Fundamentals of Transport Processes II
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Lecture - 32
Boundary Layer past a Flat Plate

Welcome to this. This is lecture number 32 of our course on fundamentals of transfer processes. We were going through fluid mechanics in this course, currently we are at the stage, where we discussing boundary layer theory.

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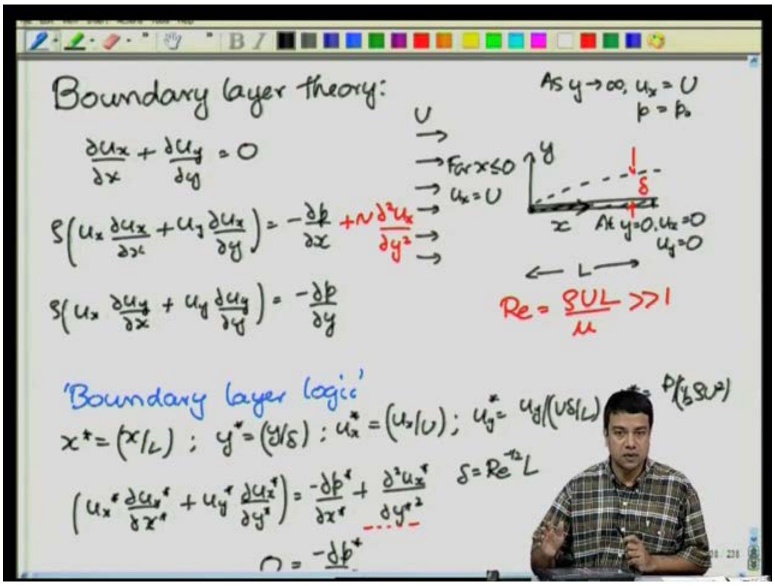

To recall the motivation for this, we first derive the equations of motion from the Navier's stoke mass and momentum equation for a Newtonian fluid, Newtonian and incompressible fluid. Then we scaled the equations and found that the ratio of inertia and viscosity is the Reynolds numbers. Then we were systematically analyzing the characteristics of fluid flow in different Reynolds number resins. First, we looked at low Reynolds numbers, a Reynolds number as you recall is rho u d by mu, where d is the characteristic length.

We were first looking at low Reynolds number where the velocity or the length scale is small, so that viscous effects are dominant. We looked at various ways of solving the Navier's stoke equations. The methods of solution are broadly similar to those for the diffusion equation, the Laplace equation, in the case of a heat and mass transfer. Then we looked at high Reynolds number resin and in this case, you can actually get non trivial solutions for the velocity field, because the momentum equation contains the pressure. In the concentration and the temperature equation, in mass and heat transfer, the solution just reduces to first order equation partial c by partial t plus u dot grad c is equal to 0.

Whereas in the case of fluids we also have a pressure which is required to enforce the incompressibility condition, so that was what we saw in the case of high Reynolds number potential flow in visit the viscosity is equal to 0. Irrotational that means that the verticity is equal to 0, there is no circulation within the fluid. So, it is all the potential flow problems. Of course, when you neglect viscosity the Navier's stokes equations reduce from a second to the first order differential equation and importantly the momentum conservation equation reduces from a vector equation to just the Bernoulli equation for the pressure.

Because of that one cannot satisfy the tangential velocity or stress conditions at bounding surfaces. One can satisfy the normal velocity or the normal stress conditions, but not the tangential velocity or stress conditions. The physical reason for that is that when we neglected viscosity, we neglected the diffusive transport of momentum. Momentum is transported both due to convection and diffusion, when we neglect viscosity there is no diffusion. Convective transport occurs only in the direction of the flow for the fluid velocity to come to a rest at interfaces or for the interface for the surface to effect the fluid velocity.

It is necessary that momentum gets transported from the surface to the fluid or vice versa. When you neglect diffusive effects there is no transport perpendicular to stream lines. The fluid at a surface if you have a no penetration condition, the fluid flows tangential to the surface. Therefore, the convective transport of momentum can occur only in the tangential direction. There is no momentum transport perpendicular to the surface. Therefore, it is not possible to satisfy the tangential velocity or stress conditions. The sheer stress in the absence of viscosity is identically equal to 0, so of course, you would expect the potential flow solutions to be valid far from surfaces.

However, as you come close to the surface if you want to be able to satisfy the tangential velocity boundary conditions. Of course any real situation this one way you have to satisfy the tangential velocity boundary conditions. If you want to satisfy these, then you have to include the effects of momentum diffusion very close to the surface. So, there is a thin layer near the surface where you have to incorporate momentum diffusion in order to be able to get a valid solution, which satisfies the tangential velocity or tangential stress conditions at the surface.

Last class we were looking at how to do this for the flow passed a flat plate the simplest configuration that one can consider? The configuration that we had chosen was just a flat pate of infinite decimal extent the thickness of this plate is assumed to be 0. It is a flat plate of infinite decimal extent of a fixed length l along the x z direction. We assume this is a two dimensional configuration, so that the plate is of infinite extent in the direction perpendicular to the board. So, we assume that it is of infinite extent in the direction perpendicular to the board.

