Fundamentals of Transport Processes II
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Lecture - 33
Stagnation Point Flow

So, this is lecture 33 of our course on fundamentals of transport processes two where we were bang in the middle of looking at a boundary layer flows. We have just analyzed the flow past flat plate in the previous lecture, and we will continue that analysis. We will finish that before going on to a different kind of flow.

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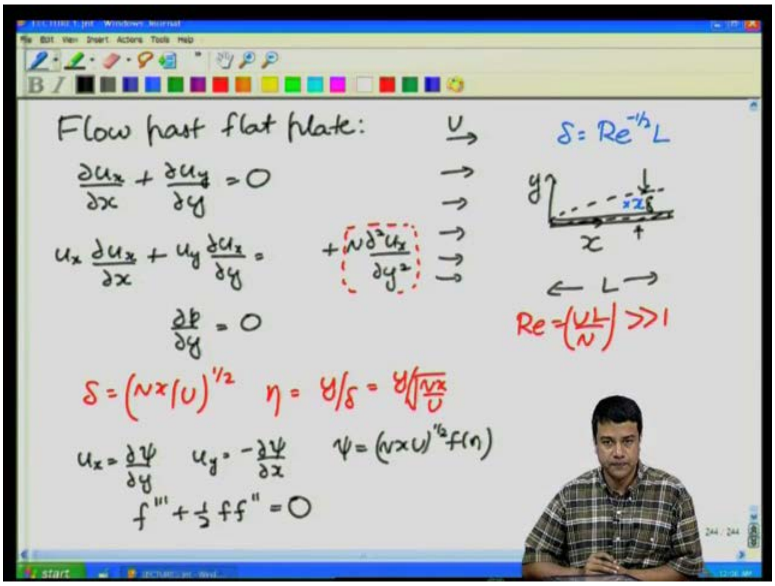

So, this was flow past flat plate, where basically we had a flat plate of infinite decimal thickness in the x y plane of some length L and there was a constant velocity far from the surface, constant velocity U far from the surface. The Reynolds number based upon the length of the plate and the free free stream velocity U, U L by kinematic viscosity, kinematic viscosity is viscosity divided by density. So, this is rho U L by nu. This was large compared to 1.

So, far away the velocity is equal to capital U at the surface itself it has to decrease to 0. If you assume Reynolds number is large and just neglect the viscous terms we reduce, we simplify the equations from a second order differential equation to a first order differential equation. And then we are not able to satisfy the tangential velocity boundary condition at the surface. In order to satisfy the tangential velocity boundary condition we postulate the existence of this thin layer of thickness delta at the surface.

The the thickness of the layer is small, so that the velocity increases from 0 at the surface to the free stream velocity capital U over a layer of thickness delta. If delta is small compared to L the velocity gradient is large and since the viscous terms are proportional to the second derivative of the velocity, inertial terms are proportional to the first derivative. If delta become small you could have a situation where the gray intense large and therefore the viscous terms become comparable to the inertial terms because of the large gradient over a small thickness delta.

We have done scaling in order to find out what this thickness delta should be in such a way that in the limit as R e goes to infinity, R e goes to infinity, delta becomes smaller and smaller in such a way that in this layer the inertial and the viscous terms, the largest viscous terms continue to be of the same magnitude. So, on that basis we found that delta is equal to R e power minus 1 by 2 times L where Reynolds number is rho U L by nu or U L by nu. Therefore, delta goes as 1 over Reynolds number power half goes to 0 in the limit as R e equals to infinity.

And in that case the conservation equations, becomes partially U x by partial x plus partial U y and partial y. So, these are called the boundary layer equations for the flow past any surface. As you can see that we have included the inertial terms in the stream wise momentum conservation equation, the pressure gradient as well as this viscous term which basically represents the diffusion of momentum perpendicular to the surface in the y direction, diffusion of momentum that is perpendicular to the surface.

The y momentum equation basically tells us that the pressure gradient in the y direction is equal to 0 that is because the flows primarily in the x direction, the y velocity is much smaller than the x velocity. The gradients are large and therefore, the pressure gradient has to be equal to 0 which means that the pressure at any location within the boundary layer is identical to the pressure at the same location far away from the, as y goes to infinity.

For our particular flat plate the pressure far away is identically equal to constant, because far way we have a constant velocity and from the Bernoulli equation the pressure itself is also a constant. Since, the pressure is a constant the pressure everywhere within the boundary layer as well; note, that the pressure at any y within the boundary layer at a fixed stream wise location is equal to the pressure in the limit y goes to infinity because partial p by partial y is equal to 0.

If the pressure is independent of the stream wise coordinate far away then the pressure is independent of the stream wise coordinate within the boundary layer as well. So, for this particular case of a flat plate boundary layer the pressure gradient turns out to be equal to 0 and the conservation equation just becomes U x, you can remove the pressure term in this conservation equation. And then to obtain similarity solutions we had to use one other piece of logic and that was that at a given location x within the flow, at a given location x within the flow there is stream wise convection; there is diffusion perpendicular to the surface.

We have neglected stream wised diffusion. Therefore, the velocity at a given location x will not depend upon the total length of the plate L. The velocity at a given location x will not depend upon the total length of the plate L. It will only depend upon the distance to the upstream edge because it is only downstream convection that is effecting the dynamics of the flow as well as perpendicular to the diffusion. Since, L is no longer a relevant parameter. The flow is going to be the same regardless of whether the plate was length L, 2 L or some other length, so long as it is larger than x because what happens downstream does not affect the flow.

So, on this basis one can define a local boundary layer thickness for the situation where the the the flow at a given length L does not depend, at a given location x does not depend upon the total length L. So, to obtain that you just substitute x for L in my boundary layer thickness definition because I should get the same flow dynamics even if that plate were only of length x because what happens in downstream does not matter. So, on that basis we had defined the boundary layer thickness, delta is equal to nu x by U power half and my similarity variable eta is equal to y by delta which is equal to y by nu x by nu.

So, this similarity variable is telling you something, it is telling you that eta is the only coordinate that matters as far as the flow is concerned. If I go to two different locations which had two different values of y, two different values of x, but eta were the same then the velocity would be the same at those two locations. So, that is what it is telling you.

So, I defined this similarity variable. This basically reduces my partial differential equation into an ordinary differential equation because both x and y are contained in eta.

My original equation was in terms of both x and y whereas, here both x and y are both contained in eta. Then we showed that since the the mass conservation equation for a two dimensional flow has to be obeyed, you can write down the velocity in terms of the stream function. And we defined the stream function in terms of a dimensionless variable f of eta times nu x times u power half. So, that is the steam function. So, f of eta which is psi by nu x by U square root of is the scaled stream function.

