Fundamentals of Transport Processes II
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Lecture - 4
Vector Calculus

Welcome to this lecture number 4 in our course on fundamentals of transport processes, where we were going through some fundamental back ground material on vectors and tensors in the last class and we will continue that in the present lecture. To briefly review what we have done in the last lecture, a vector is a quantity which has both magnitude and direction.

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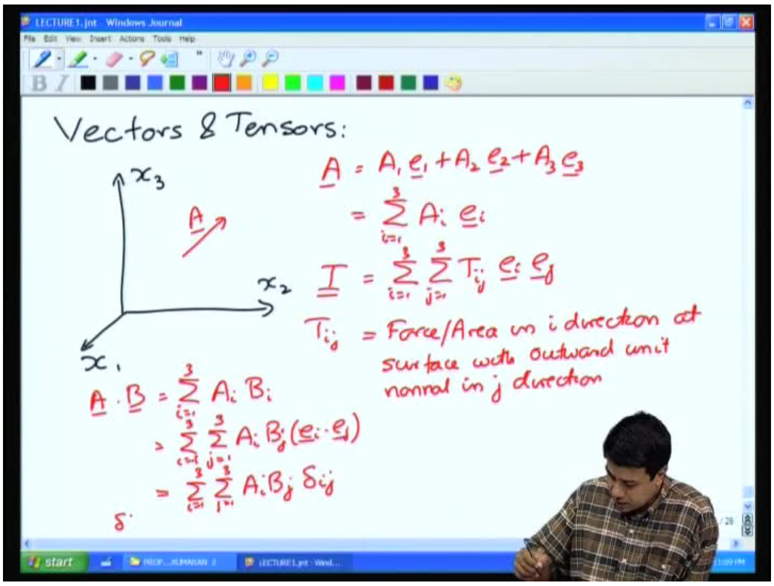

So, in a three dimensional space which I will label as x 1 x 2 x 3, in three dimensional space x 1 x 2 x 3, a vector has both magnitude and direction; it is represented by an under bar which means that it is a vector with one direction. So, this vector can be written as A 1 e 1 plus A 2 e 2 plus A 3 e 3, and I had written it for you in short hand notation as summation i is equal to 1 2 3 A i e i, where e 1 e 2 e 3 are the unit vectors in the three directions. The velocity is for example, a vector it has both magnitude and direction; force is a vector, acceleration is a vector.

We have also defined quantities which have two fundamental directions. A second order tensor an example that I had given you was the stress tensor T, which we can write it in long hand notation as i is equal to 1 to 3 summation j is equal 1 to 3 T i j e i e j. This has two directions associated with it at each point in space; one is the direction of the force, the other is the direction of the unit normal to the surface at which you are measuring the force. Of course, I cannot just write it as an arrow similar to a vector, because it has two fundamental directions and we have defined it in long hand for you in the last class. Force per area in i direction at surface with outward unit normal in j direction is T i j. So, at a given location, you can measure the force with a surface which is oriented in various ways. The force in the x direction acting at a surface whose unit normal is in the y direction, there is the surface itself is in the x i plane this T x y and so on.

So, this has two fundamental directions associated with it. You can also have higher order tensors which have three, four etcetera. We would not go through that in this course the dot product of two vectors was defined as A dot B is equal to summation i is equal to 1 to 3 of A i B i, no unit vectors in this case. Because, I am taking the dot products of two vectors and so you end up with a scalar. I also told you that we can write it as summation i is equal to 1 to 3 summation j equal to 1 to 3 A i B j e i dot e j, e i dot e j is the dot product of two unit vectors and of course. It is 1, if i is equal to j; it is 0, if i is not equal to j. So, e 1 dot e 1 for example, is 1; e 1 dot e 2 is 0; e 1 dot e 3 is 0 and so on.

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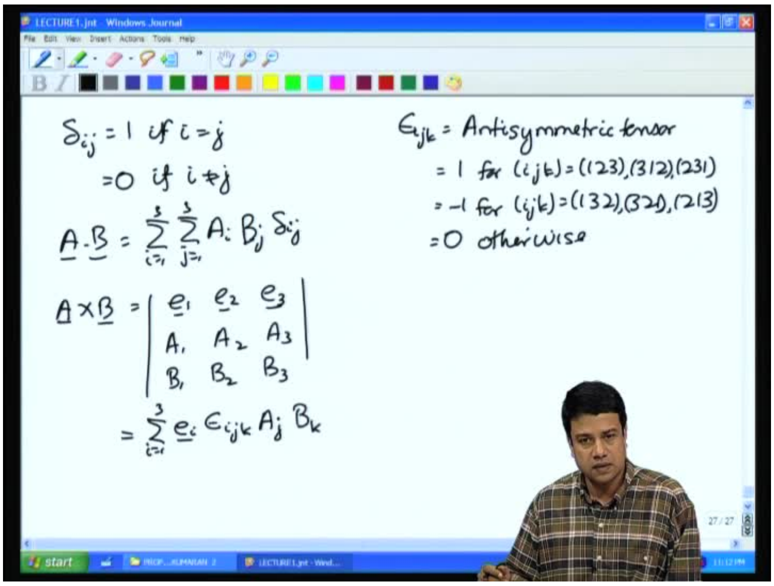

So, this can be written as summation i is equal to 1 to 3 j equal to 1 to 3 A i B j delta i j where delta i j, I defined it for you as the delta function, where delta i j is equal to 1, if i is equal to j; is equal to 0, if i is not equal to j that is the definition of delta i j. It is called the identity tensor, because you write it out in matrix form; it is it is an identity matrix so that was the dot product.

So, dot product two vectors A dot B is equal to summation i is equal to 1 to 3 summation j is equal to 1 to 3 A i B j times delta i j. We are also defined the cross product in the last lecture. We defined it differently, usually the cross product is written as A cross B it is written in matrix form e 1 e 2 e 3 A 1 A 2 A 3 B 1 B 2 B 3. The determinant of this particular matrix it is a vector, the tensor being a vector. I had showed you a different way of writing the same thing. I showed you that this can be written as summation i is equal to 1 to 3 e i epsilon i j k A j times B k, where epsilon i j k is the anti-symmetric tensor is, the anti-symmetric tensor is equal to 1 for i j k is equal to 1 2 3, 3 1 2, 2 3 1; is equal to minus 1 for i j k is equal to 1 3 2, 3 2 1, 2 1 3; is equal to 0 otherwise, called the anti-symmetric tensor, because if you interchange any two indices, it becomes the negative of itself and with this anti-symmetric tensor.

