Fundamental of Transport Processes II
Prof. Kumaran
Department of Chemical Engineering
Indian Institute of Science, Bangalore

Lecture - 38
Turbulence – Part I

So, welcome to this lecture number 38 of our course on Fundamentals Transport Processes where, we had just started the discussion of turbulent flows. As I said, turbulent flows occur primarily at High Reynolds numbers. In the case of internal flows for example like flows in a channel or a pipe at a specific value of the Reynolds number. There is the spontaneous transition from a laminar flow. The laminar flow is what we have been analyzing so far. Which the only mechanism of momentum transports cross stream is molecular diffusion. And in a channel or a pipe where, there is the pressure driven flow for example, the velocity profile is the parabolic velocity profile with straight stream lines which are aligned in the flow direction.

So, this laminar flow solution, as you can easily see by going to the equations continues to be solution of the conservation equations at all Reynolds numbers. However, in all cases, you find that at a specific Reynolds number, there is the spontaneous transition to a turbulent velocity profile. The turbulent velocity profile is characterized by a number of things; firstly, it is highly irregular that is at each point in the space. You find velocity fluctuations about a mean value. It is three-dimensional because, even though the mean flow may be two-dimensional with velocity with the flow direction and the velocity varying in only the gradient direction.

The fluctuations themselves are three-dimensional. You have parcels of fluid called Eddies with velocity is in all direction. It is highly diffusive. Because, the transport takes place due to the turbulent eddies rather than due to molecular diffusion. And for that reason, it is highly dissipative, because, transport is taking place due to momentum transfer. Momentum transfer is also much higher than that in a lamina flow. And because, of that the wall S Shear stress becomes much larger and the pressure gradient required or the friction factor is much higher than that in a laminar flow.

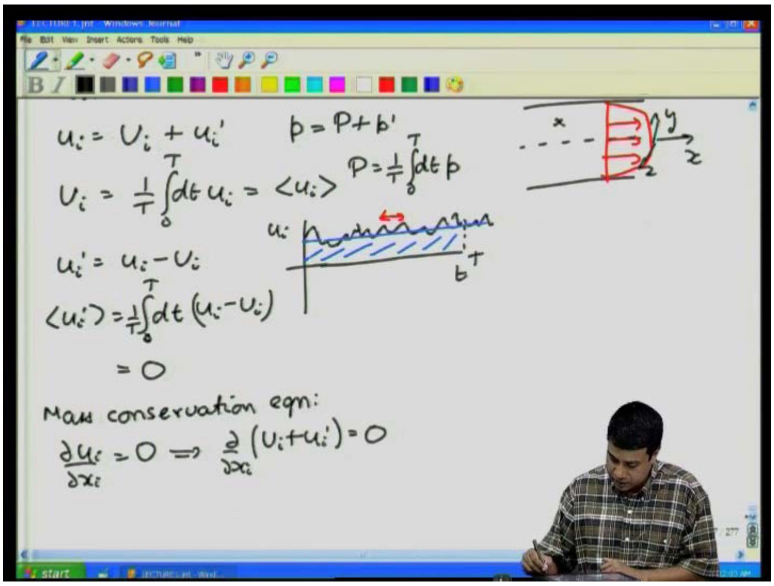
And as I said one of the important factors of a turbulent flow is that it is three dimensional. That is because in integral step in the Turbulence dissipation as we shall see is the transport of the Vorticity. Due to the stretching and bending of vortex lines this mechanism is absent for a two dimensional flow. Therefore, Turbulence itself is three dimensional. Despite all these Turbulence is still a property of the continuum Navier strokes equations. It is not that the continuum approximation breaks down the continuum approximation is still valid. And you can simulate these turbulent flows using the continuum Navier strokes equations.

However, because of the highly irregular turbulent velocity profile that you have. It is difficult to model the turbulent velocity profile simply as we did for the Lamina Profile. In case, of the Lamina Profile. We could assume that there is flow only is in one direction and gradient only in the perpendicular direction for a two dimensional flow. Now, that basis we actually derived the parabolic velocity profile for internal flows and various other velocity profile for Boundary layer flows for example.

So, the lamina velocity profile is still a solution of the equation. But, it is an unstable solution since, the flow is at High Reynolds number. The equations are non-linear and for that reason there is no guarantee that a solution exists. And even if it does exist it does not necessarily have to be unique. We do find one solution at High Reynolds number for these simple flows as the lamina solution. However, there are other solutions as well and at some point the lamina solution becomes unstable and spontaneously undergoes a transition to a turbulent solution.

So, simple way to module these turbulent flows that we started looking at in the previous lecture, was to consider the turbulent velocity profile as the sum of a mean plus. A fluctuating part the mean part in this case is defined as the time integral, as the average over time we consider steady flows for simplicity. So, that the time derivative in the Momentum Conservation equations no longer present. So, all of these mean flows are basically defined as averages over time.

(Refer Slide Time: 05:21)



So, let us first start off with our attempt at Turbulence modeling. This is the hch steady flow in a channel in which I have some mean velocity profile. And I will put a coordinate system there just for definiteness xy and z. So, the flow direction is in the x direction and y and z are perpendicular directions. I define the different components of the velocity ui as the sum of mean plus a fluctuating plus a fluctuation. Where the mean value was defined as a time integral ui is equal to 1 over T integral 0 to T. dt ui which I had also written as the average value sometimes. This is written with an over bar as well. So, these are two alternate definitions of mean, but for the present mean implies an average over time and of course, we defined ui prime is equal to ui minus capital Ui.

