Fundamentals of Transport Processes II
Prof. Dr. Kumaran
Department of Chemical Engineering
Indian Institute of Science, Bangalore

Lecture - 40
Turbulence Flow in a Channel

So, this is the 40^{th} lecture of our course on fundamentals of transport processes. Welcome to you all. Last class we were discussing turbulent flows. As I said, turbulent flows originate when the base laminar flow goes unstable the laminar flow profile is of course the solution of the equations of motion. For example, for a simple channel or tube flows it continues to be a solution of the equations of motion at all Reynolds numbers. However, as you go on increasing the Reynolds number there comes a stage at which the laminar flow profile becomes unstable and spontaneously under goes a transition to a turbulent flow.

The turbulent flow is characterized by different characteristics; one is it is at higher Reynolds numbers after the instability of the laminar flow it is highly irregular with velocity fluctuations in all directions. It is three-dimensional, even though the mean follow may be two-dimensional. You do have fluctuations at each location in the velocity in all three directions. The dissipation of energy and the stress due to these fluctuations is much larger than that due to the mean flow. For this reason turbulence flows have much higher stresses, pressure differences, friction factors, as well as much higher energy dissipation rates than laminar flows where the dissipation occurs only due to molecular or the stress, in the dissipation occur only due to molecular viscous transport.

The trans-potent turbulent flows takes place due to eddies, which are parcels of fluid of different sizes which are moving in all directions, an integral part of this transport is the intensification of the vertex lines due to the stretching and bending of I am sorry, intensification of vorticity due to the stretching and bending of vertex lines, this happen only in three dimensions. Therefore, turbulent flows are inherently three dimensional and because the the dissipation ultimately occurs at the smaller scales. As I said the smaller scales are what are called the (( )) scales.

They are characterized by a dependence only on the rate of dissipation of energy and the kinematic viscosity, so you can get out dimensional expressions for the smaller scales just based upon dimensional analysis. The (( )) scales length and velocity are much smaller than the microscopic scales. For this reason most of the kinetic energy is actually in the macroscopic scales because the velocities and lengths, there are much larger. However, the strain rate in the smallest scales is much larger than the strain rate in the large scales. Therefore, the dissipation of energy in the smaller scales is much larger than the dissipation in the large scale flow. The dissipation of energy in the small scales basically balances the production of energy in the large scale flow.

That is why the energy production in the large scale flow is actually much larger than what you would expect just based upon dissipation at the large scales itself. The flow itself creates smaller and smaller scale eddies, until you come to a length and a velocity scale at which the Reynolds number based upon the eddy length and velocity is order one, okay? So, that at that stage there is a balance between the production and dissipation of energy and it is at this scale that most of the energy is dissipated. So, we discuss the energy cascade in the last lecture of turbulent flows all the way from the macroscopic scales to the (( )) scales and this picture was related to the models. That we had discussed in the last lecture for turbulent flows, specifically the k epsilon model which is commonly used for turbulent flows.

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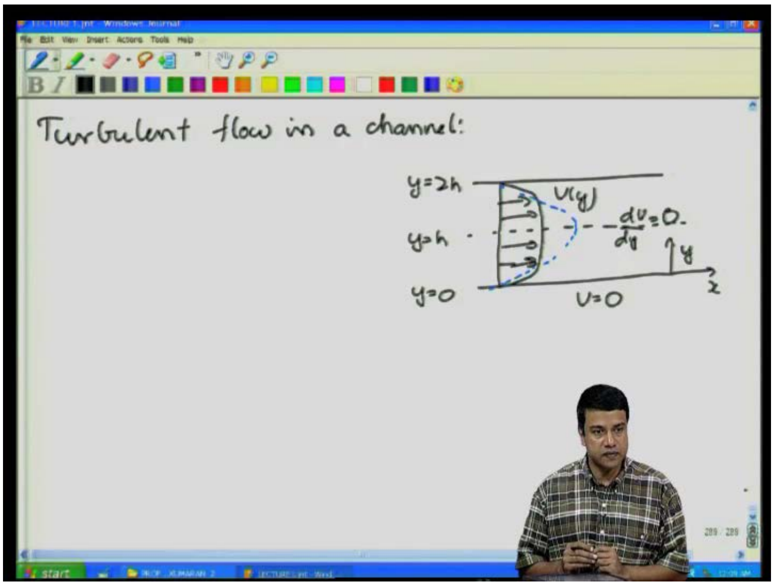

So, here in this lecture we will look at a concrete example of a turbulent flow and see how the modeling is done? So, the simplest example is actually the turbulent flow in a channel, so we have a channel of width 2 times h, I will choose an x y coordinate system in which x is along the wall and y is perpendicular. So, y is equal to 0 is the bottom surface y is equal to h is the mid plane and y is equal to 2 h is the top surface. We have a turbulent velocity flow here, so the velocity profile looks something like this. It is much flatter than the laminar velocity profile, which I would have had for a laminar flow it should be a parabolic profile at that same Reynolds number.

So, this mean velocity profile is what I called u of y and of course, there are turbulent velocity fluctuations, okay? So, how do I model the the the flow in this turbulent channel? First thing to note; I can solve the problem only from y is equal to 0 to y is equal to h because it is symmetric about that center prime. So, at y is equal to h you require from just from symmetry that at y is equal to h the u by d y is going to be equal to 0. There is the no-slip condition requires that capital u has to be equal to 0 at the bottom surface. Important to note no-slip condition requires that both the mean and the fluctuating velocity are both equal to 0 at the bottom surface.

That means that capital U is the flow in the x direction the mean velocity is equal to 0 the fluctuations are also equal to 0 u x prime u y prime and u z prime are all equal to 0 at this bottom surface. Similarly, at the top at at the mid plane divergence I am sorry, the slope of the mean velocity profile is equal to 0 in addition you cannot have a Reynolds stress a, Reynolds shear stress u x prime u y prime. We will come back to that a little bit. So, I can write down the momentum conversion equations in the x and y directions. For the mean flow these momentum conversion equations will of course, contains the Reynolds stress terms. I assume there is a steady flow so, is nothing that is changing along the x direction apart from the pressure, which of course, is required to drive the flow. So, it is nothing changing along the x direction apart from the pressure which is required to drive the flow.

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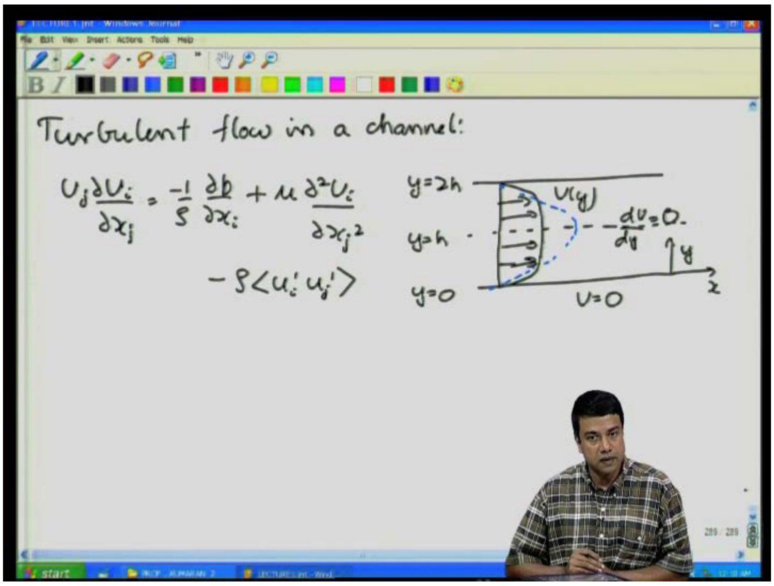

So, the momentum conversion equation for the mean velocity profile can be written as U j partial U y by partial x j is equal to minus 1 over rho partial p by partial x y minus plus the the viscosity times minus rho times u y prime u j prime. So, that is the equation momentum conservation equation where y is the direction of the momentum and j is the repeated index.