Then we have a coordinate system along this plate, we choose the origin of the coordinate system along the leading edge of the plate. We choose the origin at the leading edge. The x coordinate is along the length of the plate and the y coordinate is perpendicular to the length of the plate. So, this is the simplest configuration that one can consider its two dimensional because there is no variation in the direction perpendicular. To the x y plane the fluid is incident on this plate with a constant velocity u the fluid is incident on this plate with the constant velocity u.

So, far away from the surface the diffusion from the plate will not affect the velocity field far away because there are two mechanisms. Now, one is momentum diffusion from the plate, which takes place in all directions. Of course, but primarily the momentum diffusion that slows down the fluid at that plate is occurring perpendicular to the surface of the plate. Of course, there is also a convection the momentum is being swept backwards by the fluid because it is being swept downstream by the fluid and is diffusing from the plate for that extent. For that reason, we expect the momentum to be restricted to a thin region near the surface.

We expect the momentum diffusion to be restricted to a thin region near the surface. We will quantify what this thin region is a little later the Reynolds number based upon the density of the fluid the free stream velocity the length of the plate is large compared to 1. So, this is the high Reynolds number flow, so far away the solution for the velocity field would just be a constant velocity. Everywhere the solution for the velocity field far away is just a constant velocity field everywhere, because that satisfies the de conservation equation, the momentum equation without the viscous term. So, you require as boundary conditions since the flow is not affected by diffusion from the plate far away, how far we shall see?

But it is if you go sufficiently far away, there is going to be the region where the flow is not affected by diffusion from the surface. In that case you require that as y goes to infinity, u x has got to be equal to the free stream velocity. So, that is one of the boundary conditions. If you go sufficiently far away the velocity is equal to the free stream velocity. Also this is a high Reynolds number flow there is momentum convection downstream and diffusion from the surface. Therefore, before the fluid hits the plate itself, right there is the fluid does not feel the plate because the plate is downstream.

The only way through the fluid upstream of the plate can feel the plate is if there is diffusion of momentum in the stream wise direction. However, we have assumed that in the stream wise direction convection is large compared to diffusion that is basically what is high Reynolds number condition. Means, so if you neglect stream wise diffusion and we have done that in the last class based on physical considerations. If you neglect stream wise diffusion, then the velocity of the fluid before the leading edge of the plate also has to be the free stream velocity. So, you require that for x less than or equal to 0 u x is equal to capital U, upstream of the plate the velocity has to be the free stream velocity.

At the surface of the plate itself, you require that the velocity has to come to 0. At y is equal to 0 u x is equal to 0 and u y is equal to 0. So, these are the boundary conditions, last class we had derived the equations, the logic for that is as follows. If I consider just an in visit fluid the equations are just the Bernoulli equations or alternatively the equations momentum equation without the viscous term. Those equations the solution basically is one where the velocity is a constant everywhere the pressure is a constant everywhere we can easily verify if I put in the velocity is equal a constant. The pressure is equal to a constant into the momentum and mass conservation equations.

It will identically satisfy these equations so the mass and momentum conservation equations in the absence of viscosity are given by partial u x by partial x is equal to 0.

For this case its two dimensional, therefore I get. So, these are the conservation equations, if I neglect the viscous terms, if I put in u x is equal to capital U, u y is equal to 0 and pressure p is a constant half rho u square, you can easily verify that that identically satisfies all of these equations. But it does not satisfy the boundary condition at the surface of the plate.

Therefore, somehow we have to bring in viscous diffusion perpendicular to the plate in order to be able to satisfy the boundary condition at the surface of the plate. This boundary condition was brought in as follows, we used what is called the boundary layer logicoid, okay? If I included the diffusion term here, if I included the diffusion term in the equation plus new partial square u x by partial y square, I had neglected this diffusion term on the assumption that the viscous term is small compared to the inertial term in the limit as the Reynolds number goes to 0. Because r e is very much larger than 1, so in that case I neglected that the diffusion term in comparison to the convective.

The pressure gradient even though the Reynolds number is large, if this length is small, if the length or which the velocity varies the small length delta. Previously I had assumed when I scaled these equations to get the inviscid equation, let the relevant length scale both in the stream wise as well as the cross stream direction is the length of the plate itself. On that basis I had simplified the equations and then there was a pre factor of r e inverse multiplying the viscous term I, therefore I had neglected those viscous term. However, I could have a situation where the length scale or which the velocity varies perpendicular to the plate or the length scale or the length scale for which the diffusion perpendicular to the surface is small, compared to the length of the plate.