So, pictorially what I had said was that at a given location. If, I want to find out. What is the velocity? The mean velocity what you would do is that as a function of time you plot the velocity ui it would have some fluctuations in it. So, that there is some fluctuations in the velocity integral 0 to capital T of dt times. u is basically the area under the curve up to the time capital T.

To define a mean value you require that this time capital T has to be much larger than the time period of these fluctuations themselves have some characteristic time period. You have to make sure that the average in time that you choose has to be much larger than this characteristic time period. So, that the velocity goes through many positive and negative fluctuations about its mean value.

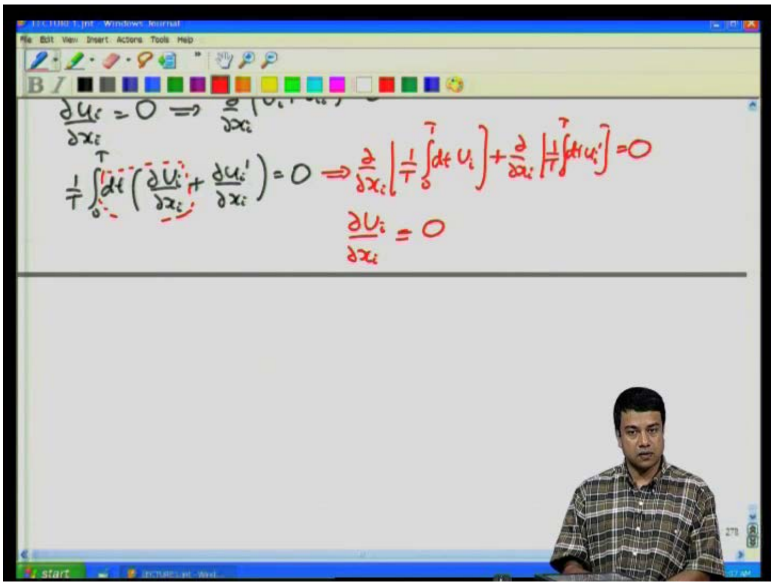
So, from the area under the curve you divided by capital T to get the mean value divided and the difference between the instantaneous velocity and the mean velocity is the velocity fluctuation. Of course, the average of the velocity fluctuations integral 0 to T 1 over T. Since, capital U is a constant in this case its already time averaged. I showed you in the last lecture that this is equal to 0.

I should note that this is not the only way this can be done. One can also do what is called an ensemble average. That is you take many realizations of the same turbulent flow with the same average velocity profile or average properties. For example, the average pressure gradient, but different instantaneous velocities and do an ensemble average over those as well for now. We will restrict our attention to just steady flows in which the averages are steady the instantaneous values are not and deal with those alone you want to take into account that time varying flows.

The pressure can be similarly written as P plus b prime. Where, P is equal to 1 over T integral 0 to T times the instantaneous value of the pressure. If, you sit at one particular location you will find that the pressure also fluctuates. And it has a mean and it has a fluctuation about that mean. So, we insert this decomposition into the Navier stokes Mass and momentum Conservation equation.

The Mass conservation equation partial ui by partial xi is equal to 0. I can separate this out into the mean plus the fluctuating part to give me partial by capital Xi of capital Uiplus ui prime is equal to 0. That would be the decomposition of the local instantaneous velocity into the mean plus the fluctuating part to get the Momentum Mass conservation. For the mean velocity alone it would be usual that as you integrate over time and divide by time.

(Refer Slide Time: 12:39)



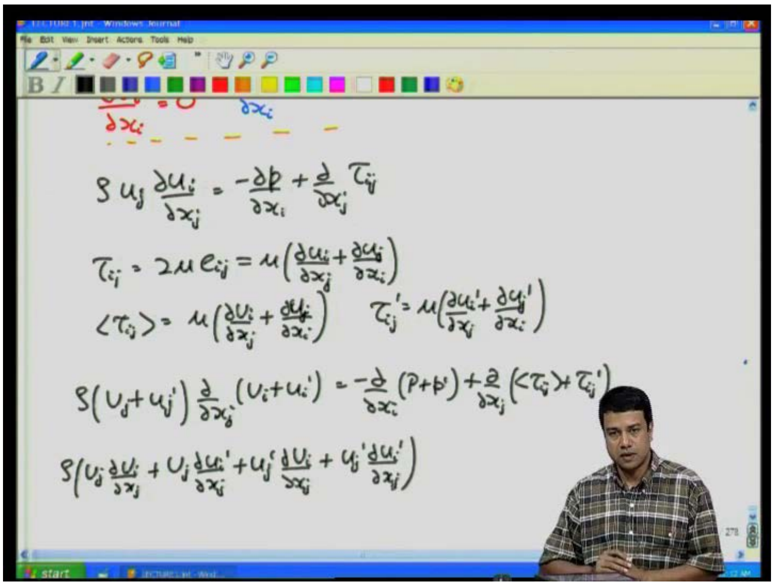
So, I can to the 1 over T integral 0 to T dt of partial ui by partial xi plus partial ui prime by partial xi is equal to 0. Note that, the mean velocity capital Ui it is independent of time. Because, at a given location I have found out what the average velocity is by integrating over time. So, its independent of time however, it could still be a function of position the mean velocity at two different locations. Which are calculated by integrating the instantaneous velocity at those two locations does not necessarily have to be independent of position it can vary from one position to the other.

