Fundamentals of Transport Processes II
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Lecture - 35
Falkner - Skan Boundary Layer Solutions-Part 2

So, welcome to this lecture number 35 of our course on fundamentals of transport processes 2, where we were looking in the last lecture at boundary layer theory.

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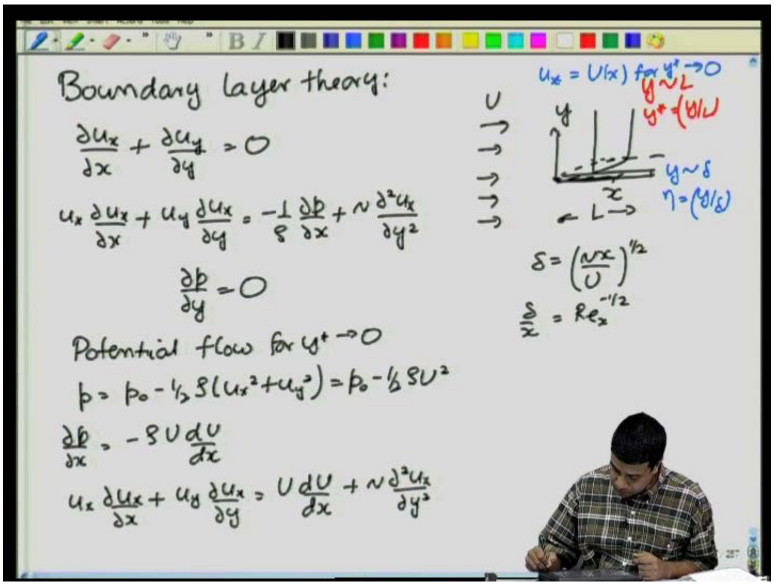

Boundary layer theory is used for the flow very close to surfaces, when the Reynolds number is high if you, the Reynolds number is high and you just simplistically neglect the viscose terms in the momentum conservation equation, then you get the potential flow equations if the flow is irrotational as well. For the for that potential flow solutions, we found that we cannot satisfy the tangential velocity boundary condition at the surface.

You can satisfy the normal velocity boundary condition at the surface, but one cannot satisfy the tangential velocity boundary condition at the surface. So, there is a correction to the outer potential flow very close to surfaces, where you have to take into account the effect of the viscose stresses in a thin layer close to the surface, in the limit of high Reynolds number the thickness of this layer becomes smaller and smaller. So, that the gradients in this layer becomes larger and larger in such a way that there is a balance between the inertial and the viscose terms in the conservation equation even as the thickness of this layer goes to 0 and the Reynolds number goes to infinity.

So, it is easier to illustrate by the flow passed a flat plate of length L where we take x as the stream wise direction and y as the cross stream direction, p the what is called the cross stream direction perpendicular to the direction of the mean flow. This is the velocity U this velocity as I, explained could in general be a function of x and there is a thin boundary layer near the surface if you are outside this boundary layer. Then viscose effects are small because the Reynolds number is large.

However, when the thickness of this region becomes smaller and smaller we have the velocity that is increasing to the outer flow velocity within a smaller and smaller region. So, as a thickness of this region becomes smaller the gradients become larger, there is the cross stream derivatives of the velocity with respect to the y coordinate become larger. The viscose terms are proportional to the 2nd derivative with respect to the y co-ordinate. So, if the length scale is sufficiently small the derivative is sufficiently large in such a way, that you can get a balance between the inertial and the viscose terms.

And, if you recall we calculated what the boundary layer thickness should be delta as you recall is equal to mu x by U power . So, this is basically delta by cross stream thickness goes as the Reynolds number based upon x to minus half. So, if the thickness goes as re power minus half then, the viscose terms are proportional to the 2nd derivative, they are re inverse larger than what you would simplistically expect, if you just scaled the boundary layer thickness, the normal the cross stream co-ordinate by the distance x. And because of that the inertial and the viscose term become a equal magnitude.

So, as I said you have an outer potential flow where you know what the velocity is, that potential flow the relevant length scale for that potential flow is the macro scale, the pipe diameter the thickness of the radius of the sphere in the case of inversed objects. In this particular case it is the length of the flat plate L. Over length scales comparable to L inertia is dominant and the viscose effects are neglected.

However, as for thin region near the surface viscose and inertial effects become of equal magnitude. So, the outer region you have y goes as L which means, that y star is defined as y by L that is the relevant length scale L. Within the boundary layer you have, the y goes as delta which means, that eta which is equal to y by delta. In the two of these solutions have to match in an intermediate region where, y is large compared to delta, but small compared to L that can always be realized because delta is re power minus half smaller than L.

Therefore, one can always find an intermediate region where y is small compared to L, y is large compared to delta the two solutions, the potential flow solution in the limit as y by L goes to 0 has to match with the boundary layer solution in the limit as eta y by delta goes to infinity. For the outer potential flow in the limit as y by L goes to 0, the normal velocity has to go to 0 because the potential flow solutions satisfies the 0 normal velocity boundary condition. Therefore, the normal velocity has to go to 0 the tangential velocity, the tangential velocity ux is equal to some function of x which we will call u of x, for y star going to 0.