If that happens as you can see, there is a second derivative acting on in the viscous term. The length is small, the gradient in that direction is large, because the velocity is varying now over a smaller length. So, the derivative is large. Therefore, I could have a situation where the viscous term is comparable to the inertial term, even though the Reynolds number is large, provided the length scale over which this velocity varies is small. Of course, if the viscous in inertial terms have to be of the same magnitude as the Reynolds number becomes larger and larger you would expect that the length scale would become smaller and smaller, as the Reynolds number becomes larger and larger.

So, on this basis we had scaled the equations by scaling the y coordinate with the lengths scale delta rather than the length scale l. What we had got was that the relevant length scale for the x direction for the stream wise direction is still the length of the plate l for the y direction. I had scaled it as the velocity u x the natural scale for the velocity u x is just capital U itself. However, for the velocity scale in the y direction comes from the mass conservation equation from the requirement that partial u x by partial x and partial u y by partial y are of both of equal magnitude, such the length scale x is l and the length scale y is delta y is much smaller than the x length scale.

You find that u y is also much smaller than u x, the relevant length scale for the y coordinate was by u delta by l u i. That was the relevant length scale for the y coordinate and the pressure turned out to be just the inertial scaling. Once you did that the conservation equations ended up producing to… Okay? So, those were the mass a momentum conservation equations to leading order we have neglected terms proportional to r e inverse. In particular we have neglected the diffusion term in stream wise direction in the x conservation equation, partial square u x by partial x square has been neglected simply because if x a, if the length scale in the x direction is l, then partial square u x by partial x square goes as u by l square.

Whereas this cross stream a second derivative goes as u by delta square, which is much larger. These conservation equations were obtained to the specific choice delta is equal to Reynolds number power minus half times. In other words the thickness of the boundary goes as r e power minus half times the length of the plate as expected as the Reynolds number becomes larger and larger. The thickness becomes smaller and smaller in such a way that the diffusion term in the x momentum conservation equation continues to be of the same magnitude as the inertial terms and that is what is required to satisfy the tangential velocity boundary condition.

The other implication of this scaling is that the pressure gradient perpendicular to the surface is equal to 0. Note, that the y coordinate in this y momentum equation has been scaled by delta. That means that the pressure is a constant in the y direction only over a thickness comparable to delta. So, that means as I go away from the surface, there is no pressure gradient as I go distance comparable to delta. There is of course, in this particular case the pressure is a constant everywhere. Therefore, there is no pressure gradient in the x direction far away from the surface as well in this particular case as y goes to infinity the pressure is just equal to a constant, because u x is equal to capital U. It is an in visit flow and therefore, p is equal to minus half rho u square, which is a constant.

So, the y momentum equation is telling me that the pressure is invariant in the y direction. However, if I go far from the surface the pressure does not change with x the pressure is constant at any stream wise position as I go far from the surface. So, there are two things here; one is the modularly equations are telling me that the pressure does not depend upon y. However, for the outer flow as y goes to infinity the pressure is a constant this implies that even within the boundary layer at any value of y the pressure has to be a constant. That is true for this particular case of the flow passed a flat plate, it is not true in general for a general flow you could have a pressure variation in the stream wise direction in the outer potential flow or the outer in visit flow in which case there will be a pressure gradient in the boundary layer as well.

We will come back to that a little later, but in this particular case, there is no pressure variation in the stream wise direction. In the outer flow, pressure is a constant independent of cross stream position. Therefore, the pressure has to be a constant everywhere within the flow that means that because partial p by partial y is equal to 0 and p is equal to p naught as y goes to infinity implies that p is equals to p naught at all values of y. Therefore, for this particular case the pressure gradient in the, a boundary layer is also equal to 0.

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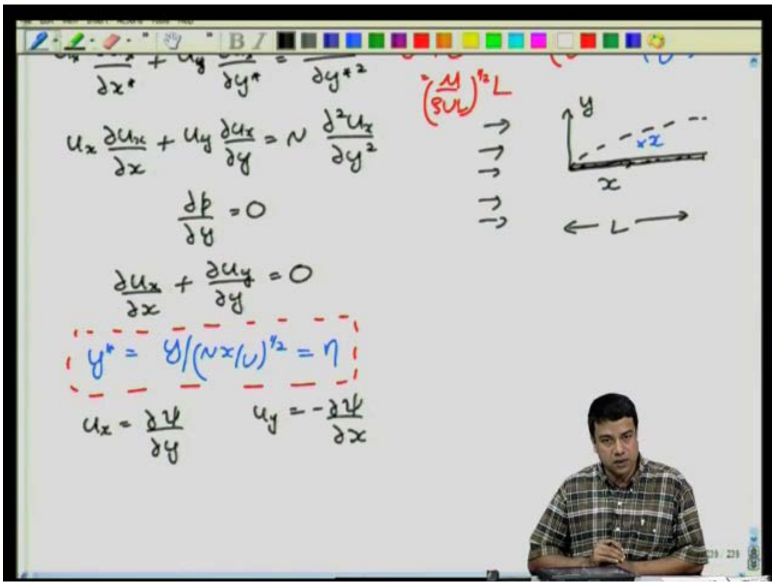

For this case the equations for this case the equations turn out to be equal to u x. This is in dimensionless form as I said the equation, the down in an terms of the equation are the same whether you express it in dimensional or dimensionless form. Because in order to get to the dimensionless form, we are dividing all terms in the equation by the same factor. Therefore, the equation turns out to be in dimensional form will be u x partial u x by partial x plus u y partial u x by partial y is equal to the kinematic viscosity nu times partial square u x by partial y square. The pressure gradient of course, is equal to 0 partial p by partial y is equal to 0, because partial p by partial y is equal to 0.