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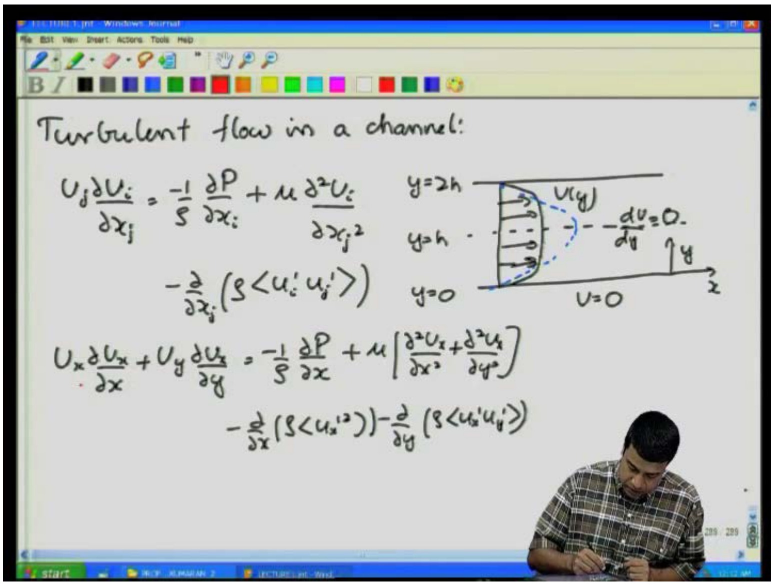

So, now if I write this equation for the x momentum equation, if I write this with the x momentum equation I will get U x partial U x by partial x. I should make a correction here. This this pressure here is the mean pressure, if you recall this is the mean pressure because I have averaged the entire momentum conservation equation over time, plus mu partial square U x by partial x square. This, one second, there is a mistake here, then I have minus partial by partial x of rho U x prime square minus partial by partial y of so, that is the momentum conservation equation. Now, we can apply a condition that there is no variation in the x direction apart from the pressure and the component of the velocity U y is identically equal to 0 because the mean flow is only in the x direction.

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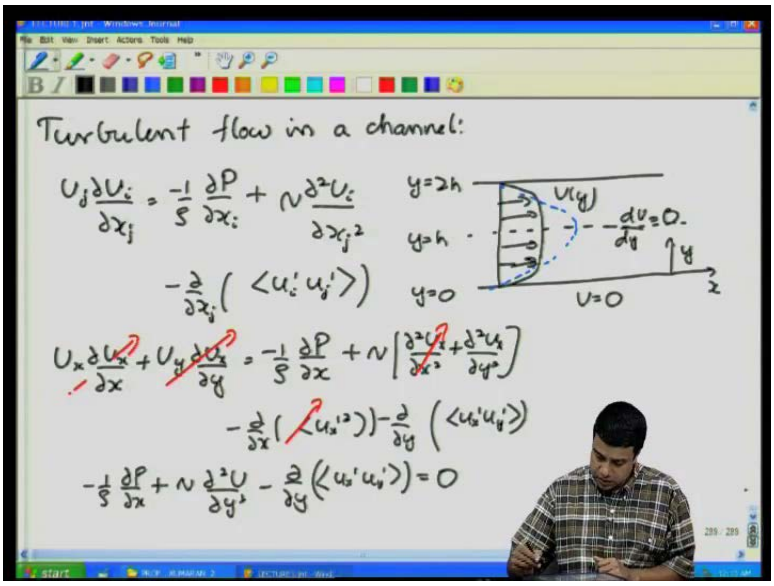

So, this there is no variation in the x direction, so that is 0 capital U y is equal to 0 because there is no mean velocity perpendicular to the main flow. There is a pressure variant of course, in the x direction this should be kinematic viscosity. This I divided throughout by the density. So, there should be a kinematic viscosity partial square u x by partial y x square is equal to 0 because there is no variation in the x direction. Similarly, there is no variation of the fluctuations as well in the x direction, because this is the fully developed flow. So, you would expect no variations in the root mean square of the fluctuating velocities in the x direction as well. So, therefore, this is also equal to 0 and my momentum conservation equation just to visit reduces to from the density here. So, you divide it throughout by the density plus nu partial square u by partial y square minus partial by partial y of u x prime u y prime.

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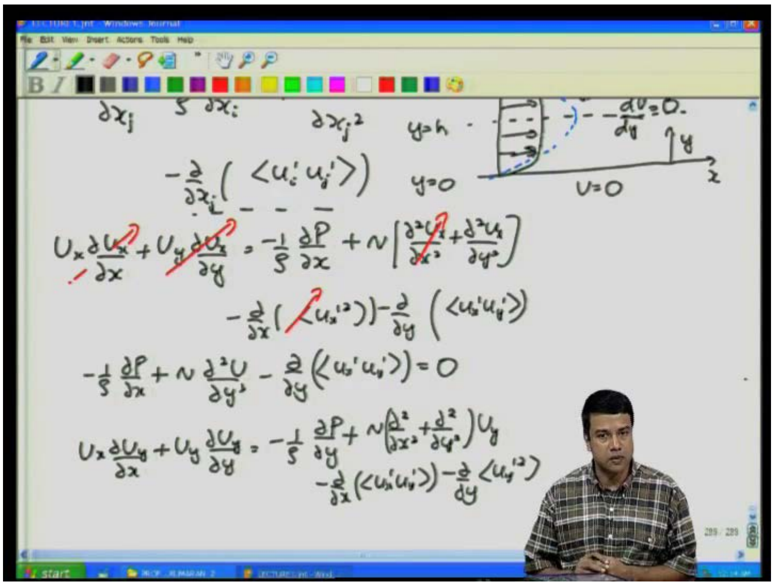

This is equal to 0. So, this is the momentum conservation equation for this channel in the x direction. For the y direction I just substitute y is equal to y I just substitute y is equal to y in my momentum conservation equation. I will get U x partial U y by partial x minus 1 by rho partial p by partial y plus. Then I get that fluctuating term that is minus partial by partial x of u x u y prime, I get this additional term here which is partial by partial x j of u y prime u j prime. So, it is partial by partial x of u x prime u y prime and partial by partial y of u y prime square.

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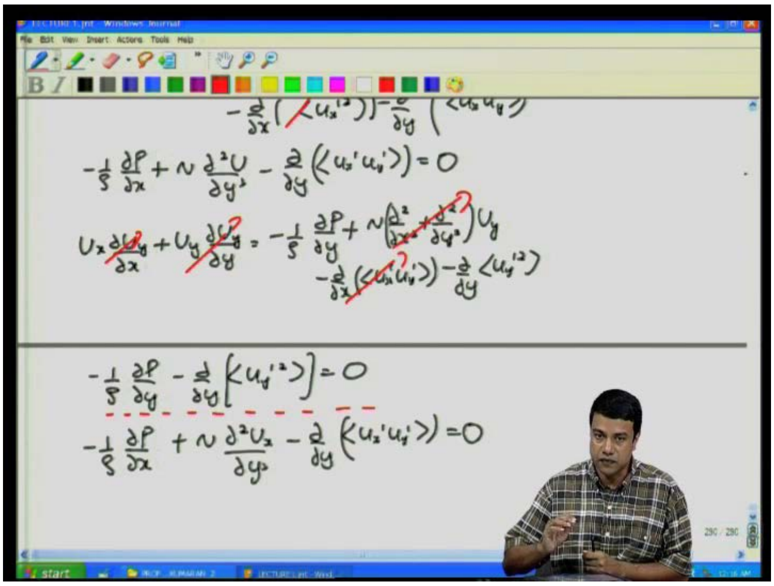

Once again, simplifications can be made in this equation. First thing u y is identically equal to 0. Capital U is equal to 0 because there is no mean velocity in the y direction. Therefore, the entire left hand side goes to 0. The viscous term goes to 0 on the right hand side, capital U is equal to 0. Since, the flow is fully developed, there is no variation in the x direction partial by partial x of u x prime u y prime this is equal to 0. Therefore, in my momentum conservation equation the y direction reduces to minus 1 by rho partial p by partial y minus partial by partial y of u y prime square is equal to 0.

So, there is the y momentum conservation equation and as I wrote the x momentum conservation equation is minus 1 by rho partial p by partial x plus mu, this is equal to 0. The y momentum conservation equation when we did it for laminar flows, we found that partial p by partial y was equal to 0. For a laminar velocity profile we found that partial p by partial y is equal to 0. That is because u y itself is 0. There was no viscous diffusion of y momentum because the velocity u y was equal to 0. Therefore, there was nothing balance the pressure term in this particular case there is transport of momentum due to the velocity fluctuations and for that reason the pressure variant is basically balancing the transport of momentum due to the fluctuating velocity in the y direction in the laminar flow there is no fluctuating velocity.

