Fundamentals of Transport Processes II
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Lecture - 34
Falkner-Boundary Layer Solutions - Part I

So, this is lecture number 34 of our course on fundamentals of transport processes. You are looking at fluid flows. The current topic that we are discussing is boundary layer theory in the limit of high Reynolds number. If you recall, we had first obtained the conservation equations, the Navier-Stokes mass momentum equations, these equations, the momentum equation reflects the balance between inertial forces and viscose forces. There are the convective terms in the transport equation and the viscose terms, and the dimensionless number, which gives you the ratio of these two is the Reynolds number.

The Reynolds number is small, the viscose terms are dominant and the inertial terms can be neglected. And we looked at various ways to solve problems in that limit; basically the equation reduces to something similar to a diffusion equation except there is a pressure gradient in the momentum conservation equation as well. And the pressure gradient basically ensures that the incompressibility condition can be enforced.

Then we moved on to convection dominated or the inertial dominated flows, we looked at a particular case of potential flows that are in viscid and irrotational. In viscid implies that the viscosity is 0 and irrotational means, that the vorticity the curl of the velocity vector is equal to 0 at all points. And we looked at how to solve those equations, when you neglect viscosity there is no way to transmit stress shear stress in the fluid at bounding surfaces. Because, the transport of momentum at bounding surfaces cannot take place due to convection. Convection always takes place along, the flow direction and there is no flow perpendicular to a surface because of the no penetration condition. The velocity of the fluid has to be the velocity of the surface in that direction in order to ensure that there is no penetration.

So, because of that one cannot satisfy the tangential velocity or tangential stress boundary condition. So, there is the flow passed a surface if that flow has to slow down to 0 at the surface itself, in order to satisfy the 0 tangential velocity boundary condition. Then the minimum requirement is that there has to be momentum diffusion from the surface perpendicular to the surface, that diffusion takes place only due to viscosity. So, when we neglected viscosity there was no way to satisfy the tangential velocity boundary conditions at the surface.

So, when we looked at boundary layer theory, the physical problem the real problem does have a 0 velocity condition at the surface. When we solved the simplified equations we found that the Reynolds number was large and therefore, the inertial terms are dominant compared or much larger than the viscose terms. So, we threw away the viscose terms altogether and just solved the problem with the inertial term alone. And when we did that we found that, we are not able to satisfy the tangential velocity boundary condition.

However, the real system the velocity does actually come to 0 at the surface. In order to resolve this inconsistency we looked at Boundary layer theory. The rational for looking at Boundary layer theory is similar to that for mass and heat transfer, even if the Reynolds number based upon the characteristic length scale of the of the flow or of the object in the case of in case of internal flows it will be. For example the channel with the pipe diameter in the case of flow around object it will be the characteristic length of the object the sphere diameter.

For example, even if the Reynolds number based upon that length scale and the macroscopic flow is large, as one comes close to the surface there is going to be another length scale called the Boundary layer thickness. Such that when the Reynolds number is defined based upon this boundary layer thickness, you will find that there is a balance between inertia and viscosity. The viscose diffusion in this case will be the diffusion only in the cross stream direction perpendicular to the flow at the surface, because the gradient is large in that direction therefore, the viscose contribution will also be large in that direction.

Therefore if I scale the cross stream direction by that small length scale, and I scale this stream wise direction by the characteristic flow length. Then I will still get a balance between the inertial and the viscose terms if, this boundary layer thickness is related in a specific way to the Reynolds number. Basically you require that in the limit as the Reynolds number becomes large the boundary layer thickness becomes smaller and smaller in such a way that, there is a balance between the inertial and the viscose terms in the equation even in the limit of high Reynolds number. So, that is the logic behind boundary layer theory.

Even though the Reynolds number based upon the macroscopic scale is large the Reynolds number then there is a smaller scale, which is set up by the flow itself, by the requirement that you have to have momentum diffusion at the surface, otherwise you cannot satisfy the tangential velocity boundary condition. That length scale is such that the viscose terms in this thin layer near the surface continues to be of the same magnitude as the inertial terms, even if the Reynolds number becomes large.

The length scale itself becomes smaller and smaller as the Reynolds number becomes larger and larger. So, that you have a thinner region near the surface where the viscose terms are important. However, the ratio of inertia and viscosity within this thin region is still finite in the limit as the Reynolds number goes to infinity. And if this ratio is preserved that means, that viscose effects have to be taken into account within this boundary layer. Therefore, one can satisfy the tangential velocity boundary condition because I have now included the 2nd derivatives in the cross stream direction. So, that is the logic. So, let us look at what we did in the previous two lectures.

