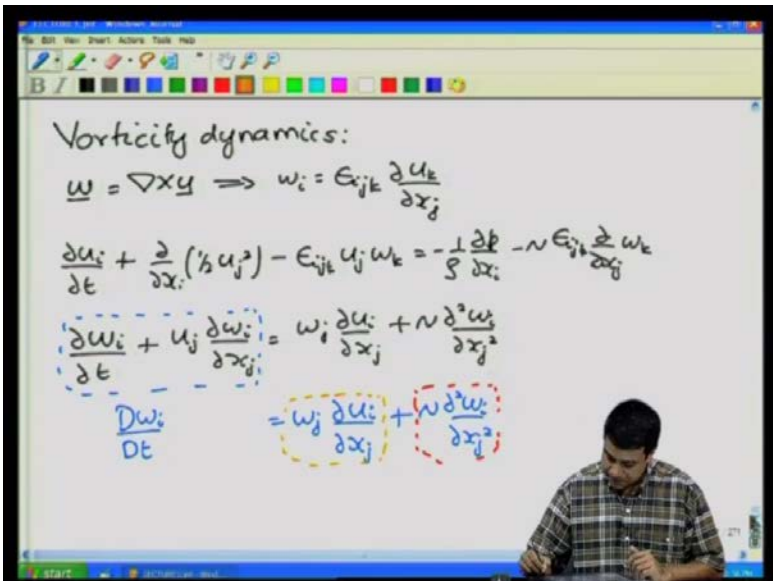
**Fundamental of Transport Process II**
**Prof. Kumaran**
**Department of Chemical Engineering**
**Indian Institute of Science, Bangalore**

**Lecture - 37**
**Vorticity Dynamics - Part 2**

(Refer Slide Time: 00:29)




So, welcome to this lecture number 37 of our course on fundamental transport processes. We were looking into fluid mechanics. And in the last lecture, we had a little bit more to do in vorticity dynamics. If you recall previously the boundary layer theory where we looked at the effect of viscosity in thin layers closed to solid boundaries. And how that could in some instances result in a separation of the boundary layer from the object and the formation of a wake, behind the object. Previously when we did potential flows, we considered the flows to be inviscid and irrational; irrational means that the vorticity is equal to 0.

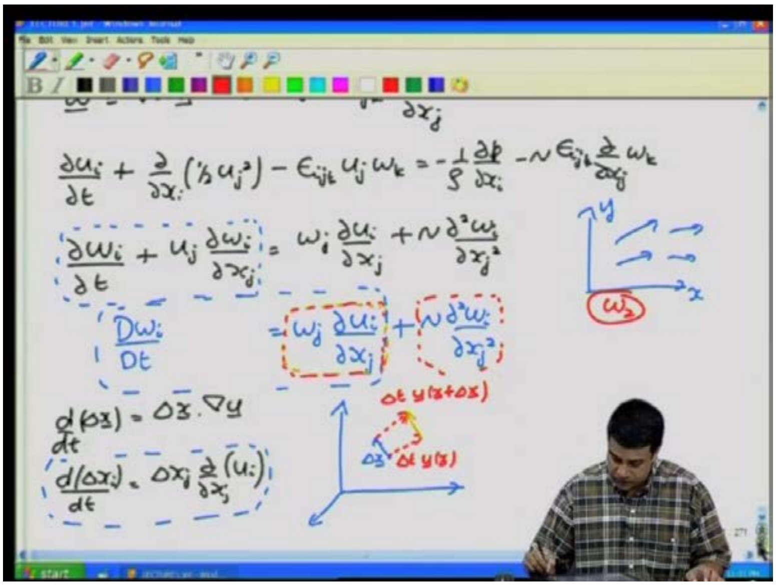
I did not really justify that the reason for doing that approximation at that stage, apart from the obvious rational that if the curl of the velocity is equal to 0. Then the velocity can be expressed as the gradient of a potential. And that greatly simplifies the formulation of the equations. In the last lecture, we had looked at the effects of vorticity in an inviscid or a viscous fluid. The vorticity is defined as the curl of the velocity or in our notation, omega i is equal epsilon i j k partial u k by partial x j, del cross u. And I showed you that the momentum conservation equation navier strokes momentum conservation equation can be written partly in terms of the vorticity.

Partial u i by partial t plus partial by partial x j x i half u j square minus epsilon i j k u j omega k is equal to minus partial p by partial x i i, should have a one over density there to make it correct dimensions and dividing throughout by density. And there was a negative sign here minus partial by partial x j of omega k, so del square u the laplacian of the velocity in the viscous term can be written as minus of the curl, of the vorticity. And in the last class, we also derived an equation a balance equation for the vorticity by just taking the curl of the entire momentum equation.

That vorticity balance equation turned out to be partial omega i by partial t plus, that was the balance equation for the vorticity the entire left hand side in that equation. The entire left hand side that I had in that equation was just the substantial derivative. And on the right hand side I have 2 terms, the second term here is of course, the useful viscous diffusion mu del square the kinematics of the diffusivity in this case is the same it is the kinematics viscosity of the fluid.

And then I have this additional term here which had no analog in my momentum conservation equation. So, this acts as some kind of a source of vorticity, it increases so if I had an inviscid fluid this viscous diffusion would be identically equal to 0. And the rate of change of vorticity is just equal to omega dot grad u this, this first term here omega j partial u i by partial x j. This we had derived in the last lecture, and I had given you a physical interpretation of this. The physical interpretation of this was if I had.

(Refer Slide Time: 05:11)



If I had within a fluid flow field, some line segment delta x vector if I had some line segment delta x vector. And this is a material line segment that is it is moving along with the fluid it is moving with the same velocity as the fluid at each point within this line segment. So, at some later time this is going to move a distance u delta t, where u is the velocity at the location x. So, this I can write this as delta t times u of x this moves some other distance delta t times u of x plus delta x.