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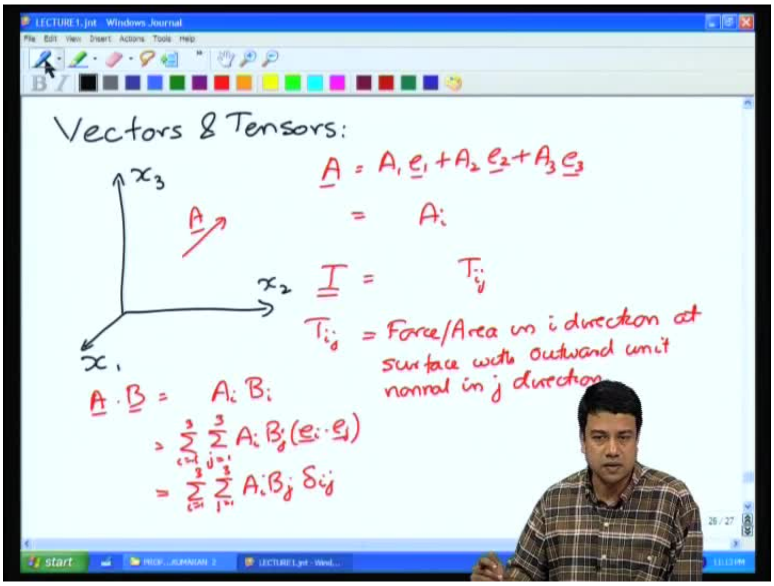

We can construct a cross product, and that was this one, we can also made a notational simplification at the end of the last lecture rather than writing this using a summation and a unit vector. We said we can just write it in terms of this alone, the factor there is unrepeated index, implies that there is already a summation and there is a unit vector. Similarly, in this case I can remove the summations and the unit vectors. T i j has two indices which are not repeated. Therefore, T i j has two summations and two unit vectors.

Now, this e i B j it has, this dot product has one index that is repeated, Therefore, if I just remove the summation, I have one index that is repeated. If it is repeated, it represents a dot product and it has become a scalar. So, there is no unit vector associated to with this particular index, because it has been repeated two times that means if the unit vectors associated with this have been dotted with each other. It had become a scalar. So, there is one summation, no unit vectors.

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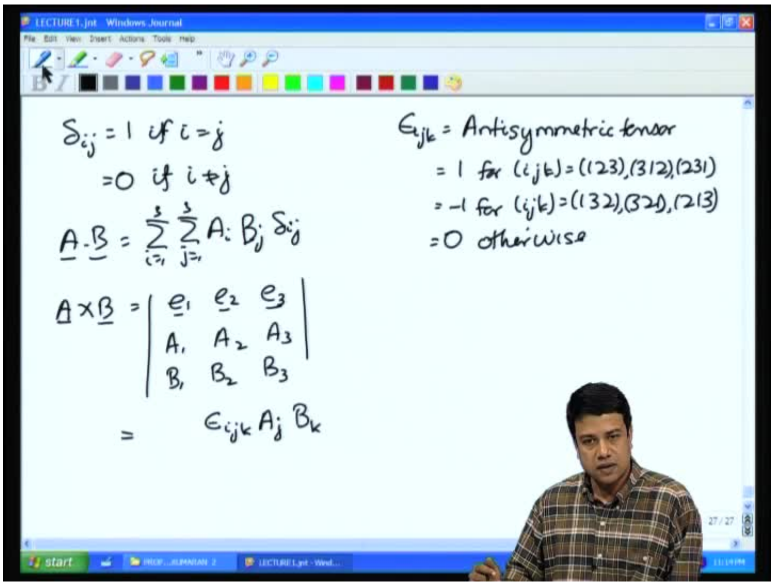

And similarly, for this cross product I could once again remove the summation and the unit vector. So, in this expression there are three indices one of them is not repeated that means; there is a summation and a unit vector associated with that two of them are repeated. So, they represent dot products therefore, the resultant is a vector itself, the other thing that we had discussed in the last lecture was the concept of real and pseudo vectors.

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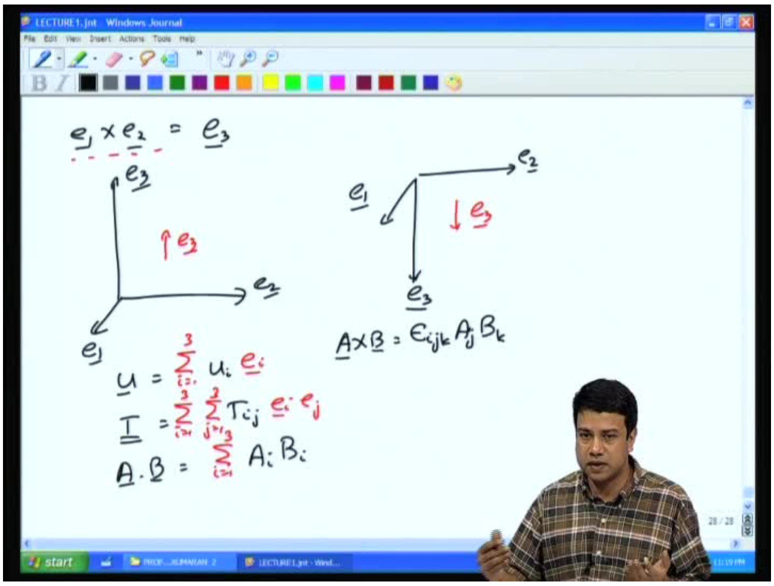

So, cross product between two vectors e 1 cross e 2 for example, you know that this is equal to e 3, the unit vector in the three direction. So, if I have using a right handed coordinate system, e 1 e 2 e 3. This cross product would be a vector in this direction, if I were using a left handed coordinate system. We are using a left handed coordinate system in that case this unit vector e 3 would be in this direction. So, the result of a cross product depends up on the right or left handed coordinate system that you are using. If it depends up on the coordinate system that you are using it cannot be a real quantity.

For example, if I have some kind of a pipe with flow going through and look at the velocity at one particular point, that velocity cannot depend up on the coordinate system that you are using to analyze the problem. It is a real quantity whereas, you finding that the cross product depends up on the coordinate system that you are used. So, it cannot be a a real quantity, it is what is called a pseudo vector. So, the cross products of two real quantities, in the last class we taken the example of distance and force or displacement and force to give you the torque. Displacement is real; you can actually measure the displacement. It does not depend up on the coordinate system that you are using. Similarly, force also is real, so it depends it is an actual quantity; you can measure velocity, you can measure acceleration and you can measure the force. The cross product of the other hand the torque does depend up on the coordinate systems that you are using so it is not a real vector ok.

So, to briefly summarize the notational simplifications at the few things that we learnt in the last lecture, you will be representing vectors and tensors mostly in what is called indicial notation; that is for each fundamental direction. I do not mean the e 1 e 2 e 3 directions but, rather directions associated with physical quantities. The fundamental direction for velocity is associated with the direction of motion. For stress, there are two fundamental directions; one with the force, the other with the direction of the unit normal of the surface at which you are measuring the force. So, if each of these two fundamental directions, you have one unrepeated index. So, it will have the velocity vector. I will just write it as u i. It is understood when there is an unrepeated index that there is a summation sign i is equal to 1 to 3, and there is a unit vector the stress for example, I will write it as T i j. There are two unrepeated indices therefore, it is understood that there is i is equal to 1 to 3 e i e j; one repeated index represents a dot product. So, A dot B, we can just write it as A I B i.

There is one unrepeated index, I am sorry, there is one repeated index. Therefore, there is one summation, there is no unit vector, the index repeat is repeated two times. The cross product was also written in terms of dot products; so A cross B. it is know is it is nothing special, it can be just written as epsilon i j k A j B k that is I have a third order tensor which I am doting with two vectors. Each dot product reduces the order by one, because one repeat repeated index means, there is no unit vector for that index. There is second repeated index; there is no unit vector for that. Therefore, there is one un repeated index and you get a vector, and think to be kept in mind is that is whenever you take cross products one has to be careful. Because you get a pseudo vector when you write an equation, the order of the vectors or tensors as well as the un repeated indices in all terms in that equation have to be the same. You cannot equate a scalar to a vector for example, so these fundamental rules we can proceed with our discussion of vectors.