So, we just get partial p by partial y is equal to 0. The y momentum equation can be integrated once with respect to y because both terms have a derivative with respect to y. So, the y momentum equation can be integrated once with respect to y.

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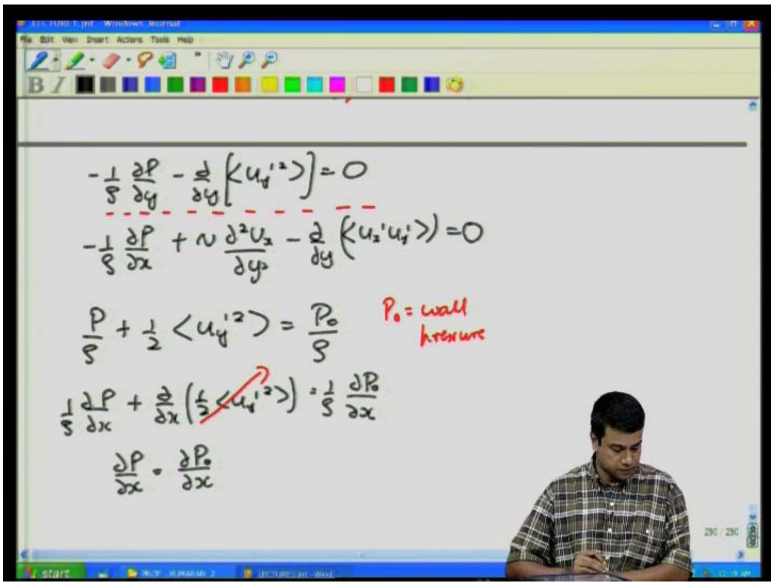

I can write this as P by rho plus 1 2 u y is equal to some constant P naught by rho. What is P naught? P naught is the pressure at the location where u y prime is equal to 0 because y have p by rho plus half u prime y prime square is equal to P naught by rho P naught by rho is the constant of integration. It is the value of the pressure at which at the location where u y prime is equal to 0.What is the location at which u y prime is equal to 0 that is the wall of course, at y is equal to 0 we have a solid wall there is no-slip condition. That means the both all components of the velocity both mean and fluctuating have to be equal to 0.

Therefore, P naught is the wall pressure, P naught is the wall pressure the pressure at the wall. The pressure at the location where u y prime the velocity u y prime is equal to 0. Further, if we take the derivative of this entire equation with respect to x, if you take the derivative of this entire equation with respect to x we get 1, by rho partial P by partial x plus partial by partial x of half u y prime square is equal to 1 by rho partial p naught by partial x partial P naught by partial x, we have a steady fully developed flow. That means that there is no downstream variation of u y prime square there is no downstream variation of u y prime square.

Therefore, this term is equal to 0 which means that partial P by partial x is equal to partial P naught by partial x. Note that the pressure P itself was the mean pressure. It was a function of both x and y P naught is a constant of integration I had one equation which has a derivative of with respect to y in it I integrated that once to get the value of P naught.

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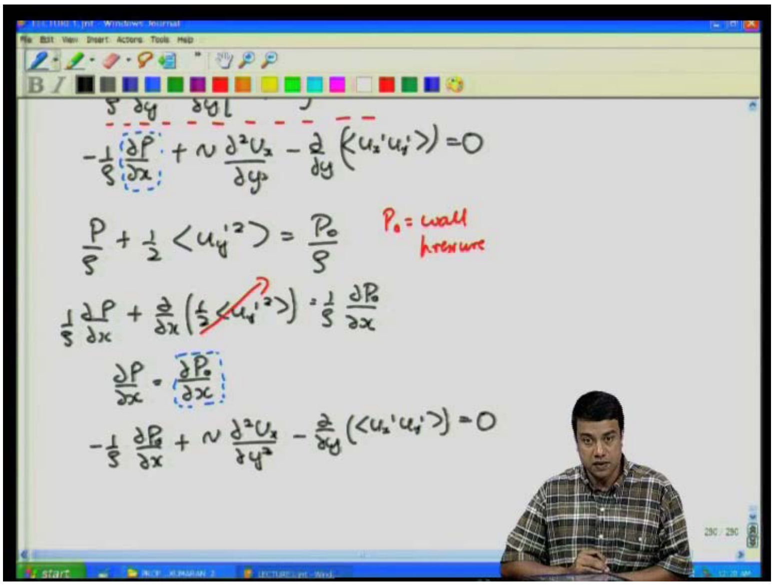

Therefore, P naught is defined only at the wall, it is not a function of y it is only a function of x, it is the wall pressure at a given location x. So, now second point that means that since partial P by partial x is equal to partial P naught by partial x, I can substitute for partial P naught by partial x here. I can substitute for partial P naught by partial x here partial P naught by partial x is now not a function of y, it is not a function of the trust induced in the channel, since it is the wall pressure itself. So, my x momentum conservation equation becomes minus 1 by rho partial P naught by partial x plus nu partial square u x by partial y square minus partial by partial y of u x prime u y prime is equal to 0. Since, P naught is not a function of x I can integrate this 1 with respect to y I can integrate this once with respect to y.

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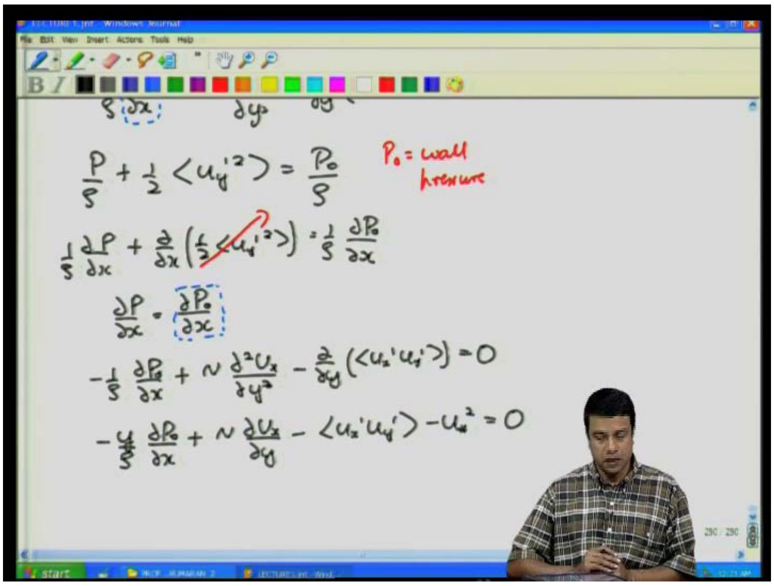

If I integrate this once with respect to y, I will get minus y by rho partial p naught by partial x plus mu partial u x by partial y minus partial by partial x y minus plus the constant of integration plus a constant of integration is equal to 0. That constant of integration I will write it as minus u star square. That is the constant of integration in the equation minus u star square is equal to 0. What does that constant of integration represent?

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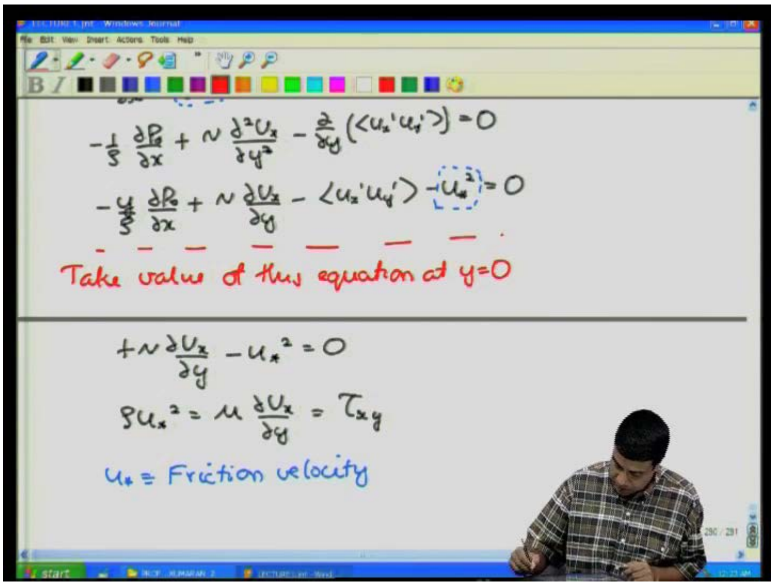

Easiest is to take the value of this equation of this equation at y is equal to 0 at y is equal to 0. Of course, I have minus y by rho partial P naught by partial x that is equal to 0. So, I will have plus mu partial u x by partial y fluctuating velocities are 0 at the wall. Therefore, u x prime u y prime is equal to 0, this minus u star square is equal to 0. This implies that if I just multiply it over by density I get rho u star square is equal to mu times partial u x by partial y is equal to the mean shear stress at the wall is equal to the mean shear stress at the wall.