So, the displacement at the time x plus t plus delta t is going to be this nu 1 which is final position minus the initial position delta t into u at x plus delta x minus u at x, which is basically equal to grad u delta x dotted with grad u u at x plus delta x minus u at x can be written as delta x times grad u. Therefore, you have the rate of change of this delta x vector, the rate of change of delta x vector. Because this is a material element that is moving with the fluid at time t, you have a certain length delta x vector, at some other time t plus delta t you have some other length delta x vector.

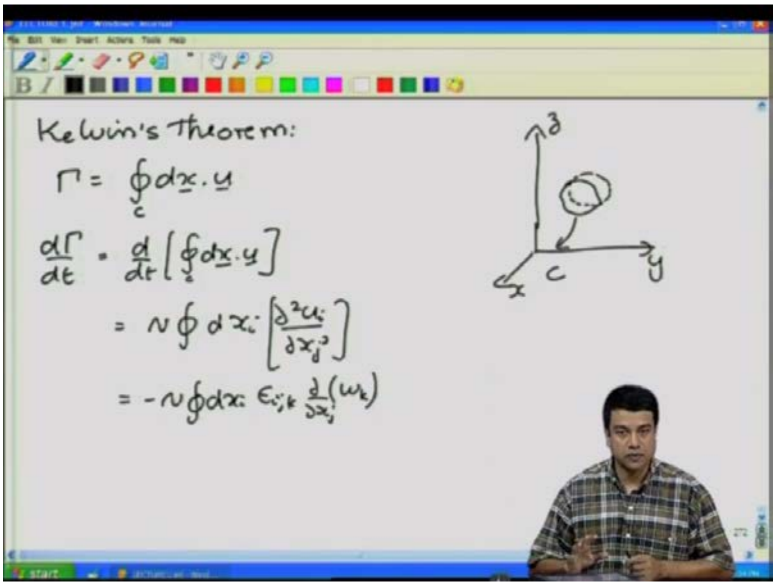
The difference between the initial and the final positions basically gives you the rate at which delta x is changing with time. And as I told you that can be written as delta x dot grad u. Note that d by d t is the substantial derivative it is in a reference frame that is moving with the fluid. Because I am considering this as a fluid material line element, that is being deformed, because the fluid is flowing. If the velocity were a constant everywhere then they would be not deformation, because both the ends of this vector are moving with exactly the same velocity. Therefore, they have exactly the same displacement and there is no change in the length of that line segment. However if the line segment if the fluid velocity varies with position, then there is a change in that line segment written in vector notation d delta x i by d t is equal to delta x j partial by partial x j of u i. That is written in vector notation delta x dot grad u the dot product is between delta x and the gradient.

Note that, this has exactly the same structure as this one this is exactly the same structure as these two put together d omega i d t is equal to omega j times partial u i by partial x j. Therefore, vorticity increases or intensifies in the flow in a manner similar to material line elements in the flow. So, this is the mechanism for intensification of vorticity due to stretching or bending of vortex lines. As I had explain in the last lecture, in case when there is a two dimensional flow, let us say the velocity is in the x y plane and it varies also in the x y plane.

So, that the flow is only in two dimensions x and y. So, I have some velocity vector in this x y plane vorticity is the curl of the velocity so vorticity, since it is the curl of the velocity it is perpendicular to this x y plane. Therefore, the only non zero component of the vorticity is going to be omega z. Because the velocity is in the x y plane the gradients are in the x y plane vorticity is the curl of the velocity is perpendicular both to the velocity direction and to the gradient direction.

Therefore, vorticity is only in the z direction and in that case this term will be identically equal to 0. So, there is no intensification of vorticity due to stretching or bending of vortex lines in two dimensional flows. It exists only in three dimensional flows, and associated with this mechanism we are actually proved a theorem which was first due to Kelvin, which basically states that, for material line for some material loop, I will call it as c a closed loop is a closed loop c.

(Refer Slide Time: 09:55)



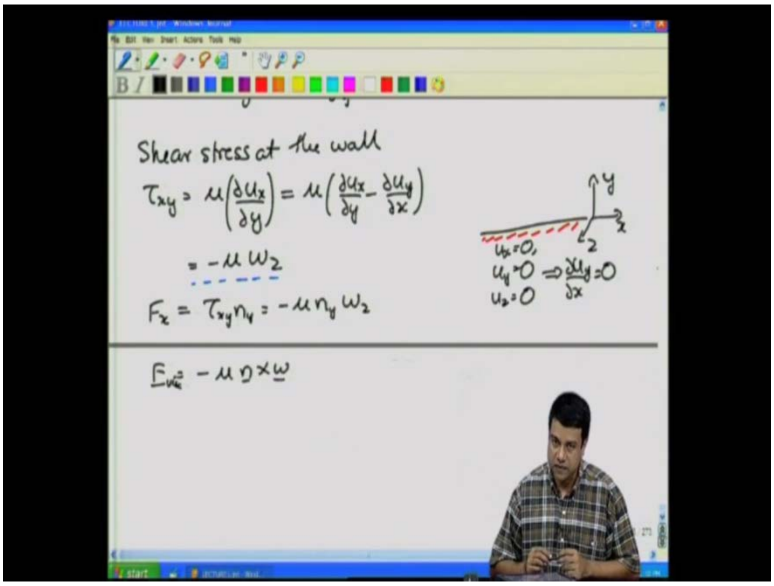

This is a material closed loop that means that every point on that loop moves with exactly the same velocity as the fluid at that particular point. So, at some later time this loop will move to some other location. The circulation on this loop is given by gamma is equal to integral d x dot u over this closed loop c. And I had shown you that this the, the rate of change of circulation, that is the rate of change of circulation on this moving loop of fluid material loop of fluid that is moving with the same velocity as the fluid velocity at that location.

