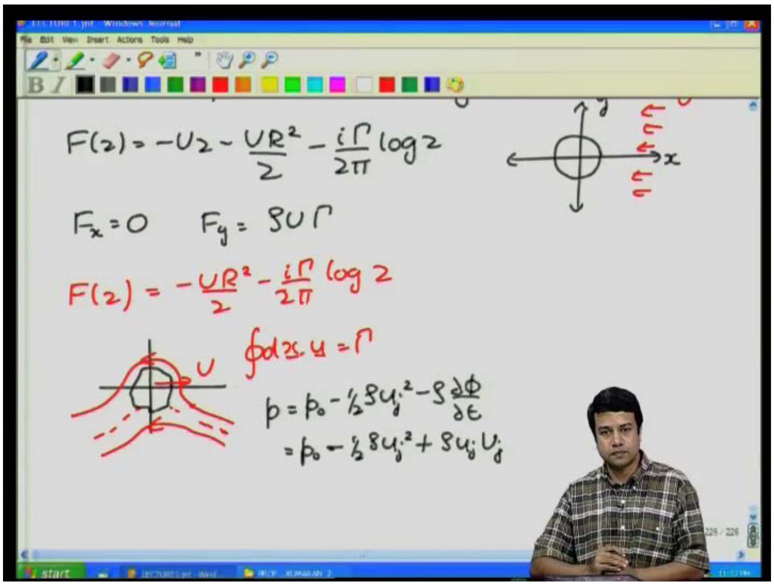
**Fundamentals of Transport Processes II**
**Prof. Kumaran**
**Department of Chemical Engineering**
**Indian Institute of Science, Bangalore**

**Lecture - 30**
**Conformal transforms in potential flow**

So, this is lecture number 30 of our course on fundamentals of transport processes, where we were discussing in the last lecture potential flows in two dimensions, and I had derived a relation for you starting with the flow around a cylinder of the forces exerted on a cylinder in potential flow, for the case where there is a net circulation on the cylinder. We had calculated the velocities, and we had shown what kinds of velocities we see around the cylinder. Then we had calculated the net force exerted on the cylinder.

(Refer Slide Time: 00:55)




So, this was the potential flow around a cylinder, and if you recall in the last lecture we had taken function F of z of the form minus u z minus u R square by z minus i comma by 2 pi log z, and I showed you that this corresponded to the flow around the cylinder where the fluid was coming in with the velocity minus u far away. And for this particular case I had got the relations for the net flow, this is x, and this is y. I got the relations for the net flow as F x the track force is equal to 0, F y is equal to rho u times gamma.

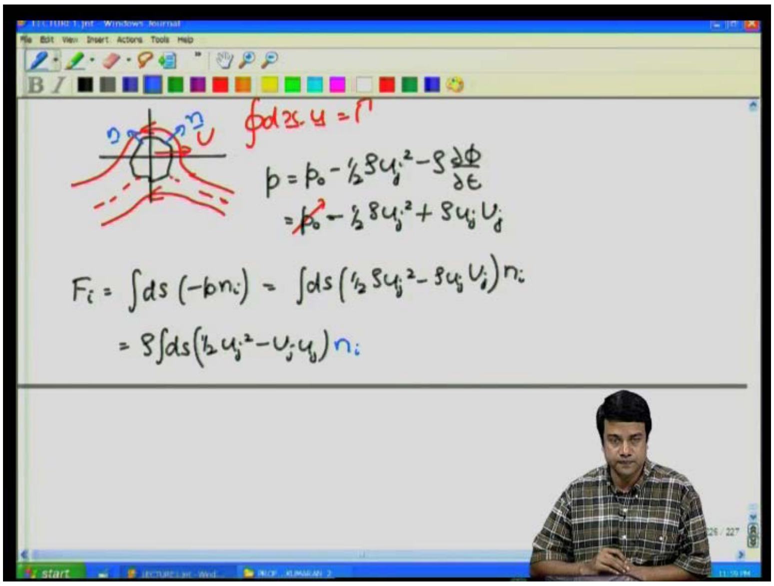
Now I would like to calculate this for a general object not for a cylinder that is moving, but that is stationary in a fluid that is; I am sorry cylinder that is at rest in a fluid that is moving far away, but rather for the case where the fluid is at rest far away, and the cylinder is moving by the constant velocity. For that particular case the function F of z just equivalent to adding velocity plus u at every point within the fluid in this particular configuration that I have shown you here velocity is minus u far away, velocity is 0 on this surface. If I want the cylinder to move I just add up velocity plus u everywhere within the flow.

In that case far away the velocity becomes 0, on the surface the velocity becomes plus u along the x direction; adding a velocity plus u everywhere is equivalent to adding potential as plus u times z, because if I take the derivative of the potential with respect to z will get an additional factor of plus u. So, this is equal to minus u r square by z minus i gamma by 2 pi log z, but let us not work it out for this specific case; let us work it out for a general case. So, let us just take the general case of an object could be of any shape a general shaped object that is moving with the velocity u in a fluid that is at rest far away, and in general there is a net circulation around this object. So, therefore in general there is a net circulation around this object. So, that integral d x dot u around the surface of the object is equal to gamma. So, that is the configuration that we would consider as object moving with the velocity u in a fluid that is at rest far away.