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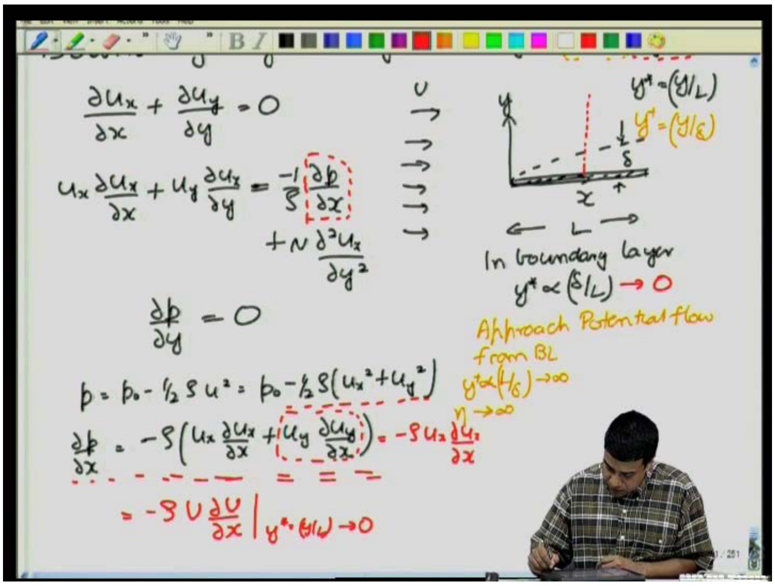

We looked at boundary layer theory, the idea was that simplest to explain this in the context of a flat plate flow. The idea was that you have a constant velocity coming in far away from this flat plate, I put a coordinate system here x is the stream wise y is the cross stream direction. I have 0 velocity conditions at the surface of this plate at the surface of this plate that is at y is equal to 0 u x is equal to 0 and u y is equal to 0.

The outer velocity the potential flow velocity basically the velocity is a constant everywhere, if I neglect viscose effects the solution that I will get is that the velocity is just a constant everywhere. That potential flow velocity does satisfy the normal velocity boundary condition, it does satisfy the normal velocity boundary condition. That uy is equal to 0 it does not does not satisfy the tangential velocity boundary condition. So, we need to take into the account the viscose effects within a thin boundary layer of thickness, we called it delta in the last two lectures at the surface itself, when you take into account viscose effects within a thin layer of thickness delta.

So, whereas, for the potential flow we had scaled both x and y by the macroscopic length in this case that length was the total length of the plate. In the potential flow we had scaled the equations by this total length of the plate, both the x and the y co-ordinate with the boundary layer theory, we will scale the length in the x direction by the length of the plate. Whereas, the y coordinate will be scaled by the small length scale delta, implication of that is that u y the velocity in the y direction is much smaller than the velocity in the x direction.

So, doing this scaling we managed to get boundary layer equations which contained the dominant effects in the limit as delta goes to 0 and r e goes to infinity. So, the equations that we got were the mass conservation equation that is, the x momentum conservation equation, we have neglected the diffusion in the stream wise direction because the Reynolds number is high and the thickness delta is small.

So, you would expect the diffusion in the cross stream direction to be much larger than the diffusion in the stream wise direction. In the cross stream, momentum conservation equation it just reduces to partial p by partial y is equal to 0 it just reduces to pressure gradient perpendicular to the direction of the flow is equal to 0, this is based upon scaling with respect to the boundary layer coordinate. So, this is valid only within a thin region near the surface of the plate.

Partial p by partial y is equal to 0 implies that the pressure is independent of the y coordinate. So, at any x location the pressure is a constant as you go in the y direction. The outer potential flow is specified in this case it is a constant velocity, the outer potential flow is specified in this case it is a constant velocity. What this equation, the y momentum equation partial p by partial y is telling you is that the pressure at any stream wise location at any x location within the boundary layer is identical to the pressure at that same location in the outer potential flow.

Therefore, there is no variation in the pressure perpendicular to the in the cross stream direction. For the outer potential flow we know what the pressure should be, if it is a potential flow then the pressure is given by minus half rho u square. Note that these ux and uy are the potential flow velocities, these ux and uy are the potential flow velocities. Therefore, at any given location x what this is saying that the pressure is given by p naught minus half rho into ux plus uy square everywhere, within the boundary layer it is independent of y it depends only up on the stream wise x coordinate.

Therefore, from that you can get what is the value of the pressure gradient in the x direction, you can get what is the value of the pressure gradient in the x direction. For the pressure gradient in the x direction is given by partial p by partial x is equal to minus rho into ux partial ux by partial x plus uy. So that, is what the pressure is given by.

Note, for the outer potential flow itself, the relevant length scale is capital L for the outer potential flow itself, the relevant length scale is capital L. So, if I scale if I define a coordinate for the potential flow y star is equal to y by L, y star within the boundary layer within the boundary layer the value of star in boundary layer, y star will be proportional to delta by L within boundary layer, y star is proportional to delta by L. For the outer potential flow relevant length scale is capital L itself. That means, within the boundary layer y star is proportional to delta by L.

Therefore, within the boundary layer y star since delta is small compared to L, we saw in the last class that is proportional to re power minus half this is approaching 0, as you approach the boundary layer region from the potential flow above as you approach the boundary layer region from the potential flow above, the y coordinate for the potential flow is going to 0. Within the boundary layer the relevant length scale, I will call it as y dagger is equal to y by delta.

