Fundamentals of Transport Processes II
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Lecture - 31
Boundary Layer Theory

So, welcome to this, this is lecture number thirty one, in our discussion of fluid mechanics. We have completed a couple of topics so far, we first derived the conservation equations for mass and momentum for a Newtonian fluid. And then we were looking at limiting cases; the first is the limit of low Reynolds number where one can neglect inertial effects in comparison to viscous effects. In that case we saw, we could derive linearised equations for the pressure and the velocity field, and we looked at various ways of solving these equations. Then we switched over to the limit of high Reynolds number, and the limit of high Reynolds number of course you neglect viscosity in comparison to inertia.

We looked at a specific case; one is in visit viscosity is zero. The second is rotational, that is the verticity is zero everywhere within the flow, and in that case we had derived the potential flow equations. Since, the verticity is zero, the velocity can be expressed as the gradient of a potential, and the pressure is given by a scalar equation, the Bernoulli equation, which relates the pressure to the kinetic energy per unit volume as well as the change in the potential with respect to time. And while discussing potential flows we had noted that since, we are neglecting the viscous terms, there is no momentum diffusion taking place there is only momentum conviction, a consequence of that is that we cannot satisfy by transactional boundary conditions at surfaces.

Mathematically, the reason is because we are neglecting the highest derivatives. The original equation was second order differential equation for the velocity vector u and that had two boundary conditions at each surfaces, that no slip surface in case of solid conditions, that is the both transactional and the normal components (( )) velocities are zero at these surfaces.

In the case of liquid gas, of course there is the gas sphere stress condition and the normal stress balance condition. Since, we neglected the viscous term in the conservation equation it was reduced to a first order equation. And for these potential flow equations, we were able to satisfy only the normal velocity or the normal stress conditions at the surfaces. Mathematically, the reason was because we reduced the equation from second order to a first order differential equation. Physically the reason is that we neglected the diffusive transport of momentum, the flow along the surface.

Since, there is the no penetration condition at the surface the flow takes place transactional to the surface. Convective transport takes place, only along the direction of flow. Therefore, convective momentum transport takes place only transactional to the surface. If you neglect momentum diffusion there is no transport of momentum perpendicular to the surface.

And this transfer of surface perpendicular to the surface is necessary. If you want to slow down the surface, if you have the free stream going at the finite velocity. If that velocity has to come down to the zero at the surface itself, there has to be diffusive transport of momentum at the surface, without that you will not be able to satisfy the transactional velocity boundary conditions of the surface.

So, the next topic is going to be boundary layer theory. In this case, we take into account that convective transport, I am sorry the diffusive transport of momentum perpendicular to the surface within the thin layer near the surface, in the limit of high Reynolds number. So, that is going to be our next topic of study, boundary layer theory.

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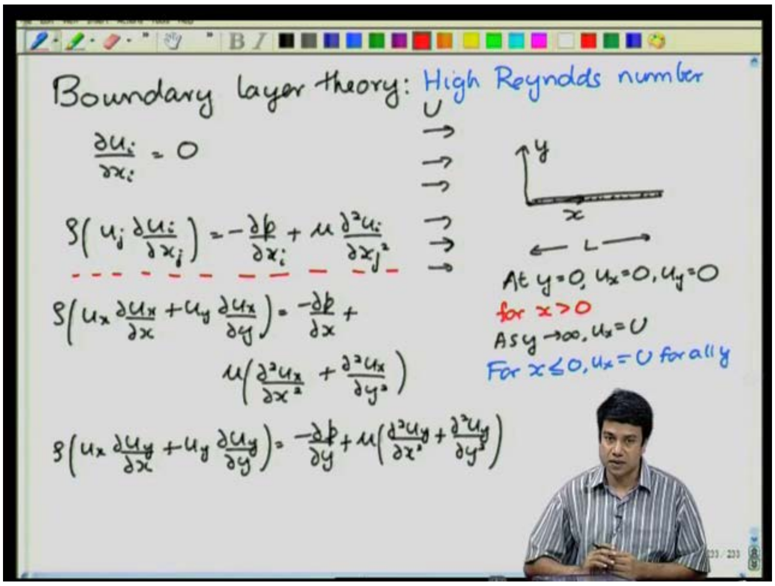

The simplest situation to consider is of course, the flow over a flat plate. Let say you have a plate, which is infinite to the thickness in a fluid and the fluid is moving in a constant velocity far from the plate. So, if I have a plate with certain length l, this is the solid plate with certain length l and I have the fluid incident this plate far away. Of course, at the surface of the plate it is satisfying the no slip boundary condition at the plate surface itself. This is the symmetric configuration, so we can without loss of generality just analyze place the flow above the plate.

Two dimensional systems, I assume that the plate is of infinite system that the direction is perpendicular to the mode. So, we used the two dimensional coordinate system, the stream wise direction is x, the cross stream direction is y. And the free stream the velocity is coming in with the velocity capital U that means, that the far from plate the velocity is equal to capital U. So, this is the two dimensional system I can analyze, I can use the two dimensional equation to analyze this problem.