So, there is in fact a time dependence in the spatial dependence. In the mean velocity as well even though, there is no time dependence because I have integrated over time. In this particular case the time and the spatial variations do commute. So, in this particular case the time and the spatial the integration over time then, differentiation over the spatial the coordinates to commute the reason is because, time and position are not independent coordinates. So, because of that I can inter change the order of differentiation and integration.

So, I taken integral over time at a given location. Taking the difference between nearby locations and because position and time are independent coordinates. The integration over time and the differentiation with respect to position do commute. And because of that I will get partial by partial xi of 1over T integral 0 to T dt. ui plus partial by partial xi of 1 over T integral 0 to T dt ui prime this is equal to 0.

Because, I can interchange the order of differentiation and integration. Because, the differentiation with respect to x the integrations with respect to time and time and position are independent coordinates in our field description of the velocity fields. This second term is the average of the velocity fluctuation. So, that is equal to 0. The second term is the average of the velocity fluctuations which is identically equal to 0. The first term capital Ui is independent of time. So, I just get 1 over T integral 0 to T of dt which gives me 1 times ui. So, my Mass conservation equation for the mean velocity just becomes partial ui by partial xi is equal to 0. So, that is the mean Mass conservation equation.

(Refer Slide Time: 15:02)



The Mass conservation equation integrate over time at a given location integrate over time at a given location. Of course, my original equation was partial by partial xi of capital Ui plus ui prime is equal to 0. And when I write the time integration I found the mean velocity equation as partial by partial xi of capital Ui is equal to 0. That implies that for the fluctuation themselves the velocity the equations partial ui prime by partial xi is equal to 0. That means, that both the mean and the fluctuating velocity, satisfy the Mass conservation equation at any the fluctuating velocity at any instant on time and the mean velocity which is time average quantity.

So, the equations for the mean and the fluctuating velocities the Mass conservation equations are partial ui by partial xi is equal to 0. And partial ui prime by partial xi is equal to 0. Next, we come to the Momentum Conservation equation. We have neglected time derivative.

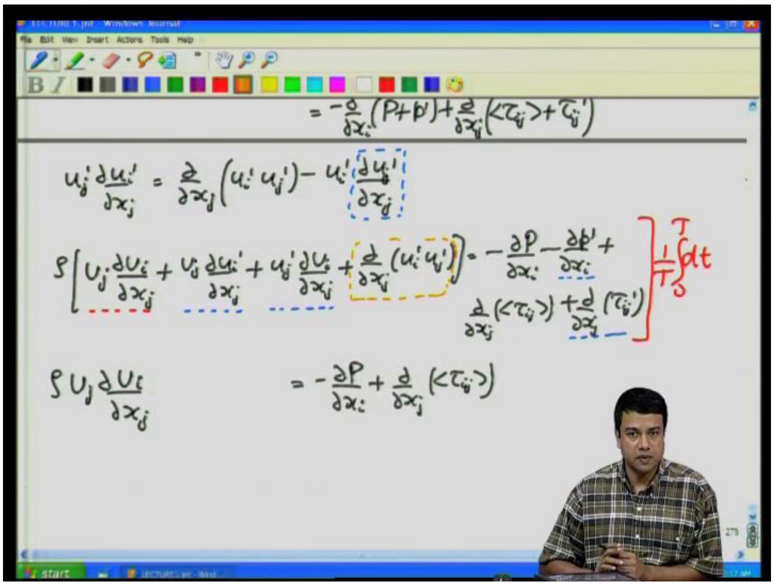
We assumed that the flows at steady state. So, my momentum conservation equation just reduces to rho uj partial ui by partial xj is equal to minus partial P. By partial xi plus partial by partial xj of tau ij. Where tau ij is the Shear stress tau ij is equal to 2 mu times Tij is equal to 2 mu. So, there is the Shear stress for an incompressible fluid.

So, now this you separate it out into two parts. One is the mean plus the fluctuation the first is the mean velocity and the fluctuating velocity. Similarly, for the pressure as well for the Shear stress. The Shear stress of course, has a mean value the mean value of the Shear stress. So, if the Shear stress is linear in the velocity fluctuations if, I just take the Shear stress integrated out in time and divide by time on the right hand side. I will just get the mean velocity gradients. So, there is a mean value of the Shear stress similarly the pressure also has a mean value.

So, if I can write as I have shown here. I can separate out the Mass conservation equation into 2 parts. 1 is for the mean velocity alone and the other is for the velocity fluctuations. If, I can do the same thing for the momentum conservation equation. Then, I could hope to solve these two equations in order to find out to what the mean velocity profile is. So, let us write down the momentum conservation equation in terms of the mean plus the fluctuating velocity.

So, momentum conservation equation becomes rho into uj. So, this is the momentum conservation equation written it as, the mean value plus the fluctuating part of the Shear stress. Of course, is tau ij prime is equal to mu to partial ui by prime by partial xj. So, that is the momentum conservation equation with the velocity expressed as a sum of a mean plus a fluctuating part similarly for the pressure and for the Shear stress. Let us simplify this a little bit. So, I will get rho into uj partial ui by partial xj plus. Which is expanding out the first term. In the term the initial term on the left hand side will just expand it out equal to minus partial by partial xi of P plus b prime plus.