So, therefore, as I approach the potential flow from the boundary layer region as I approach the boundary layer from the potential flow region I am sorry as I approach the potential flow. Rewrite that as I approach the potential flow from the boundary layer as the I approach the potential flow from the boundary layer. I will find that y dagger is equal to is proportional to L by delta because the length scale for the boundary layer is L therefore, y dagger as I approach the boundary layer from the potential region this goes as L by delta goes to infinity.

So, the requirement that the pressure in the outer potential flow, has to approach that I am sorry the requirement of the pressure in the boundary layer, has to approach the pressure in the outer potential flow in the limit as, y star goes to 0. That is I am, approaching the boundary layer from the potential flow y star goes to 0 because in the boundary layer y is delta therefore, y star is equal to y by L.

As I approach the potential flow from below y plus scaled by the boundary layer thickness goes to infinity, in that limit as I approach the potential flow from above I am sorry as I approach the boundary layer from above y star goes to 0. As I approach the potential flow from below from the boundary layer, the scaled coordinate in terms of the boundary layer co-ordinate if you recall it I had called this as eta y by delta was called as eta in the previous lectures. As eta goes to infinity, the pressure should both be the same, that is the matching condition that is required for this entire formulation to be consistent.

The quantities have to be identical as I approach, the boundary layer from above from the potential flow region, and as I approach the potential flow from below I have to get the same result. I have a potential flow solution, which is valid outside for y star going as y proportional to the length scale L. I have a boundary layer solution below when the y is proportional to delta. As y becomes large compared to delta the boundary layer solution has to match with the potential flow solution as y becomes small compared to L. So, in this intermediate region, where y is small compare to L, but simultaneously large compared to delta the two solutions have to match.

So, this is the pressure in the potential flow as y becomes small compared to L, I have a 0 normal velocity condition for the potential flow as y becomes small compared to L. As I am approaching the surface the outer potential flow solution has a 0 normal velocity condition, as I am approaching the surface from above. Therefore, as I approach the surface from above this term goes to 0 because I have a 0 normal velocity condition.

Therefore, the potential flow pressure, as I approach the boundary layer from above as y becomes small compared to L the pressure is equal to minus rho ux partial ux by partial

x. If you recall in the last lecture in this particular case capital u was just a constant, this is the tangential velocity. Note that for the outer potential flow the tangential velocity does not have to go to 0. So this terms still remains the normal velocity has to go to 0. So, uy has to goes to 0. So, that 2 nd term goes to 0 whereas, ux does not have to go to 0.

So, this matching condition basically gives me what is the pressure gradient in the boundary layer because, I know that the pressure does not vary in the cross stream direction. I know what the pressure gradient is for the outer potential flow in the limit as I approach the boundary layer, as y becomes small compared to capital L that has to be the pressure everywhere, in the boundary layer at a given x position.

So, this basically tells me that this is equal to minus rho capital U times partial u by partial x. Where this capital U is the velocity in the potential flow, in the limit as y goes to as y goes to 0 as in the limit as y becomes small compared to L. So, remember So, we are matching in this intermediate region where y is small compared to L and y is still large compared to delta, y small compared to L. Means, that this velocity in the limit as y star is equal to y by L goes to 0.

Capital U is the stream wise velocity predicted by the potential flow at the surface. The stream wise velocity is non 0 because potential flow does admit tangential velocity at the surface. So, that stream wise velocity that I get from the potential flow solution, in the limit as y goes to 0 is the velocity that I will use here. Because that is the velocity that has to be approached by the boundary layer solution in the limit as y becomes large compared to delta. So, this is the outer potential flow solution in the limit as y is small compared to L or as y star goes to 0 where y is scaled by the length scale by the outer flow.

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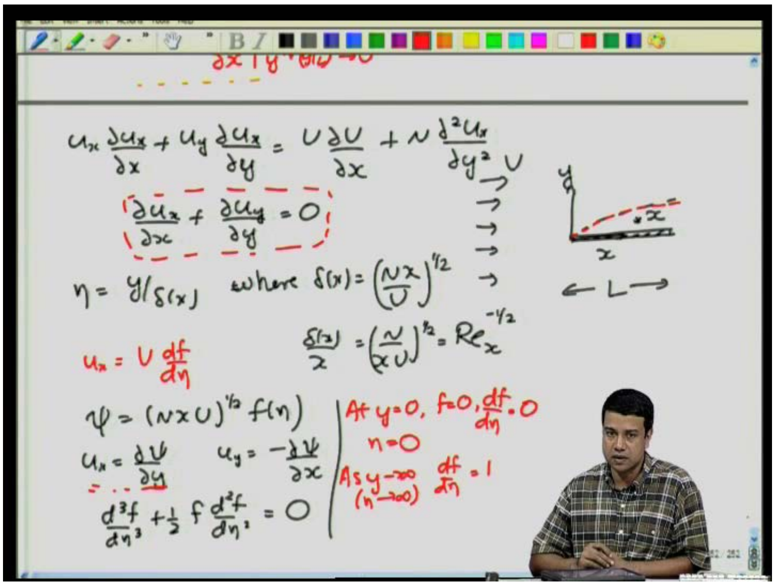

So, putting that together with this I get my outer momentum equation as ux partial ux by partial x plus uy partial ux by partial y is equal to minus 1 over rho partial p by partial x, I substitute this for partial p by partial x and I get u times partial u by partial x. Note that the stream wise velocity in the potential flow in the limit as y goes to 0 can in general be a function of x. The stream wise velocity can in general be a function of x plus mu times partial square ux by partial y square this, coupled with the mass conservation equation partial ux by partial x.