So, we express U x U y in terms of the scaled stream function, substitute that into the momentum conservation equation and get a single equation for f. If our similarity transform is correct the resulting equation should be only a, an equation in terms of eta, it should not individually contain nu x and U. So, it should be only an equation in terms of eta. It is not individually contained, nu x and U. We substituted that into this the the momentum conservation equation and did indeed obtain an equation which was only a function of eta and that equation turns out to be the Blasius boundary layer equation for f, third order equation f in terms of eta. It has to be solved subject to the boundary conditions.

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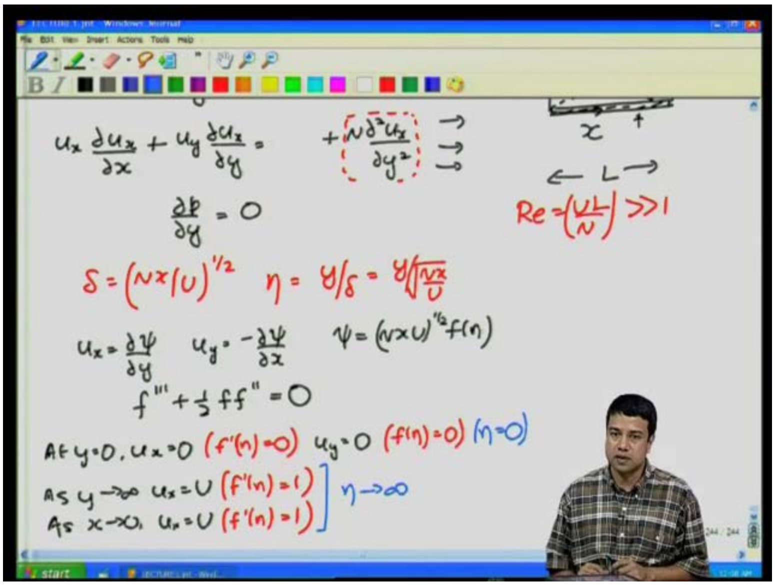

The boundary conditions for that, at the surface of the plate at y is equal to 0, u x is equal to 0 which in terms of f was f prime of eta is equal to 0 and u y is equal to 0 which when expressed in terms of eta turned out to be f of eta is equal to 0. Then I had a condition as y goes to infinity u x is equal to the free stream velocity, as y goes to infinity u x is equal to u which when expressed in terms of eta becomes f prime of eta is equal to 1. Also, as x goes to 0 as you approach the upstream edge of the plate, the flow rate has not yet felt the plate because there is only downstream convection and perpendicular diffusion.

The fluid has not yet felt the plate and therefore, you require that as x goes to 0 u x is equal to capital U which means that. And we saw that y is equal to 0 corresponds to eta is equal to 0 because y was equal to; eta was equal to y by delta. So, when y goes to 0, eta also goes to 0 and we saw that these two reduced to the same condition that is eta going to infinity. Therefore, you have three boundary conditions, two at eta is equal to 0, 1 as eta goes to infinity because the boundary conditions in both y and x turn out to be the same when expressed in terms of eta and you can solve the equations. So, I got a solution for you in the last lecture.

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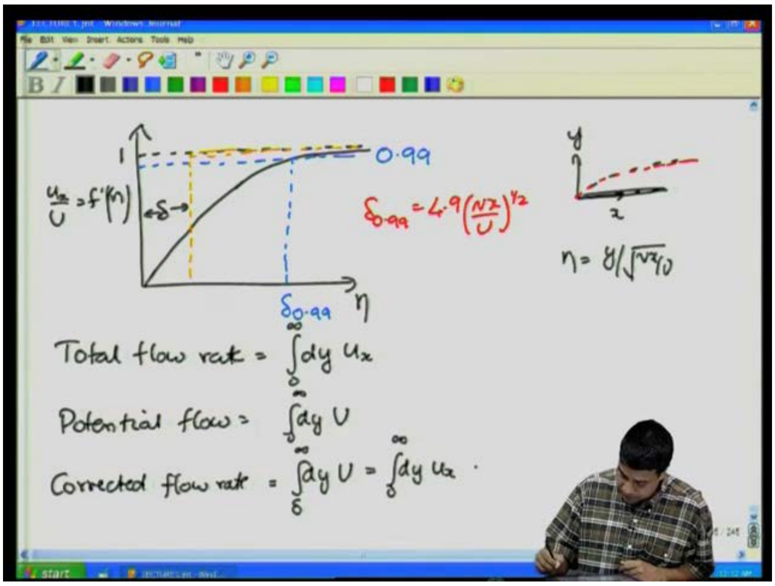

The scale velocity and this of course, as eta goes to infinity u x by U has to go to 1 because the velocity has to go to the free stream velocity U. Therefore, u x by U has to go to 1. At the eta is equal to 0 we call that at this plate, this is x and this is y and I am plotting verses eta is equal to y by root of nu x by U. This is the boundary layer there. At the surface itself u x has to go back to 0. So, the numerical solution gives you a solution that looks something like this.

The boundary layer thickness delta 0.99 is the location at which you obtain 99 percent of the free stream velocity. That is the velocity here is equal to 0 point u x by U is equal to 0.99 that is what is often referred to as delta 99, the boundary layer thickness, delta 99 and we saw in the last class that is equal to 4.9. So, the boundary layer thickness itself is proportional to x to the half. It is increasing as you go downstream, you can see form here the boundary layer thickness to equal to is increasing as x to the half as you go downstream. I had also defined displacement thickness for you in the last lecture.

The displacement thickness is the thickness by which you have to displace the boundary layer in order to get the exact same flow. Now, the free stream, the potential flow solution basically will tell you that the flow rate across any surface in the, along the y axis, the flow rate is going to be equal to the density times the mean velocity times the distance. That is what the auto potential flow is going to tell you because the auto potential flow just predicts that the velocity is equal to capital U all the way to the surface.

However, because of the no slip condition at the surface the velocity decreases to 0 and because of that there is a decrease in the flow rate. That decrease in the flow rate is written as rho U times delta where U is the mean stream velocity. So, the total flow rate is equal to integral d y from 0 to infinity times rho U x, total flow rate is equal, this is the total mass flow rate. We can work in terms of the total volumetric flow rate as well because the densities are constant. So, that is the actual flow rate. The flow rate predicted by potential flow is going to be integral 0 to infinity d y times U. Note, that this is two dimensional. So, we are always taking with respect to unit distance in the third direction. This is equal to d y times U.