So, the first thing is first what are the boundary conditions for this flow? You require that at the surface of the plate itself, both components of the velocity stream wise and cross stream has to come down to 0. So, the surface of the plate is that y is equal to 0 powers x greater than zero for x less than 0. Of course, in the coordinate system that I have used over here, there is no plate. So, the velocity can have any value, but for x greater than 0 the velocity has to come back down to 0 at the surface of the plate. That is both components of the velocity have to come for the 0 at the surface of the plate.

So, the boundary conditions are at y is equal to 0 and u x is equal to 0 and u y is equal to 0. Note that this holds only for x greater than 0 because the plate is present only when x is greater than 0. If you go far from the plate in the y direction, encount to the free stream the fluid far from the plate is not freely encounted the plate yet because it is at large distance away from the plate.

The velocity should be equal to the free stream velocity capital U. So, the second boundary conditions is that as y goes to infinity u x is just the free flow velocity u, as well as y goes to infinity u x is just the free stream velocity u. The next boundary condition these are in the y coordinate, you will see later in the y condition in the x coordinate as well, that condition is for that x less than 0.

Note that we are assuming this is the high Reynolds number flow, this flow is high Reynolds number flow. So, the fluid is swept past fluid at the velocity, fluid that is at x less than 0 is encounted the plate as yet, because it has been swept at high velocity fluid that is at x less than 0 has not encounted the plate yet. Therefore, you would require that the fluid is just the high stream velocity for x less than 0, u x is equal to capital U for all y.

So, that is the third boundary condition, equation for x is less than or equal to 0, u x is equal to capital U for all y. The fluid that is upstream of the plate, can encount at the plate only due to momentum diffusion in that direction. The point to be noticed is that the fluid upstream of the plate. If the convective effect is going to only the convective transport going to only effect the position at the downstream of the stream locations, not positions of the upstream because the convection takes place in the direction of flow it flow.

Therefore, the locations upstream of the flow can be sensed or can be effected by the plate, only if the momentum diffusion in that direction. And you will see a little later, that the momentum diffusion in the stream wise direction is small compare to the cross stream direction and we can neglected. In that case, the fluid upstream of the plate has not yet encounted the plate because there is no diffusion of momentum in that direction, that is the reason for this boundary condition. We will see little later, how this fits in naturally into the frame work that we will develop. Now, that we have the configuration and the boundary condition lets go on to equation such as.

So, the equation in the x and y directions, we can first write down the equation in the vector notation, partial u i by partial xi is equal to 0. And row this is the steady state configuration therefore, I will neglect the time velocity of the derivative of the velocity fields. Since, I am not looking at the variation of the velocity with respect to time. So, I will just take the convective term here. So, these are the vector equation one. I can write it down in terms of components because this is of only the two dimensional system.

If I write down in terms of the components of the system, I will get row into u x partial u x by partial x. So, that was the x momentum conservation equation and I have the similar y momentum conservation equation, which is row into that is y momentum conservation equation. Note that, if you want to get these equations, you have to start off with this equation and i is the free index it represents the vector direction.

So, in how to get the x momentum conservation equation, you would set I is equal to x in this indicial in the equation given indicial notation. J is repeated it is summed over, it is summed over j is equal to x and summed over I for a particular value of I, and I is x for the x momentum and I is y for the y momentum equation with sum over the values of j is equal to x and y.

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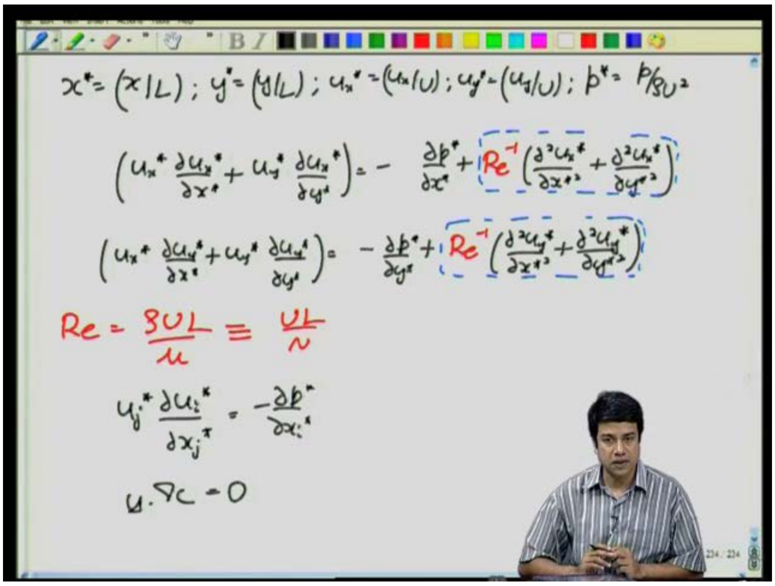

So, you can do the usual scaling in this case, the natural scale for x has to be the plate length l. Note that, there is no other length scale in the problem, plate is length infinite as for the thickness the l. So, I have naturally scaled the l, but the plate length l. Similarly, for y because there is no length scale in the problem, and if you are scaling the length the distance by l would naturally scaled velocity for the free streamed velocity capital U.