As y star goes to 0 as you approach the surface from above, the velocity ux is equal to some function of x and uy has to be equal to 0. From the Bernoulli equation we found that pressure is got to be equal to p naught minus half rho u square. So, because ux is going to u of x which is non 0 because the potential flow is predicting a non 0 tangential velocity uy is going to 0.

So, for this particular case we obtained simplified equations, when the cross stream coordinate, when the length scaled for the cross stream direction is small compared to the length scale in the stream wise direction. And these equations were the usual mass conservation equation, the momentum conservation is. So, that was the momentum conservation equation in the x direction, and the momentum conservation equation in the y direction just reduces to partial p by partial y equal to 0.

Now, the pressure for the outer flow for the potential flow for y star going to 0 the pressure is equal to minus half rho ux square plus uy square. Of course, in the limit as y star goes to 0 u x is equal to capital U and uy is equal to 0. So, the pressure just approaches p naught minus half rho u square therefore, dp by dx minus rho u du by d x. Since, the pressure gradient in the y direction is equal to 0 the pressure in the boundary layer is independent of y, it is independent to the scaled coordinate in the y direction. Therefore, the pressure in the boundary layer is the same everywhere, as the pressure in the potential flow outer potential flow in the limit as y goes to 0.

So, substituting this we got the x momentum conservation equation as ux partial ux by partial x is equal to capital U du by dx. Note that this capital U is now, only a function of x it is the y turning to 0 limit of the mean velocity in the potential flow plus. So, these are the boundary layer equations, which we had solved for different cases the flow (( )) flat plate and for the stagnation point flow. In the last lecture we were asking the question. What is the form of the mean velocity in the boundary layer thickness which will admit a similarity solution for the boundary layer equations?

If, you recall the argument for a similarity solution was as follows. If, I have a mean velocity profile and if I have a boundary layer solution at a given location x, within the flow at a given location x within the flow. The solution for the velocity profile should not depend up on the total length L because the downstream edge of the plate is downstream of this location x. Therefore, the velocity profile should depend only upon x itself as well as the mean flow velocity capital U and the kinematic viscosity mu, and on that basis we had obtained the boundary layer thickness, which was at a specific location x measured from the upstream edge as this one.

So, we had got the solution for delta just based upon dimensional analysis where u was in general a function of x. In the last lecture we had stepped back and asked the question, what are the specific forms of delta and u which will actually admit a boundary layer solution?

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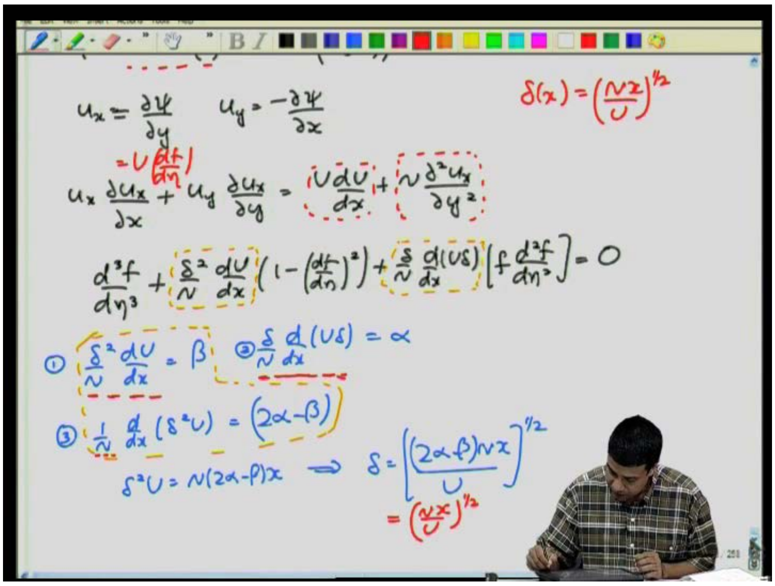

So, we had chosen for our, as a starting point. I postulate that we can write a stream function flow for the flow as U delta times f of eta, where eta is equal to y by delta of x eta is equal to y by delta of x in the stream function is u delta time’s f of eta. Note that the stream function the velocities are related to the derivatives of the stream functions. So, the stream function has to have dimensions of velocity times distance. Which this one does have, if f is dimensionless both U and delta in this case are functions of x itself. So, we chose this form U is a function of x, delta is a function of x and we used this in order to substitute this into the mass and momentum conservation equations.

And we expressed the velocity in terms of the stream function ux is equal to partial psi by partial y and uy is equal to minus partial psi by partial x. The mass conservation equation is identically satisfied, the momentum conservation equation is given by ux partial u x by partial x, if you recall in the last lecture we had substituted ux, uy and its derivatives in terms of U and delta and then, simplified to get differential equation for the function f itself. As you can see ux if I write eta is equal to y by delta of x this ux is just equal to u times df by d eta, where it acts as a similarity variable. So, ux is just equal to u times d f by d eta and I have a 2nd derivative of ux with respect to y here.

So, I will get a 3rd derivative here, with respect to of f with respect to eta. So, we substitute those equations into the momentum conservation equation calculated ux, uy and its derivatives in terms of f and substituted it in here. In order to get an equation a similarity equation for the function f of eta, the equation that we had got was d cubed f by d eta cubed plus into 1 minus df by d eta the whole square plus of U delta into fd square f by d eta square this has to be equal to 0.