This can be written as the kinematic viscosity that is integral of d x. I will take this in indicial notation that makes it easier. So, this is what we had derived in the previous section in the previous lecture and this can also be written as minus mu integral d x i epsilon i j k partial by partial x j of omega k. So, if you have an inviscid fluid in which the kinematic viscosity is equal to 0 then d gamma by d t is identically equal to 0.That means that the, the motion of the, of the, on in a moving reference frame moving with that loop of fluid of course, the length of that loop could change in general. But in a reference frame that is moving with that loop of fluid there is no change in the vorticity on this. So, vorticity is convicted unchanged through the fluid that is one reason why we could have talked about potential flows, because you cannot generate vorticity within the flow. Vorticity is transported unchanged through the fluid and the intensification of vorticity is similar to the intensification of material line elements.

So, if I had a fluid which was in initially irrotational and there was no kinematic viscosity, at later times it has got to continue to be irrotational. There cannot be a source of vorticity within the fluid itself and that is the reason why we could talk about potential flows, because we know that, that that. If the flow initially did not have vorticity it has to continue to not have vorticity internally right you cannot generate vorticity within the flow.

So, where then can vorticity be generated the answer is vorticity is generated at solid surfaces and vorticity. There is generated at the solid surfaces is then convicted unchanged through the flow the only mechanism of intensification is this mechanism of of stretching and bending of vortex lines. And of course, there is a mechanism by which vorticity is reduced the vorticity diffusion mechanism the rate of change of vorticity also has a diffusive term there which tends to attenuate to damp out vorticity variations. So, let us look at how vorticity is generated at surfaces.

(Refer Slide Time: 13:50)




If you have a surface here, if you have a surface let us call it x, let us call this y, let us call this z. We know that the, the, the shear stress at the wall tau x y is equal to mu partial u x by partial y. Now, because of the no slip condition at this wall, because of no slip condition at this wall I know that at the surface itself u x has to be equal to 0 u y has to be equal to 0. There is no normal velocity boundary condition and of course, in the third direction u z also has to be equal to 0. If there is a no slip condition at the wall u y is equal to 0 implies right u y is equal 0 at all values of x u y is equal to 0, at all values of x implies that partial u y by partial x.

This is also equal to 0, because u y is equal to 0, at every location therefore, partial u y by partial is also equal to 0. Therefore, for this surface I can write this as partial u x by partial y minus partial u y by partial x. Because we know that partial u y by partial x is equal to 0 at the wall. This of course, is equal to minus mu times omega z. We know that omega z is equal to partial u y by partial x minus partial u x by partial y. Therefore, the shear stress is equal to the z component of the vorticity divided by the viscosity at the wall itself. The shear stress can be expressed explicitly in terms of the vorticity.

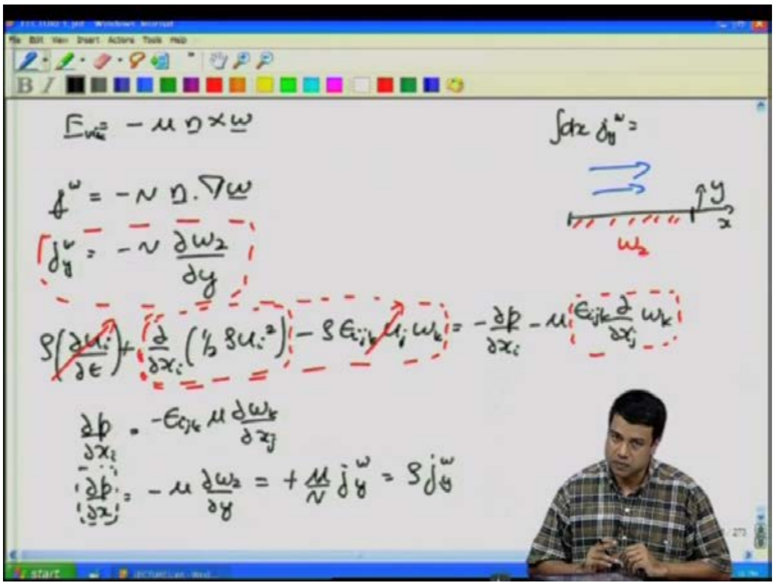
So, this basically tells you that the value of the vorticity itself at the wall is exactly fixed by the shear stress. The force exerted at this surface the unit normal is in the y direction therefore, the force exerted in the x direction the tangential force exerted in the x direction at the surface is equal to tau x y n y equal to minus mu n y times u z. This is the viscous shear stress tangential to the surface, this can be generalized in this particular case I have taken a coordinate system with my x axis along the surface and my y axis perpendicular to the surface.

But, I can write this in a most general coordinate system as f vector is equal to minus mu n cross omega. So, this works regardless of which orientation the, the surface is in the coordinate system you can see that this, this should be omega z. So, this basically tells us this, this f vector is the viscous force, this basically tells us that the value of the vorticity at the surface itself is equal to the viscous force divided by the viscosity tangential viscous force divided by the viscosity.

So, this is like at the surface itself the vorticity has a fixed value to that is determined by the shear stress that is exerted at the surface. Similar to for example, if I were doing a heat transfer problem I would be able to fix the temperature at the surface. The temperature would have a finite value at the surface and that it would change as you go, within the fluid in agreement with the conservation equation which contains the convective and the diffusive term.

Similarly, in this case the vorticity has a specific value at the surface and of course, it varies as you go into the flow depending upon the convective, the viscous. And we have this additional vorticity intensification term, so in in accordance with these three terms the vorticity would vary as you go within the fluid. However in order to find out how much vorticity there is in the flow it is not sufficient to know what the vorticity at the surface is you have to know what is the flux of vorticity, what is the flux of vorticity that is taking place in the direction perpendicular to the flow. In the conservation equation if you recall for the vorticity, we had a diffusion term in that diffusion term the diffusion coefficient was identical to the kinematic viscosity of the fluid itself. Recall this diffusion term, here this diffusion term the diffusion coefficient is identical to the kinematic viscosity of the fluid itself, therefore the flux the flux of vorticity.