I was able to neglect the pressure gradient in the stream wise momentum conservational equation the pressure gradient that was supposed to be there. Here in the stream wise momentum conservational equation the pressure gradient that was supposed to be there, here in this stream wise momentum conservation equation could be neglected, because partial p by partial y is equal to 0. The pressure goes to a constant in the out of flow. Then of course, I have to satisfy the mass conservation equation that is partial u x by partial x is equal to 0. Now, the equation that I have is a second ordered differential equation in the coordinates x and y.

I had derived for you that one I did the scaling the boundary layer thickness delta, okay? Pi is equal to r e power minus half times l is equal to nu by rho u l power half times, so that was the value of delta I had got for the boundary layer thickness. That is the value of delta for a plate of length l; that delta is the value of the boundary layer thickness for a plate of length l. So, I have my x coordinate and my y coordinate and the flow coming in. If I scale my x coordinate by the length scale l for the length of the plate, then the boundary layer thickness becomes r e power minus half times l. You can see that I can also write this as nu x by nu l by u power half mu by rho is the kinematic viscosity nu.

Therefore, I get nu times l by nu power half is the boundary layer thickness delta. This is for a length of plate l scale, this way the equations are partial differential equations with variations in both x and y. Therefore, you need to solve a partial differential equation in order to get the complete solution subject to boundary conditions velocity is a constant. Far away the velocity is equal to 0 on the surface. However, one can make a simplification here and that is as follows the convective effects in this flow are large compared to the diffusive effects. Therefore, if an, and as you can see in these equations, I have neglected stream wise diffusion as I said partial square u x by partial x square has been neglected.

The reason is because in this stream wise direction, convection is much larger than diffusion. Therefore, stream wise diffusion has been neglected. Of course the cross stream diffusion has been included because over thickness, over a small flow boundary layer of thickness at the surface. When you include only stream wise convection, if I am sitting at a particular location x within the flow, if I am sitting at a particular location x within the flow the velocity field at that location x cannot be effected by locations downstream. Because convection is sweeping through a downstream at a neglected stream wise diffusion. Therefore, the velocity field at a given location x cannot be effected by…

What happens in downstream locations alternatively? The velocity field at this location x cannot depend upon the total length l, because that total length l is downstream of this location x the velocity field at a location x can only depend upon upstream conditions. That is the distance of the location x from the upstream edge of the plate as well as the distance from the surface y. What that implies is the boundary layer thickness at this location x does not depend upon the total length l, whether I have a plate of length l or two l, the velocity field at this point is going to be the same, simply because I do not have stream wise diffusion, which transmits information about the downstream locations.

Therefore, the velocity of the field at this particular location l I am sorry, at this particular location x cannot depend upon l, it has to depend only upon x the distance from the upstream edge, because here located our coordinate system at the upstream edge. Therefore, a boundary layer thickness that arrived in terms of this at a location x, if I define the boundary layer at given location x that cannot depend upon total, upon the total length l a boundary layer thickness; that I write can only be depend upon the location x. The only way to write that is to write this as nu x by u power, so at the given location x the boundary layer thickness can depend only upon x.

The kinematic viscosity which determines diffusion from the surface and the mean velocity which is sweeping the momentum diffused. So, this is the boundary layer thickness that I can use in order to find out what is the velocity field at a given location x. So, once I have done this right then my scaled y coordinate my scaled y coordinate becomes y star is equal to y by nu x by u power half. You can see that the scaled y coordinate it is now a combination of y and x scaled y coordinate is now a combination of y and x. If you recall what we did in heat and mass transfer, this is a similarity variable because this is a variable that contains both the x and y coordinates just based upon similarity arguments.

We manage to say that the flow field at a given location x depends upon x and y which cannot depend upon the total length l. The length scale the boundary layer thickness that comes out is nu x by u power half. Therefore, I get only one variable which contains both y and x a in heat and mass transfer. When we did this, we did this in the terms of time because in that case the time variable a, we got the dimensionless variable just based upon dimensional analysis. Later on when we did boundary layer theory, we got a variable which is very similar to this one and except it had a one third power.