So, that was the equation the equation that we got for f, after substituting for the velocity ux, u y in terms of f into the momentum conservation equation as also for substituting for this particular velocity and if you recall, we divided throughout by the pre factor of the highest derivative in order to get a dimensionless equation.

Obviously, this equation admits a similarity solution only if these two functions or both simultaneously equals to constants. If they are not equal to constants, if they do depend upon x then the final equation that I get is not an equation of f in terms of eta alone, its only if these two are constants to I get a final equation in which, this can be solved for getting f in terms of eta thereby getting the stream function in the velocities.

So, therefore, in the last lecture we had said that delta square by nu du by dx is equal to some constant beta, and delta by nu d by dx of u delta is equal to some constant of alpha. So, this was the final solution that we got, both of these functions delta is a function of x capital U is a function of x. However, both of these functions given here, have to be constants and we were looking for solutions that that satisfy this. So, rather than writing in terms of alpha and beta alone, we had shown you in the last lecture that 1 by nu d by dx of delta square u is equal to 2 alpha minus beta rather than using these 2, my original equation I had to solve for these 2, this 3rd equation that I have is a combination of these 2.

So, it is a linear combination of the 1st two equations there, I have equation this 1st equation 1 the 2nd equation 2. This 3rd equation is a linear combination of equations 1 and 2. So, rather than solve 1 and 2 I can as well solve 1 and 3 because 3 is a linear combination of 1 and 2. So, therefore, I will solve these 2 equations in order to get 0 the velocity profiles for which a similarity solution can be obtained 1st thing is 1^{st}, this equation number 3 basically gives you a relationship between delta and u, this equation basically stays at 1 over nu d by dx of delta square U has to be equal to a constant.

Which means, that delta square U is equal to 2 alpha minus beta times x plus C just by integrating once, and as I told you this constant C is can always be set equal to 0 from the requirement that the boundary layered thickness has to go to 0 at x is equal to 0. Which means at the that the location of x is equal to 0 corresponds to the upstream edge of the boundary layer, with that requirement I can always set c equal to 0 without loss of generality.

And this basically gives me u is equal to I am sorry delta is equal to I have a factor of nu here. Note that there is a factor of nu here which basically comes out of this kinematic viscosity. So, therefore, delta is equal to so, this equation basically gives me delta in terms of u and x. Note that u is also a function of x of course, it does not have any additional information compared to what we already had just based upon dimensional analysis, we had written down the delta of x is got to be equal to nu x by u power half.

We had already said that delta of x has to be equal to nu x by u power half of course, we delta is just a boundary layer thickness it is, something that is used to define a dimensionless variable. I can always change delta by a constant factor, the solution expressed in terms of the scaled variable eta will change, but their solution expressed in terms of the original variables will be unchanged. So, I can always define delta with respect to an unknown constant if you, recall in fundamentals of the transport processes 1 we had actually done the calculation by putting in an undetermined constant into the expression for the boundary layer thickness. Then showing that it does not matter in the end.

So, therefore, I can without loss of generality set two alpha minus beta equal to 1, u is positive in this particular case because the flows in the x direction we can always align our x axis with the flow and therefore, 2 alpha minus beta can always be set equal to 1, without loss of generality because delta is known only to within a multiplicative constant scaling that we are using for the y coordinate. So, this basically gives me back the familiar result nu x by u power half it gives me a relation between delta and capital U. I have 1 more relation which is of course, equation 1 which I need to solve to find out what is the form of x that emerges.

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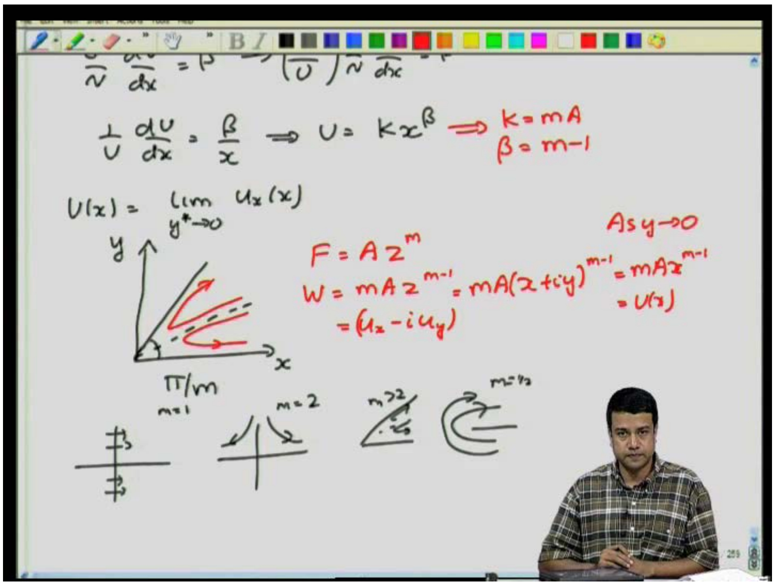

So, the 2nd equation is delta square by nu du by d x is equal to beta, substituting my expression for delta is delta square is nu x by U delta square is nu x by U and I have 1 over nu du by d x is equal to beta. This tells me that 1 over U du by d x is equal to beta beta by x therefore, log U is equal to beta log x or the solution for this is U is equal to beta times x, K times x power beta. So, that is the solution for velocity fields.