(Refer Slide Time: 19:15)



The flux of vorticity can be written in a manner similar let me write this as the flux of vorticity, the flux of vorticity can be written in a manner similar to the flux of heat. For example, as minus nu times the n dot grad omega. So, in this particular case where I have a surface with x and y coordinate. The flux of vorticity in the y direction is equal to minus nu time n y this equal to partial omega z by partial y. So, this is the vorticity that is coming out of the surface the flux the amount of vorticity coming out per unit area per unit time into the fluid.

So, that is a vorticity flux that is taking place from this surface turns out you can write an expression, for this vorticity flux from the momentum conservation equation, which was expressed in terms of the vorticity itself. If you recall the momentum conservation equation as expressed in terms of the vorticity is partial u i by partial t plus partial by partial x i of half rho u i square minus rho epsilon i j k u j.

Omega k is equal to minus partial p by partial x i plus minus mu epsilon i j k partial by partial x j of omega k. You recall we had written the laplacian of the velocity vector in terms of the vorticity here. And similarly, we had simplified this convective terms in terms of the vorticity. If you apply this equation at the surface itself, if you apply this equation at the surface itself, we have a no slip condition at the surface, we have a no slip condition at the surface. And therefore, all of these terms the all terms proportional to the velocity will go to 0. The first terms goes to 0, because the, if the flow is at steady state is identically equal to 0. The next two terms this term goes to 0, because the velocity is equal to 0, at the surface. This is no, no slip condition and if you are looking at the variation in the vorticity.

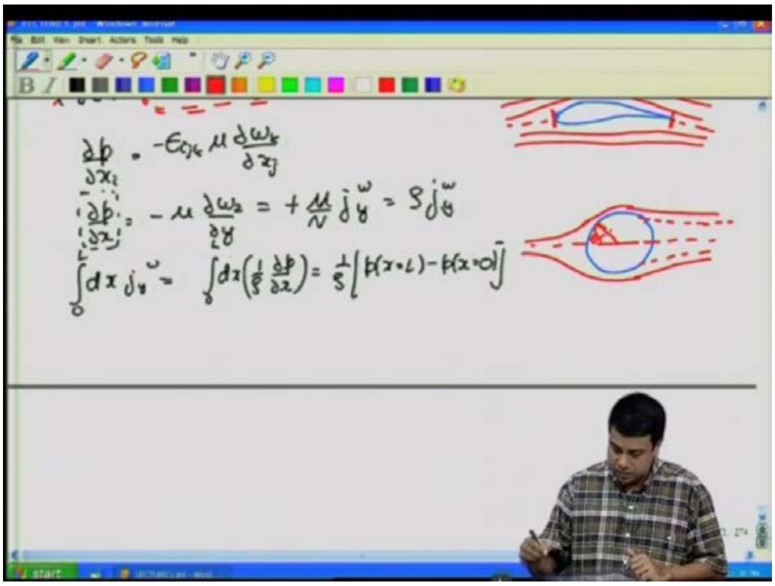
In the, in the x momentum conservation equation this term is also equal to 0, because I know that the velocity is equal to 0. Therefore, the gradient of velocity along the flow direction is equal to 0. So, for this particular case you find that the equation becomes partial p by partial x i is equal to epsilon i j k mu partial omega k by partial x j with a negative sign. And if you simplify for this particular configuration x y and z where the vorticity is the only non zero component of the vorticity is omega z.

The only non zero component of the vorticity is omega z, because the flow is in the x y plane flow, is in the x y plane the only non zero component of the vorticity is omega z. In that case you will get partial p by partial x is equal to minus mu partial omega z by partial y, because x is I and therefore, partial omega k by partial x i j will basically give you partial omega z by partial y, which this is equal to minus mu by nu times j z omega as I have defined this is j y omega. From this equation so this from this equation j y omega is equal to minus nu partial omega z by partial y.

Therefore, this is equal to plus mu by nu times j y omega, so this is just equal to rho times j omega y. So, this implies that the vorticity flux coming from the surface depends upon the pressure gradient at the surface, only when there is a pressure gradient will you get a flux of vorticity coming out of this surface. If there is no pressure gradient there will be no vorticity flux. Therefore, for a flow where there is a pressure gradient along the surface the vorticity flux is just equal to minus 1 is just equal to 1 over rho times the pressure gradient.

The net vorticity flux between two locations here integral over d x of j y omega integral over the length, as you recall when we did boundary layer theory, we integrated the force over the length in order to find out how much vorticity is actually coming out of the surface. We can integrate the vorticity flux over the length, note that this vorticity flux can be positive or negative depending upon whether the pressure gradient is positive or negative. It is negative if the pressure is decreasing downstream and it is positive if the pressure is increasing downstream, so the net vorticity flux that is coming out.

(Refer Slide Time: 25:15)



Integral d x j y omega from between two locations let us call it as 0 and l the start and the end of the surface. This is equal to integral between 0 to l d x of 1 by rho partial p by partial x, which is equal to 1 by rho into p at x is equal to l minus p at x is equal to 0. So, the net amount of vorticity coming out from this stretch of surface of course, this is a two dimensional flow. So, the vorticity is flux is defined only per unit area perpendicular to the plain of the board. But, per unit length perpendicular to the plane of the board the total amount of vorticity coming out of this surface is proportional to the difference in pressure between the two ends of the surface.