If you recall, so this provides my similarity variable for solving the equations the original equations as expressed are functions of x and y on physical arguments of argue that a given location x. The flow dynamics depends only upon the location from the upstream edge not the total length of the plate. Now, that basis have got a local boundary layer thickness which depends only upon x when I defined a scaled variable, a scaled cross stream variable by scaling y by this boundary layer thickness you get a variable that contains both x and y. It is a combination of both x and y this variable eta that is a similarity variable.

As with similarity solutions if I substitute this into the governing equations, I will finally, get a single equation which depends only upon eta and which does not depend individually on x and y. So, that is going to be my check that the similarity solution for these boundary layer equations is actually consistent. So, we will take the similarity variable express the velocities in terms of this, put it into the equations and then verify that the final equations that we get. Both the final equations as well as the boundary conditions depend only upon eta and not individually on x y, the velocity u or the kinematic viscosity nu.

Now, we have got a similarities variable. The next step is to simplify the equations for the velocity field themselves. So, I have pressure gradient is equal to 0 in the y direction mass conservation equation and a momentum conservation equation two dimensional flow i can make use of expressing the velocities in terms of the stream function so that the mass conservation equation is identically satisfied. Therefore, I used the stream function formulation u x is equal to partial psi by partial y and u y is equal to minus partial psi by partial x where psi is the stream function. Both x and y will now be expressed in terms of eta through this similarity transform.

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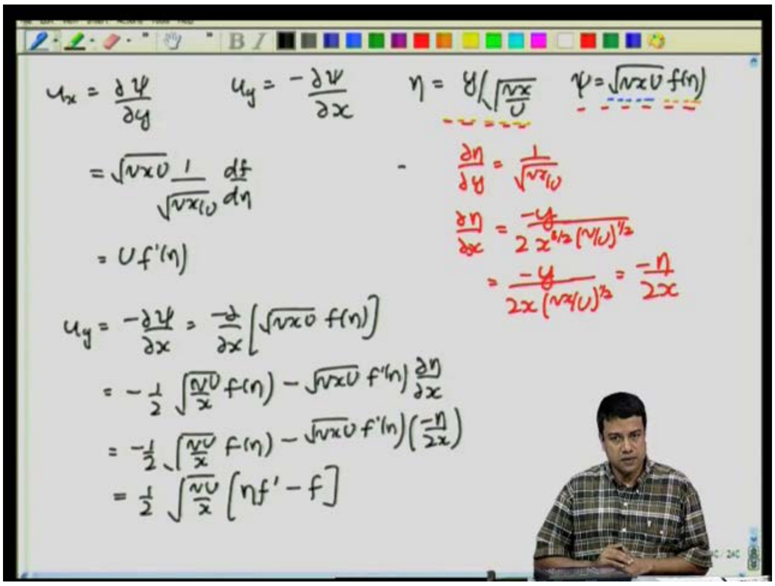

So, let us do that, so I have stream function psi is equal to I am sorry and u y is equal to minus partial x, so first I have to define a dimensionless stream function. You do it as follows i scale the x coordinate, I am sorry, I am sorry i scale the u x the velocity u x and you know that the stream that similarity variable is y by root of nu x by u. So, i express the velocity u x in terms of the scaled velocity and the similarity variable eta, I can define u x star is equal to u x by u is equal to 1 by u partial psi by partial y.

If I express y in terms of eta, I will get a factor of nu root of nu x by u coming out. So, this becomes 1 by u times 1 by root of nu x by u partial psi by partial eta is equal to 14 by root of nu x u. You can see from this that if I scale psi by root of nu x u, I can get an stream function that is dimensionless because u x star on the left hand side is dimensionless on the right side eta is dimensionless. Therefore, psi by root of nu x u has to be dimensionless, so for this purpose it is convenient to define the scale stream function psi is equal to root of u x u times some function of eta psi. Psi is equal to root of nu x u times some function of eta.

So, I will use this scale stream function in my equations in order to find a solution for the momentum equation. Note that this stream function automatically satisfies the mass conservation equation. So, there is no need to satisfy mass conservation equation in that case, so in terms of this stream function u x is equal to partial psi by partial y, which is equal to square root of nu x u times 1 by root of nu x by u partial f by partial eta just expressing psi in terms of f. By using this expression and y in terms of eta using this one and this should be a total derivative, because of my postulate that the f is only a function of eta. So, this will end up being as you can see u times f prime eta where I am using the primes for differentiation with respect to eta.

This is the total derivative because I postulated that f is only a function of eta, okay? So, therefore u y is equal to minus partial psi by partial x and psi depends upon x through two terms one is this, first term here which has an x dependence in it and then there is a second term here, which depends upon eta. Eta in turn depends upon f, so for this reason it is convenient for us to write ahead of time the derivatives of eta with respect to x and y. We will use these expressions later partial eta by partial y is equal to 1 by root of nu x by u and partial eta by partial x is equal to minus y by 2 x power minus half.

Therefore, when I differentiate, I will get minus 1 by 2 x power 3 by 2 1 by 2 x power 3 by 2 into nu by u power half. This is a common factor of nu by u power half, I can also write this as minus y by 2 x into nu x by u power half and y by nu x by u power half is just eta again. So, this is just equal to minus eta by 2 x, okay?