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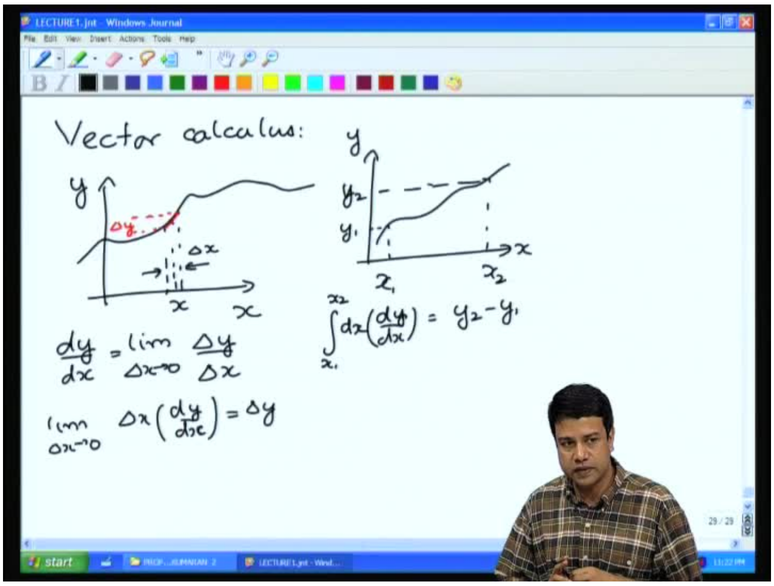

The next step is to go to look at vector calculus in vector calculus; there are quantities which are defined derivatives and integrals which are defined similarly, to what they are in scalar calculus. So, let us briefly review the kinds of how you define the derivative in just a scalar function. If I have some function y as a function of x. It is a single valued function that means that each value of x, there is only one value of y; have some function and if I want to find out what is the derivative at one particular point in it. I take a small interval, so I want to find out what is the derivative at this particular location x. I take a small interval, delta x around this point x.

For this small interval delta x i, find out what is the difference in y when I have notice travel a small distance delta x. I wrote the difference in the y coordinate, and if I take the ratio delta y by delta x around this point. I keep making the interval smaller and smaller, and in the limit as the interval goes to 0 as delta x goes to 0; delta y will also go to 0. But, the ratio itself will have a finite value. So, if I take the limit as delta x goes to 0; delta y will also go to 0 but, the ratio will have a finite value that is the let derivative d y by d x, so that is how the derivative is defined. Now, the equivalent integral relation in, in simple calculus just to reverse it that if i have two, two locations x 1 x 2 y 2 and y 1. I know that the integral the area under the curve integral d x times the derivative between x 1 and x 2 is equal to y 2 minus y 1. So, that is the integral equivalent of this derivative. In the case of vectors and tensors, we would like to derive similar relationships for derivatives and integrals except that; you know have vectors which are varying in three spatial coordinates. The relationships themselves will be similar. So, what this is saying is in the limit of delta going, limit of delta x going to 0 of delta x times d y by d x that is the derivative times the distance traveled, it is equal to the change in the function y. the derivative times the distance traveled is equal to the change in the distance y.

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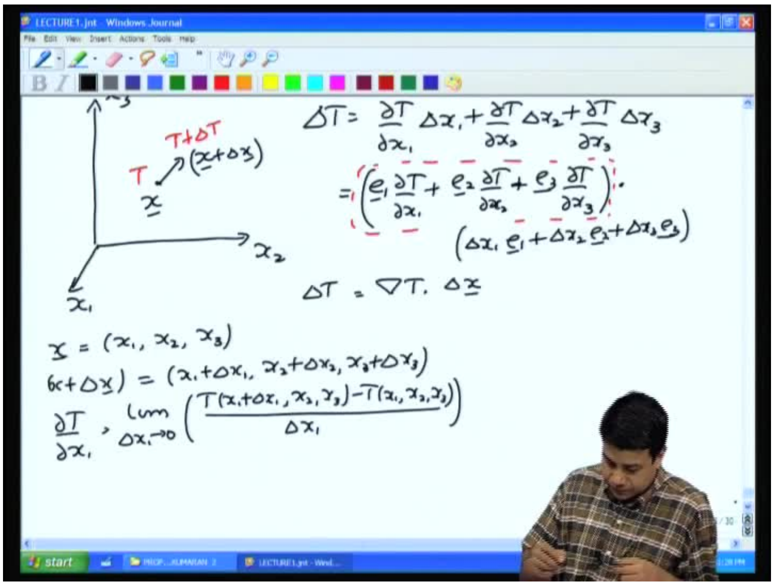

So, let us look at our first vector derivative. Let us say that I have a 3 dimensional space in which there is a temperature, which is varying in somewhere with position in this three dimensional space. The temperature in a room for example, it is it is varying at; it has different values at different locations. Let us say I am sitting at the point x, x vector and from this point I go a small distance in away; keep to a new location x plus delta x, where x is equal to x 1, x 2, x 3 that is three components x 1, x 2 and x 3, and x plus delta x also has three components that is x 1 plus delta x 1, x 2 plus delta x 2, x 3 plus delta x 3.

Now, at this particular location, the temperature has some value T and at this new location, it has some other value T plus delta T. There is a temperature at T at the location x is T, and the temperature of the location x plus delta x is T plus delta T. Similar to what we had in the previous example, the value of the function y has one value at x and it has some other value at x plus delta x in one dimension. So, what is the difference in temperature as you go a small distance delta x. It is an easy thing to do. Delta T is equal to partial T by partial x 1 delta x 1 plus partial T by partial x 2 delta x 2 plus partial T by partial x 3 delta x 3. There is a difference in temperature when you gone a small distance delta x in one particular direction. Note partial derivatives may take the derivative with respect to x 1. Let me just write it down out for you here partial T by partial x 1 is equal to limit as delta x 1 goes to 0 of T of x 1 plus delta x 1 x 2 x 3 minus T of x 1 x 2 x 3 the whole thing divided by delta x 1. So, when you take the partial derivative, you keep the other two coordinates exactly the same.

So, that is meaning of the partial derivative. So, this is delta T, I can write it in vector form in this particular manner e 1 partial T by partial x 1 plus e 2 partial T by partial x 2 plus e 3 times dotted with, dotted with delta x 1 e 1 plus delta x 2 e 2 plus delta x 3 e 3. It is a dot product of this displacement vector, delta x vector dotted with this quantity here, and this is what is called the gradient of the temperature; gradient of the temperature dotted with delta x.

So, this is delta T, note that it is very similar to what we had in one dimensional calculus, delta x times d y by d x is equal to delta y; delta T is equal to grad T dotted with delta x. So, this basically tells you, if am looking, if I am sitting at some particular position in space and I move a small distance in some direction. What is going to be the change in temperature when I move that small distance? Note that this grad T itself is a vector which is defined that each and every point in space, grad T itself is a vector which is defined at each and every point in space in this particular case, because I was using this coordinate system, I had defined it in terms of the derivatives in this coordinate system however this vector grad T so.