So, it defined u x star is equal to u x by U and u y star is equal to u y by U. Put that into the equation, you put that into the equation and I would not go into the details, and it's quite easy to obtain. What you will get is the row? U square by l into u x partial u x by partial x. So, that is what for the x equation and similarly, for the y equation you will get sorry that is what you get for the x and y equations, there should be a row u square by l here.

Since, I am considering the limit of high Reynolds number I am scaling by the inertial scales in, which case my scaled pressure p star, is equal to p by row u square. My scaled pressure p star will be equal to p star row u square. Once, if u done that you divide throughout by this p factor here, divide throughout by this p factor here, row u square by l on both sides of the equation. When expressed in terms of p star this this lens scale will get disappear, and when I divide throughout by row s u square by l, this just becomes inverse of Reynolds number. Where the Reynolds number is defined as the row u l by mu, which is also equal to equivalent to u l by mu. This also Reynolds base number on the free stream velocity and the length of the plate.

If we consider the plate of the high Reynolds number then of course, these two terms go away the consider the limit by high Reynolds number, then no long the I have the diffusive terms viscous terms. Therefore, my equation just reduces to first order equation of the form is equal to minus partial p by partial xi star, this reduces to first order differential equation.

In this particular case, the flow around the flat plate the pressure for the entrance are to be 0 the pressure for the entrance are to be zero because far away, the equation are in visit I just have a constant present everywhere. So, the pressure for the entrance is to be 0 I just find that grad u is equal to 0. If you recall when we did mass and energy conservation, we had the similar expression u dot grad c is equal to 0.

In the case of high peculated number flows numbers, which are conviction dominated. The equivalent in this case, in the absence of pressure radiant u dot grad u is equal to 0. I told you that when u dot grad c is equal to 0, it implies that the concentration does not vary along the stream direction lines. Similarly, in this case u dot grad u is equal to 0 the velocity does not varies along the stream lines the velocity constant along stream lines.

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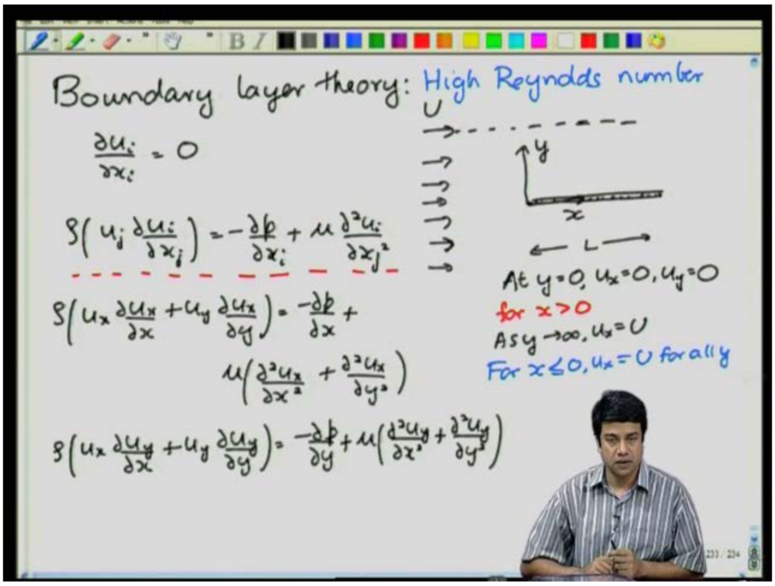

What that employs is that the velocity? Along any stream line is just independent of the stream line, along with the as you go with down the stream line. The velocity independent of the stream line, with that there is no way to satisfy the 0 velocity boundary conditions at the surface itself. That is because as I said, I have neglected the momentum diffusion and when I neglect momentum diffusion, the there is no mechanism for the velocity of the fluid to be influenced by the surface itself.

There is no way for the surface to exert the transactional stress on the fluid, but of course, in a real system the velocity does have to come to 0 at the surface. In a real flow, the velocity does have to come to 0 at the surface and the reason it comes to 0 is the following. If you recall in this equation, I have neglected I have neglected these two terms I have neglected these momentum diffusion terms that is because the Reynolds number is large.

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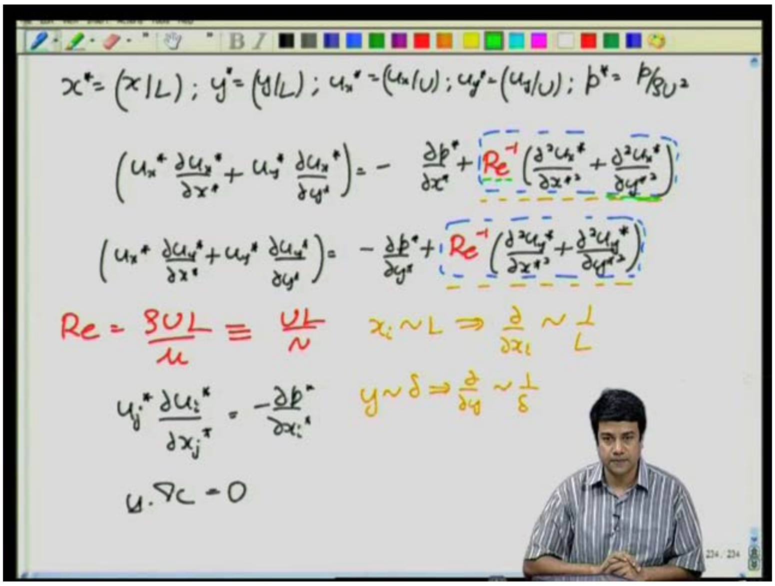