So, this enables us to express the velocities u x and u y in terms of eta itself. So, let us do that so minus partial psi by partial x is equal to minus partial by partial x of root of nu x u f of eta. First I take the derivative with respect to square root of nu x u. When you take that derivative, you just get minus 1 by 2 root of u u by x f of eta minus root of nu x u f prime of eta times partial eta with respect to partial x. So, this is minus half an f of eta minus root of nu x u f prime of eta times minus eta by 2 x, because that is the derivative of eta with respect to x.

So, this I can combine as you can see I get a common factor of square root of nu x by u times the half outside. So, I get half square root of nu u by x into eta f prime minus f. So, that is the velocity u y as you recall for the x momentum conservation equation from the previous slide, we also require these two partial u x by partial x partial u x by partial y in the second derivative. So, we will calculate those as well, okay?

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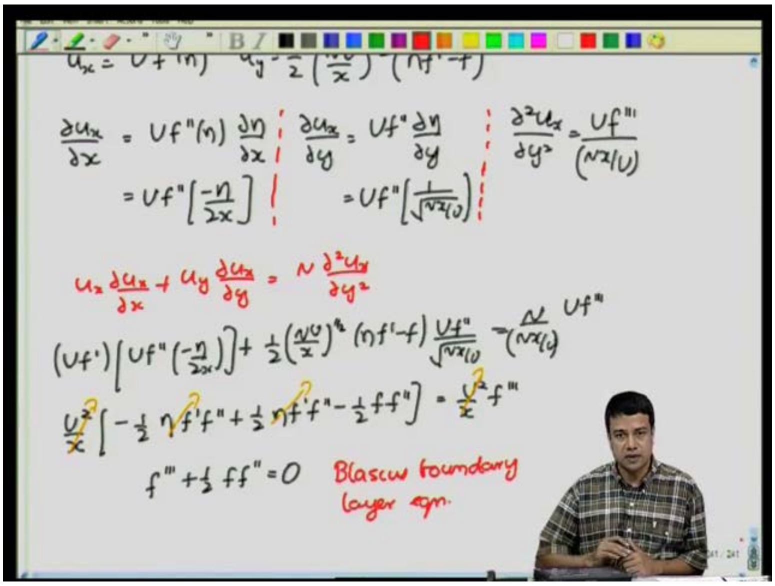

So, I have u x is equal to u times f prime of eta u y is equal to half square root of nu u u by x power into eta f prime minus f. Next need to calculate partial u x by partial x is equal to u f double prime the second derivative with respect to eta times partial eta by partial x. As you recall this was equal to u f double prime into minus eta by 2 x because partial partial eta by partial x was equal to eta by 2 2 x minus eta by 2 x here. So, that is the velocity partial u x by partial x, then I also have partial u x by partial y, which is equal to u f double prime partial eta by partial y is equal to u f double prime into 1 by root of nu x by u power half.

There is partial u x by partial y and the last thing that I need for the right hand side is the second derivative, which is quite easily to easy to obtain partial square u x by partial y square is equal to u f triple prime by u x by u, because when you take the second derivative, I just get a factor of 1 over square root of nu x by u times f triple prime. Multiplying this term here, note that this is f triple prime, so it is a third derivative with respect to eta. It is the third derivative with respect to eta. Now, I put this into my conservation equation u x partial u x by partial x my partial y square.

So, this becomes u f prime into u f double prime into minus eta by 2 x plus u y, which was half nu u by x power half into eta f prime minus f times partial u x by partial y, which was equal to u f double prime by root of nu x by u. On the right hand side, I have the kinematic viscosity by nu x by u times u f triple prime. If you done everything correct there is a common factor of u square by x on all the terms in the equation. If you done everything correct, there is a common factor of u square by x on each and every term in the equation. You can verify that each term has a pre factor of u square by x, so this will give me u square by x into minus half f prime f double prime plus eta.

Actually minus half f f double prime is equal to u square by x into f triple prime. Now, you can see that this common factor of u square by x cancels out on both sides and i end up with an equation, which is independent of the velocity u the kinematic viscosity nu or x. It depends only upon eta, so that is a check that our similarity solution actually works the resultant equation that I get is independent of x nu and u. It depends only upon the similarity variable eta, in addition these two terms also cancel out you can see minus half eta f f prime f double prime plus half eta f prime f double prime. Finally, you get the equation for the scaled stream function of the form f triple prime plus half f f double prime is equal to 0.

This is called the Bessel’s boundary layer equation and this equation has to be solved subjected to boundary conditions, in order to find out what is the velocity field. So, this for the stream function the scaled stream function as you recall this a the scaled stream function psi is equal to nu x u power half times f of eta. So, what are the boundary conditions here?