This is not correct because the velocity is coming down to 0 at the surface itself. So, this is is over predicting the flow rate. So, in order to correct the potential flow rate you displace the boundary layer by a distance delta. So, correct it. It displace the boundary layer thickness by a distance delta while retaining the same outer free stream velocity. So, this is the displacement thickness delta which I need to displace the boundary layer in order to get the same flow rate. So, this has to be equal to the actual flow rate 0 to infinity d y U x. This has to be equal to the actual flow rate.

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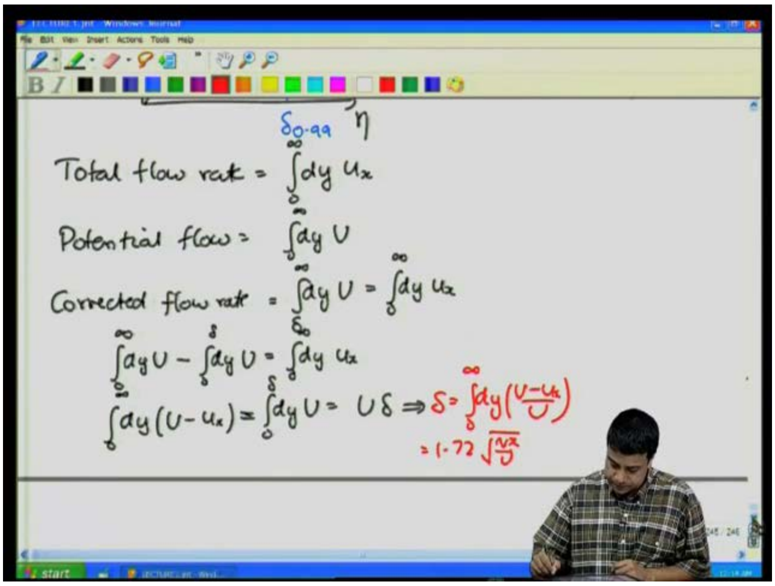

This will give me the definition of delta, left hand side I can write it as integral 0 to infinity d y U minus integral 0 to delta d y U is equal to integral 0 to infinity d y U x. So, I take the second term to the right hand side, I just rearrange this equation to get integral 0 to infinity d y U minus U x is equal to is equal to U times delta. This gives us the expression for delta itself; this gives us the expression for delta. This implies that delta is equal to integral 0 to infinity d y U minus U x by U.

So, that is the expression for delta that you get because capital U is a constant. It is just the free stream velocity and this as I told you in the last lecture, it turns out to be equal to about 1.72 root of U x by U. It is about a third of delta 0.99, delta 0.99 is about 4.9. This is about a third of them, there is another often used quantity, it is called the Von Karman momentum thickness.

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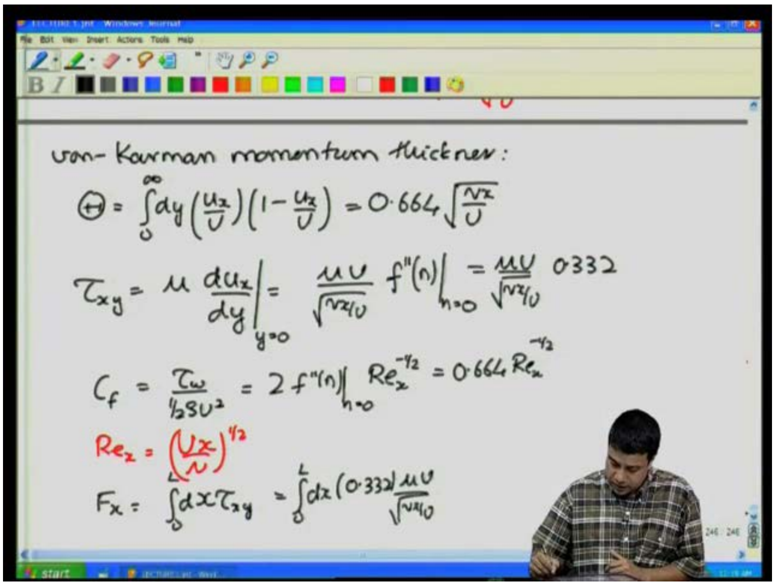

It is basically measures the momentum flux deficit rather than the flow rate deficit, it actually mentions the momentum flux deficit. I would not go into the details of that. I will just give you the final expression, this momentum thickness is defined as integral 0 in to infinity d y of… As you can see U x by U is the rate at which momentum is being convicted down and 1 minus U x by U is the momentum deficit, if the flow actually had a velocity capital U then the momentum deficit would be 0, but the velocity is actually less than capital U and therefore, you have a momentum deficit and this is 0.664.

Finally, we can also calculate the shear stress acting on the surface. As I said you have a plate (( )) fluid flow flowing past that surface. The fluid is has a velocity U faraway. Near the surface that flow has been slowed down because of the presence of that surface because the momentum transfer from the surface to the fluid that results in a decrease in the momentum of the flow rate and of course, that also exerts a force on that surface. We can calculate both the stress as well as the force on that surface.

First the shear stress mu times d U x by d y this has be calculated at the surface y is equal to 0 and now I can express d U x by d y in terms of f and eta which I had calculated earlier, y is equal to 0 corresponds to eta is equal to 0. So, if I do that I will get mu u by the boundary layer thickness that is root of nu x by U times f double prime of eta at eta is equal to 0 and this numerical coefficient f double prime of eta at eta is equal to 0 has to be measured from the, has to be obtained from the numerical solution.

This f w prime of eta at eta is equal to 0, the value of this coefficient has to be measured from the numerical solution. This turns out to be approximately equal to 0.332 times 0.332. So, that is the shear stress acting at the surface. Note, that the shear stress decreases as x power minus half, the length scale is increasing as x power plus half, the shear stress is the ratio of the velocity and the length scale, so there is decreasing as x power minus half.

The second friction coefficient is a dimensionless quantity which is defined as tau w by half rho u square and if you put that in you will get 2 times f double prime of eta at eta is equal to 0 times the Reynolds number based upon x to the minus half which is approximately 0.664 R e x to the minus half where R e x is the Reynolds number based upon the local distance from the upstream edge where R e x is equal to U x. So, local distance from the upstream edge and finally, the drag coefficient, the total force exerted per unit length perpendicular to the surface with the total force exerted on this entire plate per unit length perpendicular to the surface.