So if I if I sit at one particular location and move a small distance to some other location delta x; the delta T that I get should be independent of coordinate system that I am using to analyze the problem. Therefore, grad T itself the vector should be independent of the coordinate system that I am using to analyze the problem. In some other coordinate system, partial T by partial x 1 partial T by partial x 2 and partial T by partial x 3 will change. But the vector grad T will remain the same. It will have magnitude and direction. The magnitude is independent of coordinative system; the direction is independent of coordinative system. Therefore, this is a vector property of this temperature field, the gradient of the temperature field so for our definition of gradient.

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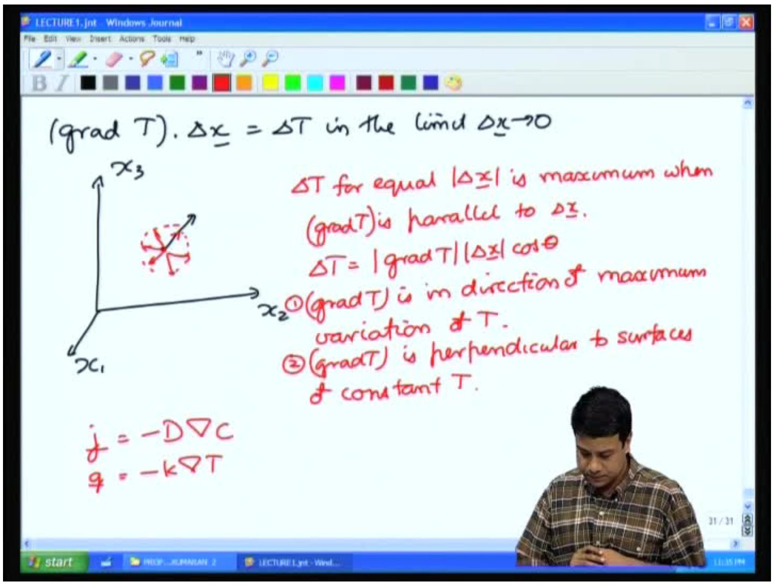

The gradient is defined as grad T. This is a vector dotted with delta x is equal to delta T. In all directions for any delta x, so why did I take the limit as delta x goes to 0; in the limit as the delta x vector, the magnitude of the vector goes to 0; grad T dot delta x is always equal to delta T. Delta T the change in temperature when I go a small distance is equal to the gradient of T. The vector which is defined at each point in space dotted with the displacement that I have under gone. So, there is a first of all vector derivates, the gradient, the gradient is is the first of all vector derivatives. It has certain implications; so as I said the grad T is a vector.

So, it is a vector at each location pointing in some direction; it has dimensions of temperature divided by displacement, temperature divided by distance. It is grad T times of distance is equal to change in temperature. It has some direction, and let us say exact at this particular point and I fix the distance that I am going to travel; I fix the distance that I have going to travel; I fix the distance time that I am going to traveled but, not the direction. So, I fix the distance that I have going to travel and I travel in various directions; equal distance in various directions. I travel in equal distance in various directions, and I measure what is delta T when I travel that distance that T at the final position; minus T at the initial position. What is the different delta T?

Now, this difference delta T, this difference delta T for equal magnitudes of the distance traveled; delta T for equal magnitudes with the distance traveled is maximum when grad

T is parallel to delta x. You know that that the delta T is equal to grad T dotted with delta x. Let me write this as the gradient here system to avoid confusion; grad T is a vector when it is parallel to delta x. You get the maximum change in temperature; because obviously, delta T is going to be equal to magnitude of grad T, magnitude of delta x times cos theta, where the two are parallel; theta is equal to 0, and therefore, delta T is a maximum. That means that grad T is in direction of maximum variation of T that means that grad T is in the direction; it points out the direction in which the temperature variation is a maximum so that is a physical significance of grad T.

As I go in various directions at a point, I go in equal distance in all these various directions. In the direction, where the displacement vector and grad T are in the same direction, you will get the maximum variation in temperature. Of course, in one direction, you get the maximum positive variation when theta is equal to 0; other direction, you will get the maximum negative variation when theta is equal to pi. So, that cos theta is minus 1. But, anyway gives you the direction in which temperature is varying by a maximum amount. So, that is the physical significance, the first one physical significance grad T. The second is that if I go along directions which are perpendicular to grad T; if I go along the direction which is perpendicular to grad T, I will get 0 variation; that means that the temperature is a variation is 0 in the plane perpendicular to grad T. Because, grad T dotted with delta x is equal to 0; that means the temperature is a constant in the plane perpendicular to grad T.

If in a other way, grad T is perpendicular this I am sorry is perpendicular to surfaces of constant T. So, grad T is perpendicular to surfaces of constant T. If you are travel in the direction perpendicular to grad T. At a given location, the direction perpendicular is is this a plane but, grad T itself can vary with location, and so there therein the direction that the the surface perpendicular to grad T is the surface on which temperature is a constant. So, these are the physical significances is of the gradient of a function. It need not be temperature; it can be any function, pressure, concentration, any function.

The gradient of that function is along the direction of maximum variation of that function and it is perpendicular to the direction is along which that function is a constant, particularly useful for constitutive relations that we had seen in part one for the mass and heat transfer. If you recall we had that the flux mass flux j is equal to minus D grad c. The vector direction grad c is the direction along which there is maximum variation of concentration and it is perpendicular to to to surfaces of concentration; that means if the mass flux is taking place along direction of maximum variation of the concentration field. Similarly, the heat flux q is equal to minus k grad T. Each flux is a vector, it was parallel to the gradient of temperature; the vector which is the gradient of temperature so that is the gradient of a function.

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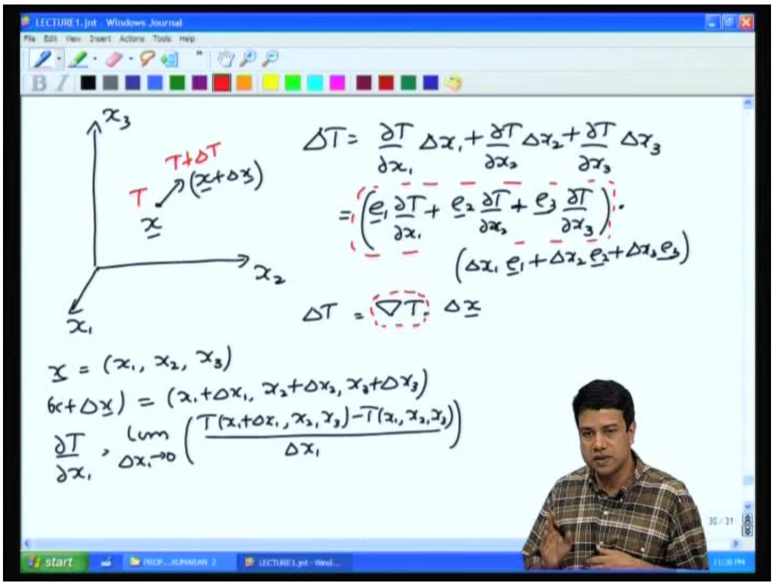

If I write it in long hand notation here is just equal to e 1 partial T by partial x 1 plus e 2 partial T by partial x 2 plus e 3 partial T by partial x 3. In this orthogonal coordinate system but, this quantity itself has a identity independent of coordinative systems here. So, vector which always points in the, it is a single valued vector which always points in the direction of maximum variation of that function, and it is perpendicular to a surface of constant value of that function.