So, there is the physical interpretation of this velocity u star square, there is a physical interpretation of this velocity u star square rho u star square is the shear stress at the wall for that reason u star is often called as the friction velocity. It is a velocity scale that is defined not based upon the mean velocity itself, but rather from the stress from the requirement that rho u star square is equal to the wall shear stress. The other thing we can do is to write down this equation, take the value of this same equation.

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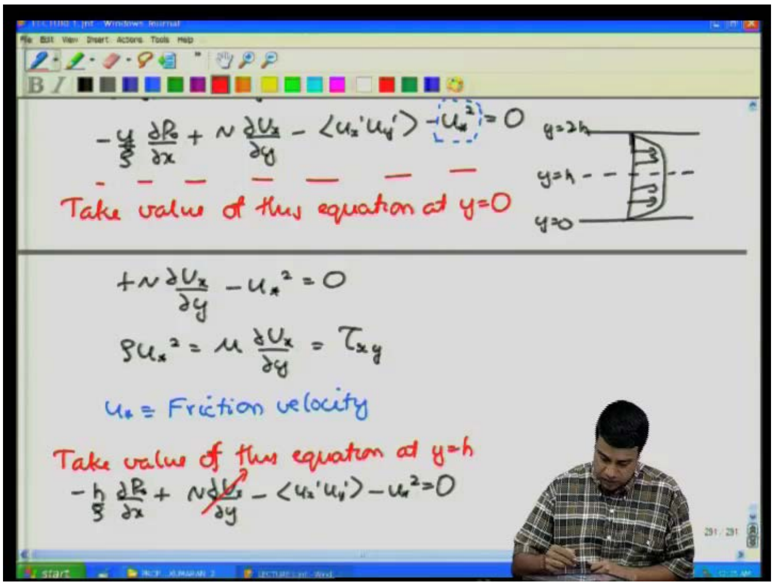

This equation at y is equal to h as you recall y is equal to h is the center of the channel. I will have the velocity profile; that looks something like this, can we write that, be better for you. The velocity is of course symmetric about the center of this channel y is equal to h that means that at y is equal to h. I require that the velocity gradient the mean velocity gradient has to be equal to 0, because the mean velocity has to go through a maximum at y is equal to h. So at this particular location the mean velocity itself the gradient of the mean velocity is equal to 0. The velocity goes through a maximum so therefore, in my equation at y is equal to h I have minus h by rho partial P naught by partial x plus mu partial u x by partial y minus u x prime u y prime minus u star square is equal to 0.

At y is equal to h the velocity gradient is equal to 0, in addition symmetry requires that u x prime u y prime is equal to 0, because you require that at y is equal to h itself the configuration is perfectly symmetry. So if I have a parcel of fluid moving with a fluctuating velocity plus u x prime at this location alone, it should have equal probability of having the velocity in the plus u y direction or minus u y direction, because at this plane at this symmetry plane the velocity has to be symmetric with respect to plus and minus y in y directions.

Therefore, u x prime u y prime as to be equal to 0 for a given velocity in the x direction, there should be equal probability of the parcel of fluid moving either in the plus y direction or in the minus y direction at this symmetric plane along.

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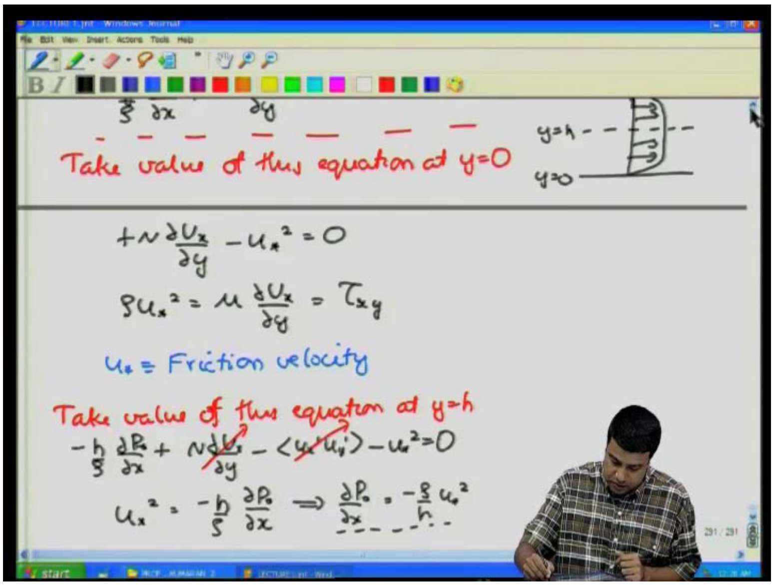

So, this also becomes equal to 0. So, this gives me another expression for u star and that is that u star square is equal to minus h by rho d P d x or alternatively the wall shear stress the the wall pressure variant is related as minus rho by h u star square. Therefore, substituting for this for the pressure gradient from this expression in terms of the friction velocity, I substitute for the pressure gradient in my momentum conservation equation which is here.

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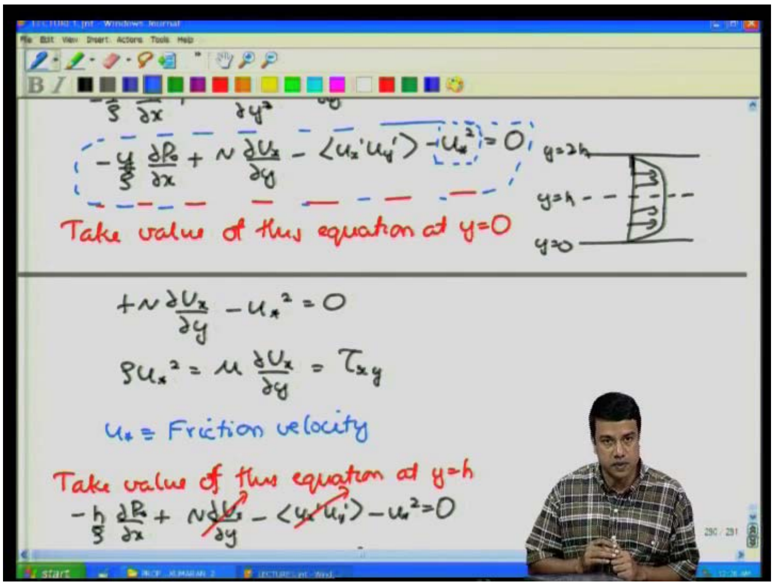

This was the reduced momentum conservation equation that I had.

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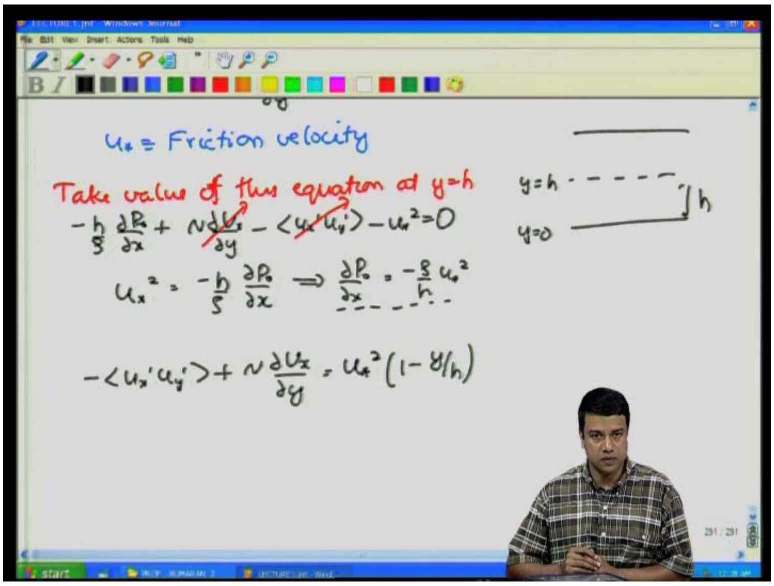

Based upon the condition at y is equal to h I would also obtained an a relationship between u star. The pressure gradient substituting that you get an equation for the x momentum conservation equation, which goes as plus nu is equal to u star square into 1 minus y by h. So, this is the expression that I get where u star was equal to the rho u star square was equal to the wall shear stress.