The force in the x direction is equal to integral 0 to L d y times tau x y. For this actually stress times area, but since this is a two dimensional system we have to calculate the force exerted per unit length perpendicular to the surface. I am sorry this is d x. Integral of the stress along the stream wise coordinate gives me the total force. So, this is equal to integral 0 to L d x times the shear stress which is basically let me just write in terms of 332.

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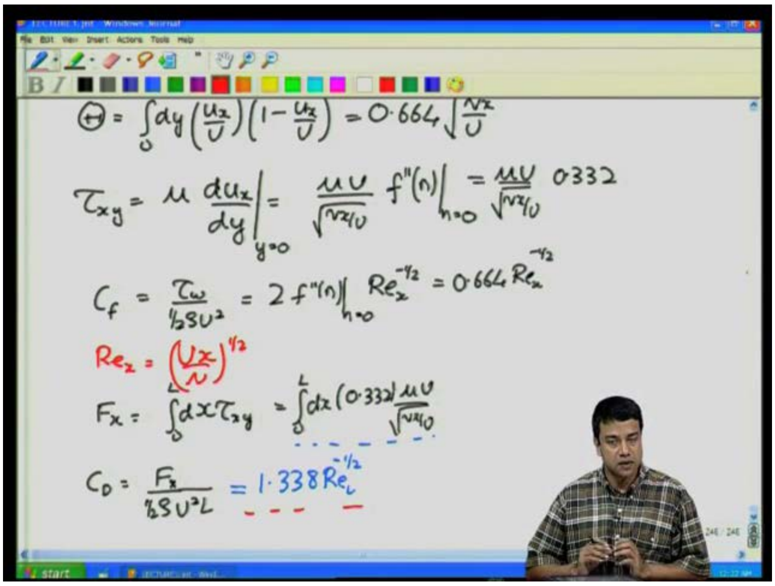

And then you can define the drag coefficient C d is equal to the force in the x direction y half rho U square L, that is the traditional definition of the drag coefficient. And this turns out to be equal to, if you go through this calculation, do the integration over the x coordinate, calculate the force and then divide by half rho U square L. I will leave that as an exercise for you. If you do that you will get this is equal to 1.338 Reynolds number L to the half, R e l to the minus half.

So, this basically tells you that the force scaled by the inertial scales, half for U square L is an inertial scale for the force, it goes as R e power minus half. The total force due to inertial effects has two scalars rho U times the length that therefore, this tells you that the drag coefficient goes to 0 as Reynolds number goes to infinity. That is because the drag coefficient is scaled by the inertial scale for the force. And for a potential flow itself we showed that this C d has to be equal to 0, for a three dimensional object in potential flow the net force has to be equal to 0 if there is no circulation.

This flat plate is a symmetric configuration, there is no circulation. So, we just calculate the force using potential flow calculations. I will get 0. However, when I do take into account the effect of viscous diffusion in a thin boundary layer near the surface, I find that the drag coefficient goes as R e l power minus half or the force scaled by the inertial scales goes as Reynolds number power minus half. The skin friction coefficient goes as the Reynolds number based upon x to the minus half where R e x is the local Reynolds number based upon the distance from the leading edge.

So, this is the correlation that is often used for flat plate flow and using boundary layer theory simplifying the boundary layer equations and then using, defining a similarity variable we are able to get this correlation exactly. So, that that is a power of the boundary layer theory. Just using a simple argument, a simple scaling arguments for insisting that inertial and viscous forces have to be of equal magnitude in the limit SRE goes to infinity, we get the scaling for the boundary layer variable and then a simple piece of logic that what happens the flow at a given location x cannot depend upon what happens downstream.

We manage to get the similarity solution and that similarity solution has given us back this correlation for the flow, for the drag force on the flat plate. So, this is simple configuration where we solved the boundary layer equations. However, the equations themselves are more general. They can be used for any configuration. So, we look at one other configuration for which we can get an exact solution and that is what is called a stagnation point flow.

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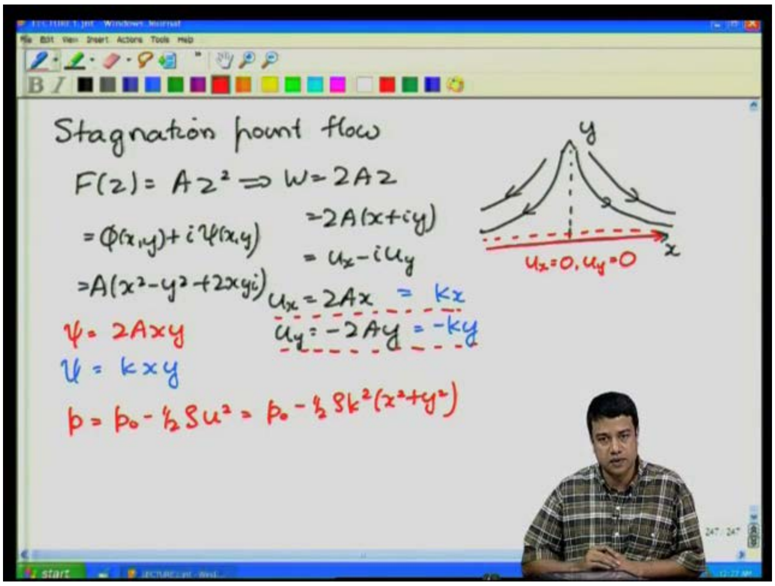

We have already seen this in our potential flow solutions, we have already seen this kind of flow, the flow in a corner, flow that is approaching a surface and it has, I will call this as the x and this as the y. This flow we have already seen in potential flow solutions, if you recall potential function F of z in that case was equal to A z square which implies that w is equal to 2 A z which is equal to 2 A into x plus i y. If we recall the complex velocity was related as U x minus i U i to the real components or the real velocities which means that for this particular case U x is equal to 2 A x, U y is equal to minus 2 A y. The potential function itself, the complex potential has as this real and imaginary parts phi of x y plus i psi of x y.

And if you take z square and expand it out you will get x square minus y square plus 2 x y i which means that the stream function for this is equal to 2 A x y. So, we will call this constant 2 A as some constant k, I will just call it for simplicity, I will write this as k x, u y is equal to minus k y and psi is equal to k x y. So, partial, therefore, U x is equal to partial psi by partial y and u y is equal to minus partial psi by partial x. So, this is the outer potential flow solution. Also, for this flow p the pressure satisfies the Bernoulli equation p naught minus half rho U square is equal to p naught minus half rho k square to x square plus y square.