(Refer Slide Time: 19:31)



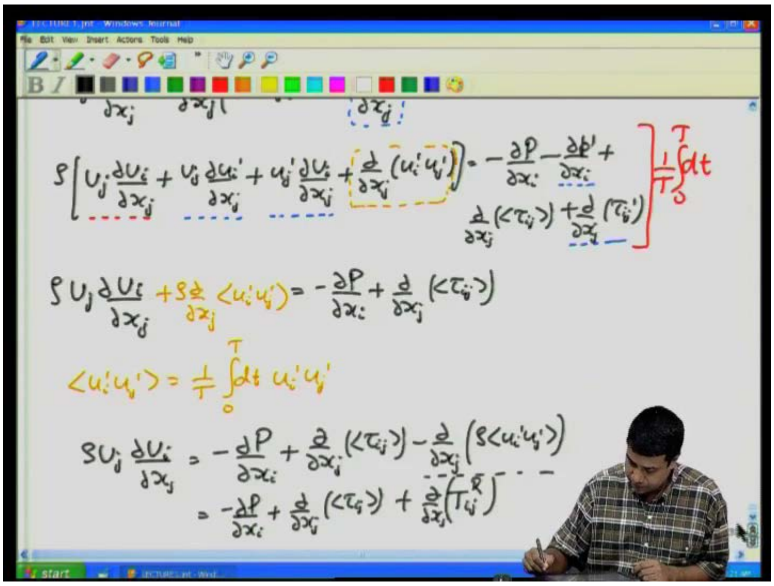
And now, for this momentum conservation equation. I will take the average 1 over capital T integral 0 to T of dt of this whole thing. But, before that lets just briefly rewrite this term in a form that will turn out to be more convenient. Later on I can write uj prime partial ui prime by partial xj is equal to partial by partial xj of ui prime uj prime minus ui prime partial uj prime by partial xj just using the chain rule for the differentiation.

This term here is the divergence of the fluctuating velocity partial uj prime by partial xj is the divergence of the fluctuating velocity. Which we know has to be 0 because, both the mean and the fluctuating part individually satisfy the incompressibility condition. So, my equation becomes. So, that becomes my equation for the momentum conservation equation. And now, what you do is you do the average that is you take this entire equation and you do the integral for this entire equation. You do 1over T write here on this entire equation. You do 1 over T integral 0 to T dt of the entire equation. When you do that if you look at this first term here involves only capital U. And capital U is independent of time does depend on position but, it is independent of time.

So, it take 1 over T integral 0 to T of dt of capital Uj partial ui by partial xj. I will get the same thing back because, this is independent of time. So, I will get rho uj partial ui by partial xj. The second term here it is linear in the fluctuating velocity it is capital Uj partial small ui prime by partial xj. So, it is linear in the fluctuating velocity. When you take a term that is linear in the fluctuating velocity. And integrate over time I should get

0 because, the time average of fluctuations is 0 capital Uj is of course, the mean velocity. But, its time independent uj prime is the fluctuating part that is time dependent. Bu,t it integrates out to 0. So, all terms that are linear in uj prime in the fluctuating quantities will end up averaging out to 0. You can see this term is linear the pressure gradient this term is linear partial P prime by partial xi. Prime was defined such that its time average is equal to 0 similarly, this term is also linear all of these terms will average out to 0. And I will get an equation that goes as minus partial P by partial xi plus partial by partial xj of the average value of the Shear stress gives the average of the Shear stress. What about this term here when I integrate this over time and divide by capital T? This term is the product of ui prime times uj prime this does not in general average out to 0 the reason is as follows.

(Refer Slide Time: 25:13)



If, they have something that is fluctuating in time with the 0 average. If the something that is fluctuating in time with a 0 average this is my time coordinate and this is ui prime for example, if I have something that is fluctuating in time with a 0 average.

And if, I take the integral overtime there is equal amounts of positive and negative area. And therefore, I will end up with something that is 0. However, if I want to plot ui prime square the square of this quantity is always constrained to be positive. Because, the square of a negative number is also to positive. So, square is always positive. For that reason the average of the square is not in general 0 it is in general non 0. So, partial by partial xj of ui prime uj prime contains a product of two fluctuating quantities. If, those two fluctuating quantities for independent of each other then of course, it would be 0. But, if they are correlated the two fluctuating quantities are correlated to each other the average in general does not have to be equal to 0.

So, for that reason this term is in general non 0. And I will have in the conservation equation a term of the form plus rho partial by partial xj of ui prime uj prime. Let me write it more precisely. It is the average of this of the average of ui prime uj prime where I have defined the average of ui prime uj prime minus 1 over T. This is in general non 0. I can rewrite the question a little bit and write this I will partial xi plus partial by partial xj of tau ij the average value of the shear stress this is the mean Shear stress minus.