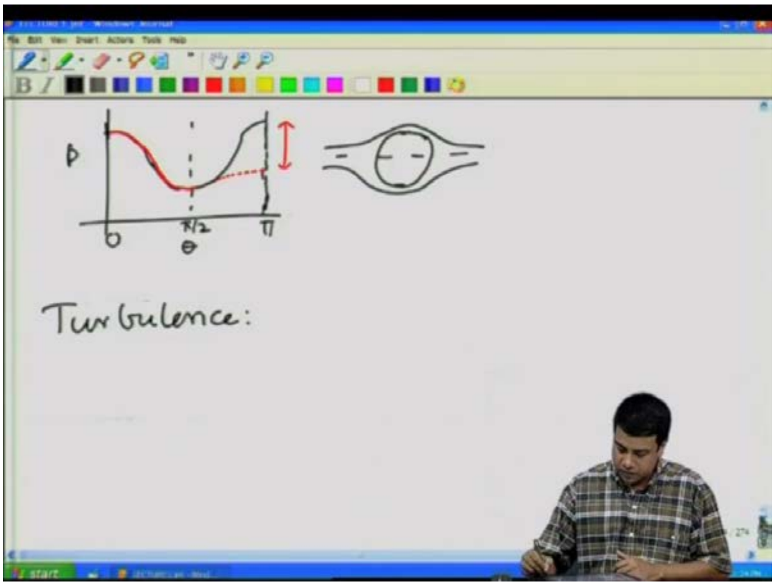
So, only in the case where you have a pressure driven flow, where there is a pressure difference between the two ends will you have a net vortictiy flux coming out of the surface. A net amount of vorticity if the pressure at the two ends is the same, even though there may be pressure gradient in between if the pressure at the two ends is the same. The net amount of vorticity that is coming out has to be equal to 0 even though at certain locations you may have vorticity coming out and other locations it goes back.

In such that the net vorticity is equal to 0, our familiar example of cylinders or airplane wings. If you we solved the potential flow problem for an cylinder and we also motivated the potential flow problem for an aero plane wing as the fluid goes around this, as a fluid goes around this. If you look at the top surface alone, as the fluid is going around the surface, initially the stream lines are being compressed and then they expand again. Because the, the flow is being compressed, above and below so initially the stream lines are being compressed.

And then they expand again for this case initially the velocity increases you have an accelerating flow. Then the velocity decreases again you have a decelerating flow, when the flow accelerates the pressure of course, decreases because p is equal to minus half rho u square plus p naught. The pressure initially decreases and then as the flow decelerates the pressure increases again. However you know that the pressure at these two ends has to be the same because both ends are open to the atmosphere. So, the pressure at the two ends has to be the same what that implies is for this particular case the net amount of vorticity that is generated from the surface is going to be equal to 0. Because the pressure on the two ends has to be the same.

Therefore, if you, if you have even if you have a boundary layer as long as that boundary layer is confined boundary layer. The pressure at the two ends is exactly the same and for that reason there will be no net vorticity generation from the surface that is not true for a bluff body. For this bluff body we found that what actually happens is that you have the flow accelerating as it goes around the cylinder on the upstream side on the downstream side there is one location at which the separation takes place. If you recall that is where the parameter beta that that is the analogous parameter for the flow in a corner goes below minus 0.0904 this is this is true for a cylinder. And this is true for a sphere, as well so in the accelerating region if I plot, if, if I plot the function of this angle theta if I plot the pressure if I plot the pressure as function of this angle theta.

(Refer Slide Time: 29:27)




Around the surface at theta is equal to 0, you have the upstream edge, upstream edge the velocity has the stagnation point there the velocity is very small. Therefore, I have the pressure is largest at this point p is function of theta; theta goes from 0 to phi, just by symmetry for the potential flow solution alone. If I have the potential flow solution alone, the potential flow solution actually goes all the way around. And I showed you just by symmetry that for the potential flow solution there is symmetry between the upstream and the downstream side. The pressure is equal to minus half u square minus rho u dot capital u.

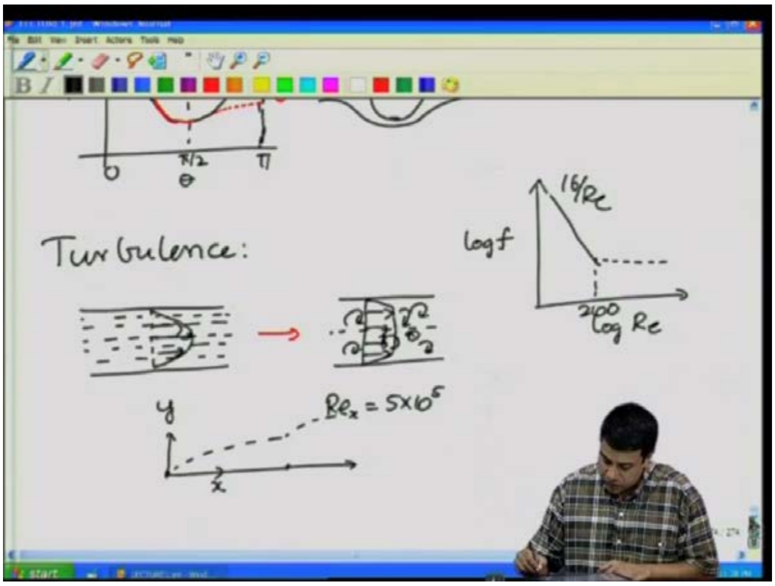
And, because of that there is, symmetry between the upstream and the downstream side. That means if the pressure has to be large on both sides, because the velocity is 0 at the stagnation points. And it comes down in between it comes down in a symmetric manner in between, this axis a symmetry corresponds to pi by 2 which is the top and the bottom of the cylinder. This is what the potential flow solution predicts but because of boundary layer separation I showed you that there is actually a, a wake region that forms behind the cylinder. In this wake region the pressure is actually much lower.

And so if you, if you plot the pressure difference along with boundary layer separation or define there is that it, it approximate the potential flow solution the upstream side. Because the boundary layer is attached to the surface on the downstream side at some point there is separation, and the pressure never recovers back to the original value that had on the upstream side. So, if you plot, if you, if you plot the pressure going around the cylinder you find that it decreases as in potential flow on the upstream side. But, it does not recover back to the original pressure predicted by potential flow on the downstream side.