So, that is the set of boundary layer equations which have to be solved. We solved this for two specific cases one was the flow passed the flat plate. So, the 1st case that we solved was the flow passed a flat plate we added we used this additional piece of logic there, if you recall in addition to the boundary layer scaling and the requirement that the viscose and the inertial terms have to be of the same magnitude in the limit as the Reynolds number goes to infinity.

We used this additional piece of logic that at a given location x at a given location x within the boundary layer, the velocity cannot depend upon the total length of the plate L. That is because in our conservation equations we have stream wise convection along the flow cross stream diffusion. There is no diffusion in the stream wise direction convection, in the limit of high Reynolds number is sweeping the velocity field downstream, diffusion is taking place only in the cross stream direction.

Therefore, there is no mechanism for information or for influence of downstream conditions on the velocity at the location x. The velocity at the given location x should not depend upon what the total length of the plate is because the downstream edge of the plate is downstream of the location x, should not depend upon the total length of the plate you should get the same velocity whether the plate is of length 2 L or something else. So, long as it is larger than x.

So, because of that the velocity field at a given location x has to depend only upon x, on that basis we had simplified the equations by defining the similarity variable. In this particular case it came just simply out of dimensional analysis, we defined y the similarity variable eta is equal to y by delta of x where, delta of x is equal to mu x by u power half. I could scale y by x itself to get a dimensionless variable I could scale y by x itself to get a dimensionless variable, but we know physically that is not correct because the length scale in the y direction is much smaller than the length scale in the x direction. Based upon dimensional analysis and the fact that the only dimensional quantities are the kinematic viscosity mu the mean flow velocity u and length scale x. This is the only other possible scaling just based up on dimensional analysis.

So, we would use this scaling not to define a length scale delta in the cross stream direction. Note that this delta if I write down this in terms of the ratio delta by x delta of x by x is equal to mu by xu power half, which is the Reynolds number based up on x to the minus half. So, the implicit assumption here, is that the Reynolds number based up on the value of x from the upstream edge of the plate is large compared to 1. Obviously, is not going to be valid as you come closer and closer with the leading edge because the edge smaller and smaller there is going to be some points at which the Reynolds number becomes small.

But, as long as you are sufficiently far from the leading edge, this delta is always much smaller than the location x itself and we had defined we had solved this equation by defining the stream function. A stream function in this case in this two dimensional flow case we can always define a stream function, which will ensure that the mass conservation equation is identically satisfied. The stress scale stream function was defined as the psi is equal to mu xu power half times f of eta, where f is some function only of the dimensional, dimensionless variable eta and mu xu power half is the scaling factor that is used.

And based up on this we got the conservation equation basically you express the velocities in terms of the stream function that is ux is equal to partial psi by partial y and uy is equal to minus partial psi by partial x. Express these velocities in terms of the stream function and substitute into the momentum conservation equation and we got one equation for f, which was d cubed f by d eta cubed plus half f d square f by d eta square is equal to 0, these had to be solved subject to boundary conditions, at the surface of the plate we found that ux and uy have to both be equal to 0.

Therefore, the boundary conditions were at the surface of the plate y is equal to 0, f is equal to 0 and df by d eta is equal to 0. And as y goes to infinity this also corresponds to eta is equal to 0 as y goes to infinity which corresponds to eta going to infinity, we found that df by d eta is equal to 1 because we just solved for the x velocity ux is equal to u times df by d eta because we take the stream function and then substitute that into the expression for the velocity.

What you find is that ux is equal to capital u times df by eta and these were solved subject to boundary conditions and it discussed various features of the boundary layer solution, for you the boundary layer thickness goes as x power plus half. So, it increases as you go down stream the boundary layer thickness increases that means, that the gradient decreases the gradient decreases as x power minus half. That means, that the shear stress also decreases as x power minus half and we got correlations for how the shear stress and the net force vary the skin friction co-efficient and the drag co-efficient for this particular case.