So, the pressure satisfies the Bernoulli equation. So, this is the pressure for potential flow and the velocity for potential flow. Problem is this velocity does not satisfy the 0 of tangential velocity boundary conditions at the surface. At the surface itself, this surface is the stationary surface, at the surface itself you require that U x is equal to 0 and U y is equal to 0. The outer potential flow does satisfy the the condition that U y is equal to 0 because U y is equal to minus k y, does not satisfy the tangential velocity boundary condition U x is equal to 0.

Therefore, in order to satisfy this condition we have to postulate in the existence of boundary layer very close to the surface. That boundary layer has a thickness which is determined in such a way that the inertial and the viscous effects are of equal magnitude within that boundary layer even in the limit as R e goes to infinity. In this particular case the velocity, the velocity of the outer potential flow is actually changing since it is a function of x, this velocity is equal to k times x whereas for that flat plate problem we had a constant velocity far away.

In this case it is increasing as a function of x. So, one has to postulate that there will be a thin boundary layer near the surface whose thickness is such that there is a balance between inertial and viscous forces within this boundary layer. Of course, we do not postulate a fixed length l in this case. We just consider a stagnation flow and we assume that it is a infinite extent, the flat plate case we had postulated a fixed length and then we had said that at a given location x, the flow dynamics should depend only upon the distance x and not on the total distance L.

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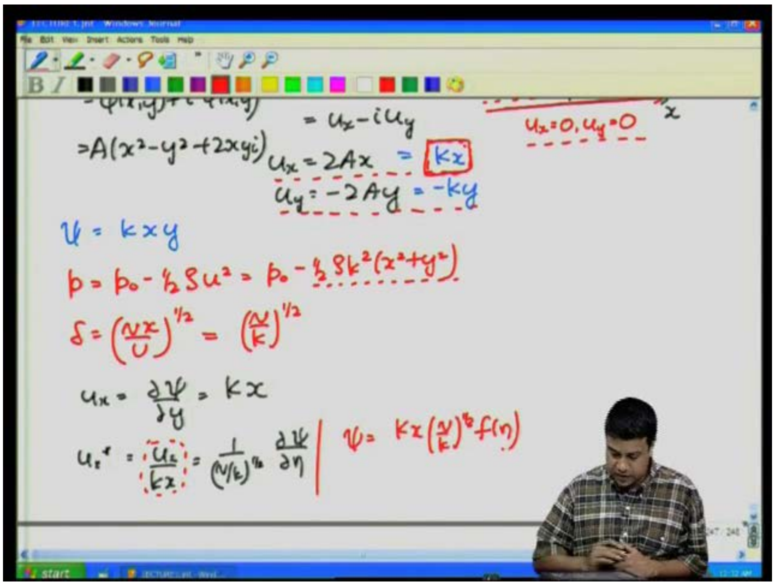

On that basis we had got a boundary layer thickness delta is equal to nu x by U power half. We use the same logic here; we use the same logic here. Capital U is actually equal to k times x, capital U is equal to k times x because the the velocity far away, outside the boundary layer has to be given by k times x. If I just simply substitute and this turns out to be correct I simply substitute U is equal to k times x, I get a boundary layer thickness which goes as U by k power half.

Note, that this boundary layer is actually independent of x, it is just a constant value. The reason is because the mean flow velocity itself is increasing as function of x. The boundary layer thickness will be determined by a balance between convection that is sweeping momentum downstream and diffusion which is transporting momentum from the surface into the flow. For the Blasius boundary layer past a flat plate the connective velocity, mean flow velocity was a constant, there was diffusion from the surface.

Therefore, I got a boundary layer which was increasing as we went downstream. In this particular case the mean flow velocity is increasing proportional to k x, there is, this is the increase in the mean flow velocity as you go downstream. So, the mean flow velocity itself is increasing that is diffusion from this surface, the balance is such that the boundary layer thickness itself turns out to be a constant. So, this comes out just simple scaling arguments in this particular case because the velocity is increasing, velocity increases the pressure also increases, pressure is also increasing proportional to x square within the boundary layer.

Both velocity and pressure are increasing and therefore, the the boundary layer thickness in this particular case is remaining independent of the position x. So, the outer flow stream function is given by psi is equal to k x times y. Now, what about within the boundary layer? Of course, the the velocity field has to come to 0 at the surface. So, at the surface itself I need to be able to satisfy U x is equal to 0 and U y is equal to 0. So, I modify this as psi is equal to k x times f of eta, you require that f of eta has to go to y as y goes to infinity.

Let us let us go back and define our stream function carefully. U x is equal to partial psi by partial y is equal to k times x. This is in the limit as y goes to infinity and I can define my stream function U x star, U x by k times x is equal to 1 over the boundary layer thickness, 1 by nu by k power half times partial psi by partial eta. So, in this equation u x in the left hand side is dimensionless. If I define U x star is equal to U x by k times x eta is dimensionless on the right hand side.

Therefore, if I divide throughout I should be able to get a stream function that is dimensionless provided I define my stream function as as psi is equal to k x by mu by k power half times f of eta. I am sorry psi x into nu by k power half the velocity times the line scale k x is the velocity scale because u x is equal to k times the x times the length in the y coordinate times f of eta is the stream function.

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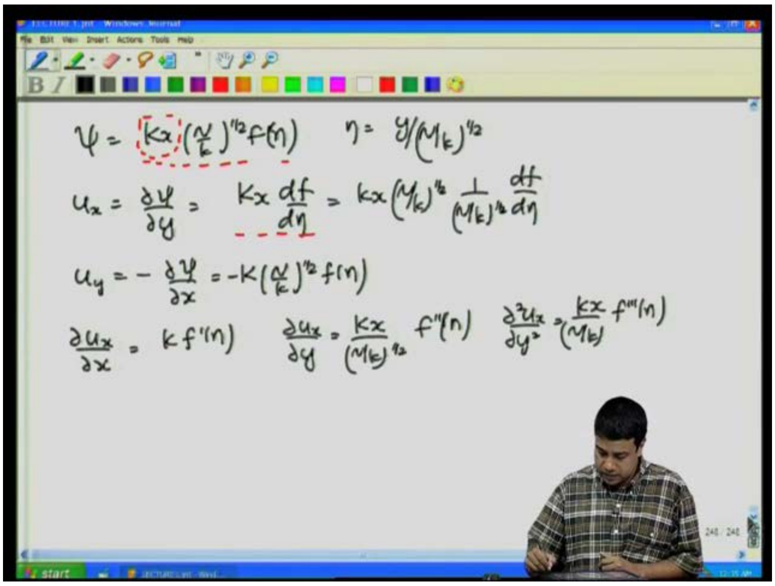

So, I define psi is equal to wait actually I can have f of eta where eta is equal to y by u by k power half. Note, that k itself has dimensions of inverse time and therefore, nu by k power half has dimensions of length. Then I can get my equations u x is equal to partial psi by partial y is equal to k x d f by d eta. If I will take partial f by partial y I get 1 over nu by k power half times partial f d f by d eta. So, I just get U x is equal to k x times d f by d eta.