So, this is my final expression for the velocity profile as you can see there is a fluctuating momentum transfer and there is the mean momentum transfer. The sum of these two is equal to minus u star square into 1 minus y by h. We can scale these equation their nature length scale for the y coordinate is h the length scale the half width of the channel h is the natural length scale for the y coordinate. What about for the fluctuating velocity you have a natural scale. Here that is u star there is a natural scale for the fluctuating velocity.

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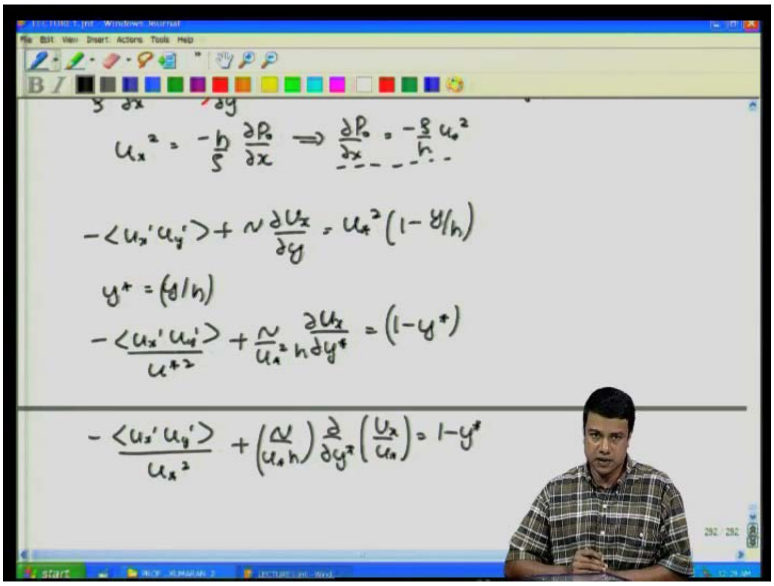

So, if I scale it this way if I scale the length scale by h and the fluctuating velocity by u star, the dimensional less equation I will define y star is equal to y by h. The dimensional less equation becomes minus u x prime u y prime by u star square plus nu, now I have to scale u x by u star. I have to divide throughout by u star square, so I get nu by u star square partial u x by h times partial y star. Because I defined y star is equal to y by h, so I get a factor of h coming out is equal to 1 minus y star. Rearranging these terms a little bit, what I will get is minus u x prime u y prime by u star plus nu by u star h partial by partial y star of u x by u star is equal to 1 minus y star. Nu by u star h is a Reynolds number based upon the frication velocity.

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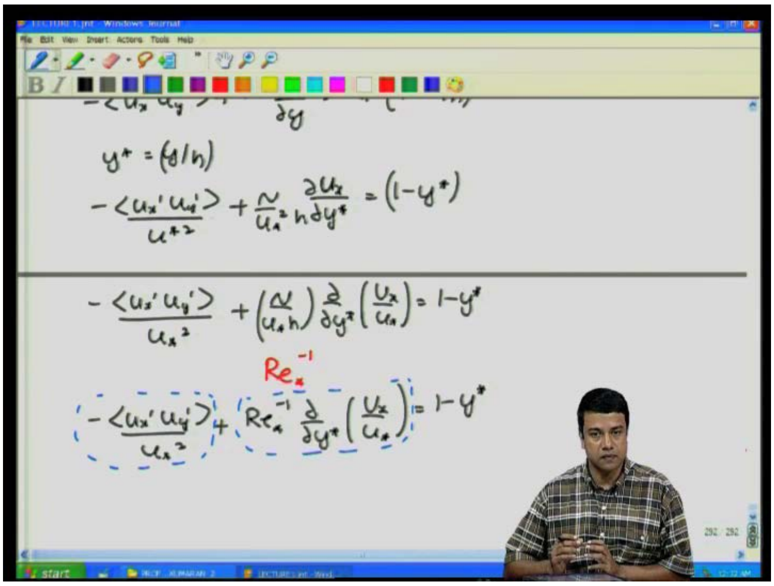

Because u star is the frication velocity h is the length scale the half width of the channel. So, when nu by u star h is the length scale based up on the nu by u star h is an inverse for a Reynolds number based up on the frictional velocity. So, if I have expressed this in terms of that Reynolds number, I will get minus u x prime u y prime by u star square plus Re star inverse is equal to 1 minus y star. Clearly in the limit of high Reynolds number you would expect the Reynolds number based upon the friction velocity also to be large. That means that this term is small compared to this term. Basically it reiterates what we said for a turbulent flow the transport of momentum due to the Reynolds stress during the fluctuating velocity is much larger than the transport of momentum due to the mean velocity gradient.

So, in this case in the center of the channel there is a balance between the fluctuating velocity transport of momentum due to fluctuating velocity and the pressure gradient there is a pressure gradient that actually gave us this u star square term the wall pressure variant. So, this seems to indicate that the mean velocity then, then the transport of energy due to the mean velocity is not important. However, at the wall itself fluctuating velocities have to go to 0 and transport very close to the wall ultimately has to happen due to the viscous diffusion. The diffusion of momentum due to the mean flow because the fluctuating velocities themselves have to go to 0.

However, the mean velocity gradient will still be there we know the fluctuating velocities go to 0. u x prime and u y prime go to 0. But however, there will be a mean velocity gradient at the wall and the stress due to that. That mean velocity gradient is is important very close to the wall as the fluctuating velocities go to 0. So, clearly there has to be a reason very close to the wall where the mean velocity gradient is important and the fluctuating velocities go to 0. What is the length scale appropriate for that? Clearly that length scale is not the length scale h because that is a channel half width and the Reynolds number based upon that is large.

So, the length scale that you get you can depend only upon the friction velocity, which basically it is the is measure of the shear stress of the wall and the kinematic viscosity. From the friction velocity in the kinematic viscosity, we can get only one length scale that is nu by u star because the kinematic viscosity has dimensions of length square for time, frication velocity is dimensions of length per time. Based upon that, you can get only one length scale by balancing out the kinematic viscosity by taking the ratio of the kinematic viscosity in the frication velocity.

So, that is the viscous length scale near the wall we would expect over that length scale. The fluctuating velocities have to go to 0 because there to be identically 0 at the wall. There will still be a mean velocity gradient and the shear stress due to that and that region the mean velocity gradient the shear stress to the mean velocity gradient has to balance the pressure gradient. So, let us see what you get for that region.

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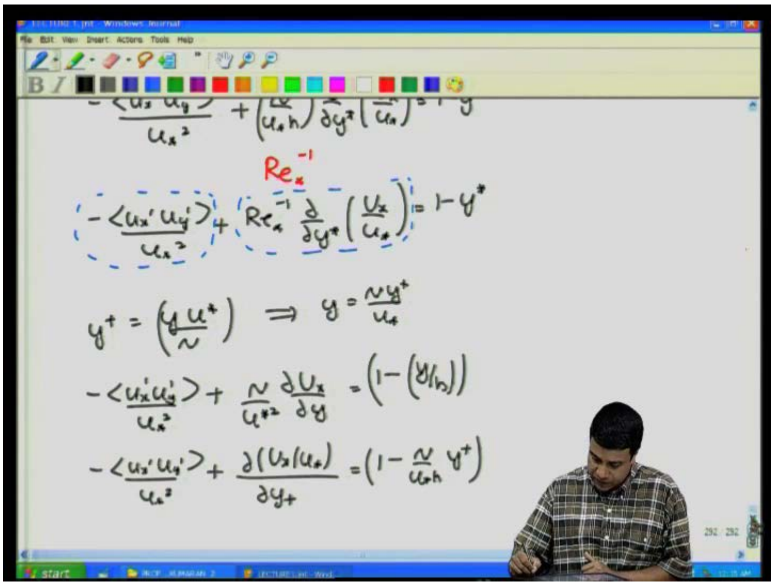

Therefore, I have to rescale my y coordinate as y plus. It is, it is called the inner of the viscous coordinate is equal to y u star by nu. So, there is the scaled velocity the scaled y coordinate y divided by nu by u star. So, if I define y plus in this way and I express once again in my equation of motion, what I will get is minus u x u y prime by u star square plus this kinematic viscosity divided by this is explicitly is equal to 1 minus y by h. I express therefore, y is equal to nu y plus by u star. With this you will get minus u x prime u y prime by u star square plus this will just give me partial of u x by u star by partial y plus is equal to 1 minus mu by u star h y plus.