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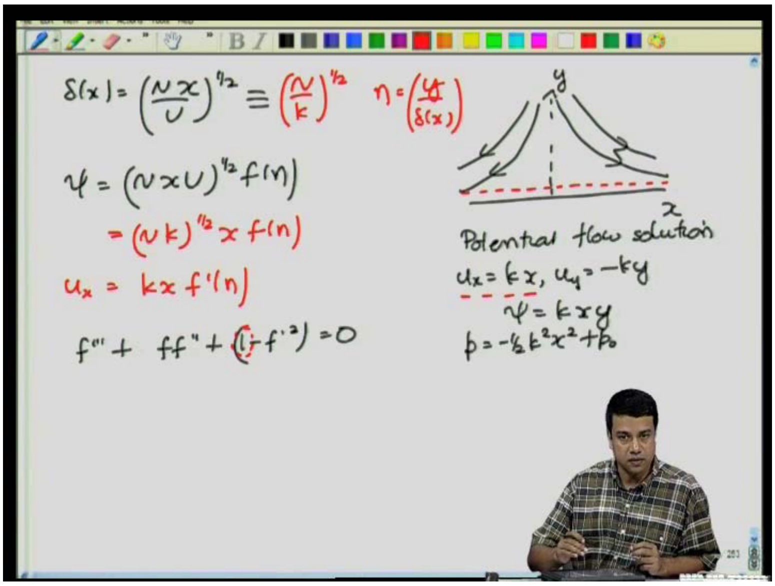

Next, we had also discussed the case of stagnation point flow, for this particular flow the outer solution potential flow solution, ux is equal to k times x and uy those were the potential flow solution for the stream function in the two components of the velocity. And we had proceeded in a similar manner in the 1st problem, we got the boundary layer thickness as delta of x is equal to mu x by u power half in this particular case the outer flow velocity actually increases proportional to x, the outer flow velocity increases proportional to x. So, if I substitute the potential flow velocity capital U is equal to kx then, I get this is equal to mu by k power half and based upon this predicts that the boundary layer thickness is nearly a constant, the boundary layer thickness is nearly a constant.

Similarly the scaled equation for the stream function psi is equal to mu xu power half f of eta, where this eta is defined as y by delta of x. In this particular case delta is independent of x. So, I substitute the expression for y here, for capital U here as k times x and this becomes equal to mu k power half xf of eta. So, that is the equation for the stream function, and my expression for ux becomes equals to once again U times kx times f prime of eta.

So, this is the substitution that we use for the velocity and the stream function in this case and we got the equation for the for the stream function as f triple prime plus f f double prime plus 1 minus f prime square is equal to 0. Let me just check if that expression is correct, yeah this is there is no half here. And then we had solved this in this particular case we found out that the boundary layer thickness is a constant.

Note that the pressure in this case the pressure that we got for this case for the outer potential flow was p is equal to minus half k square x square plus p naught. This is the pressure of the potential flow in the limit as y goes to 0 as always. And because of that I get an additional pressure term in this equation. This term here, if you recall this one here reflects the scaled pressure.

So, once again we had solved this equation and got a similarity solution once again I told you I can do it only numerically, you cannot do it analytically. But the basic features of the flow are easily accessible analytically because the pressure is increasing the velocity is increasing downstream the potential flow velocity is increasing proportional to x.

So, this is an accelerating flow, and for the accelerating flow you have diffusion from the surface convective, velocity increasing downstream and the balance between these two results in a constant boundary layer thickness. In the flat weight case the velocity field was constant, and there was diffusion from the surface. So, once again we had solved this particular obtained the solution for this particular case and I looked at the displacement thickness and the so, on.

Now, let us ask the opposite question what are the kinds of velocity profiles that we can obtain a boundary layer solution? For what are the forms of u of x. Note that capital U of x is the potential flow velocity in the tangential direction for the potential flow in the limit as y goes to 0 because it is in that matching condition for the potential flow y goes to 0 for the boundary layer eta goes to infinity, in that region you are matching with two solutions.

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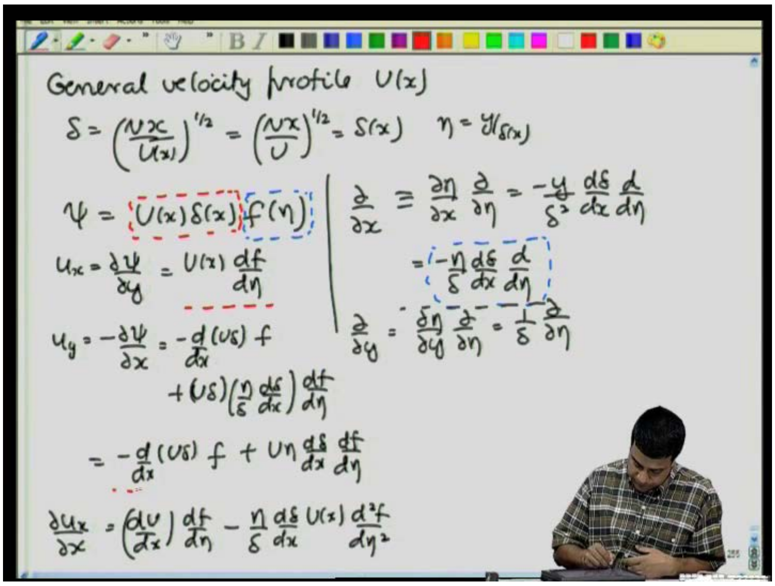

So, let us consider a general velocity profile. This is the potential flow velocity in the limit as y goes to 0 for the potential flow as, you are approaching the surface in the outer potential flow. The boundary layer thickness for this velocity profile we had already calculated we saw it a couple of times therefore, the boundary thickness delta is equal to mu x by U of x power half for the flat plate U was a constant and for the stagnation point flow U was proportional to x.