The velocity U of x has to be a power log, in x in order to be able to obtain a similarity solution for this particular case. What are those solutions the power log solutions? Recall that U of x is equal to limit for the outer potential flow, limit as y star goes to 0 of ux of x. Is the limit of the outer flow potential velocity in the limit as y, going to 0 is capital U of x and that has a power log dependence upon x.

What are those velocity profiles which satisfy this condition? We have already seen them, when we solved the potential flow equations. If you recall when we did potential flow, the flow within a corner the flow within a corner of angle pi by m, flow within a flow within a corner of angle pi by m has, complex potential F is equal to A times x power m has a complex potential F is equal to A times x power m and the complex velocity is equal to m A times x power m minus 1.

I am sorry I should write these terms of the complex variable of z m A z power m minus 1. So, there is a complex velocity, this is equal to ux minus i uy is equal to u x minus i uy. So, A z for a complex potential basically corresponded to a constant flow, for the for m is equal to 1 if, you recall we just had a constant velocity profile for m is equal to 2 we had a stagnation point flow this for m is greater than 2, you have flow in a corner that looks like this, m greater than 2.

You can also have m less than 1 and the minimum value of m is the half and m is equal to a half correspond to a flow that went like. This we had already seen this in our potential flow solutions. So, the complex velocity is m A z power m minus 1 which x minus i y, I am sorry is equals to ux minus i uy you could also express it as ur minus i u theta times e power minus i theta.

So, what is the velocity capital U for this case? The velocity capital U is equal to the limit as y goes to 0 of ux my complex potential is given by m A times z power m minus 1. I can also write this as m A times x plus i y power m minus 1, because that is just x plus i y in the limit as y goes to 0 in the limit as y goes to 0 this just is equal to m A times x power m minus 1. In the limit as y goes to 0 as y goes to 0.

So, this is capital U of x this is capital U of x. As you can see it is as a power log m A times m x y power m minus 1. So, this corresponds to this velocity provided K is equal to m times A and beta is equal to m minus 1 beta is equal to m minus 1. So, these are the flows the flow in a corner for which the velocity u x in the limit as y goes to 0 has a power log dependence upon the x co-ordinate. For these particular velocity fields we can get a similarity solution, and for that similarity solution the equation that has to be solved.

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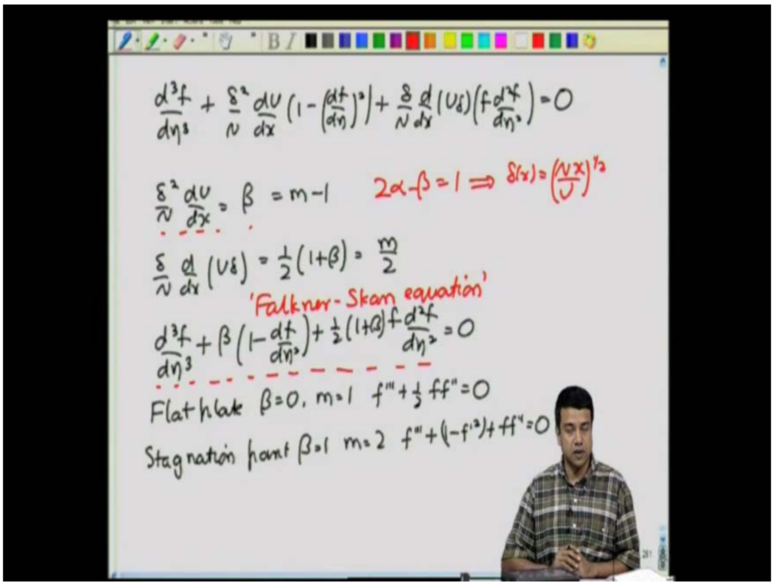

And as you recall is, d cubed f by d eta cubed plus delta square by nu du by dx into 1 minus df by d eta the whole square plus delta by nu d by dx of u delta into f d square f by d eta square is equal to 0. As you know, we have already solved for delta square by nu du by dx is equal to this constant beta, is equal to m minus 1 for the corner flow if we if we correlate the exponent beta, to the exponent m for the complex potential in the flow through a corner then beta is equal to m minus 1. And you can easily verify the delta by nu d by dx of u delta delta by nu d by dx of U delta will turn out to be half because so, I will step back a little bit.

So, this is equal to beta we also know, that we have set two alpha minus beta is equal to 1 because that entered in the definition of the boundary layer thickness, this gave us the delta of x is equal to nu x by u power half. So, if you use this 2 alpha minus beta is equal to 1 and this is equal to beta. You can easily show that this is equal to half of 1 plus beta which is equal to m by 2.

So, with these constants the equation for the boundary layer, can be written as d cubed f by d eta cubed, plus beta 1 minus df by d eta square plus half into 1 plus beta into f d square f by d eta square is equals to 0. So, for different values of m beta is equal to m minus 1 and you substitute the value of beta in here, and you will get a solution for the boundary layer equations.

If you recall, for the flow passed a flat plate for the flow passed a flat plate you had a constant velocity. Therefore, beta was equal to I am sorry m was equal to 1, and beta which was m minus 1 was equal to 0. And for that particular case the equation for flat plate beta is equal to 0 and m is equal to 1 and we got f triple prime plus half ff double prime is equal to 0, beta is equal to 0 and m is equal to 1. For the stagnation point the velocity was increasing linearly with x so, that means, that beta is equal to 1 or m is equal to 2 recall that flow in a corner the complex potential was equal to A times z square.