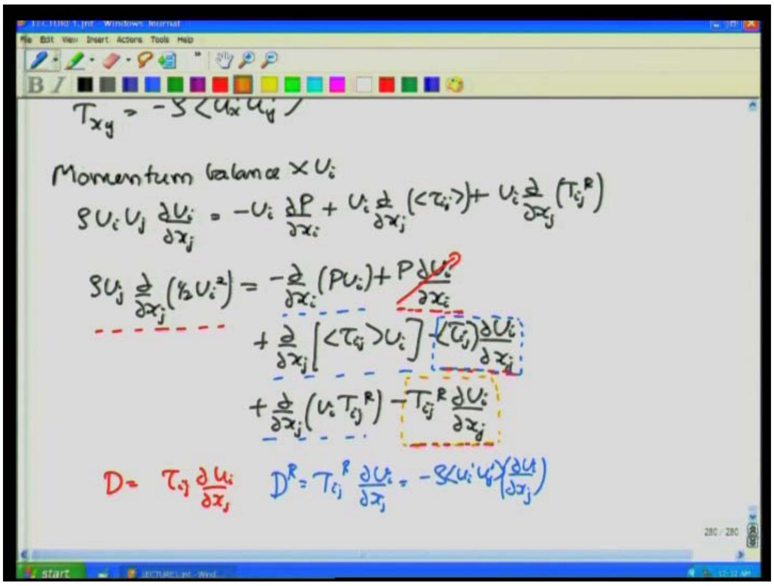
So, this is the pressure gradient. I should be careful here this is mean pressure. Gradient plus the viscose stress minus this additional term. Here this additional term is also the divergence of a second order tensor. Second order tensor is rho times ui prime uj prime. So this in the momentum equation this acts in a manner. Similar to the stress tensor. I could write this as minus partial P by partial xi plus partial by partial xj of tau ij plus partial by partial xj of TR ij. Where capital TR ij where is equal to minus rho ui prime uj prime it is called the Reynolds stress.

This is the transport of momentum transport of momentum due, to the fluctuating velocity of the fluid not due to the molecular velocity. The Shear stress tau ij is the transport of momentum due to the molecular velocity fluctuations. However in a turbulent flow in addition to molecular velocity fluctuations. You also have fluid velocity fluctuations fluid eddies that are transporting momentum in various directions. So, this is the transport of momentum due to the fluid velocity fluctuations is called the Reynolds stress and of course. As I said in a turbulent flow in the channels and pipes and so on the friction factor is much higher. Than what you would expect for a laminar flow the friction factor for a laminar flow is what is expected from molecular diffusion.

So, therefore, if the friction factor is much higher; that means, that the dominant contribution to diffusion in a turbulent flow. Is due to this eddie diffusion or the Reynolds stress tensor is much larger than a fluid molecular stress tensor. And this brings us to an important point here. We said we would like to simplify the equations by actually taking a time average in order to get an equation. For the mean velocity alone the time average local velocity alone. When we tried to write down an equation for this mean velocity alone. We ended up getting an additional term which contained a velocity fluctuation in the average of the velocity fluctuations in it.

So, because of this we cannot just de couple the mean velocity from the fluctuating velocity. Since there is substantial transport of momentum due to velocity fluctuations it is important to be able to model this turbulent Reynolds stress tensor. Because if you do not do that then you would neglect the largest contribution to the stress. So, why is there this additional turbulent Reynolds stress tensor. That is because they are ends up being co relations in the velocity fluctuations.

(Refer Slide Time: 30:23)



So, for example, the turbulent stress T xy is equal to minus rho times ux prime uy prime. If both ux prime and uy prime were independent variables then this average would be identically 0. However, in the turbulent flow there are correlations between the fluctuations in the x and y direction. If we have a parcel of fluid with a velocity in the x direction, it has a greater probability of having fluctuating velocity. In the minus y direction and because of that these parcels of fluid transport Momentum cross the flow. And result in an increased stress, an augmented stress which is this Reynolds stress. They also result in a dissipation of energy because there is an additional mechanism for the energy dissipation in the turbulent flow.

Let us just look at, that by writing down the energy balance equation in this case. So, in order to write down to the energy balance equation I take the Momentum balance. And multiply it by the velocity U i itself.. So, this becomes rho Ui, Uj partial Ui by partial xj minus Ui partial p by partial xi plus. So this is I take the Momentum balance equation multiplied by the mean velocity. So, there is a mean energy conservation equation, and I can simplify the various terms in this. So, first term as you can see it can easily be written as rho Uj partial by partial xj ofhalf Ui squarehalf rho Ui square is the kinetic energy per unit volume. So, this is the convicted derivative of the kinetic energy per unit volume.

This first term here, can be written as minus partial by partial xi of P y plus P partial Ui by partial xi just by using integration by parts. The second term is plus partial by partial xj of tau ij, Ui minus tau ij partial Ui by partial xj. And then, you have a last term here, which is plus partial by partial xj of Ui times a Reynolds stress. So let us, look at these 3 terms in series this first term. Here, is the convicted derivative of the kinetic energy itself. convicted derivative of the kenotic energy itself. The three terms that you see on the left hand side. Where all the divergence of something divergence of a vector or the divergence in all cases there is a divergence of a vector.

If you have, the divergence of a vector integrated over a volume that is equal to n dot that vector integrated over the surface. So, if I write down the energy equation for a given volume of fluid, these three terms. On the left hand side basically represent the flux of energy into or outer that volume. The first term there partial by partial xi of P uy represents the flux due to the pressure forces. The flux of energy into or outer the volume due to, the pressure forces the one below is the flux of energy due to the shear stress. The mean shear stress and the last one is the flux of energy into out of a differential volume due to the Reynolds stress.

So, these terms do not represent a net addition or dissipation of energy in the fluid volume itself, it just represents fluxes on the surface of the volume. The other 3 terms on the right hand side, the other 3 terms on the right hand side are all energy production. Or dissipation terms they represent a net addition or dissipation of energy from the flow. The first term on top there the P times is partial Uy by partial xi is the pressure work done of course. For this particular case it will be identically equal to 0 because the flow is incompressible the divergence of the mean velocity is equal to 0. The next term below is the shear stress times partial Ui by partial xj. If you recall, when we did the energy balance argument. We said that this was for the fluid velocity. This was the viscose rate of dissipation of energy this was equal to the dissipation of energy equal to tau ij times partial Ui by partial xj. Where Ui was the fluid velocity itself. That was the dissipation of energy that is always positive. So, that always removes energy from the system therefore, the frictional viscose dissipation is always removing energy from the system.