So, I will just call this as nu x by u power half where it is understood u itself is a function of x. So, both delta and u are functions of x, the stream function if you recall, we had got it as nu x u sorry nu and some function of eta, why do we choose the stream function of this form? The reason is because, you know that ux is equal to partial psi by partial y, which is equal to partial psi by partial eta partial y this turns out partial eta by partial y because eta is defined as y by delta of x. So, partial eta by partial y just gives you 1 over delta. So, this ends up being equal to u of x times df by d eta. As you know u of x has to approach capital U as y goes to infinity and therefore, this basically gives you the solution the df by d eta has to approach 1 as y goes to infinity.

So, with this particular choice of the stream function, I get an equation for the scaled velocity which is ux by capital U, which is just dimensionless f prime of eta. So, that is the reason that we choose psi is equal to u delta times f of eta, if you recall for the flat plate clays delta was equal to nu x by u power half. So, I got nu xu power half times f of eta for the stagnation point flow case once again delta was at constant. So, I just got u of x which was kx times this delta which is at constant.

So, this is the solution the choice for the stream function, and this choice for the stream function substituted into the momentum conservation equations in order to find out what is the form of the velocity capital U, which will admit a similarity solution. So, basically if I take this form substituted into the momentum conservation equation. I should end up in with an equation that depends only upon eta and not individually upon capital U nu or x. So, that is there requirement that will determine, what is the form of the solution that admits. What is form of the mean velocity that admits a similarity solution?

So, we have to basically substitute this in to the equation and get them and solve the resulting equation. So, 1st partial by partial x in terms of the similarity variable this equal to partial eta by partial x partial by partial eta, you have to do it by chain rule because I am going to substitute for all x and y derivatives in terms of the similarity variable eta.

So, since eta is equal to y by delta I will get, minus y, y delta square d delta by dx times f prime of eta. Differentiate using chain rule this will be equal to minus eta by delta. So, minus eta by delta times d delta by dx times df by delta is the d by delta. Similarly the derivative with respect to y partial by partial y is equal to partial eta by partial y partial by partial eta is equal to 1 over delta.

So, those are the derivatives in the x and y direction. So, I will use these to calculate what the velocities are, and their derivatives are if you recall U x is equal to parcel psi by parcel y that I had just derived for you this was equal to U of x df by d eta, uy on the other hand is equal to minus partial psi by partial x psi has 2 parts. One is this pre factor which depends upon on x and the 2nd is the function f itself which depends upon x through the dependence of eta upon x.

So, both of these have to be differentiated separately. The 1st part is just minus d by dx of u delta times f, is the derivative of u delta with respect to x times f the 2nd part is due to the differentiation of f with respect to x, partial f by partial x is given by this expression here, I use this for expressing the derivative with respect to x in terms of the derivative with respect to eta. So, this there is a negative sign here. So, you will get plus I have the pre factor U delta into eta by delta d delta by dx, df by d eta, and I can simplify this to write minus d by dx of u delta times f plus u eta d delta by dx df by d eta.

And then if you recall for the momentum conservation equation, we have sorry partial U x y partial x partial U x by partial y in the 2nd derivative in the viscose term. So, partial ux by partial x once again contains 2 parts 1 is because of the derivative of U with respect to x. So, this is the 1st term is dU by dx times d f by d eta plus U of x then I have to differentiate df by d eta with respect to x. So, at this becomes and you get an minus sign here as you can see partial by partial x has a negative sign in front of it. So, I will get minus eta by delta, d delta by dx, u of x d square f by d eta square. So, that is partial U x by partial x. The derivatives with respect to y are quite easy.

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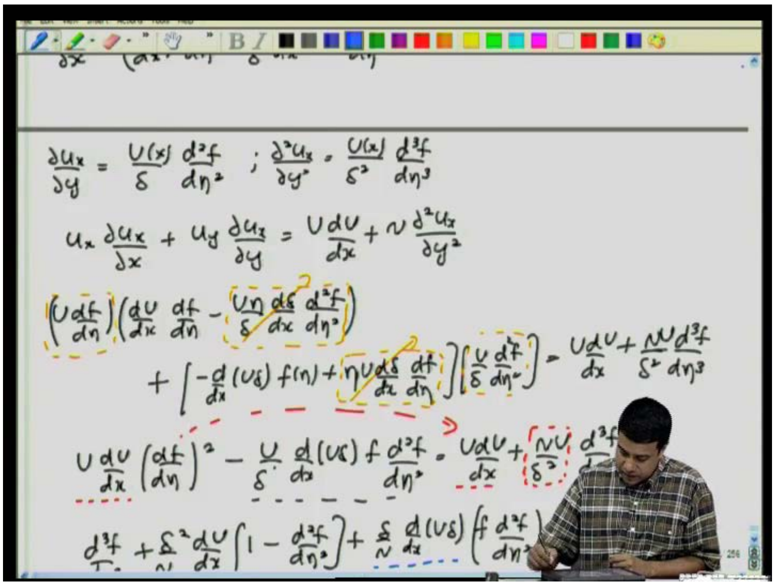

Partial ux by partial y since capital U does not depend on upon y, the derivative partial ux by partial y just becomes u of x by delta d square f by d eta square. In the 2nd derivative partial square ux by partial y square is equal to. So, those are all the quantities that we need to substitute in to our momentum conservation equation.