So, beta is equal to 1 or m is equal to 2 and the equation we got was f triple prime plus 1 minus f prime square plus ff double prime is equal to 0 is beta was equal to 1 or m was equal to 2. So, these are special cases this more general velocity profile which, for which one can get similarity solutions. Of course, for each particular case this equation has to be solved numerically in order to find out what the solution for the velocity profile is. So, for each particular case for each corner angle beta is equal to m minus 1, where m is the exponent in the complex potential, and for that particular velocity profile this equation has to be solved numerically to get the solutions. We would not go through the numerical solutions here, but rather we will just look at the qualitative features of the flow.

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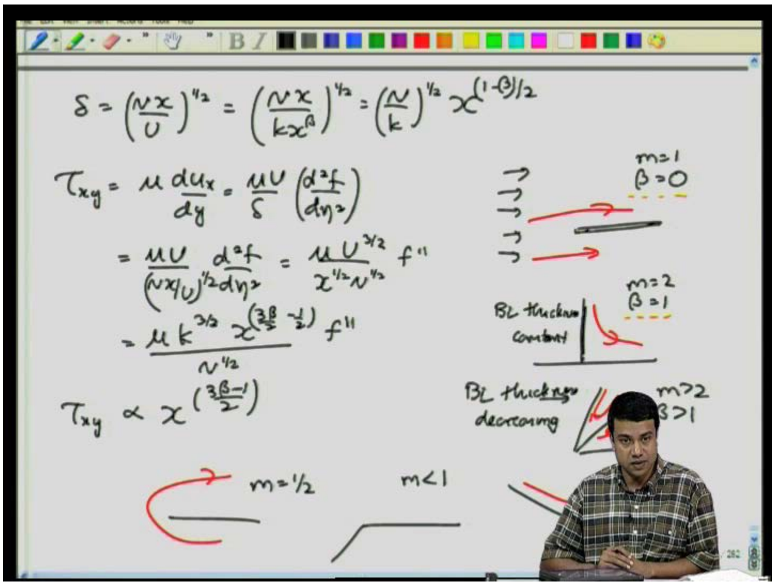

Recall that, delta was equal to nu x by u power half for these solutions I should point out here. That these are called the this is called the Falkner Skan equation. The Falkner Skan Boundary layer equation for velocity profiles in which capital U vary increases as an exponential as some power of x these are Falkner Skan equations for which one can get Boundary layer solutions. For these solutions delta is equal to nu x by u power half which is equal to nu x by k x power beta power half. So, this is equal to nu by k power half into x power 1 minus beta by 2.

So if, you recall for a flat plate flow for the flow passed a flat plate, m is equal to 1 and beta is equal to 0. Because the velocity was not increasing exponent to 0 because, the velocity was invariant with the x coordinate. For the stagnation point flow, we found that m is equal to 2 beta is equal to 1. When you have corner flows with acute angles for which m is greater than 2 and then, you have obtuse angles which is like this for which and then you can also have the flow that goes something like this. Then you can have a flow that goes something like this, m less than 1 then you have this limiting case m is equal to half.

So, those are the different kinds of flow profiles that you can have in a corner. So, we only looked at the these 2 the flat plate as well as the stagnation point flow a special cases for the more general flow profile. The boundary layer thickness goes as x power 1 minus beta by 2 that means, when beta is equal to 1 the boundary layer thickness is a constant. That is what we got for a stagnation point flow for beta is equal to 1, or m equal to 2 the boundary layer thickness is just a constant for beta is equal to 0 or m is equal to 1 you have a boundary layer which is increasing as you go downstream, the thickness goes as x power plus half.

Now, in between these two you will find that the boundary layer thickness goes as 1 minus beta by 2. So, this m is equal to 2 means the beta is greater than 1 here. So, in this case where beta is greater than 1 where m is equal. In this case when beta is greater than 1, when beta is greater than 1 you are having a boundary layer which is decreasing with downstream distance, in this case you have BL thickness decreasing. This case BL thickness is a constant, and when m becomes a beta becomes less than 0 or m becomes less than 1, you have a boundary layer thickness which is increasing with downstream decreasing is increasing with downstream distance.

So, boundary layer thickness depends upon this parameter beta here, the boundary layer thickness depends upon the value of beta that you have. Of course, if the Boundary layer thickness increases or decreases then of course, the shear stress goes as 1 over the boundary layer thickness the shear stress taw xy is equal to mu dux by dy is equal to mu u by delta square, d square f by d eta square.

So, there is a boundary layer thickness this goes as mu u by mu delta goes as nu x by U delta square goes as nu by u times d square f by d eta square. You recall that ux was equal to u times df by d eta which means, that partial ux by partial y is equal to u by delta d square f by d eta square. We should have delta here, is half is equal to mu u power 3 by 2 by x power half mu power half f double prime.

So, this is the expression for the shear stress capital U goes as k times x power beta, and I have x power half in the denominator. So, this becomes mu k power 3 by 2 x power 3 beta by 2 minus 1 by 2 by mu power half f double prime. That means, the shear stress goes downstream as x power 3 beta minus 1 by 2. So, therefore, the shear stress increases downstream for 3 beta greater than 1 beta greater than one-third whereas, it decreases downstream for beta less than eta.