Therefore, Re inverse is small, Re was calculated assuming the relevant length scale of the length is the plate itself, and the Reynolds number was calculated assuming that. The relevant length scale is the length of the plate itself that means, which the relevant radian goes as the mean velocity free stream velocity, divided by l. So, what I have assumed is that, the length scale is that the length scale xi goes as l, which implies that the partial by partial xi goes as one over l. That is the radiant's per scale is the quantity divided by l, that is the variation in the quantity takes place over distance comparable to length of the plate l because that is the only lens scale in the system.

On that basis we find that the diffusion basis is re inverse or smaller than the convection terms. However, that does not satisfy the boundary conditions the velocity has to come to 0. What is the answer to this paradox? The answer is that, if the velocity variation takes place over a length, which is smaller than l. If the velocity variation takes place over a length scale, which is much smaller than l, then the gradients are much larger than one over l. If the variation velocity for example, in this case we are interested in diffusion perpendicular to the flow.

So, if the length scales for the y direction is some length delta, which is smaller than l, it will employ that partial by partial y goes as one over delta, which is much larger than one over l. And I have the second derivative on the right hand side with respect to the y coordinate. So, I could have a situation where delta is turning much smaller than l. So, one over delta square is much larger than one over l square.

So, I have the second derivative on this term, second derivative on this term that multiplies this re inverse Reynolds number is large. So, this re inverse is small, that was based up on the length of the plate. On the velocity I have the second derivative the lens scale is small, if it goes to delta the second derivative delta goes as one over delta square. And I could have the situation, where the product of these two one over delta square is the large number Re inverse is the small number.

I could have this situation with the product of this situation, of the same magnitude of the same of other terms in the momentum conservation equation. In particular the product of these two is the same magnitude of the convective terms, if that happens I have a balance between the convection and diffusion. And then I can satisfy the boundary conditions. This balance has to hold as the limit as I go to infinity, as I make Re larger and larger.

The Reynolds number based upon the plate length becomes larger and larger, the length scale delta becomes smaller and smaller. In such a way that, this second derivative times Re inverse is always of the same magnitude it does not go to infinity, it does not go to 0. In the limit is Re goes to infinity, this remains finite and it always balances the inertial terms that is the only way, you will be able to get a situation, and satisfies the boundary conditions in the transactional velocity.

So, what do we do now? We rescale our equations; partial at the length scale perpendicular to the surface of the plate is some small length delta. I require that because ultimately unless the diffusion terms balance the convection term, there is no way to satisfy the boundary conditions. I partial at there is the length delta and I will get what is the length scale delta? From the condition in Re goes to infinity this term remains finite, and of the same magnitude of the inertial terms. So, that is the condition that I will use.

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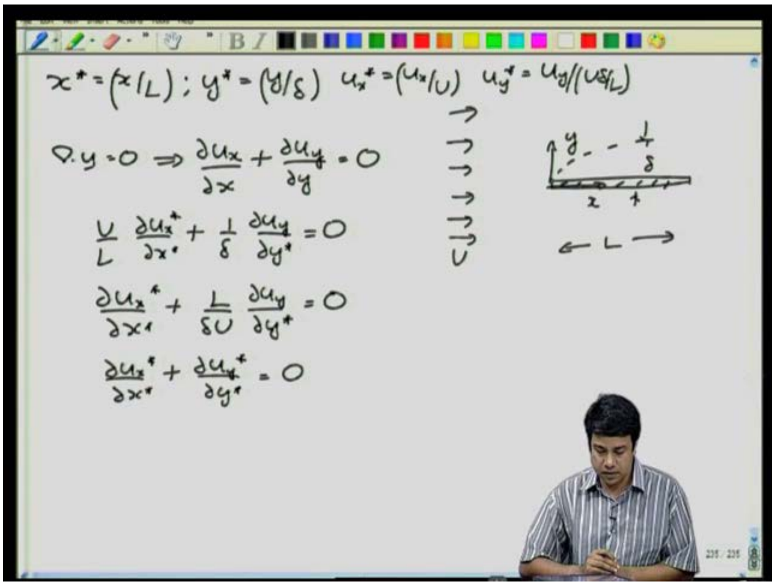

So, let us do that rescaling. So, basically I have plate of total length l, I assume that there is some small length delta here, which is much smaller than l. This is the x and this is the y direction and this stream fluid coming in. So, at the small length delta, the length scale in the stream wise direction is still l because that is the length scale, which varies in the stream wise direction. That is the plate length is l that is length scale varies in the steam wise direction.