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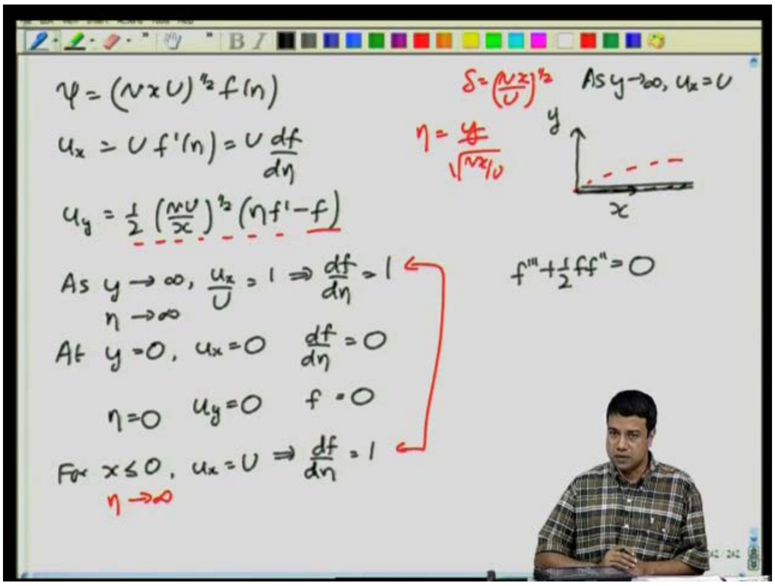

So, we had the flat plate y x there was a boundary layer there whose thickness was increasing proportional to x power half the boundary layer thickness was delta is equal to nu x by u power half. The solution psi was the solution for the stream function psi is equal to nu x u power half times f of eta and u x is equal to capital u times f prime of eta just u d f by d eta and u y was equal to this expression here. So, these are the boundary conditions, we have to impose as y goes to infinity u x is equal to capital U as y goes to infinity u x is equal to capital U; that means that as y goes to infinity u x by u is equal to 1, which implies that d f by d eta is equal to 1.

So, the slope d f by d eta has to go to a constant value 1 as y goes to infinity at the surface itself at y is equal to 0, u x is equal to 0, u x is equal to u times f prime. That means f prime has to be equal to 0 d f by d eta is equal to 0. In addition you also require the normal velocity has to be 0 normal velocity contains two parts nu x by u power half times eta f prime minus f. This has to be 0, we know that f prime is already 0, because u x has to be 0, which means that f has to be equal to 0, y is equal to 0. Corresponds to recall that eta is equal to y by root of u x by u, so y is equal to 0 corresponds to eta is equal to 0, y goes to infinity corresponds to eta goes to infinity at constant x.

If you recall, I also had another condition that is at the edge of the plate. The fluid has not felt the flat plate yet because I have a high Reynolds number flow where momentum is been convicted across the plate. At the upstream edge the fluid has not felt the plate yet. Therefore, the velocity is this undisturbed for x less than or equal to 0 u x is equal to capital U, which implies that d f by d eta is equal to 1 at x is equal to 0. What is the value of eta as you can see at x is equal to 0, eta is equal to infinity because eta is equal to y by the root of nu x by u.

So, this corresponds to eta going to infinity x is equal to 0 corresponds to eta going to infinity. You can see that this either the upstream edge or far away from the plate as y going to infinity. They are the same in the eta coordinate the boundary conditions are also the same these two boundary conditions are at different locations in x and y, but they are the same location in the similarity variable eta. The equations are telling me that the solutions; the boundary conditions that has been satisfied is also the same in terms of this similarity variable eta.

So, I have two conditions at y is equal to 0 or eta is equal to 0, one condition as eta goes to infinity three conditions at third order differential equation as you just saw f triple prime plus half f f double prime is equal to 0, okay? So, this equation can be solved subject to the boundary conditions to get a solution for f this is a non-linear equation a. So, it is not a linear equation it is not as easy easy to solve analytically. It is a non linear equation, but never the less one can get a solution for this non linear equation by numerical methods and the solution for this equation turns out to be of this form.

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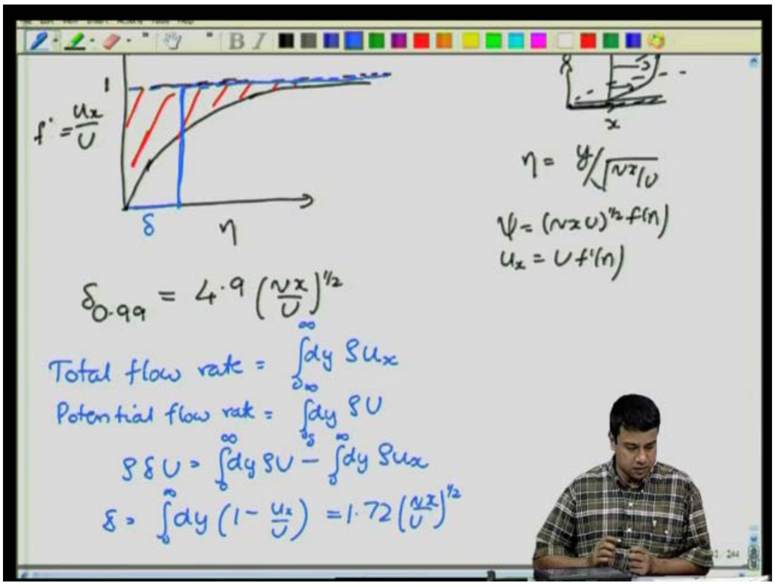