So, between the upstream and the downstream stagnation points there is this pressure difference. And because there is a pressure difference over the entire surface there has to be a net vorticity that is generated. That vorticity is effectively is what is generating the circulating flow in the wake behind even at high Reynolds number. So, this completes our discussion of vorticity dynamics vorticity cannot be created or destroyed within the flow. It can be attenuated if viscous effects are included but if you neglect viscous effects vorticity is convicted without there can be no creation within the flow itself.

It is generated at solid surfaces and it is convicted and it is amplified by stretching or bending of vortex lines within the flow. And the net amount of generation of vorticity at a surface depends only upon or the net flux at the surface depends only upon the difference in pressure, between the two edges of the surfaces. So, this leads us naturally to our next topic and that is turbulence. That is the, the flow in the wake of these objects or in the flow in the pipes after transition takes place. So, highly chaotic flow is called turbulent flow, and turbulence is said turbulence is difficult topic simply, because the flow is so irregular but first what happens, how is turbulence generated, for example for internal flows for a flow in a pipe.

(Refer Slide Time: 33:19)




We have solved the velocity profile on various occasions, we know that the fully developed laminar flow in a pipe is a parabolic velocity profile. However as you keep increasing the Reynolds number at some point there is this spontaneous transition, to what is called a turbulent flow. The turbulent flow the velocity profile is much flatter the velocity profile it will smoke plug like for the same, for the same average velocity the maximum velocity is lower. So, it is more plug like velocity profile where as in a laminar flow you have nice straight stream lines.

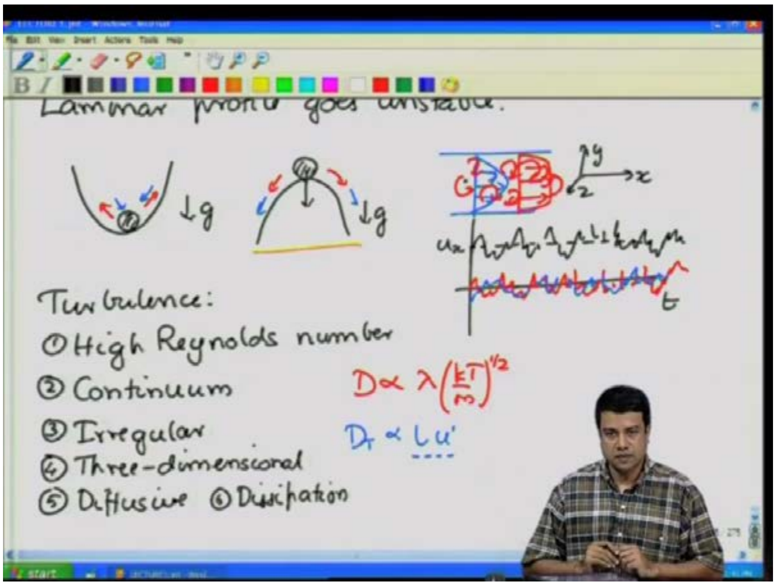
And the transport of momentum across the flow takes place only due to molecular diffusion. In this case you have highly irregular stream lines and you have velocities in all 3 directions velocity fluctuations in all 3 directions at any instant in time at any location. So, the velocity is even though the mean flow takes place only in one direction. The local fluid velocity is in general three dimensional it has fluctuations in all directions and fluctuations.

There are parcels of fluid called eddies which, which have velocities in all directions and there are eddies of various sizes within the flow. And this is reflected in for example, in, in flow through pipes and channels for, a for the specific case of the flow through a pipe. If you plot the log of reflection factor versus the log of the Reynolds number, we know that the laminar profile satisfies a log law of the form f is equal to 16 by R e at a Reynolds number corresponding to about 2100. There is a spontaneous transition that takes place to a turbulent flow with a much higher friction factor than what you would expect for a laminar flow. Reflection factor is much higher than 16 by R e therefore, this flow is much more dissipater than the the 16 by R e law critics. So, this happens for external flows as well for example, the boundary layer flow, we got the velocity profile for a laminar boundary layer the blasius profile for a laminar boundary layer. At some point this boundary layer itself will become unstable.

The point is when the Reynolds number based upon x is equal to five into ten power five about 500000 when the Reynolds number reaches about 500000 that the, the boundary layer profile itself becomes unstable it is no longer laminar. And you go on what is called a turbulent boundary layer which follows a different law for the boundary layer thickness is a function of Reynolds number. And it is much more dissipater, in all of these cases it is not that the laminar profile is no longer a solution of the equations. Similar thing happens for example, in when you heat a fluid from below there is a, a, an instability, because of the density gradient.

Similarly, in the case of rotating fluids you find an instability which leads to eventually to turbulence. In all of these cases it is not that the laminar velocity profile is no longer a solution. If you insert the laminar velocity profile into the navier stokes equations the equations are satisfied at all Reynolds numbers. For example, the parabolic flow in a pipe that parabolic velocity profile satisfies the navier stokes equations at all Reynolds numbers. So, the parabolic profile is still a solution of the equation except that it is not a stable solution. So, the reasons there is a transition is…

(Refer Slide Time: 37:26)




Because the laminar profile goes unstable it is still a solution of the equations but any small disturbances will make it go to some other solution. To draw an analogy if I had, if I had a ball in a cup in a gravitational field this is a stable configuration. Because if I displace this cup a little bit, if I displace this ball a little bit either on the left or on the right of this in axis symmetric cup, I can displace it in any direction. This will just come back to its original state, because gravity is acting downwards if I turn this upside down and I place the same thing over here, at this point exactly all forces are balanced.