We can expand this out as k x nu by k power half into 1 by, 1 by nu power half and that will get cancel out. U y now is equal to minus partial psi by partial x. Note, that eta is independent of x, eta is independent of x. So, the only dependence on x of the stream function itself comes in through this term here. The only dependence on the stream function comes in through this term k x here. So, this becomes equal to k x, I am sorry k times that is, that is the velocity U y and then if you recall for the momentum conservation equation I require partial U x by partial x, partial U x by partial y and the second derivative.

So, partial U x by partial x is equal to k times f prime of eta, just take it by differentiating this with respect to x because eta is independent of x. Then partial U x by partial y is equal to k x by U by k power half f double prime of eta. I differentiate once with respect to y, so I get 1 by U by k power half times f, d f by d eta derivative. So, I get f double prime and the second derivative differentiate once again, differentiate it once again. So, these have now got to be substituted into the equation for the the momentum conservation equation for the flow in the boundary layer.

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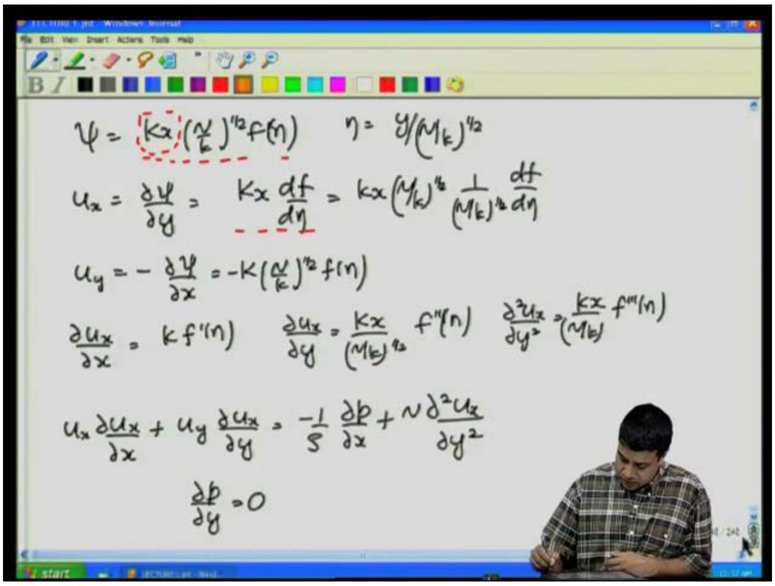

Momentum conservation equation is U x partial U x by partial x and put the density in there. Yes, that is the equation for the boundary layer. We still have not got the pressure gradient yet; we still have not got the pressure gradient yet. The pressure gradient is obtained by the condition that partial p by partial y is equal to 0. That means that as in the case of a flat plate boundary layer the pressure within the boundary layer is identical to the pressure far away from the surface because there is no variation in the pressure perpendicular to the surface.

The pressure within the boundary layer is equal to the pressure in the free stream far away. What is the pressure in the free stream? We got that from the Bernoulli equation here. Therefore, partial p by partial x at any location within the boundary layer has to be the same as the pressure gradient far away simply because the y derivative of the pressure is equal to 0. What is the pressure gradient far away?

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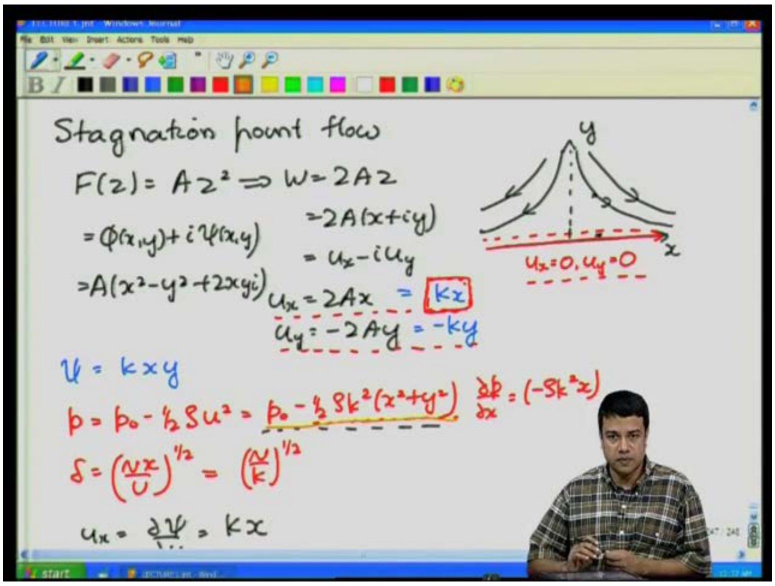

The pressure gradient far away is given by, far away from the surface the pressure gradient if I take the derivative of this with respect to x I will get rho k square x with the negative sign minus rho k square x. That is the pressure gradient far away. You can see if you just take this expression, just take this expression differentiate it with respect to x I get minus rho k square times x far away. Since, the pressure variation is 0 in the y direction this partial p by partial x is the same within the boundary layer as well.