So, I define x star is equal to x by L, that is the length in the stream wise direction and the scale y velocity will y by delta. I do not yet know what deltas? I will choose it from the requirement of diffusion of the viscous, and the inertial terms cutting you to be the same magnitude in the limit as re goes to 0. In other words, as Re becomes larger and larger delta has to become smaller and smaller, in such a way that the second derivative of the velocity times Re inverse remains of the same magnitude. The length scale, the scale for the velocity is equal to u x by u because far away from the plate, the velocity is capital U at the surface it has to come down to 0. Therefore, it has to vary between 0 and capital U.

So, U is the free stream velocity, what about the velocity perpendicular to the plate in the cross stream direction. The scaling obviously, the velocity of perpendicular with u expected to be 0 far away from the plate because far away from the plate. We just have the unidirectional flow with the constant velocity capital U, you would expect to be small far away from the plate.

What about the velocity U? At the surface of plate itself, the velocity scale for that is not capital U, one has to be careful, when evaluating this velocity scale. The way you get it is going to the mass conservation equation, and scaling all term in the mass conservation equation. So, the mass conservation equation is del dot u is equal to 0, which implies that partial u x by partial x plus is equal to 0. I express each of these in terms of the scale variables. So, u x is equal to capital U times u x star and x equal to capital L times x star.

So, I will get u by l partial u x star by partial x star plus, I do not yet have scaling for u y yet, but I scale delta and then determine what is the scaling for u y. It becomes one by delta partial u y by partial y star is equal to 0. Divide throughout by p factor of the first term, that is first thing you can do, you will get partial u x by partial x plus l by delta U partial u y this is equal to 0. Now, all terms in the equations here, dimensionless except the u y term.

So, it a natural to define a scale term, u y star is equals to u y by U delta by l. And with that, the mass conservation equation just becomes partial u x by partial x plus partial y is equal to 0. Note; that the velocity scale for u y, the velocity scale for u y is u delta by U delta by l, the velocity scale for u y is u delta by l. So, with partial related to delta was smaller compare to l.

Therefore, the velocity in the y direction is smaller compared to the velocity in the x direction. So, it is a narrow natural consequence of the mass conservation equation. The mass conservation equation is telling you, the scalar by u x scale for x has to be same as, the scale for u y has to be scale for y. So, there is the natural consequence of the mass conservation conditions. So, that is the scale mass conservation equation, which is given as a scaling for the velocity u y.

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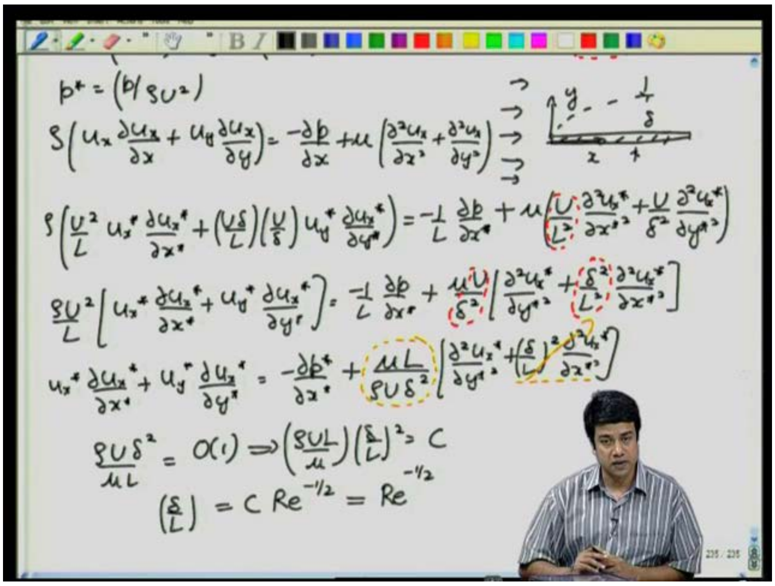

Next, let us go to the momentum conservation equation. So, the momentum conservation equation in the x direction, first we take the x momentum conservation equation because it is the x velocity, that we were having difficulty with the boundary condition. We had required that the x velocity has to come to 0 at the plate; the transactional velocity 0 has to come to 0 at the plate. The potential flow equations were not equal to the giving us time, because we did not satisfy the transactional boundary conditions. Therefore, we first analyze the x momentum equation; it is somehow, we have to get the diffusion out in the viscous. And the inertial terms would be comparable in order to be able to satisfy this boundary condition to be x velocity.

Now, we express each term in this equation, in terms of the scale variables. So, first term it has to u x, the u x is equal to capital U times u x star. So I have the u square and x is x star times l. So, I will get u square by l, u x star partial u x. The second term, u y u delta by l delta times u x, which is u divided by delta. So, you will get u delta by l into partial u x by partial u y will be u by delta.

The pressure you have, in the scaling of the pressure yet, the pressure scaling will come out of the requirement of some scaling, all the other terms in the equations. We will just leave the pressure for the moment, this is equal to minus 1 over L, partial p by partial x star plus. The viscosity into the scaling for u the u x is capital U, the scaling for x square is l square, partial square u x by partial x square plus U by the scaling for y is, now delta square. Therefore, the scaling for y is delta, the partial u x by partial y square will be U by delta square partial square u x star square by partial square y star square.