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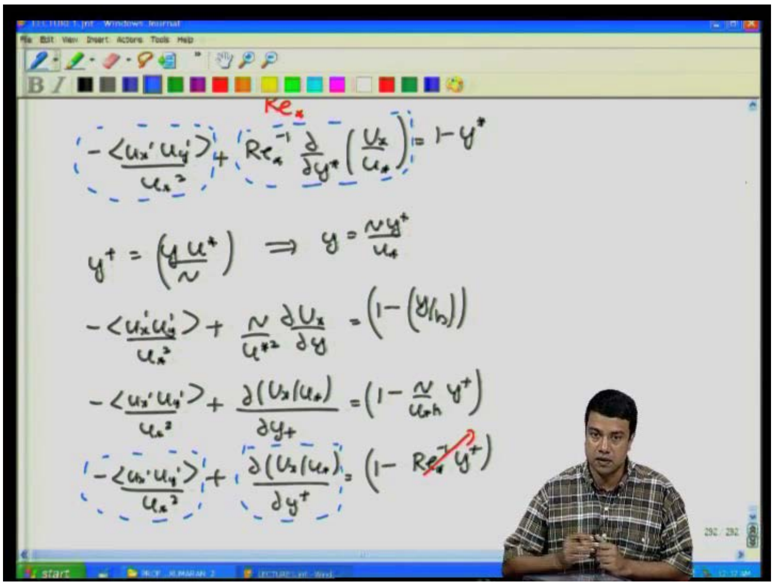

Express again in terms of the velocity y plus and you can easily see that this can be written as… 1 minus Re star inverse y plus. Since, I have a factor of Re star inverse here, that means that in the limit of high Reynolds number this goes to 0 in the right hand side just becomes 1. So, in this wall viscous sub layer of thickness y plus I am sorry, nu by u star. In this wall viscous sub layer of thickness y plus by nu star, there is a balance between the means viscous stress and the fluctuating Reynolds stresses. So, we have two regions in the flow, the central region where of course the mean viscous transport the viscous transport due to the mean flow is much smaller than the momentum transport due to fluctuations.

However, as you come close to the wall, fluctuations have to go to 0 because they are identically 0 at the wall. The velocity gradient is still there. Therefore, you could have a balance between the viscous stresses due to the mean flow and the Reynolds stress. So, you have 2 equations in 2 regions, one is where y is comparable to h the thickness of the the half width of the channel, the other is in the viscous sub layer where y is comparable to u by u star. The ratio of these two length scales is of course, 1 over the Reynolds number. So the viscous sub layer is Re star inverse smaller than the channel half width h.

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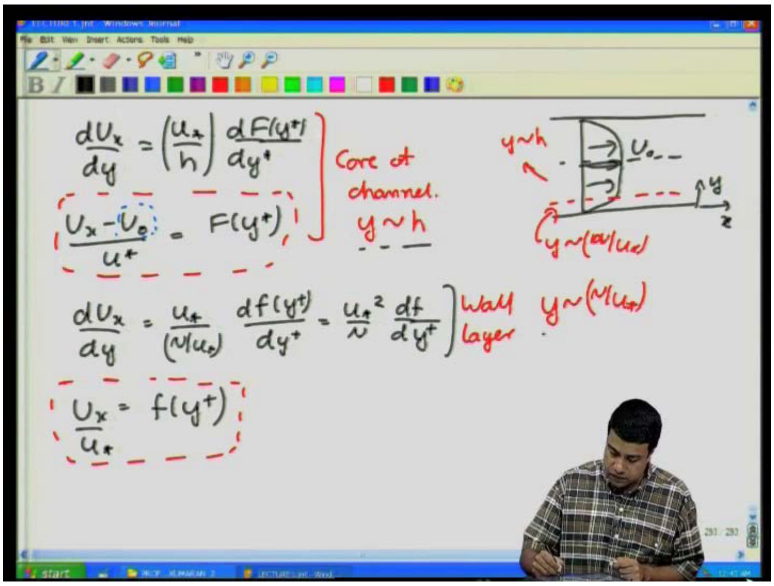

So, now how do we can we say something about the actual velocity profiles in these two regions in the outer region and in the wall region? So in the in the center of the channel in the center of the channel, let me call it the core of the channel that is on the distance from the wall becomes large compare to the viscous sub layer thickness. The only relevant length scale for the mean velocity profile should be the length scale h itself. The only length scale for the mean velocity profile should be the length scale h itself. That means that I should be able to write I should be able to write the velocity gradient the only length scale is h itself.

The frication velocity is still there because that term is the mean shear stress on the pressure gradient at the center. So the both the pressure gradient at the center and the wall shear stress are both equal to the frication velocity. So, if if we write the pressure the gradient of the mean velocity, just based upon dimensional analysis, this has to be of the form u star by h. So, these are the relevant length and the velocity scales times some dimension less functions d f by d y star where this f is some dimension less function of y star f is a function of y star.

If I integrate this from the center of the channel to any location y if we integrate the this from the central of the channel to any location y, because I am looking at the velocity profile in the core of the channel I would like to go towards the wall. Of course, but there the large different as I will just show you because the length scale relevant length scale is mu by u star, so I have to integrate from some location within the domain for which this velocity is written down and that is the center of the channel. So therefore, U x minus U x naught is the velocity profile which is the value of the velocity at the center of the channel.

Let me just write this velocity is equal to U naught. It is a constant this has got to be divided by this is just equal to F of y star this is the scale velocity this is the law that applies in the core of the channel. What about as you go close to the wall? As you are close to the wall the relevant length scale is nu by u star as you force the wall the relevant length scale is mu by u star. That means that very close to the wall I should have d U x by d y is equal to u star by nu by u star d of f of y plus, because that is the only length scale where f is some dimensional less function of y. So, is equal to u star square by mu d f by d y plus u star square by nu d f by d y plus.

So, therefore I have these two laws for the velocities in these two different regions. This is at the wall this is in the wall layer. So, this is for y let me y goes as h in the core of the channel, when y is proportional to h itself. This is in the wall layer where y goes as nu by u star. So, I have one particular law in the core of the channel and the distance from the wall is large compare to the viscous sub layer thickness. There were other one very close to the viscous sub layer. So, this I can integrate once to get the velocity U x is equal to u star U x by U star is equal to f of y plus I just integrate this once and I will get U x by u star is equal to f of y plus.

So, I have these two laws one valid in the core of the flow where the turbulent fluctuations are large compared to the viscous momentum diffusion of stress, the other in the wall region where both the viscous diffusion viscous stress and the Reynolds stresses are comparable. Now this so let us divide the domain into three regions, 1 is here I have y goes as nu by u star. I have this region here where y goes as h. Obviously, these are two approximations for the velocity profile in two different regions these are two approximations for the velocity profile in two different regions. These are two approximations have to converge to the same value in the intermediate region, where y is large compared to nu by u star and y is small compare to h.

In this intermediate region both of these velocities have to merge. Of course, I can obtain an agreement for the mean velocity itself by just adjusting the value of velocity of the center. So, the mean velocity itself agreement can be obtained if you shift the center line velocity. However, you require that the velocity gradient the gradient in the velocity in both cases have to converge to the same value. So we have one law for y going as h another law for y going as nu by u star.