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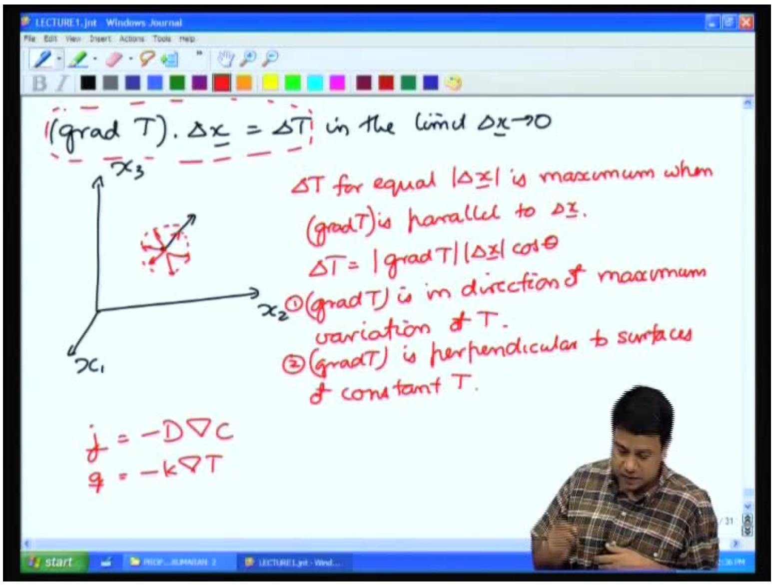

So, this is the derivative, the definition of the derivative, the definition of the integral is the inverse of this one.

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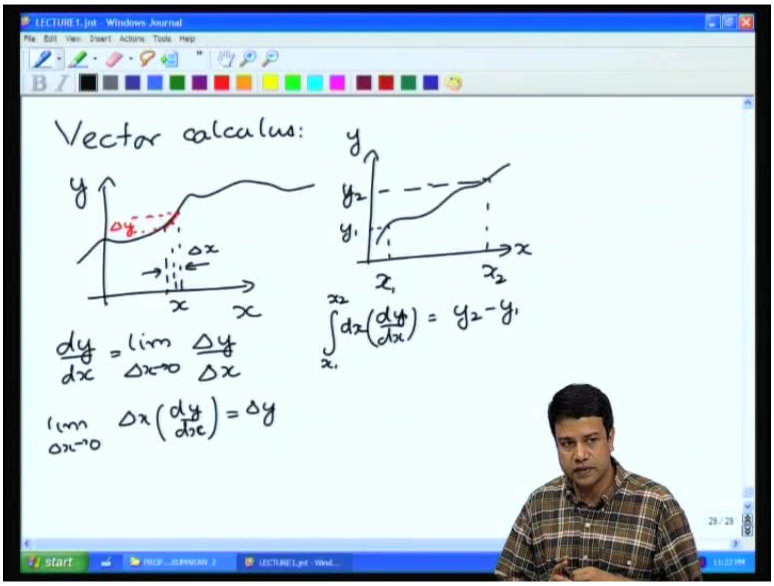

So, if you recall when I made the analogy for the derivative for a single valued function I said that d y by d x is equal to limit as delta x goes to 0 of delta y by delta x. The inverse of that is the integral the difference in the value between two end points, difference in the value between two end points is equal to the integral of this derivative.

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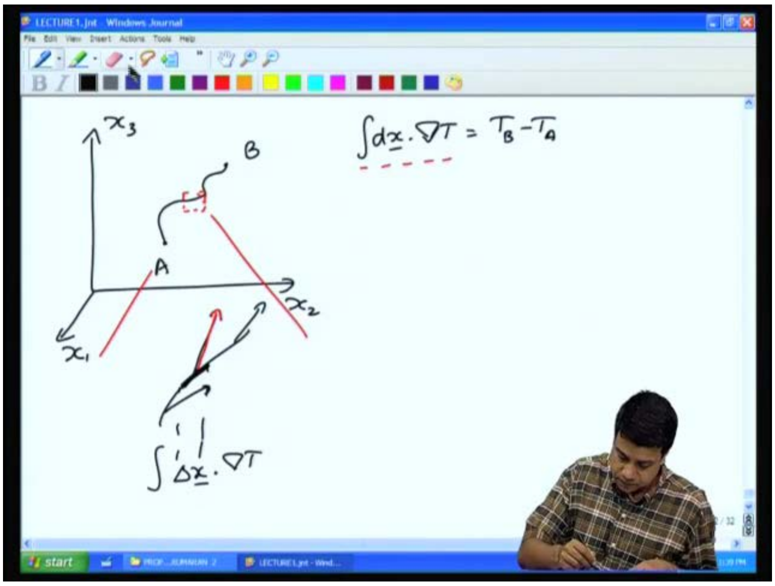

Similar, thing can be done for gradients and the integral equivalent of this is as follows. So, if I have two points A and B, and if I along these two points by some path, go by some point path from A to B. What the integral relations says is? That integral along this path of d x dot grad T is equal to the difference, the temperatures between these two end points; that is if I take so, what is what is this left hand side mean just look at that; so I am taking some path, and along at some location along this path. I have this line element this. If I just increase this, I have this line element along this path; I will make in black; I have this line element and everywhere along this path.

I also have the gradient of the temperature field that is defined. So, gradient of the temperature field, here maybe in this direction, here may be in this direction and so on. So, at this particular point, I take a unit vector displacement, this is the vector displacement delta x vector along this point. I take the unit the vector displacement delta x vector dotted with grad T. So, this is a displacement vector; this is the grad T vector. So, this is the grad T vector and this is the displacement vector, and I take x delta x dotted with grad T then sum that up, all the way from the initial to the final location and I get T B minus T A. So, that is the integral relationship for grad T.

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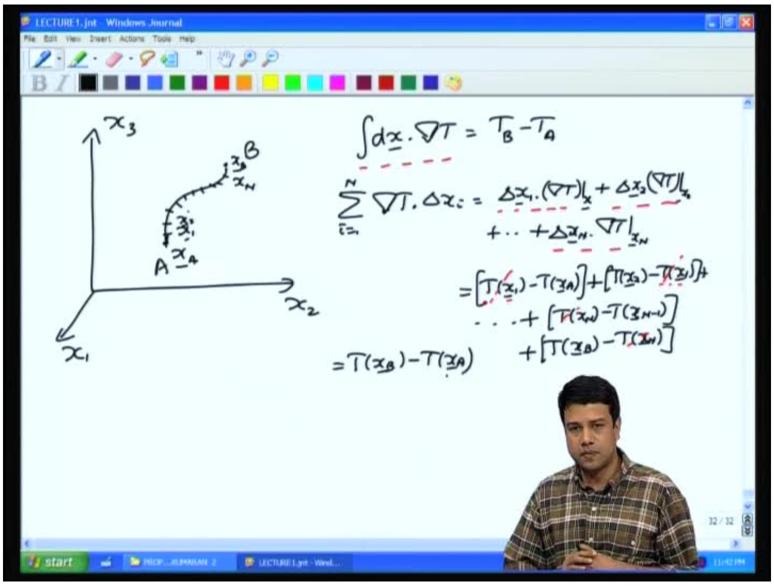