Momentum conservation equation as you recall is U x partial U x by partial x, plus U y is equal to U dU by d x, plus nu partial square ux by partial y square. So, I substitute the expressions that I have just derived into this equation. U x is U df by d eta and partial u x by partial x is d U by d x df by d eta minus U eta by delta d delta by d x d square f by d eta square, plus u y plus u y u y as you recall is given by minus d by d x of U delta times f of eta plus eta U d delta by d x d square f by d eta square into partial u x by partial y is going to be equal to U by delta df d square f by d eta square, partial U x yeah d square f by d eta square.

I am sorry this is df by d eta the expression to y eta there, and I have just use the expression for Uy that I derived here. I just used the expression for U y that I derived here plus u eta d delta by dx times d f by eta, and on the right hand side I have plus nu by delta square d square nu U by delta square. So, that is the final expression that I get for the momentum conservation equation. This can be simplified this can be simplified one can verify that the terms that are proportional to this 2nd term here multiplied by the pre factor is equal in magnitude in opposite in sign to this term, multiplied by this term in both cases you get eta U square by delta d delta by dx df by d eta d square f by delta square.

And so, these two terms actually cancel out these two terms actually cancel out and the expression that I get is U du by dx into df by d eta the whole square minus U by delta into d by dx of U delta f d square f by d eta square is equal to U du by dx plus nu U by delta square d square f by I am sorry this is a 3rd derivative, please correct that this is a 3rd derivative because the ux itself is proportional d f by d eta. So, there will be a 3rd derivative.

So, now, to non-dimensionalize the equation I can divide throughout by the factor of any one term because if I divide throughout by the pre factor of one term. I will get an equation that is non-dimensional, it is most convenient to divide by the pre factor of this term that was the most convenient choice. So, then if I divide throughout and take all the terms to one side I will get d cubed f by d eta cubed plus delta square by nu du by dx. That is the pressure gradient term into 1 minus d square f by d eta square there is have U du b y dx over here that divided by nu u by delta square.

Similarly in the left hand side I have u d u by d x times d square f by d eta square. So, if I take this to the right hand side, I will get one minus d square f by d eta square and the final term is due to this 1 which will be plus, delta by nu by dx of U delta into f d square f by d eta square this has to be equal to 0.

So, that is my scaled momentum conservation equation, we had asked the question in the beginning, what is the velocity profile for which we can get a similarity solution for the boundary layer flow? Obviously, we can get a solution only if, these two terms are just numbers independent of x, independent of nu and independent of capital U, k they are just numbers. So, that is the requirement that we have a boundary layer solution in this particular case, only those velocity profiles for which both of these simultaneously are constants do admit a boundary layer solution.

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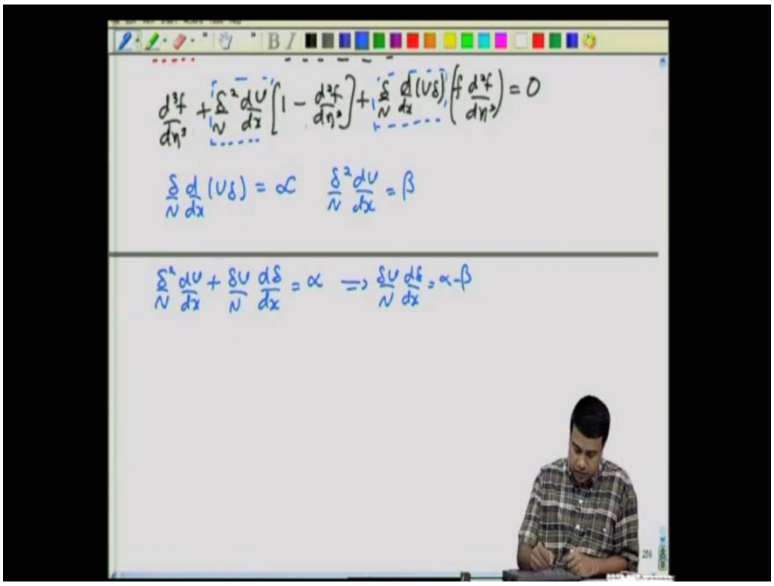

So, what are the velocity profiles? So, let us look at what the velocity profile have to be. I require that this term here delta by nu d by dx of U delta is equal to some constant, I will call that as alpha in addition the 2nd term also has to be a constant, delta square by nu du by dx is equal to some constant beta. So, both of these have to be equal to constants if I take the number. So, this delta by nu d by dx of u delta is equal to delta square by nu du by dx plus delta u by nu d delta by dx this is equal to alpha. Which means that delta u by nu d delta by dx is equal to alpha minus beta because, delta square by nu du by dx is just the factor beta.