So, this is a solution of the equations of motion for this particular case it is a valid solution. The problem is if I put a small disturbance in either direction this will not come back it will continue to go, continue to go until it reaches some other solution. So, same thing for the flow transition in the flows and channels and pipes. The laminar solution is a stable solution at low Reynolds number, at at all Reynolds numbers. The problem is when the Reynolds number increases beyond a certain value any spontaneous disturbance will make the flow go to a turbulent state. This brings us to an important point here and that is that if you recall in the limit of low Reynolds number. We had said that, we neglect the inertial terms and because of that the equations end up being linear that means that solutions exist and they are unique.

So, for low Reynolds number you can have only one particular solution. However in the high Reynolds number limit the inertial terms become important and therefore, the solution is no longer unique. Because the solution is no longer unique there could be multiple solutions for the navier stokes mass and moment conservation equations for a specific configuration. Laminar solution still exists the problem is that if you have a flow in a pipe with a laminar flow. Parabolic velocity profile and I put a small disturbance anywhere alright just I put a small disturbance for this particular case. The solution goes to some other solution and that other solution is the turbulent flow.

So, the first thing turbulence, first thing it is a high Reynolds number phenomena. The first thing it is a high reynolds number phenomenon, because we know at low Reynolds number equations become linear and the solutions are unique. So, you require high Reynolds number for in order to have a turbulent flow. So, it is, it is a phenomenon of high Reynolds number, the second important point is that it is a continuum phenomenon it is obtained by solution of the continuum equations that is the continuum navier stokes equations it is not that the continue approximation itself breaks down. So, it is a continuum phenomenon even though there are large fluctuations in a turbulent flow it is still a continuum phenomenon. The third thing is that it is highly irregular as I said the velocity profile in a turbulent flow, looks more plug like it is not parabolic, where as it is flatter than a parabola at the center and it has a sharper gradient at the walls.

There is only the mean velocity profile if I average the velocity over a long period of time. The local velocity profile actually has fluctuations of various length scales called eddies in various directions at any given point, in time if I have to plot any component of the velocity. So, for example, if I have to sit at one particular location, if I have to sit at one particular location and plot as a function of time the velocity component u x say. So, let us put our coordinate system here, x is the flow direction and y and z are the gradient are the two directions perpendicular to the direction of flow. y is the direction of the gradient the direction in which the mean velocity varies, if I have to sit at one particular location and I have to plot the velocity u x. Of course, it has an average value the mean value at that location this is the, mean value at that location. However if you plot the instantaneous velocity you will find that it actually fluctuates, it fluctuates around the mean, it has a highly regular irregular fluctuations about the mean.

So, this is u x in addition you will find that there are velocity fluctuations both in the y and the z directions as well. It is not that you just have fluctuations in the x direction alone you have fluctuations both in the y and the z directions as well. So, the y velocity will also fluctuate even though the mean y velocity is equal to 0. And you will have fluctuations in the z velocity as well, if it is a highly irregular flow with fluctuations in all three directions at each point in space.

And as I said the fluctuations are three dimensional, you cannot have two-dimensional turbulence even though the mean velocity may be profile, may be only two dimensional the fluctuations in general occur in all three directions. The reason is a little complicated but it has got to do with the fact that a, an important, an important step in turbulent energy transfer through to this fluctuations is the, is the intensification of vorticity. Due to the stretching and bending of vortex lines, if you recall when we did vorticity dynamics, I told you that vorticity of course, diffuses by mechanism similar to viscous diffusion. However it is also stretched and bend due to the intensification it is also intensified due to the stretching and bending of vortex lines. In a manner similar to, the change in material line elements due to the gradient in the velocity.

That mechanism if you recall I had said that it does it in two dimensions that is identically equal to 0, because I have a two dimensional velocity profile, with two components u x and u y varying, in the x and y directions. The vorticity vector is perpendicular to both of these so it is in the z direction. And because of that that mechanism is absent in two dimensions, because I have omega z and the strain rate is in the x and y plane.

So, in that particular case it is equal to 0, therefore, you need to have a three dimensional flow in order to be able to intensify vorticity by stretching and bending of vortex lines. That means the turbulence has to be three dimensional. In addition it is highly diffusive, the diffusion coefficient in a turbulent flow are often orders of magnitude larger than that in a laminar flow. The reason for that is because this mixing that is taking place, is taking place due to the transport by these eddies, these parcels of fluid formally an eddie is defined as a parcel of fluid in co-related motion.

These little packets of fluid, as I said they have velocities in all three directions. So, they are moving both stream wise, and cross stream an, an Eddie that moves in the cross stream direction will take along it is temperature. For large distances in the cross stream direction, in a laminar flow this diffusion occurs due to molecular diffusion, due to the transport of molecules across a surface which results in mixing. And that diffusion coefficient, if you recall in a gas it is, it is the same whether the, the it is for heat mass or momentum transfer, because the mechanism is the same it takes place by the actual motion of molecules.

And this mechanism this, this transport scales roughly as the mean free path times the fluctuating velocity. A fluctuating velocity in a gas is equal to the square root of k T by m, half m v square for the molecules; the molecular velocity fluctuations half m v square is equal to 3 by 2 k T. Therefore, the velocity scales as k t by m power half that velocity times the length scale, the mean free path is what constitutes the diffusion coefficient. This we had done in the detail in the beginning of fundamentals of transport processes one where we had looked at diffusion processes in great detail.