Let us justify how that works? Why do you get this integral relationship for grad T? So, I have this path between the two locations and I can divide into small little bits. So, this path I divide into small little bits. So, this is x A, this is x 1, x 2 etcetera x N in the next bit, I divide into small little bits. This integral of delta x dotted with delta T. I can write as a summation of grad T dotted with delta x i, i is equal to 1 to N, where delta x i, where each of these intervals; delta x 1 is between x 1 and x A x, delta x 2 is between x 2 and x 1 and so on. So, this equal to delta x 1 dotted with grad T at the location x 1 plus delta x 2 dotted with grad T at the location x 2 plus etcetera, dotted with grad T at the location x N. Now, each of these individual quantities, each of these individual quantities is related to the difference in temperature between the end points. By this relationship grad T dotted with delta x at any point is equal to the difference in temperature between the final and the initial location ok, grad T dotted with delta x, if I moved to some displacement, so difference in temperature between the final and the initial location.

So, this can be written as T of x 1 minus T of x A that is for the first interval; for the second interval I get T of x 2 minus T of x 1 plus etcetera plus plus T of x N minus T of x N minus one plus T of x B minus T of x N. So, I have just expanded out and you can see that in this first term I have T of x 1; second term I have minus T of x 1. So, these two will cancel out, then the second term I will have T of x 2 that will cancel out T with T of minus x 2 for the third interval, T of x 1 will cancel out to T of x N minus N for this finial interval, and finally, I will just be left with T of x B minus T of x N that is the difference in the temperature at the two end points. Let me just write it clear and clear for you that is the difference in temperature between the two end points, very similar to the integral expression that I had previously ok.

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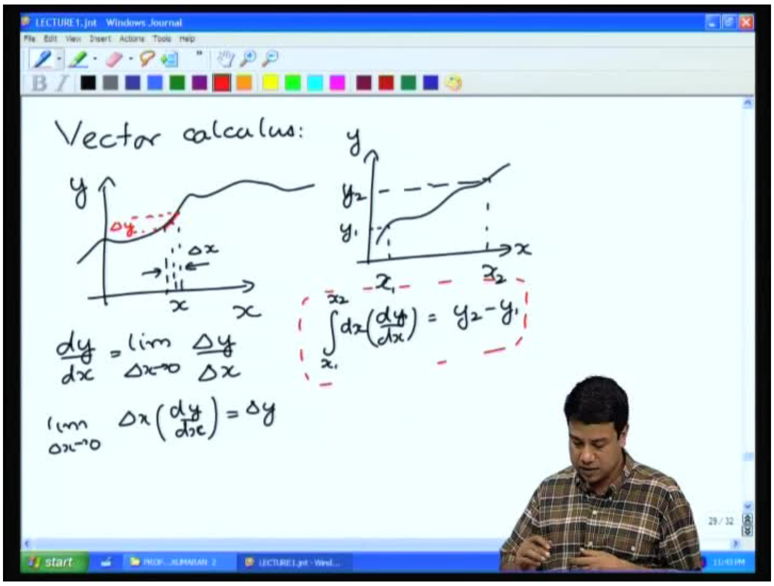

Very similar to this one, I am sorry; very similar to this integral expression except that what we have derived now is in three dimensions, for three dimensional displacements what we have derived is delta x dotted with grad T between two end points and that has certain consequences.

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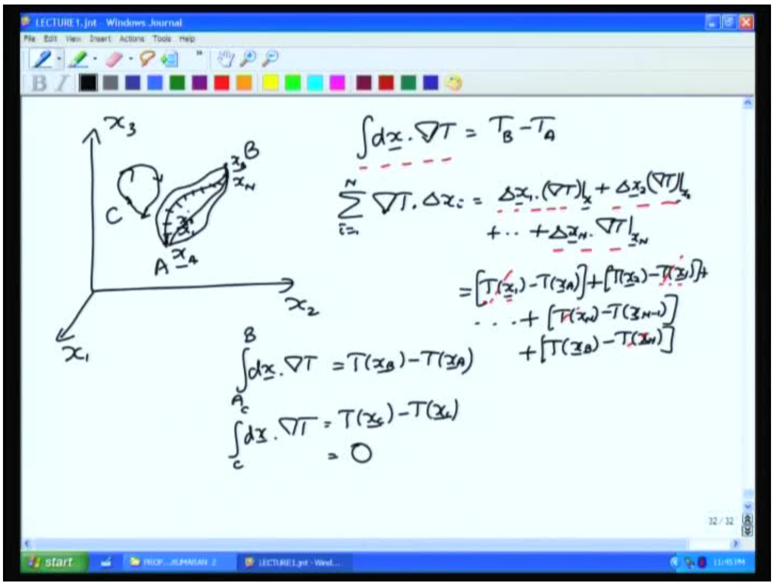

One of the consequences is, So, the relation that I have desired for you was that integral d x dot grad T is equal to this between A and B. One of the consequences is that T of x B minus T of x A depends only up on the end points, and that means that this integral d x dot grad T has to be the same, if the end points are the same regardless of the path that you take, that means that I have to get the same result, whether I go this way or I go this way or I go that way or I go by some other path. So, I have to get the same result regardless of what path that I take for this integral so that is one consequence.

The second consequence is that if I start somewhere and go around and come back to the same location. If I started some point C, go around and come back to the same location, integral between the same end points is T at x C minus T at x C. This has to be 0, by this is second consequence. If you go in a path that ends at the same location that you started integral d x or grad T has to be equal to 0. So, that is a second consequence of this, so the this is in general more powerful than just the one dimensional gradient vector, I am sorry one dimensional derivative.

So, to briefly summarize the gradient of a vector, I am sorry the gradient of a, of a scalar quantity is a vector; directed in the direction of maximum variation of that quantity perpendicular to surfaces on which that quantity is a constant to a temperature field, concentration field and so on; and the integral relation for that is that if you go around, if you go from location A to location B along any path and take the integral of d x dotted with grad T along that path. It is always equal to the difference T B minus T A. So, the difference in temperature is equal to the integral along that path is the same on any path that you take, and if you start from one location and come back to the same location; integral of d x dot grad T has to be equal to 0.

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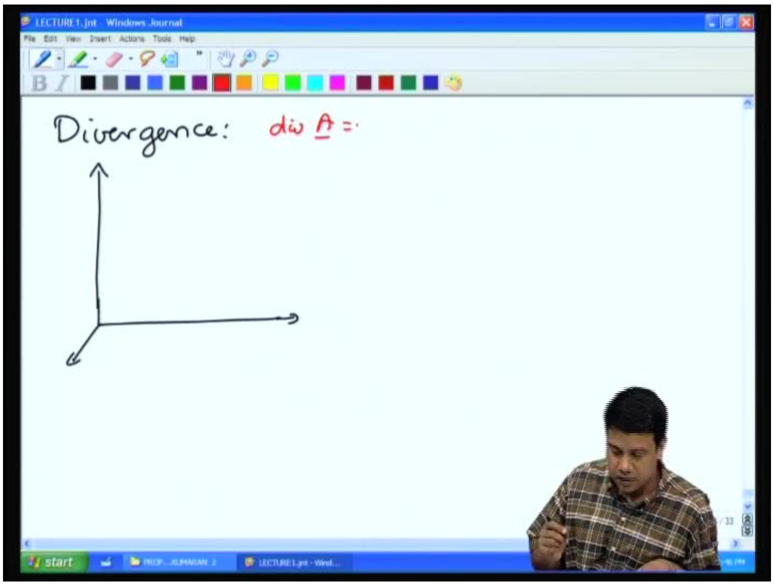

So, the next quantity that we will do is the divergence. The divergences as you know action of scalar, I am sorry action of vector, divergence of A and it gets your scalar and how do you define the divergence.

