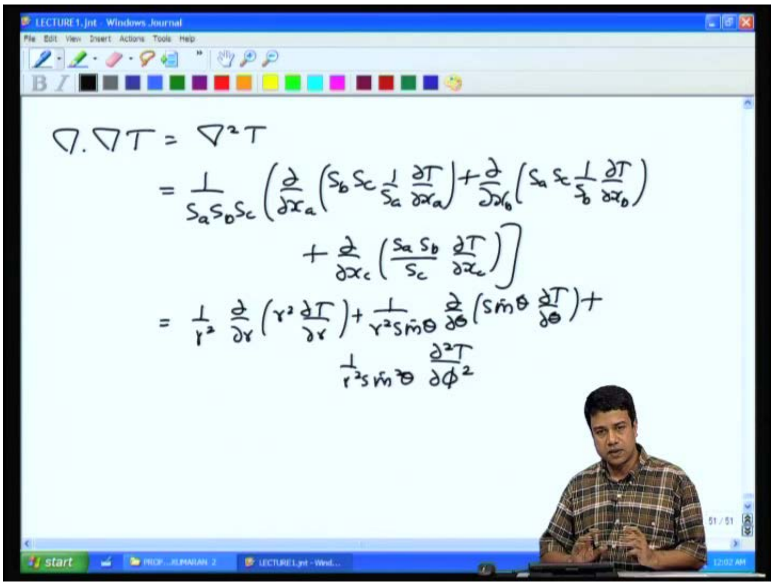

You can calculate this for a spherical coordinate system, this becomes 1 by r square d by d r of r square partial d by partial r plus 1 by r square sine theta, exact same expression that we got when we did shell balances. So, go through this the way we have done it in this class compare it with what we have done in the previous fundamentals of transport processes 1, where we did the same thing on the basis of shell balances.

We managed to get conservation equation sine spherical coordinate system, you will find that the divergence operator, the Laplace an operator in both of these cases are identical. If I able to compare those you get a better understanding of how we derived these in this of course, we made no reference to the basic to the underlying reference coordinate system. And in this case, we have managed to derive these operators independent of coordinate systems depending only up on the scale factor, in the variation of unit vectors with respect to position. We will continue this in the next class, while we will try and apply some of these concepts to the velocity field in fluids, we will start fluid mechanics in the next lecture.