You can see that, the first term in the equation, the convective term it has the common factor o u square by l, it has the common factor o u square by l. So, let us take that out, I have row U square by l into partial u x by partial x, which is minus 1 over L partial p by partial x. In this second term, in this viscous term I have two terms, which is the stream wise second derivative of the velocity and the cross stream the second special derivative of the velocity. You can easily see that, the cross stream second derivative is much larger than the stream wise, because the cross stream goes as u by delta square stream, goes as u by l square and we portialited delta the much smaller than l. Therefore, 1 by delta square is much larger than 1 over l square.

So, I will take 1 over delta square as the common factor out. So, this is becomes mu U by delta square into partial square of u x by partial y square, plus a second term, which goes as delta square by l square, partial square u x by partial x square. You can see this term here this term here, multiply it by u y delta square here. You see back U by L square, which is the factor partial of u x by partial of x square. I just taken one common factor out, I am taking the largest factor as one in front, because I expect the largest term to be one balances the be initial terms.

So, I divide throughout by row U square by L divide throughout by row u square by l and once you do that, give you scaling for the pressure. If I divide row square by L the pre factor of pressure reduces to just 1 over row U square. So, I define p star is equal to p by row U square, and my questions becomes u x partial u x by partial x is equal to minus parcel p by partial x plus u L by row U delta square okay. Divide throughout by row square by L and the pre factor of the viscous terms becomes u l by row U delta square.

So, this illustrate what I had try explain a little earlier, in this particular case of the lens scale in the stream wise direction, is large compare to the length scale in the cross stream in the direction. The diffusive term, in the stream wise direction second terms direct to stream wise coordinate is much smaller than the diffusion terms and the cross stream direction. Therefore, I can neglect stream wise diffusion, in comparison into cross stream diffusion because the smaller thickness and the larger gradient are in the cross stream direction.

So, neglect this, and obviously there is going to be balance between diffusion, and a between viscosity and an initial or between convection. And diffusion in the limits Re goes to infinity, only if this term continues to the main of order one in the limits Re goes too infinitely. So, I require that the boundary layer thickness delta should be such that, this term continues to be finite it does not go to 0, it does not go to infinity, it goes as Re to the 0th power. It is independent of Reynolds numbers in the limit of Re going to infinity.

So, as the Reynolds number goes to infinity, this term has to be independent of the Reynolds number. So, I require that row U delta square by mu L equal to some finite number. I will call it is order one; it goes neither Re to a positive power nor Re to negative power, it goes Re to negative power, it will go to 0. The viscous term will become small compare to the initial term, it goes to the Re positive power, it will go to infinity, and still do not have a balance because there are nothing to balance this consist.

I can rewrite this, as row U L by mu into delta by L whole square is equal to some numbers, equals to some constant. It is a number that means, finite as Re goes to infinity that means, that row U L by mu that Reynolds numbers itself row U L by mu that Reynolds numbers itself. That means, the delta by L whole square as to be b equal to C times Re minus half, that is delta by L has to inc decrease to 0, as Reynolds number to the minus half power in the limits of Re goes to infinity.

Then will be viscous term continue would be a same magnitude as an initial term, even in the limits Re goes to the infinity. Let us look at the little bit, so what this is saying have a module layer thickness. The thickness becomes smaller and smaller proportional to Re power minus half in the limit is Re goes infinity. In such a way ever, that the balance in the initiate for the viscous term continues to balance terms, even as the Reynolds number becomes larger and larger. This delta itself is just a lengths scale that reduce scaling my equation delta, of course you require the remain constant.

In the limit as Re goes to I am sorry constants times minus half in the limit as Re goes to infinity. However, the constant just a lengths scale it can be said to an any value without lost a generality. This constant can be set to any value the equation is written in the terms of scaled variable will change with a changes in the value of constant. However, the equation written in the terms of un scales variables not changes, because the delta only length scale, I am using to non-dimensionalised the coordinate.

So, without loss of generality, you can set this constant as 1 and write delta by L is equal to Re power minus half. Any value of the constant will change, this solution expression term of the scale variable, because I am using delta to scale variable. However, it would not change the solution into the terms of the un scaled variable, ultimately when I convert back to the un scaled variable, I will put the same solution once again the equation self has only one solution. So, this is given as the depends of delta on a Re, on such a way that the viscous inertial terms continue to balance each other, even the limit as Re goes to infinity. So, this was the x momentum of conservation equation.