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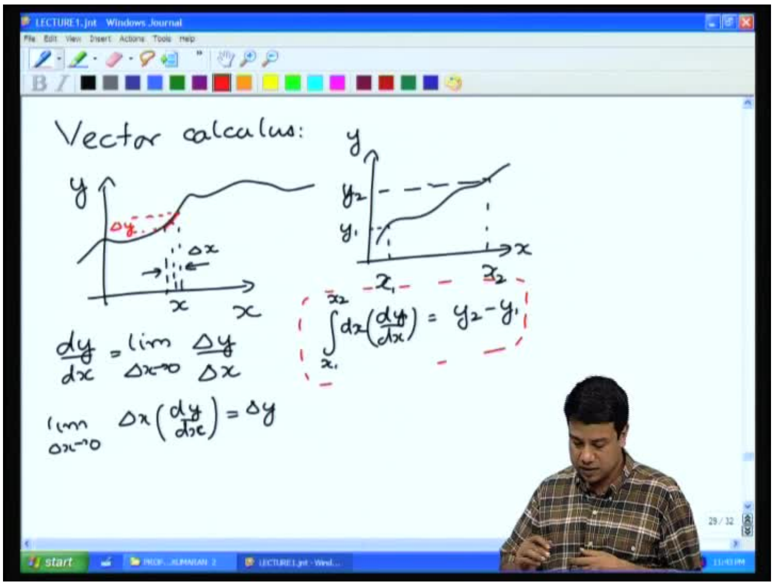

In one dimensional calculus, we defined the derivative by taking the smaller interval moving the small distance; taking the difference in the dependent variable as a function of the distance of the of the interval in the independent variable.

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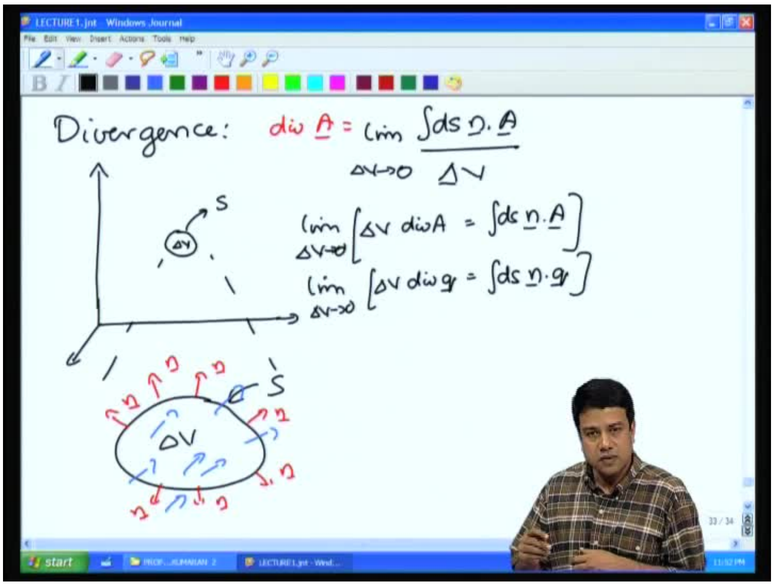

The divergence is formally defined as, what I need to do is? I need to construct a small differential volume delta V with a surface of this volume is S. So, let me just expand it out little, by the scalar what I mean expand it out, I have this differential volume V. The surface of this volume is S. At each point along the surface, I have some unit normal vector perpendicular to the surface. This unit normal is defined as the outward unit normal. It is directed outward to the surface, and this divergence is defined for this vector as integral over the surface of n dot A divided by delta V; in the limit as delta V equals to 0. So, this is a quantity which is defined at each point within the field.

So, for example, if I have a flux vector or a velocity vector, the divergence of that flux vector is defined at each point within the field. Divergence construct a small volume at that point; the volume has a surface S; on that surface at every point on the surface is defined the unit normal outward, unit normal n. You take that outward unit normal multiplied dotted with this vector A. This vector A is once again defined at each point during the field. It dotted with this vector A integrate over the surface and divide by the volume. In the limit as a volume become smaller and smaller this quantity will converge to finite value. The volume itself will go to 0; surface area will also go to 0 but, the ratio of these will converge to a finite value and that is what is called the divergence of A. Note that I have taken n dot A, therefore, what I end of is with is a scalar divergence of a is a scalar.

The other thing to notice on the numerator, I have d S n dot A that means that the numerator has dimensions of surface area times A, the denominator has dimensions of volume. Therefore, the ratio has dimensions of A divide by length. Physically this is of course, this divergence so, if I write this in another way to make the physical interpretation clear. In the limit as delta V goes to 0, I have delta V times divergence of A is equal to integral d S n dot A, integral d S n dot A is equal to delta V times the divergence of A. So, for this particular differential volume, if A were for example, the heat flux, if A were for example, the heat flux integral d S n dot A, n dot A is the flux outward along the outward unit normal at the surface; that is the amount of material coming out of the surface per unit area, that I am integrating over the entire surface.

So, let me just write this to give a better physical understanding, delta V divergence of q is equal to integral d S n dot q, q is the heat flux; heat coming out per unit area per unit time; n dot q is the total heat coming out of the surface. It is the component of q that is parallel to the surface does not come out. Only the component that is perpendicular to the surface is leaving the surface. Therefore, n dot q is the amount of heat coming out of the surface per unit area per unit time. I have integrated this over the surface area. So, integral d S n dot q is the total amount of heat coming out of the surface per unit area per unit time. Total amount of heat coming out per unit area per unit time is equal to the divergence of q times the volume itself.

So, that is a physical interpretation, the divergence of q is equal to the amount of heat coming out multiplied, so divergence of q multiplied by the small differential volume. It is equal to the total amount of heat that is coming out of this surface. You will see an integral relation of this a little later which will make the physical interpretation. So, how do we relate this to the partial derivatives that we had in the definition of the gradient value. So, in order to find out the formula for the divergence in specific coordinate system what we need to do is to construct the differential volume in that coordinate system. Calculate the divergence for that differential volume.