If I plot the velocity as a function of eta just for your reference this is my plate this is my x this is my y coordinate. The boundary layer this eta coordinate is a scaled distance from the surface is equal to y by root of nu x by u. This eta is a scaled distance from the surface the velocity has to increase as y increases and outside the boundary layer it comes nearly equal to the free state velocity u within the boundary layer. Of course it has to decrease to 0, the solution within the boundary layer is what we just obtained in terms of the stream.

Function psi is equal to nu x by x power half times f of eta and we obtained u x is equal to u f prime of eta that means as a function of this distance eta. I can plot u x by u which s f prime that f prime has to go to one far away because u x is equal to capital U. So, that is go to one far away because u x is equal to capital U at eta is equal to 0. It has to decrease to 0 because the velocity has to decrease to 0 is that means that f prime. This is equal to f prime, this is equal to f prime, this has to decrease to 0. You get a profile that goes something like this, you get a profile that goes something like that, let me just plot it a little better starts at 0.

Then it approaches one asymptotically; that means the limit as eta goes to infinity f prime is equal to 1 at any finite eta f prime is always slightly less than 1, but it approaches one far away. So, this momentum diffusion thus takes place all the way, but the effect is very small as you go far away the the boundary layer thickness delta is often referred to as delta 0.99 is the distance at which the velocity is equal to 99 percent. the free stream velocity the distance at which the velocity is equal to 99 percent. The free stream velocity that is what is taken as the boundary layer thickness commonly, so this is delta is equal to 0.99. The numerical solution will tell you that this is equal to 4.9 times nu x by u power half.

So, that is the boundary layer thickness 4.9 times nu x by u power half is the distance at which the velocity approaches 99 percent of its free stream value. There are other definitions that are often used for boundary layer thickness; one is the displacement thickness that is if this entire velocity profile. If this entire velocity profile that I had I had, so this is the velocity u x by u that I actually have for the actual flow my potential flow solution. Basically says that the velocity is just equal to a constant the potential flow solution. Basically says the velocity s equal to just this constant value.

So, the actual flow rate pass, this plate is actually smaller than the flow rate you expecting the potential flow because the potential flow is assumed that this constant velocity one. The actual flow rate has a lower velocity. Therefore, there is a deficit in the flow rate corresponding to this area under the curve this much is the deficit in the velocity, because this plate is is slowing down fluid close to the surface. There is this much deficit in the velocity because of the presence of this plate, okay?

The displacement thickness basically is the thickness by which we have to shift the velocity displacement. Thickness is the thickness by which we have to shift the velocity in such a way that the flow rate obtained with this displacement from the surface of the potential flow is identical to the actual flow rate that you get. In other words, if I displace my potential flow by this displacement thickness delta, this deficit and velocity is equal to delta times capital U. So, that is the displacement thickness, the displacement thickness as you can see the total flow rate is given by is given by integral 0 to infinity d y rho times the actual velocity rho times u x.

The potential flow rate is equal to integral 0 to infinity d y rho times capital U, which is the external flow field. This I can write it as integral 0 to delta d y rho u plus integral, so this can be written as y a or I may write it other way. So, I have the total flow rate is is exceeding the potential flow rate by this that is by rho u times delta. So, the displacement thickness is defined as rho delta times u is equal to integral 0 to infinity d y rho u minus integral 0 to infinity d y times rho u x. There is a potential flow velocity minus the actual velocity and from that you can easily verify that the displacement thickness is given by delta is equal to integral 0 to infinity d y into 1 minus u x by u.

So, this basically measures the deficit in the flow rate in this region, because the fluid is been slowed down by the presence of the plate. So, this if you calculate from the numerical solution you cannot calculate exactly the displacement thickness as we calculate from the numerical solution. This turns out to be equal to 1.720 nu x by u to the power half, okay? So, this is the displacement thickness the boundary layer thickness is commonly taken as the distance at which the boundary layer reaches 99 percent of the free stream velocity.

So, we look at a few other features of this Bessel’s boundary layer. In the next lecture before we go on to the boundary layer for another configuration that is the stagnation point flow kindly go through this the logic behind this this exercise. Before you come to the next lecture, because we will stop here this logic is identical to that of all other kinds of potent boundary layer flows. This will be used repeatedly when we look at different types of boundary layer flows. We will continue this briefly in the next lecture before going to boundary layer for another kind of flow and we will see you then.