So, m is between 1 and 2 that means, that beta is between 0 and 1, and m less than one beta is less than 0 and m is equal to half means that beta is equal to minus 1 by 2. So, beta which is m minus 1 goes all the way from minus a half all the way to large values as the value becomes larger and larger you get the flow in a corner of smaller and smaller angle, because the angle is equal to pi by m which will be pi by beta plus 1.

And depending upon the angle you get either a flow that is accelerating downstream because x power beta where beta is positive. That means, that the velocity increases downstream you have an accelerating flow. If beta is negative the velocity decreases downstream that means, it is a decelerating flow. For a constant flat plate flow the boundary layer thickness increases as x power half, for a stagnation point flow it is a constant, if the angle is less than 90 degrees the boundary layer thickness will decrease as you go downstream which is greater than 90 degrees, the boundary layer thickness will increase as you go downstream. And of course, in between you have stagnation point flow for which it is a constant.

If beta is greater than a 3rd if beta is greater than a 3rd than the than the shear stress increases downstream where as if it is less than a 3rd the shear stress decreases downstream. So, this is the class of solutions for which you can get boundary layer solutions, and I went through this in some details in order to show that the boundary layer equation has a very specific form for all of these. This form can be solved exactly only for the case, where the potential flow corresponds to the flow in a corner. However, these solutions the Falkner Skan solutions for the boundary layer equations are important because, they give us some intuition into to the flow in other cases, it give us inside into the flow in other cases and let us go through that in little bit of detail.

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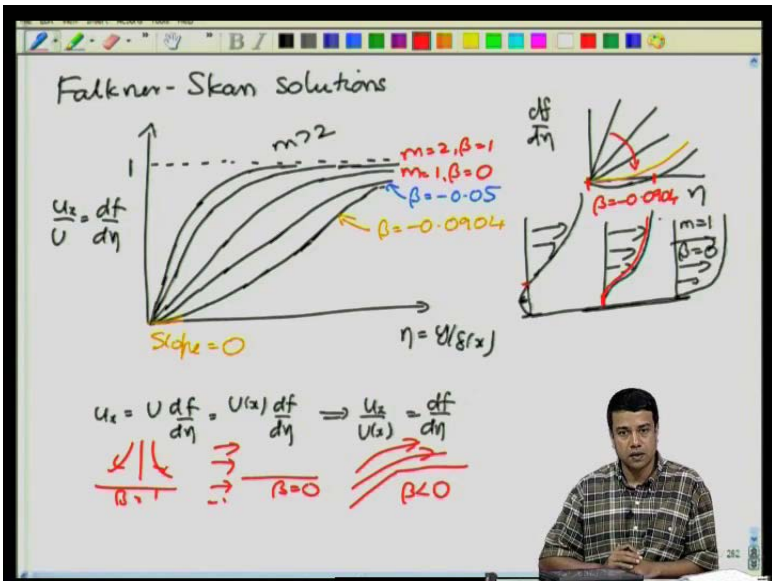

So, you get these numerical solutions for the Falkner Skan equations, similar to what we had done for the flow passed a flat plate in the stagnation point flow. I can plot, eta is equal to y by delta of x on the x axis that is m this particular case I have x and I have y. I could have a corner flow, but I still look at, the distance from the y axis scale that by delta to get eta. And I know that ux is equal to U times df by d eta, where U is in general a function of x, capital U is the velocity of the outer potential flow in the limit as y goes to 0.

Which means at the scale velocity ux by u of x is equal to df by d eta. So, I can plot ux by capital U is equal to df by d eta. Far away you have to attain the free stream velocity capital U that is in the limits as eta goes to infinity, the velocity has to go back to the potential flow velocity eta goes to infinity while y star goes to 0 simultaneously. If ux is equal to capital U that means, the scaled velocity is just equal to 1.

So, far away it has to reach 1, and we look at various solutions here all of them depend upon the value of these df by of the value of the parameter beta that we had which was equal to m minus 1. For the blatious flat plate boundary layer where m was equal to 1 and beta was equal to 0, we got the solution here, which looks something like this. Note that the shear stress is proportional to d square f by d eta square that means, that the we have to numerically calculate the value of d square f by d eta square as in order, to calculate what the shear stress is. So, this let us call it as m is equal to 1 I am sorry beta is equal to 1 no m is equal to 1 and beta is equal to 0 this is the flat plate blatious solution for the boundary layer.

If plot this for the this stagnation point flow you find that it goes something like this, it of course, approach as one, but it approaches it faster, the reason is that the boundary layer thickness is a constant therefore, because the flow is accelerating downstream. So, this becomes for m is equal to 2 and beta is equal to 1. The stagnation point flow and as you increase even further you will get something like this, this is for m greater than 2 what happens if m becomes less than 1 that corresponds to something that is that is that does not have a constant velocity far away. If you recall, m less than 1 corresponds to flow that looks something like this, m less than 1 or beta less than 0.

If I took a solution for m is equal to qualitatively if I take for beta is equal to minus 0.05 I will get something like this, it approaches one much slower I am exaggerated here, but it approaches one much slower. At a value of m is equal to minus 0.0904 at the value of beta is equal to minus 0.0904 something important happens, at the value of beta is equal to minus 0.0904 the solution is such that. The solution is such that the slope at the origin the slope at the origin note that this is df by d eta the slope of this will be the 2nd derivative the slope at the origin is equal to 0.