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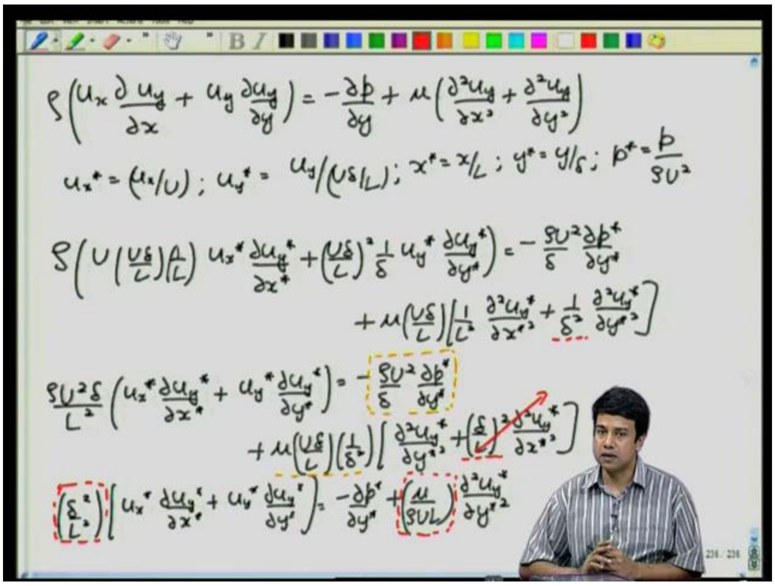

Next, we go to the y momentum conservation equation. Next, we go to the y momentum conservation equation, the y momentum conservation equation is u x partial y times row is equal to minus parcel p by parcel y plus mu. Once again, we use the expressed in terms of scaled variable u x star equal to u x by U, u x star equal to U delta by L, x star equal to x by L, y star equals to y by delta, p equals to P by row. So, express my equation and term of this. So, first term row, U x as the factor of U, U y has the factor of delta L and there is 1 over L derivative with respect to x.

The second term, the second order in u i times u y times u y. So, you will get U delta by L the whole square into derivative of with respect to y. So, this is becomes 1 over delta pressure scaling this row U square y is delta. So, this become minus row square by delta plus, there is the viscous term the U y is U delta by L into 1 by L square, partial square U y star. As you can see, for the convictive term, the total pre factor is U square delta by L square. After, you multiply all terms of there you get the common factor of U square delta by L square. So, you get row U square delta by L square plus the viscous term.

So, just write the down here, once again, the larger derivative term is variable in the cross stream direction, because 1 over delta square is much larger than 1 over L square. Therefore, I will take that as common pre factor, because I have to take the largest term common of the pre factor, which is the momentum conservation equation of y direction. As usual, I can may be deflect stream ways fusion with respect terms in the fusion, because the lengths steam in the direction is much smaller.

You can see that here, as well the cross stream ways the diffusion can terms contain factor of delta by the L whole square. That can be neglected with respect to the cross stream diffusion terms, direction always neglect stream by stream diffusion with respect of the cross stream diffusion. I divide throughout by the largest term in this equation, which is largest term in the equation; no it is not the convictive term. You can see the convictive term goes as row U square delta by L square, the pressure goes row U square by delta. Pressure gradient contains 1 over delta, where as the convictive delta by L row square by delta by L.

So, the largest term is actually this one, contains the delta and a denominator and the delta is the smaller number compare to L. So, I divide throughout by the pre factor of these terms, to get the dimensionless equation; do out by the pre factors. What able get the somewhere on the left side, I will get delta square by L square into u x, and for the second term; what I will get is this term here, contains mu U by U L by delta it contains mu U by L delta.

If you divide row square U by delta, you can easily verify the resulting pre factor, that you get should be mu by row U L partial y square. You can see by this term contains the pre factor delta by the L whole square; delta by l whole square goes Re minus 1. So, in the limit is as Re goes the infinity delta by L, the whole square goes the Re minus 1, which goes to 0. This inertial term this viscous term here, contains the 1 over Reynolds number, Reynolds number based on the plate on L is also small. Therefore, this is also small, what ultimately the equation for the cross stream moment reduces to that the pressure gradient in the cross stream direction equals to 0.

So, cross stream diffusion equation, basically reduces to partial p by partial y equals to 0. So, this case be the simplified momentum equation in the x and y direction. In the y direction in the equation tells me, partial p by partial y is just equal to 0, and I had a momentum conservation equation in the x direction. I derive this equation by scaling and I obtain reduced scaled equations.

However, if this are the dominate terms in the scaled equation, they will also be the dominance term in the original un scaled equation. Because I get the scaled equation by dividing all terms by some factors, all terms factors are divided by the same factor. Therefore, if the scaled equations contain this as a dominance terms. Then the un scaled equation will also contain this as dominant terms.

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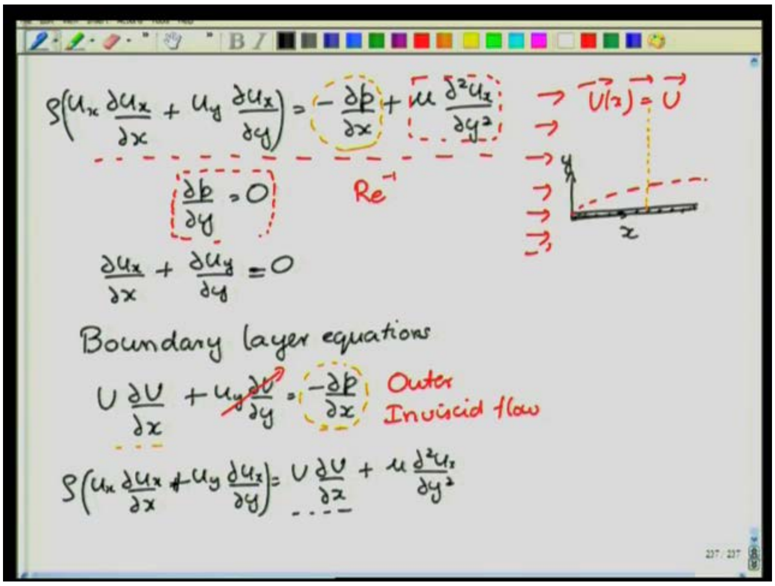