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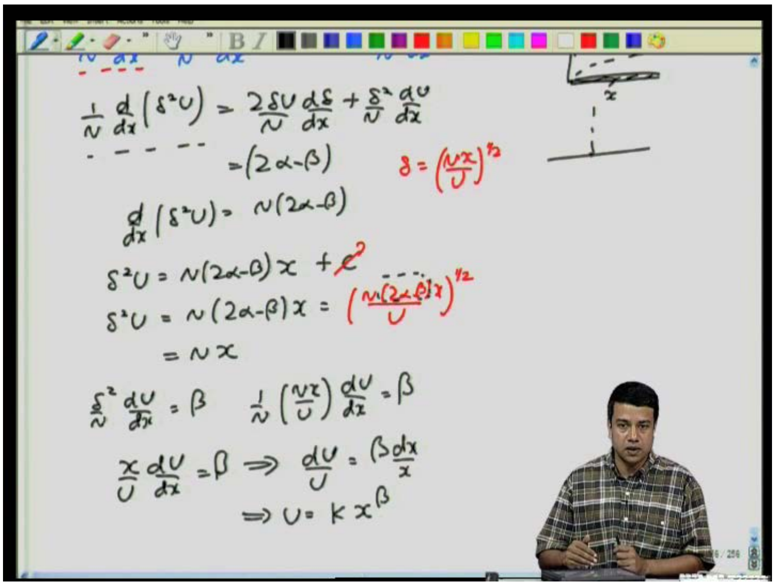

If I take if I take the number 1 over nu d by d by dx of delta square u, if I look at this particular number this is equal to 2 delta u by nu delta by dx plus delta square by nu du by dx this equal to 2 into alpha minus beta plus beta itself. So, this is equal to 2 alpha minus beta. So, this has to be a constant 2 alpha minus beta which means that d by dx of delta square u is equal to nu into 2 alpha minus beta. So, boundary layer solution exists only f delta square times U is equal to nu into 2 alpha minus beta into x plus a constant c. There is an undetermined constant here because I am solving a 1st order differential equation.

However, I can always set this constant equal to 0 by stating by putting the origin, at the upstream edge of the boundary layer. So, if I put the origin at the upstream edge of the boundary layer. I know that the boundary layer I know that the boundary layer thickness has to go to 0, at that upstream edge therefore, by putting the boundary layer the origin at the upstream edge the constant c is always equal to 0. Note that I can always choose the origin of my coordinate system regardless of what coordinate this is I either this 1 or this 1 by choosing the upstream edge of my boundary layer as the location at which the x is equal to 0. So, then I get delta square U is equal to nu into 2 alpha minus beta into x. Note that I had said that delta was equal to nu x by u power half. Note that in the beginning, I had postulated that delta was equal to nu x by u power half.

So, this definition of delta it had an undetermined constant in it. So, this solution is basically telling us, that this definition of delta it has this undetermined constant in it when I define delta this way I had said the constant in that equation to be equal to 1. In this particular case this constant is turning out to be equal to 2 alpha minus beta because nu x by u power half if I divide throughout by capital U I will get that delta is equal to nu x by U power half and the equation that I get for delta will be equal to nu into 2 alpha minus beta x by U power half. As I said this is a constant numerical factor this 2 alpha minus beta is a constant numerical factor to be consistent with my definition of the boundary layer thickness as nu x by u power half I have to set this equal to 1.

So, without loss of generality you can set 2 alpha minus beta equal to 1. So, that delta square nu is just equal to nu times x. So, it gives us one condition if I said delta square times u is equal to nu times x it gives me one condition for a constant value for the boundary layer thickness. The 2nd condition of course, comes from any one of these equations I have two equations here, the equation that I got here was a combination of those two.

So, I can choose any one of those equations in order to get the 2nd condition I can chose delta by nu, delta square by nu du by dx is equal to the constant beta and I know what is delta. Therefore, I can use the definition of delta to get, u by x d I am sorry x by U 1 by nu into delta square is nu x by u into du by dx is equal to the constant beta.

So, this basically gives us x by U du by d x is equal to beta, which implies that dU by U is equal to beta dx by x which implies that U is equal to some constant times x power beta. So, only those velocity profiles for which the mean flow potential flow velocity in the limit as y goes to 0 is of this form x power beta are the only 1s that does the do admit a boundary layer solution. These are the only velocity profiles that will admit the boundary layer solution, those for which the outer of flow the potential flow velocity increases as a power of x as you go downstream in the flow.

So, this has told us what are the kinds of velocity profiles for which we can get a boundary layer solution, what do these correspond to, we will look at in the next lecture. We will find that these actually corresponds to potential flow solutions that we had got previously, which are basically the flow in a corner of various angles. Kindly go back and revise that the potential flow solutions for the flow in a corner for various angles. So, only for those solutions that we can get a boundary layer similarity solution.

So, this has basically given us an expression for a similarity solution for the boundary layer profile. Next, lecture we will look at what this physically represent. So we will continue this in the next lecture kindly go through the derivation so far as well as the potential for solution for flow on the corner and we will continue this in the next lecture. We will see you then.