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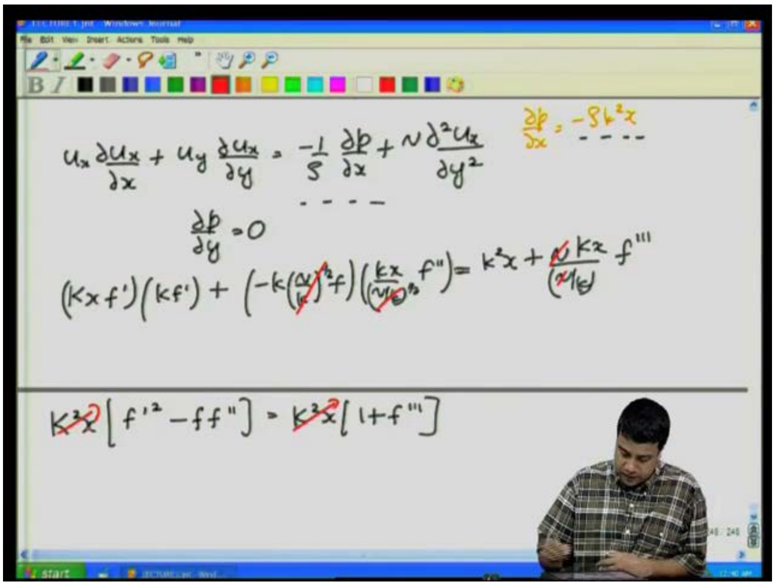

So, I can substitute for partial p by partial x as minus rho k square x and put that into the equation after substituting the value of the velocity U x U y and its gradients. So, let us put that in. So, u x is given by k x times f prime, partial U x by partial x is k f prime plus U y is given by minus k nu by k power half f, partial U x by partial y is and partial U x by partial y is given by k x by nu by k power half f double prime. Partial p by partial x is minus rho k square x.

So, I just get plus rho k square here. I am sorry I just get plus k square x here. Now, partial p by partial x is minus rho k square x and I have minus 1 by rho partial p by partial x here. So, that basically just gives me plus k square x plus kinematic viscosity into the second derivative, kinematic viscosity into the second derivative. So, the kinematic viscosity into the second derivative is given by this expression nu pi into k x by nu by k into f double prime, the third derivative.

And as you can see each term in the equation is multiplied by k square x. In this expression these two will cancel out to give you a factor of 1. In this expression here nu will cancel out to just give me k square x. So, I finally, get an expression which has k square x on each term. f prime square plus minus. And I can cancel out k square x on each term, I can cancel out k square x on both sides, divide throughout.

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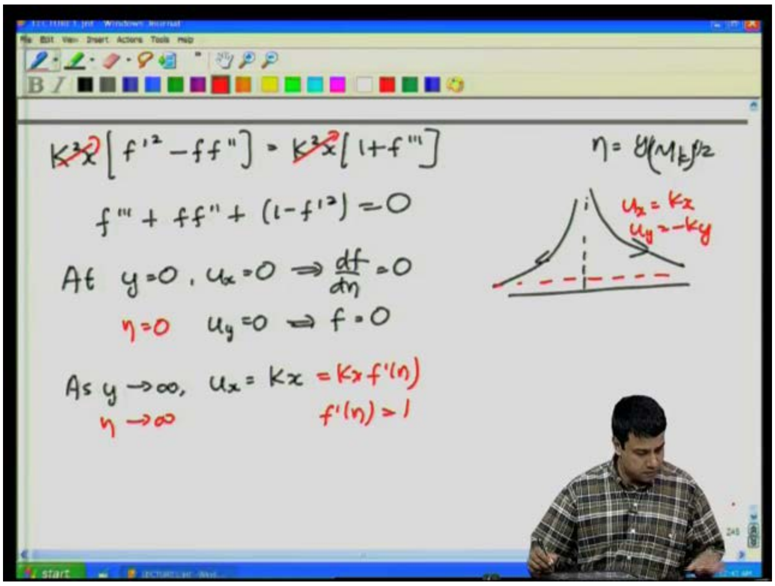

And my boundary layer equation becomes f triple prime plus f f double prime plus 1 minus f prime square is equal to 0. So, that is the boundary layer equation for this particular case, the stagnation point flow, the flow towards the surface. This is once again a third order equation. It has to be solved subject to boundary conditions at the surface itself. At y is equal to 0 U x is equal to 0 which implies that d f by d eta is equal to 0. You also require that U y is equal to 0 which means that f itself is equal to 0.

This y is equal to 0 also corresponds to eta is equal to 0 because we know that eta is equal to y by nu by k power half. So, y is equal to 0 also corresponds to eta is equal to 0. What about as y goes to infinity? U x is equal to capital U which implies that I am sorry U x is equal to k x. As y goes to infinity as we recall we have to recover the auto flow solution. So, I have a boundary layer, a thin boundary layer here. In the limit as y goes to infinity, as as y goes to infinity I have to recover the auto flow solution here which has U x is equal to k x, U y is equals to minus k y.

So, as y goes to infinity U x is equal to k x and we know that U x is equal to k x times f prime of eta in terms of the function f. Therefore, this requires that as eta goes to infinity f prime of eta is equal to 1. So, clearly this equation has to be solved subject to these boundary conditions in order to find out what is the solution for the velocity field. And once again these can be solved, these these solutions can be obtained.

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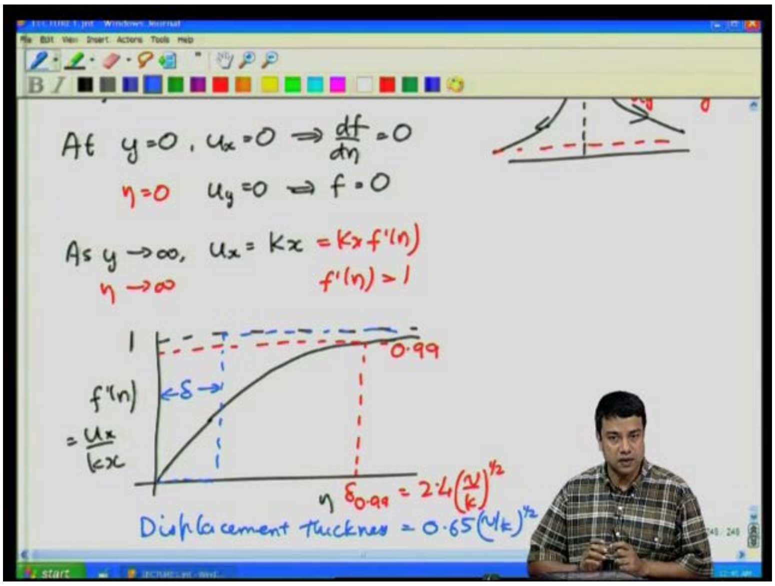

But you find once again like the Blasius boundary layer if you plot the velocity field eta times f prime of eta is equal to U x by k x. You have free stream velocity. Note, that now U x is been scaled by the factor that is dependent on x. So, in the limit as eta goes to infinity I should recover the value of 1 here. The actual numerical solution that you get will look something like this. If you try to use the same measures that we had earlier delta 0.99 as the value of the velocity, of the eta at which U x by k x is equal to 0.99 that is u x reaches 0.99 of its free stream value.