Slope at the origin is equal to 0, the slope is proportional to the shear stress because the sphere stress is equal to dux by dy which is one over delta times dux by d eta. So, that shear stress is equal to 0 at the surface the slope is equal to 0. So, what this implies is that as I, if I expand it out if I expand out the phenomenon here. If I just look closely at the original alone if I just look closely at original alone.

For m is equal to 2 I get a slope like this, m is equal to 1 I get a slope like this, m is equal to 0.05 the slope comes here. At m is equal to minus 0.0904 the slope itself is 0 at the origin for the numerical solution form is equal to minus 0.0904, the slope itself is equal to 0 eta versus d f by d eta. So, the slope is decreasing as beta decreases, at comes down to 0, once its comedown to 0 or if you decrease beta any further beyond minus 0.0904 the only solution is that the velocity has to go negative.

Pass the slope goes to 0 if you decrease it further the velocity has to go negative. What does a negative velocity mean? Note that positive velocity that means, that you have flow in the stream wise direction along the plate. Negative velocity implies that there is flow that is going opposite to the main flow within a small region, there is flow that is going opposite to the main flow within a small region.

So, let us plot that lets plot the velocity in this. So, this was the velocity for, m is equal to 1 or beta is equal to 0. At beta is equal to minus 0.0904 the slope is equal to 0 at the origin, at that value the slope is equal to 0 at the origin you can see that the slope is equal to 0 at the origin. You should decrease beta below that what has to happen is that the velocity profile has to look something like this. That means, there is a region close to the surface where the velocity is actually opposite to the mean flow. And then you have the 2nd point at which you have 0 velocity.

So, that is what the numerical solution for the velocity field for these Falkner Skan solutions tells you beta is equal to 1 is the blatious flow passed a I am sorry beta is equal 0 is the blatious flow passed a boundary layer flat plate in our Falkner scan solutions pi by 2 correspond to beta is equal to 1 as you, opened up the corner you ultimately reached a flat surface for which beta was equal to 0 corresponds to m is equal to 180 degrees. If beta going below 0 corresponds to the angle going below 180 degrees. So, beta.

So, if you recall this was beta is equal to 1 beta is equal to 0 was just this 1 and beta becoming less than 0 basically correspond to a flow that was something like this, m less than 1 or beta less than 0. So, for beta less than 0 you find that there is a velocity profile which has a backward circulation very close to the surface. So, beta less than 0 when beta reaches minus 0.0904 the shear stress becomes 0, as decreases below that you have a region where there is a backward circulation very close to the surface.

So, what that means, is that there is a separation of the boundary layer from the surface, there is 2 points at which the velocity is equal to 0, this at the surface itself there is an intermediate point within the flow, at which the velocity is equal to 0 and in between the velocity is going backwards. That is when you have a boundary layer in which the velocity is decelerated. Note that for the flat plate the velocity is the constant, for the stagnation point flow the velocity is accelerating downstream. For the plate it is a constant, when you make the angle greater than 180 degrees the velocity actually decreases because beta is negative u goes as x power beta which goes as negative power of the downstream distance. So, this is a decelerating velocity profiles.

So, this are specific solutions for the boundary layer equations for the particular case, where the velocity had a power log dependence upon the x co-ordinate, how can this be use to understand flow passed a general object. So, let us look at the simplest case I will take in this particular case the flow passed a cylinder.

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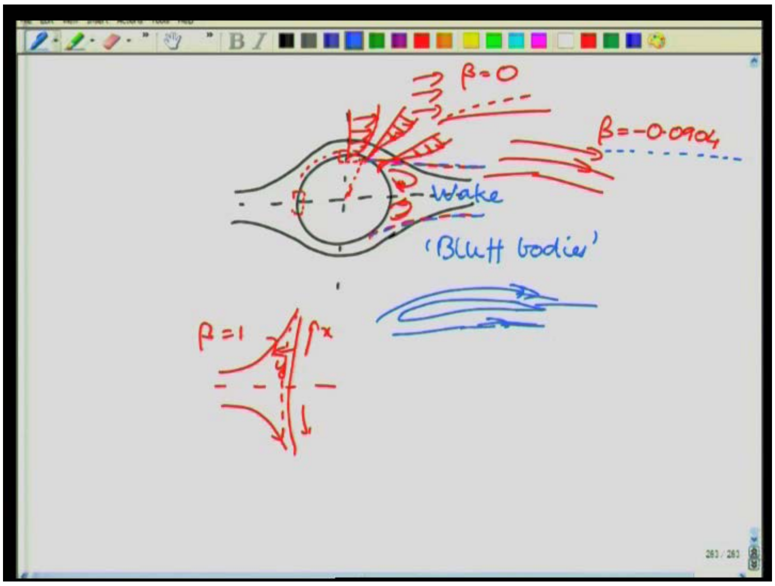

Simplest is to consider a two dimension cylinder my potential flow solution predicts a symmetric velocity profile, which we had obtained using complex variables previously the potential flows predicting something like this, the velocity profile that is symmetric about, the this plain of symmetry. What about, boundary layer theory? I can calculate the boundary layer thickness locally based upon the local, mean velocity and whether it is locally accelerating or decelerating.