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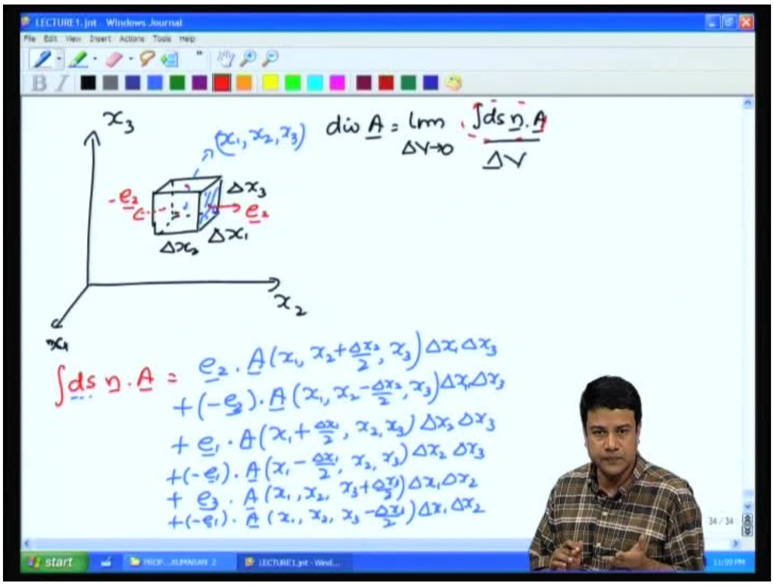

So, let us do that first for the Cartesian coordinate system, I construct the small differential volume with surfaces along directions of constant coordinate. So, it has delta x 1 in the x 1 direction; delta x 2 in the x 2 direction; delta x 3 in the x 3 direction. Divergence of A is equal to limit as delta V goes to 0, integral d s n dot A divided by delta V. For this particular case, delta V is equal to delta x 1 times delta x 2 times delta x 3, and this cubic volume has six surfaces; one two of which are perpendicular to the x 1 direction; two are perpendicular to the x 2 direction, and two are perpendicular to the x 3 direction, and I have to calculate n dot A over each of these.

So, further surface that is perpendicular to the x 2 direction. There are two surfaces that occur to in each direction for the surface that is perpendicular to the x 2 direction, the outward unit normal is in the plus x 2 direction. So, we have to normalize e 2. So, if I calculate this integral d s n dot A for this surface in the plus x 2 direction. So, I am constructing my volume around the center point here. Let us call this center point as x 1 x 2 x 3. So, because the center point is at x 1 x 2 and x 3; the surface on the right is at x 1 x 2 plus delta x 2 by 2 and x 3.

So, this surface here is at x 1 x 2 plus delta x 2 by 2 and x 3. So, that from the right hand side, I have e 2 dotted with A vector at x 1 x 2 plus delta x 2 by 2 x 3 that is for the surface on the right times the area itself. The area of the surfaces delta x 1 delta x 3, the area of the surface delta x 1 delta x 3, because it is perpendicular to the x 2 direction. For the surface on the left, the unit normal is in the minus e 2 direction, because the unit normal for that surface. The outward unit normal is pointing in the minus e 2 directions. For the surface on the left, the outward unit normal is pointing in the minus e 2 directions. So, I will have minus e 2 dotted with A at x 1 x 2 minus delta x 2 by 2 x 3 times delta x 1 delta x 3.

So, the six surfaces that this volume has, this is for the first two surfaces perpendicular to the x 2 coordinate; they have unit normals in the plus e 2 and minus e 2 direction. And then you have two surfaces which are perpendicular to the x 1 direction that is the front and the back; along the front surface, the front surface is located at x 1 plus delta x 1 by 2 x 2 x 3; the back surface is at x 1 minus delta x 1 by 2 x 2 and x 3. For the front surface, the unit normal is along the e 1 direction dotted with A at x 1 plus delta x 1 by 2 x 2 x 3 times the area perpendicular to the x 1 direction. So, the area is delta x 2 delta x 3 and then I have the back surface at which, the unit normal is in the minus e 1 direction at the surface of the back.

So, I will take plus minus e 1 dotted with A at x 1 minus delta x 1 by 2 x 2 x 3 and then I have top and bottom surfaces, they are perpendicular to the x 3 plane. So, the unit normal of the top surface is plus e 3; surface is located at x 1 x 2 x 3 plus delta x 3 by 2. Similarly, the bottom surface, the unit normal is in minus e 3 direction, surface is located as x x 1 x 2 and x 3 minus delta x 3 by 2. It is the straight forward extension of what I have just direct field. So that is only this path alone. Now, I have to divide by delta V. I can simplify this as you can see this e 2 dot A is just e 2; the component in the x 2 direction of the vector A. Similarly, minus e 2 dot A is minus e 2. So, I can simplify that.

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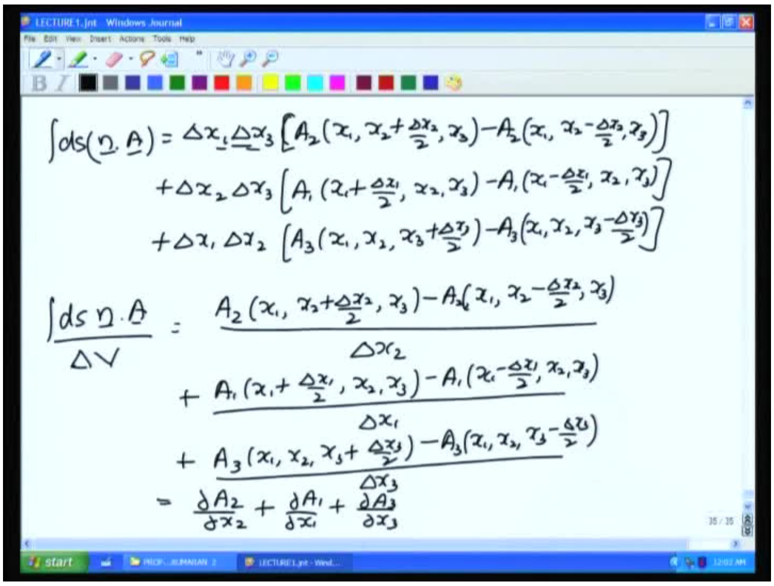

So, this becomes is equal to delta x 1 delta x 3 into A 2 at x 1 x 2 plus delta x 2 by 2 x 3 plus delta x 2 delta x 3 into A 1 at x 1 plus delta x 1 by 2 plus delta x 1 delta x 2 A 3. Now, I have to divide by a volume. Volume is delta x 1 delta x 2 delta x 3. So, divide the whole thing by delta x 1 delta x 2 delta x 3. So, I get integral d s n dot A divided by delta V is equal to and I divide throughout by delta x 1 delta x 2 delta x 3; the delta x 1 and delta x 3 over here will cancel out. We get this whole thing by delta x 2 plus. So, when I take the limit delta x 1 delta x 2 delta x 3 going to 0. This just becomes partial A 2 by partial x 2 plus partial A 1 by partial x 1 plus partial A 3 by partial x 3 so that is the divergence in a Cartesian coordinate system.

Now, I have to derive for you integral relation for this the divergence that we will continue in the next lecture. In the mean time please go through out, we have done in this class before coming for the next class; I will briefly revise what has been done here. So, that there is some continuity and then we will proceed from here to defined divergence, its integral relation curl and its integral relation as well we will derive these formulas in a Cartesian coordinate system for the present. But as we proceed, I will also show you have to derive it in other coordinates systems, we had done it previously when we did mass and energy conservation equations, we did these things, we did balances and we got out certain quantities and said these. Those were the divergences is gradients, and so on this is a more systematic way to end it and we will go throw this in detail before we proceed to to the fluid mechanics. So, we will see in the next lecture.