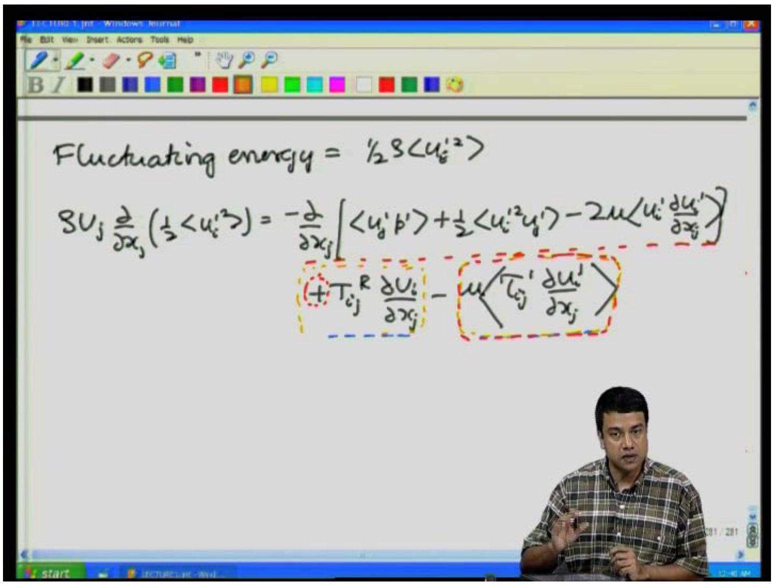
So, this is the average of that averaged over time the average shear stress times the average velocity gradient. This last term here, is the energy dissipation due to the Reynolds stress. As I said the Reynolds stress is due to the fluctuating velocity of the fluid locally at each point the fluid has a fluctuating velocity. Because, of that there is additional moment of transport because these fluctuations are correlated, and that causes the Reynolds stress. So, in addition to the viscose dissipation mechanism that I had for a normal fluid the average value of stress times the average rate of deformation. I also have this additional dissipation due to the Reynolds stress times.

So, this dissipation due to the Reynolds stress term is equals to Tij. Reynolds partial Ui by partial xj is equal to minus rho Ui prime Uj prime. Now, for a turbulent flow we said, it is highly dissipative. So, the rate of dissipation of energy is much larger than what you would expect for a laminar flow. That rate of dissipation of energy that you would expect for laminar flow, is given by this average value. The average shear stress times the average rate of deformation, for a turbulent force highly dissipative high Reynolds number that means, that this rate of dissipation of energy is much smaller than the convective terms which are pumping energy into the flow the term on the left for example, is a convective term as well as the pressure gradient term they pump energy into the flow.

Reynolds number is high that means, that this dissipation is small compare to the production of energy. Due to the mean pressure gradient to mean convection however, the flows at steady state and at steady state there has to be a balance between production and dissipation where that implies is that the dissipation of energy in the mean momentum conservation equation. Due to, the Reynolds stress is much larger than molecular diffusion. Therefore, in the momentum in the energy conservation equation the dominant diffusion is due to the Reynolds stress term. That is what is balancing the production of energy. Due to, the mean flow or due to the mean pressure gradient. When we did momentum when we did the energy conservation equation you recall that I said that when the energy is dissipated from the mean velocity mean flow, it ultimately goes into thermal energy for heating the fluid. So, the net loss of energy mechanical energy in the flow equals a net gain of thermal energy. Due, to the molecular due to an increase in the molecular velocity fluctuations. In this particular case it does not happen right. Because, the Reynolds stress does not contain the molecular fluctuations in it there is no molecular diffusion on the Reynolds stress. There is only the turbulent velocity fluctuation in the Reynolds stress.

Therefore, the energy that is dissipated from the mean flow. Due, to or that is removed from the mean flow due to the Reynolds stress terms cannot directly go into molecular fluctuations. Basically it is enhancing the fluid velocity fluctuations and not the molecular fluctuations. So, where does the energy go. In order to find out where the energy goes. You have to taken the total energy conservation equation that is a equation for half rho Ui square. Where u is the fluid fluctuating velocity, and from that remove or subtract out the energy. The mean energy conservation equation in order to get a conservation equation for the energy of the velocity fluctuations. It involves a little bit of the algebra. So, I would not go through that in detail, but I will just give you the final result.

(Refer Slide Time: 40:42)



The final result is that if you write a write down an equation for the Fluctuating energy itself the Fluctuating energy is, given by half rho times Ui prime square. That is the fluctuating velocity square average times half rho. That is Fluctuating energy per unit volume. If you write down an equation for the fluctuating energy, that is you write down the equation for the total energy mean. Plus fluctuating subtract out the mean energy equation that I just had. And you will get an equation for the Fluctuating energy. And the equation for the Fluctuating energy is rho Uj partial by partial xj of is equal to minus partial by partial xj.

So, this is the equation for the Fluctuating energy. Where all averages as I said, are 1 over capital T integral 0 to capital T of dt times that quantity. So, the time averages as usual there are various terms in this equation. But, I would like you to focus on just 2 of them. These first few terms here are of course, the gradient the divergence of something these are all the divergence of some quantities fluctuating quantities averaged. Divergence some quantities that means, when I integrate over a volume is equal to the flux of that quantity across that surface.