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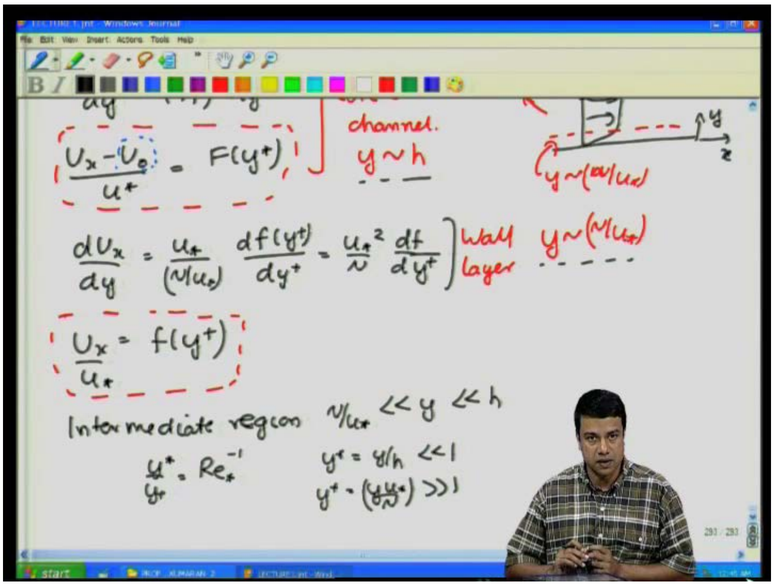

In the intermediate region, that is nu by u star much smaller than y much smaller than h. What is it mean nu by u star much smaller than y much smaller than h? Since, y is much smaller than h, y star this is equal to y by h is much smaller than 1, since y is much larger than mu by u star, I have y plus is equal to y by u star by nu is much greater than 1. So, as I take the limits of y star small compare to 1 and y plus large compared to 1, the ratio of course, you know? y y star by y plus will be equal to Reynolds number. The ratio of the Reynolds number y star by y plus is equal to the Reynolds number.

Therefore, as I simultaneously take, y star becoming small compared to 1 actual, this is yes as I simultaneously take y star becoming small compare to 1 and y plus becoming large compare to 1. The two large for the two velocities in these two regions have to be identical. The two large for these two the velocities have to be identical as I said the absolute value of the velocity itself contains u naught in it which still has to be determinant so that is unknown. However, the velocity gradients do not contain any constants; the velocity gradients themselves do not contain any constants. Therefore, you require that the in this intermediate region the velocity gradients, where you, whether you approach the outer region from the wall layer or you approach the wall from the outer region. These two velocity gradients have to be the same. So in this intermediate region you require that.

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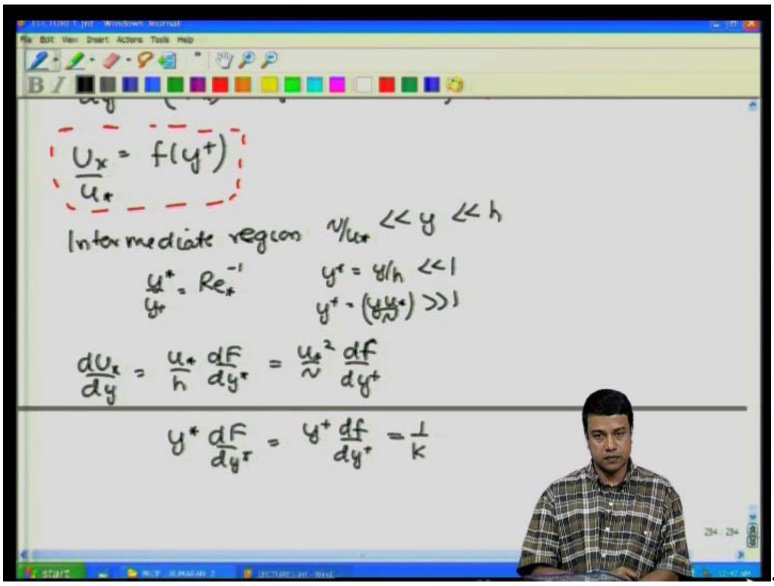

The U x by d y, this is equal to u star by h d f by d y star. This has to be equal to u star square by nu d a small f by d y plus u star square by nu d small f by d y plus. I can cancel out 1 u star here and multiply both sides of the equation by y should multiply both sides of the equation by y. What you get is that y star d f by d y star is equal to y plus d f by d y plus. This has to be true in the limit as y plus goes to infinity and y star goes to 0. Simultaneously, for any value of the Reynolds number this has to be true for as y plus goes to infinity and y star goes to 0.

Simultaneously, for any value of the Reynolds number if I keep h a constant and change the Reynolds number, I will change the right hand side of the equation, but not the left hand side on the other hand. If I keep the Reynolds number the frication Reynolds number a constant and change h, I will change the left hand side, but not the right hand side. The only way that this equality will continue to be satisfied in all of these cases, since the both of these are equal to constants that constant is traditionally written as 1 over kappa. So, you will follow that notation here that means that these two are equal to constants 1 over kappa.

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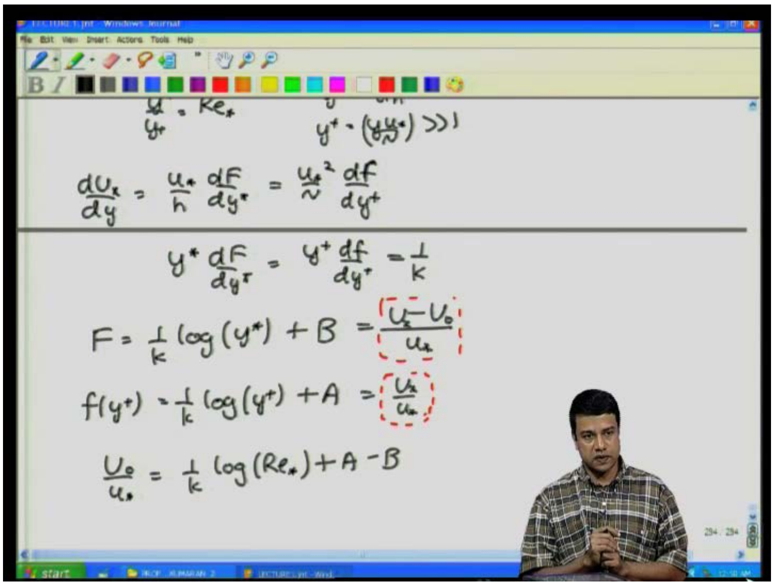

So, these equations can be solved. So the solution for f basically is f is equal to 1 by k log y star plus a constant b and f of y plus is equal to 1 over k log of y plus plus a constant. So, just this matching condition that both the velocity profiles have to go to a common value, in an intermediate region in between the outer core flow, where the Reynolds number stresses and the viscous sub layer, where the Reynolds stresses are balanced by the viscous stresses just that requirement alone tells me, that both of the velocity profiles in both of these regions at we get by logarithmic profile. In this intermediate region of course, in the viscous sub layer there has to be departure from this, because that y plus is equals to 0, the velocity has to go to 0.

But, in the intermediate region as you approach the outer flow from the wall, we should get a logarithmic velocity profile which matches with the velocity profile that you get as you approach the wall from the core of the flow. Beyond this, one cannot obtain any further information just from analysis alone. One has to go to experiments to see what these constants are 1 over kappa A and B. As I told you this f is equal to u minus u not by h and this is equal to u x by h. So, this from these two equations, from these two equations we can infer I am sorry. This is u star, we can relate the mean velocity to these constants, okay? That is we get u naught by u star the mean velocity at the center of the channel divided by the frication velocity is equal to 1 over kappa log of Re star plus A minus B.

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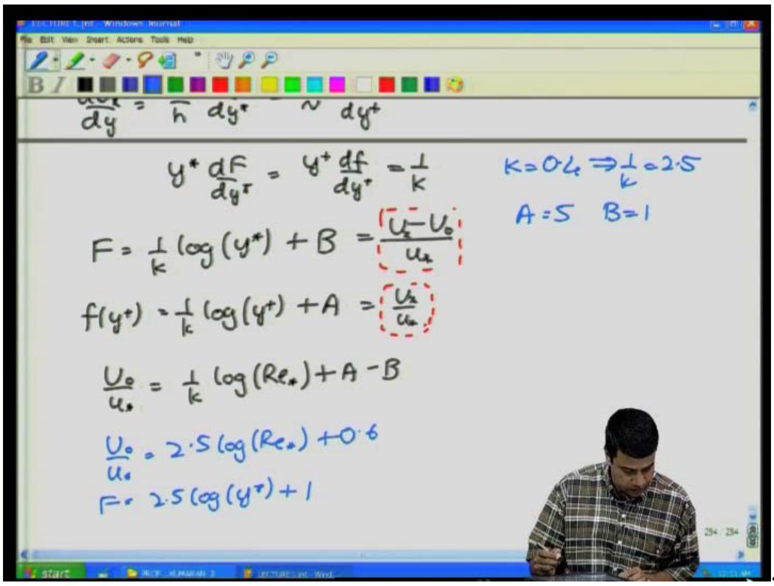

That gives us a relation between the mean velocity and the frication velocity. Beyond this, one cannot obtain information just from analysis alone. One has to go and fit the data from experimental results. And a lot of data from experimental results which are been fitted gives the values of kappa is equal to 0.4, which implies that one over kappa is equal to 2.5 is equal to 5 and b is equal to 1. So, these are called the log loss for the wall the logarithm loss for the velocity profile in the wall layer and in the intermediate layer between the the viscous sub layer, in the bulk of fluid.