As I showed you the mean velocity here, corresponds to effectively a stagnation point flow. A stagnation point flow for which beta is equal to 1 or m is equal to 2 for which the velocity increases proportional to x as you go downstream, in this particular case this is the x co-ordinate this is the x coordinate and this cross stream direction is y. So, long as the boundary layer thickness is small compare to the radius of curvature this corresponds to a stagnation point flow.

So, this the velocity increases proportional to x, as I go downstream that acceleration decreases of course, at the upstream edge it has to accelerate to go around the object. As I go around the acceleration decreases until I come to this point, where this basically corresponds to the flow passed a flat plate, because the flow is tangential to the surface the velocity is nearly a constant. So, this corresponds to a flow passed a flat plate for that you have a boundary layer, which is increasing proportional to x power half in this case the boundary layer is constant.

So, this basically corresponds to beta is equal to 1 this corresponds to beta is equal to 0 flat plate. If I go a little bit downstream, if I go little bit downstream of the top and bottom of the cylinder. I will come to a flow which basically looks like this 1 they will come a region at which beta is equal to minus 0.0904 they will come a region that is slightly ahead of the upper and lower surfaces at which beta becomes negative, at the upstream edge stagnation point it is 1 at the top and bottom it is 0, if you go little bit downstream it will become negative.

And once it becomes negative the shear stress is going to go to 0 and you are going to have. So, here you have a velocity profile which is basically a blatious boundary layer profile. So, if you go little bit downstream once beta becomes minus 0.0904 the shear stress at the surface goes to 0. If you go little further, you will have a region where there is a re-circulation at the back where the boundary layer has come of the surface, as the region where the flows is in the opposite direction very close to the surface, and then there is another point at which the velocity is equal to 0.

So, there are 2 points at which velocity is equal to 0 one is at the surface itself and the other is a little distance away in the decelerating portion behind this sphere. And if I plot the locust of all points at which the velocity is equal to 0, I will get something that goes like this, and within this region because the flow is going opposite to the mean flow I am going to get a velocity profile that goes something like this, and I get the velocity that goes something like this, which is re-circulating in this region.

So, at this point where the shear stress is equal to 0 that is where you see the start, of a re-circulating region behind the sphere. And because this boundary layer comes of the surface there is another location at which the velocity is equal to 0, and within the region within the region that is traced out by these two, where the velocity is equal to 0 you have the flow that is going in the opposite direction, this region is what is called the wake, where the flow is going in the opposite direction.

The reason is because behind the sphere you have decelerating potential flow velocity in a decelerating potential flow velocity if the value of beta goes below this value of minus 0.0904, there is are circulation at the back and the boundary layer separates, from the surface we have another surface on which the mean velocity is equal to 0, which goes all the way into the fluid and between these two regions you have a wake region where there is a re-circulating re-circulation of the flow, the flow is going in the opposite direction.

That is why, the potential flow solutions are not applicable for these kinds of bodies, where you do have a separation in the decelerating flow, the boundary layer solution is telling us that, when you have deceleration at the outer velocity the boundary layer separates from the surface potential flow solution is no longer valid there. That is the wake region where the velocity is there is the recirculation in the velocity and the velocity close to the surface is opposite to the direction of the mean flow.

And the reason that you have non 0 drag on the surface is that the pressure in the wake is actually very small, pressure in the wake is much smaller than the potential flow pressure upstream. If the flow or potential flow both upstream and downstream, the pressure would have been equal, but because there is awake where the velocity where there is are circulating region pressure in that region is much smaller and because of that you have a drag force is called the form drag. This boundary layer separation always occurs in what are called blutt bodies, bodies which have a non 0 cross section ahead of the flow.

If, you want to prevent boundary layer separation you have to use what are called slendor bodies at air craft wing. For example, a slendor bodies they are optimized in such a way that there is no boundary layer separation or there is boundary layer separation very far downstream very far downstream. The flow around it there is no, the flow around it such that there is no deceleration anywhere along the flow a deceleration is sufficiently small that beta never reaches this value and if beta never reaches this value. You will never have a separation at the boundary layer the shear stress on the surface will always be non 0 you would not have separation at the boundary layer.

So, this is the additional piece of this is why boundary layer theory is important, even though potential flow predicted that there is a solution everywhere. Boundary layer theory tells us that at certain locations, if the velocity is decelerating the boundary layer is going to separate from the surface and your potential flow equations are no longer valid in that separated region behind in the wake, in that in that wake you have to use you have to analyze it separately because the potential flow solution is no longer valid. And because the pressure is very small in the wake basically, the drag force that you get is due to the pressure exerted by the potential flow on the upstream side.

So, this completes our discussion of boundary layer theory. We managed to get solutions for a very specific case for a particular case, which corresponded to potential flows which are flows in a corner only for this particular case we are able to get analytical solutions for the boundary layer equations. But, the intuition that we developed is more general thickness of the boundary layer dependence on the kinematic viscosity and the mean velocity profile.

And what it tells us about boundary layers attachment to surfaces and the separation in a decelerating flow these are all far more general concepts which are important because they tell us about, the limitations of potential flow. So, this completes our discussion of boundary layer theory and we will in next class, we look at some additional features of flows in which there is vorticity. So, we will briefly discuss vorticity dynamics before we go on to looking at turbulent flows. So, we will see you on the next lecture.