So, these do not represent a net increase or decrease in the fluctuating energy, but rather just transport across surfaces. The important terms are here this one and this one. So, left hand side is just the time derivative the substantial derivative of the Fluctuating energy. I should put this is half Ui prime square here, of the fluctuating quantities the rate of change of the fluctuating energy. That is the energy in the fluid velocity fluctuations. There are two terms here, one is Tij times partial Uy by partial xj. You can see that, this appears is the term that appears in the mean energy conservation equation over here.

This is exact same term that appears in the mean energy conservation equation over here, except here this appears with a negative sign here, it appears with a negative sign and over here, and it appears with a positive sign.

So, whatever energy is being removed from the mean flow is going into increasing the kinetic energy of velocity fluctuations. So, this there Reynolds stress times the mean velocity gradient. The Reynolds stress dotted with the mean velocity gradient acts as a mechanism for the transfer of energy from the mean flow to the fluctuations. So, that is how fluctuations are generated. Because, there is this mechanism which transfers energy from the mean flow to the fluctuations, and that mechanism which is Reynolds stress times. The mean velocity gradient it appears with a negative sign the mean energy conservation equation it appears with a positive sign in the conservation equation for the fluctuating Energy.

Of course, at steady state the net energy has to be equal to 0. So, this has to balance by the dissipation of energy. That is there in the last term over here. See this last term here, is the fluctuating stress dotted with the fluctuating velocity gradient tau ij prime times partial Ui prime by partial xj. We had discussed this in the context of the fluid velocity when we did the energy conservation equation and I told you that this term always has to be positive.

So, it always removes energy same as to for the fluctuations. This term is always positive. So, it always removes energy and that is how, fluctuations are sustained by a balance between the input of energy into fluctuations. Due to this Reynolds stress term, the input of energy into fluctuations this actually increases the fluctuating energy that is balanced by the dissipation of energy. So, to summarize as follows. When you wrote an equation for the mean velocity profile right, we had of course, the flux terms which were the divergence of some quantities. Then, we had the viscose dissipation in the mean flow that is the mean stressed or double dotted with the mean velocity gradient. Then, you had this Reynolds stress term that I said, was the dissipation due to the Reynolds stress term was much larger than the dissipation due to the viscose dissipation of a mean flow.

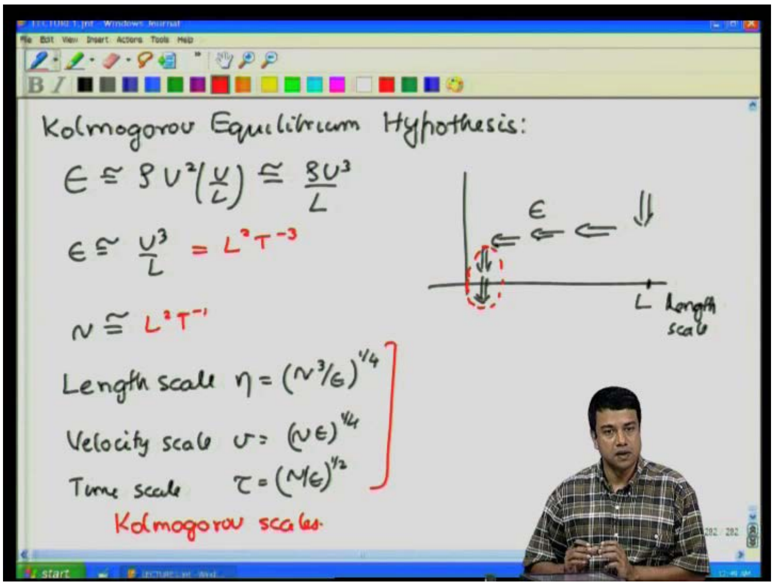
So, that Reynolds stress term transfer's energy from the mean flow to the fluctuations. It acts as sink in the equation for the mean velocity profile acts as a source in the equation for the fluctuating kinetic energy. So, it is a sink in the mean kinetic energy equation a source in the fluctuating kinetic energy equation. In the equation for the fluctuating kinetic energy this appears as a source, there is a sink due to the stress double dotted with the fluctuating velocity gradient. And these two balance each other. A corollary of this as since the Reynolds stress term balances the energy dissipation due to the fluctuations in the fluctuating energy equation. On the other hand it was much larger than the dissipation in the mean energy equation. That means that the dissipation due to, the fluctuating velocity is much larger than the dissipation due to the mean velocities.

So therefore, most of the kinetic energy which is being generated by the mean flow gets transferred to the fluctuations. That is where it will dissipation in the mean flow the kinetic energy gets transferred to the fluctuation and it gets dissipated. Due to, the viscose stresses acting on the fluctuating velocity profile, the dissipation due to the mean velocity profile is very small.

So, because of this mechanism there is energy. They transfer to fluctuations which is sustaining the fluctuations in the turbulent flow, there is a constant production of energy in the mean flow. That is the Reynolds stress term acts as a production of energy it is source of energy which transfers energy from mean flow to the fluctuations. Where it ultimately gets dissipated due to fluctuations for the mean flow the Reynolds number is large. So, because of that the dissipation in the mean flows actually small compared to convective transport. And if the dissipation is small, you would expect the velocity to have very little frictional loss. However, there is this turbulent Reynolds stress mechanism, which is acting as the dissipative mechanism for the mean flow. Which is comparable to the mean flow velocity the production in the mean flow it transfers energy to the fluctuations where ultimately it gets dissipated in the fluctuations.