This delta of 0.99 is equal to 2.4 times nu by k power half. In contrast to the Blasius boundary layer this, the boundary layer thickness in this case is a constant, that is because the boundary layers accelerating downstream. It is not remaining, the boundary layer is not remaining at a constant value, but rather it is accelerating downstream. Now, can I also calculate the displacement thickness? How much do you need to displace this boundary layer by, what is the displacement thickness by which you need to displace the boundary layer in order to get the exact same flow rate as you got for the potential flow?

That is if I displace the potential flow by a constant thickness, how much would I have to displace it by in order to get the same flow rate that I am getting in the boundary layer? And this displacement thickness in this case turns out to be this is 0.65 times nu by k power half. So, the exact same calculation that we did earlier can be done in this case as well. I took a particular example of a flat surface with fluid that is incident on the surface, turns out that this is applicable more generally in other cases as well.

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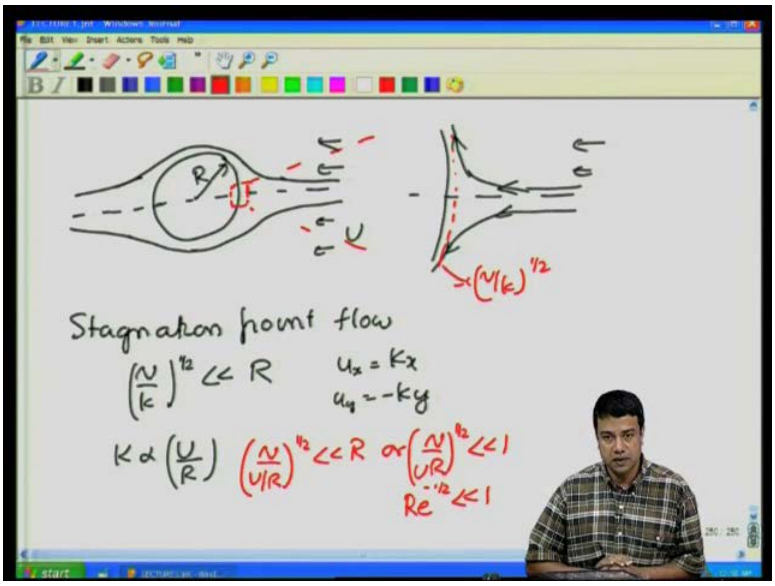

If you recall if you have for example, a flow past a cylinder, if you have a flow past a cylinder the flow at the front edge of cylinder goes around and comes back at the real edge. The, this is the potential flow solution for the flow past a cylinder. It goes around the cylinder. If you focus attention on a thin region over here and expand it out what it looks like is surface like this with flow that is incident on the surface. So, in two dimensions this looks very much like a stagnation point flow.

For this stagnation point flow I have a thin boundary layer of thickness here which goes as nu by k power half. For this configuration to approximate the configuration of the flow past a flat surface I require that this boundary layer thickness has to be very much smaller than the radius of curvature. I require that this boundary layer thickness has to be very much smaller than the radius of curvature. So, for this to approximate stagnation point flow nu by k power half has to be much smaller than the radius of the sphere. Otherwise, the curvature will affect the flow near this point.

Therefore, for this to approximate the stagnation point flow nu by k to the half has to be much smaller than the radius of the sphere. What is k itself? k is a strain rate, it has dimensions of inverse time because if you recall we got U x is equal to k x and U y is equal to minus k y. k itself has direction, has dimensions of inverse time. So, it is something like the strain rate. The strain rate for this flow will go as the mean flow velocity past the cylinder; strain by itself will be proportional to the mean flow velocity divided by the radius itself because the radius is the only large scale in the flow.

So, this requirement that nu by k power half becomes much smaller than R, basically translates to nu by less than R or if I divide throughout by R, I will get nu by U R power half, nu much less than 1. nu by U R power half is inverse of the nu by U R is the inverse of the Reynolds number for the flow past the cylinder. Reynolds number based upon the radius and the mean the flow velocity. Therefore, I require that the Reynolds number power half minus half, this minus less than 1.

So, Reynolds number power half basically gives you the ratio of the boundary layer thickness at the surface to the radius of curvature of the cylinder. And if this is small enough for the Reynolds number is large enough the flow near the (( )) stagnation point can be well approximated by an extensional flow of the kind, the, by the flow in a corner. So, this holds for the two dimensional objects. A similar calculation can be performed for the three dimensional objects.

We can also do a similar calculation for the flow past the cylinder for example, of a, of a sphere for example, using a cylindrical coordinate system. That is a slightly different calculation where it still yields to a very similar result as well. I would not go in to the details of that, but I will just restrict attention to two dimensional flows. So, we have calculated boundary layer solutions for the velocity profile using the similarity transform for two cases.

One is the flow past a flat plate; the second is the stagnation point flow, the flow in the corner of angle 90 degrees pi by 2. One can ask the question the other way around, what are the kinds of mean flow velocity profile which will admit a solution which is of the boundary layer type. You will see that in the next lecture, but let me just say that not all velocity profiles can be reduced to boundary layer solutions. When we looked at heat and mass transfer we found that any velocity profile does have a boundary layer type solution for the concentration of the temperature field.

That we have got as an exact result in that, in heat and mass transfer. In this particular case it turns out that the momentum conservation equation is non linear than the velocity field and for that reason not all boundary layer, not all mean velocity profiles can be reduced to a boundary layer solution. There are only specific forms of the velocity profile that can be reduced to a boundary layer solution. So, in the next lecture we will address that question.

What are the mean velocity profiles for which I can derive a similarity solution for the boundary layer equations? It will turn out that there are only specific forms and for that case we will see that there are various kinds of solutions, some in which the boundary layer thickness increases with the stream wise direction, in the case of a flat plate it increased. In this particular case for a stagnation point flow it was a constant. There will be some velocity profiles for which it actually decreases for the downstream distance.

In general if the velocity profile, if the pressure gradient is 0 as in the case of flow past a Blasius flat plate the velocity, the the boundary layer thickness increases as x power half. For this case where there is a pressure gradient, the pressure increases as you go downstream because we saw that p is equal to minus rho k x, rho k square times x. p is equal to minus half rho x square k square x square. Therefore, d p by d x is equal to minus rho k times x. That is an accelerating flow.

The velocity is increasing proportional to x as you go downstream. For this particular type of acceleration the boundary layer thickness was a constant. Next lecture, we will try to derive general relations about how the boundary layer thickness increases or decreases depending upon whether the mean flow is accelerating or de accelerating. We will continue this boundary layer thickness in the next lecture. We will see you then.