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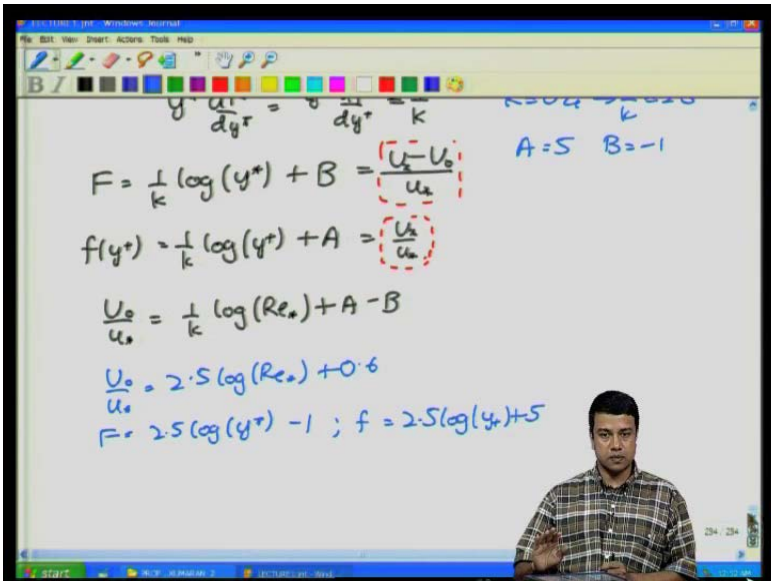

And on this basis you find that U naught by u star plus 0.6 and F is equal to 2.5 log of y star plus 1. I am sorry, B is equal to 5 B is equal to B is equal to minus 1 and minus 1. F is equal to 2.5 log of y plus plus 5. So, these are the constants that are obtained on the basis of analysis of a large amount of data. So, this tells it is some indication of, how these turbulent flows can be modeled? Basically, we written down the equations for the mean momentum in the x and y directions including the Reynolds stress terms, and we use some fairly simple considerations. There was no variation in the x direction apart from the pressure you should can have a non-zero pressure gradient and symmetry conditions for both the mean, and the fluctuating velocities.

Then, there was one important additional ingredient that we used and that was that whatever velocity profile that there in the core of the channel depends only upon that length scale h, right? It does not depend upon length scale relevant very close to the wall. The velocity profile very close to the wall depends only upon the length scale nu by u star which is the viscous sub layer thickness. Because the length scale relevant for the entire channel is not relevant for the flow very close to the wall. So, we wrote down lost just based upon dimensional analysis for those two for the mean velocity variation in those two regions, then the requirement that the velocity gradients have to match in this intermediate region between the viscous sub layer and the fully developed turbulent flow on top.

Here was a form for the velocity profile, I told as the velocity profile just based upon this similarity. Argument has to be logarithmic in that intermediate region and also give us relations between that logarithmic velocity profile, and the mean velocity of the centre of the channel. So, using rather simple considerations, we were able to obtain a specific form for the velocity log the logarithmic velocity log. In the intermediate region of course, that law had constants which then have to be fitted for the very fact that in experiments you find the same. Logarithmic velocity law implies that these are very powerful techniques simple dimension analysis along with (( )) expansions that is matching of the velocity profile in two different layers.

The velocity in the outer core is of course, known only two within 1 unknown constant which is the velocity at the center the velocity in inner region the constant there is just the wall velocity. However, the velocity gradients in the intermediate region if you come from towards the wall from above or if you go towards the turbulent core from below these two gradients have to be identical the same. That gave us the form for the velocity profile. So, in that sense it is a very power full argument and of course, the that the constants in that in that in those (( )) velocity profiles have to be obtain from experiments. But, never the less you do find that those velocity profiles are actually away in experiments.

So, this is given you some basic idea of how turbulent modeling this term in these in homogenous flows. The previous lecture we consider just a homogenous turbulent flow itself without reference to any spatial variations. But in this case we actually consider the flow in a channel and seen how we can model the velocity fluctuations that are taking across the channel. So this completes all the discussion that I have on turbulent flows we started our discussion on fundamental of transport processes two by basically starting focusing on deriving the conservation equations. The navies stoke mass and momentum equations, we also derived the equation for the energy in the angular momentum.

These were done using the fundamentals of vector calculus that we develop that gradient divergences and the curl. And once we derived those, the equations we found that the stress tensor has to be symmetric in order to satisfies the angle of momentum conservation equation and just one simple considerations of symmetry and the dependence of the stress on the rate of deformation. We decompose the flow fields into three different parts radial expansion of; compression, rotation and extraction of strain, corresponding to the isotropic anti symmetric and symmetric trace less parts with the rate of deformation tensor.

And we said that the that the viscous stress should depend upon the symmetry trace less part. Because the rotation cannot effect the cannot generate internal stresses and the radial part is 0 for an incompressible flow. On that basis we had got the navies stokes mass and momentum conservation equations for an incompressible fluid. The mass conservation equation basically said that the divergence of the velocity is equal to 0. Everywhere so, at the radial component of the radial deformation tensor is equal to 0. Then we had the momentum conservation equation. The rest of the course was dedicated to different ways of solving these equations.

The equation themselves have a structure similar to those for (( )) energy and mass transport they contain convective and diffusive terms, inertial and viscous terms the only addition is the pressure that is required to enforce incompressibility, because we now have one mass conservation equation 3 momentum conservation equation. Because the it is it is a vector and therefore, you need 4 variables 3 components the velocity and one pressure. The ratio of the inertial and the viscous terms is the Reynolds number and we use that to advantage.

We looked at the limit of low Reynolds number, where the viscous terms are dominant the solution of these equations basically, reduce to solution of Laplace equations for the pressure and the homogenous velocity. We looked at various ways to solve these you know the interpretation of of vector harmonics by just taking gradients of the fundamental source solution was ways to solve for the velocity fields around a sphere. For the diet pole due to rotation of the sphere or due to the straining, we also looked at surfaces close to each other the lubrication problem.

How do we take advantage of the fact that large amount of fluid has to flow through a thin gap, thereby generating large stresses? In order to solve the problem and get analytical solution, which told us that the force increases as 1 over the distance between the surfaces. Then we went on to potential flows, once again in that case invicid, irrotational, no viscosity, vorticity is equal to 0. However, you do get non trivial solutions, because you have a pressure there in mass and energy conservation equations you we do not have a pressure. So, we not get (( )) not trivial solutions. In this case you have a pressure because it is irrotational, the velocity can be expresses the gradient of a potential the momentum conservation equation reduces to 1 scalar equation, for the pressure the Bernoulli equation.

We looked at different ways to solve problems both in two in three dimension using our vector spherical harmonics. As well as in two dimensions using the method of complex variables, important results the force on an object in three dimensions is 0 of its at steady flow it is moving with a constant velocity. If the velocity varying it goes as the half the added mass times the acceleration. In three dimensions you could have a force a lift force even for a steady flow provided there is a net circulation. Then we looked at module layer theory, how does one incorporate the effective of viscous stress is very close to surfaces?

We have got an important result there and that is that when we have an decelerating boundary layer, the pressure is decreasing, the velocity is decreasing, as a function of distance module layer separation takes place behind bluff bodies and the potential flow solutions are no longer valid there. However, if you have an accelerating flow you have a confined module layer. Therefore, one can talk of an outer region where the potential flow is valid and a thin boundary layer near the surface (( )) power minus half where viscous effects have to be taken into account. We looked at the dynamics of vorticity, which happens after this boundary layer separation or vorticity is regenerated somewhere within the flow ok.

Vorticity cannot be produced within the flow, it can be dissipated due to viscous effects. It can also in general we convicted and intensified due to the stretching or bending of vertex lines. Finally, we looked at turbulent flow in the past few lectures I explained to you the spectrum of turbulence the energy cascade why eddies are created of many of various sizes, the dissipation of energy at the smaller scales. Finally, I also explained to how one would go about modeling a turbulent flow of practical interest in this particular case the flow in a channel. So, that briefly summarizes all the all the work that we have done in this particular course. I hope you found it very useful and I wish you all the best.