So, the flow itself is generating small scale fluctuations. Such that, the dissipation in those fluctuations balances the production of energy into the mean flow. So, this is the mechanism by which energy is transported from the mean flow to the fluctuations and the dissipated at the fluctuation scale. So, now the next question that one can ask is. What are the length and time scales which characterize these fluctuations? Of course, the largest fluctuations you would expect would be comparable, to the mean flow velocity and length scale. Because, you have eddies that are transporting across the entire distance of that flow. However, what is the smallest length and time scales that characterize these fluctuations? Because, as I told you earlier a turbulent is a continuum effect. Therefore, these fluctuations have to take place on length and time scales that are much longer than the molecular length and time scales. So, the length and time scales for these fluctuations are determined by what are called the Kolmogorou Equilibrium Hypothesis.

(Refer Slide Time: 50:09)



So, it is called as the Kolmogorou Equilibrium Hypothesis. So, the idea is that there is a large scale of flow. Let us call this as the length scale there is a large flow scale L and there is some small scale. There is energy that is put into the turbulent flow. At this large scale of course, a Reynolds number based upon this large scale is large. Therefore, there can be no dissipation at the scale the dissipation at the scale is very small. However, the energy goes through a series of length scales until it is dissipated at a small scale. What is the small length scale? The total amount of energy that is put in at the large scale has to be dissipated at the small scale. So, this is called the energy dissipation rate epsilon. Whatever, energy is put in at the large scale has to go through these length scales and get dissipated at the small scale. The total amount of energy that is passing through is what is called Epsilon, is the energy that is passing through per unit time per unit volume. So, that epsilon normally you would write as an energy per unit time per unit volume.

So, you would write as the density times U square. Where U is a velocity per unit time scale for the large scale flow is U by L. Because, as the inverse of the strain rate for the large scale flow. So, it writes as u by L. So, you would expect it to have dimensions of rho U cubed by L. However, rather than defining it per unit volume you could as well define it per unit mass. Because, the density is the only thing that contains the mass dimension here. So, I will define epsilon is commonly defined as just U cube by L and I take out the density because that the only one that has the mass dimension by work dimensions of length and time.

So, this is dimensions of this for the macroscopic flow with the macroscopic velocity capital U and macroscopic length scale capital L. You would expect the rate of production from the mean flow to be U cube by L. This also comes out the fact that for the mean velocity this gives me this term is the production term from the mean velocity. This term is the production term you can see it has dimensions of U cube divided L because, it has one gradient time Uj times partial by partial xj ofhalf U square. And I have taken out the density which is just a constant.

Now, for the small scales themselves for small scales themselves, the length scale and the time scale of the eddies are expected to be much smaller than the macroscopic scales. Therefore, these length and time scales are not affected by the macroscopic length and time scales capital L and capital U this is much smaller. What can they be affected? By one Is of course, the dissipation rate epsilon. Because, that is the amount of energy that is coming in the other is of course, kinematic viscosity. Because, that is what is dissipating energy ultimately and converting it into heat.

So, therefore, the other variable that can be depending upon is the kinematic viscosity. This has dimensions of length square time inverse I, could have work to the viscosity. Of course, I just multiply this by the density. But, since I defined epsilon without a density. I will define the kinematic viscosity also without the density. So, this has dimensions of length square T power minus 3 nu has dimensions of length square T inverse. And these are the only two things that the macroscopic length time and velocity scales can depend upon.

Of course, you can now get the length velocity at times scale just by dimensional analysis just from the assumption that, the small scales do not depend directly on the velocity in the length scale of the large scale flow. But, only if the amount of energy that is a coming in which has to be dissipated here. So, that is the only thing that the small scales depend upon. On this basis you can easily see that the length scale, eta for the small scale flow has to be equal to U cube. By epsilon power one by four the velocity scale is equal to mu times epsilon power one by four time scale tau is equal to nu by epsilon power half these are what are called the Kolmogorou of scales. So, this is an important result in turbulent flows it tells you what is the smallest scale of the eddies that are generated by turbulent flows. The smallest scale depends only upon the amount of energy that is being by the generated by the mean flow. And has to be dissipated ultimately by the small scale flows and the kinematic viscosity. Because, the viscosity which ultimately dissipates just on that basis one can infer what are the length the velocity and the time scale of the smallest scale eddies in the flow.

So, this result we will continue in the next lecture there is a little bit more to be done. We can for example, calculate the ratio of the Kolmogorou scale to the macroscopic length scale or the velocity scale to the macroscopic length scale. And try to see, why these scales are generated? And from that we will look at simple ways of modeling the turbulent flows. So, this is an important point in turbulent flows to summarize what I have done. So, far I took the momentum conservation equation average it over time and I got an additional term due to fluctuations.

The Reynolds stress term that is inevitably present in all the turbulent flows and much larger than the molecular stress term for the viscose stresses. We looked at the energy conservation equation for both for the mean flow and the fluctuations. And found out why the fluctuating velocity profiles are actually the Reynolds stress term. Times the mean velocity gradient is much larger than the dissipation in the mean flow. This acts as a mechanism for transform of energy from the mean flow to the fluctuations. And further fluctuating kinetic energy equation there is a balance between the energy coming in from the mean flow due, to the Reynolds stress and what is dissipated due to viscose dissipation.