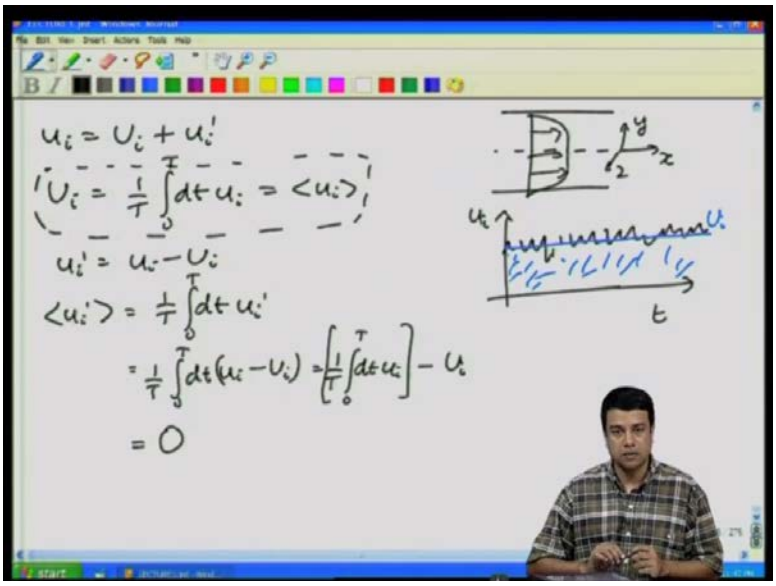
So, this is the mechanism that results in diffusion, due to molecular diffusion in a gas, in a liquid of course, instead of the mean path you will have a molecular length scale molecular diameter. However in the case of turbulent diffusion in addition to this you also have diffusion, due to turbulent eddies. And that diffusion, due to turbulent eddies the turbulent diffusion is proportional to the length scale of an Eddie. The, the spatial extent of an eddie times the velocity fluctuation of an Eddie itself, turns out that this is much larger than the molecular diffusion coefficient in a turbulent flow.

And for that reason turbulent flows are highly diffusive the diffusion coefficient is much larger than the molecular diffusion. And associated with high diffusion is also high dissipation, the shear stress at the surface is represents the flux of momentum perpendicular to the surface. That is balanced by the pressure gradient at the two ends in the case of an internal flow. The flux of momentum if it were just molecular would be just given by the viscosity times the gradient of the velocity.

However, you have this additional mechanism of transporting momentum across the flow due to turbulent diffusion. So, that turbulent diffusion is much larger than the molecular diffusion. Therefore, the momentum flux from the surface in a turbulent flow is much larger than what you would expect for a laminar flow of that same average velocity. Because of that the wall shear stress also is large and therefore, the pressure difference that is required is also much larger than that for the laminar flow that much higher pressure difference also results in a much higher friction coefficient.

In a turbulent flow that is the reason that the friction coefficient you see is at the transition point there is a, there is a jump from the laminar friction coefficient or the friction factor to the turbulent friction factor, which is much higher than 16 by R e. So, of course, if you have a highly regular irregular velocity profile it is difficult to model the velocity profile, because we usually assume that quantities are very smooth in space. So, how do we model a turbulent flow? A simplest stake approach would be to say that we do not worry too much about the fluctuations themselves, but let us just try to write down the equations for the mean velocity profile. So, let us look definitely at one particular configuration.

(Refer Slide Time: 50:33)




Let us say I will just take for example, make a reference to a simple pipe flow, let us take a steady state velocity profile. So, that there is no time derivative in the conservation equation. I know that locally at each point the velocity is fluctuating rapidly. However on average, if I average it out over some period of time the mean velocity is independent of time you get a mean velocity that is independent of time, if you average it over some period of time, in this particular case the flow is only in the x direction.

So, the mean velocity is only in the x direction, there is no mean velocity in the y and z directions. So, simplistically I could say that, I want to separate out the total fluid velocity into this, into a mean capital u i plus a fluctuating part u i prime. The mean velocity I will define as an average over time, in this particular case I am considering the flow that is statistically steady, even though there are local time fluctuations in the velocity. If you average over long period of time all quantities have averages which are independent of the time period over which you average over provided it is sufficiently large.

So, I define capital u i this is how it define the average velocity. So, I have some, some time period t and I plot the local fluid velocity, at some location, I plot the location fluid velocity at some location one particular component of that fluid velocity. It is fluctuating value it it shows fluctuations, if I take integral 0 to capital t d t of u i i get the total area under this curve I get the total area under this curve. That is equal to capital u i times t where capital u i will turn out to be the average velocity. So, this is the average velocity times t times capital t is equal to integral of the velocity over time. As I said in the y and z directions this velocity will be average velocity will be equal to 0. Because there is no net more in the y and z directions, however I can write it in this fashion and then if I do it this way then I get u i prime is equal to u i minus u I. If I take the average so I will also write this average as u i i have used this symbol for the average.

If I take the average of u i prime, if I take the average, of u i prime is equal to one over t integral 0 to t d t of u i prime is equal to small u i minus capital u I. Note that capital u i is independent of time its already a time average, small u which is the fluid velocity does of course, depend upon time but capital u is the average velocity which is independent of time. So, I am basically integrating a constant capital u i from 0 to t and then dividing throughout by t. So, this just becomes equal to 1 over t integral 0 to t d t u i minus the velocity capital u i itself. Because capital u is independent of time and of course, I know from this relation, I know from this relation that capital u i is equal to 1 over t integral 0 to t d t of u i. So, this means that the average of the fluctuation has to be equal to 0. The average of u i prime has to be equal to 0.

So, I can separate out the velocity into a mean part plus a fluctuating part you do the same thing for the pressure, you do the same thing for the pressure separate on to a mean part and a fluctuating part. In this particular case I have a study state flow and therefore, I can separate out the, I can, I can separate out the integration over time with the with the spatial differentiation that I will do in the conservation equations. If we had something that was time varying that would be much harder, in that case you would have to do what are called ensemble averages. In this particular case since I am focusing only on a study state flow I can do this averaging explicitly.

So, what I will do next is to take this separation into capital u and u i prime insert it into the momentum conservation equation. And try to see if I can get some closed form equation for the momentum conservation equation for the mean velocity alone. So, if I can do that then I can solve for the turbulent flow mean velocity alone, without worrying about the turbulent fluctuations. As we will see in the next lecture you cannot do that quite. So, simply and we will look at the reasons for that and continue our discussion on turbulent flows in the next lecture we will see you then.