So, these are boundary layer equation for the flow pass any surface. In this particular case considering just a flat plate, but for the flow past any surface. These are the boundary layer equation these are the boundary layer equation for the boundary layer very close to solid surfaces, when the boundary layer is thickness is restricted to a very small distance in the limits of high Reynolds number. This are the boundary layer equations and this equation contain additional viscous term here, contains the solid cross stream, because that is what is required to reduce a transitional velocity to 0, And the surface in the stream wise momentum equation.

The cross stream velocity conditions already were satisfied by the potential flow in a normal velocity condition. What we need to do satisfy the stream wise velocity condition? That is taking care of adding this additional term, and a common feature of all boundary layer equation is that is pressure gradient. In the cross stream direction is always equal to 0, reason is because if you have to generate the pressure gradient in that direction, there has to be some flow in the direction.

The velocity in the y direction, in the all of these is boundary layer problems is always very small compared to the velocity in the x direction. Therefore, there is not thing to balance in the pressure gradient in the contain direction. And leading an approximation, you always find pressure gradient in cross streamed direction is always equal to 0.

The terms neglected here, are Re inverse smaller then the term that is retained, the terms neglected in this equation, Re inverse smaller and that the term is retained. Now, if you want to solve the boundary layer equation. We required that in the limit, y goes to infinity the solution is for the equation naturally, goes over to the solution for the out of flow, out of potential flow.

So, enough plate case for example, here we have flat surface over here, with two coordinates x and the stream y in the cross stream direction. And I have some kind of boundary layer close to the surface, where viscous effect are important, where viscous effects are comparable to inertial effects. The outer potential flow was specified with no viscosity in it, out of potential flow verified with no viscosity in it.

So, If consider the out of potential flow without the co-operating, the boundary layer moment in diffusion as some velocity U f x. In this particular case, this was the constant velocity, the specific case you are solving you required that the velocity U was a same everywhere, far away from the surface. But, as we will see in other cases, where there may be either x or acceleration or deceleration of the free stream flow far away from the surface in the x direction. So, for the out of flow, you required that the flow has satisfy to the equation for the out of flow, there will be a smooth transition from the boundary low to the out of flow. If the out of flow, when viscosity was neglected also satisfied this same equation.

So, for the outer flow itself, I require that capital U, partial U by partial x here, is the free stream velocity along the flow direction plus the cross stream velocity is equal to minus partial p by partial x. However, when I go far from the surface, the velocity u y goes to 0 and velocity u y is identically to 0 far from the surface. Therefore, this equal to 0 for the outer flow, let me call it as inviscid flow. So, minus partial p by partial x is equal to U time, equal partial U by partial x there capital U is the outer potential flow. However, we know that partial p by partial y is equal to 0, at a given location x.

The pressure does not depend the y regardless of what the distances from the surfaces the pressure invariant that the direction. So, for a given x, the pressure independent of y, in the limitation y goes to infinity the pressure given by this, the pressure is independent of y. That means, any y pressure given by this because I have the condition, the pressure is independent of y, in the limit y goes to infinity does given by partial u by partial x. That means, the pressure has given by u times partial u by partial x at any value of u of y. Therefore, I can substitute this for the pressure at any value of y, because I know the pressure independent of y. And my equation, becomes u x or partial by partial x u y is equal to U plus U partial square u x.

So, those are the boundary layer question, expressed in terms of the free stream velocity far from the surface. Far means, distances from the surfaces large compared to delta because distance comparable to delta, that the momentum diffusion to predictable to convective terms. And the distance from the surface becomes large compared to delta; momentum diffusion becomes small compared to conviction. For our particular case for a plate, this equal to 0 because capital U just the constant.

So, free stream constant the velocity far away therefore, the capital U has the constant therefore, the terms is equal to 0. From the next lecture, we will solve this equation as one more little information, that we need before solve this equation. And that is that the velocity at given location x does not depend up on the total length L, but on the distance x itself, because total lengths downstream of the plate.

We will use that in the next lecture, in order to get a similarity, solution for this flow (( )). As you can see the logic, that we have used here, is very similar to the logic that we have use previously in boundary layer theories for heat and mass transfer. (( )) use full to go back and look at those calculation, to see how we derived the scaling and those cases, the scaling in this case also very similar. Kindly, go and have look at that once again, and we will continue to get proceed into the similarity solution, similar to what was done in heat and mass transfer in the next lecture? We will see you back.