So, how do we calculate the force on this object? Of course, first we calculate the pressure, and then we calculate the force as integral of p dot n d s with the entire surface of this object. So, the pressure p is equal to p naught minus half rho u j square minus rho partial phi by partial t. In this particular case we are considering an object that is moving with the steady velocity capital u, and therefore, the time derivative of the potential is not 0, because as I had shown you in couple of lectures ago if you have an object which is moving and your coordinate system origin is at the center of the object; the potential at a fixed observation point will change, because the origin of the coordinates system is changing. That has to be taken into account.

Then there are two reasons why the potential at an observation point changes. One is because there is the object velocity itself may change. The second is because the origin of the coordinate system is moving. In this particular case we are considering an object that is moving with a constant velocity. So, there is no change in the velocity; however, there is a change in the potential at a fixed point, because the origin of the coordinate system is changed. And that if you will recall we calculate it to be minus half rho u j square plus rho u j times capital U j, rho u dot capital u where capital u is the velocity of the object.

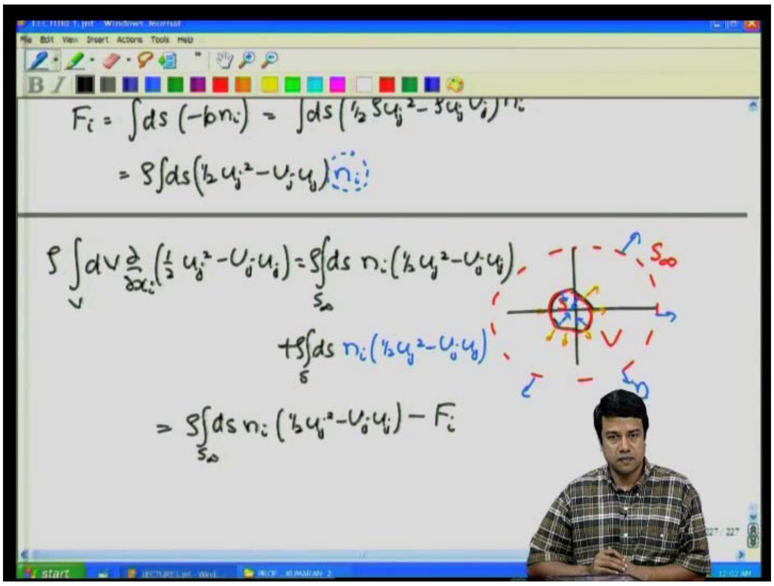
(Refer Slide Time: 06:21)



Of course, without loss of generality I can set p naught is equal to 0 because that is just a constant pressure that is far away when the velocity goes to zero. So, I get p is equal to minus half rho u j square plus rho u dot capital u. Therefore, the force F i is equal to integral of the surface of minus p times the u naught. So, this is equal to integral d s of half rho u j square minus rho u j times capital U j, and I will simplify this a little bit. To write this as rho integral over the surface of half u j square minus times n. Note that this unit normal n i has defined here; it is the outward unit normal to the object if normal n I is the outward unit normal to the object.

Now we will do the calculation in a manner similar to what was done for a three dimensional object. What I will do is I will reduce the integrals to just integrals over infinity, and since I know how the velocity is decay as r goes to infinity, from that I will be able to calculate what is the force rather than doing it over the surface of the object, because we do not currently know what the shape of the object is or anything about the object. You want to calculate the force independent of those things.

(Refer Slide Time: 08:16)

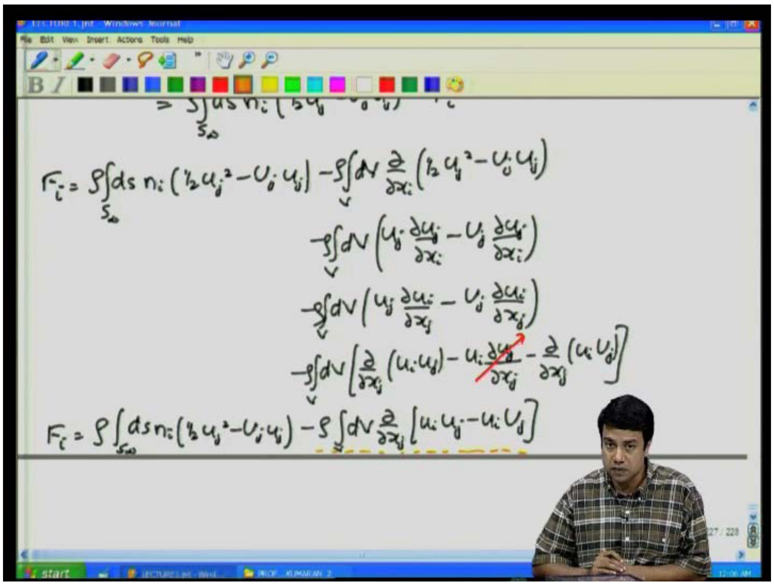


So, if I have this is my object, and I consider the fluid in between this object, and the surface at infinity. This s is the surface of the object; s infinity is the surface at infinity. For the fluid in between the surface object and the surface at infinity I can write using the divergence theorem that rho times integral over the volume partial by partial x i of that has to be equal to this is the divergence of something integrated over the surface and be written as the unit normal times that thing integrated over the, I am sorry the divergence of something integrated over a volume over the volume v where this is the volume v of this object partial by partial x i of half u j s. This is going to be equal to integral over the surface at infinity of n i times half u j square minus u j u j plus the integral over this surface plus the.

So, this is the integral of the surface at infinity where my unit number is defined this way. This is n for the surface at infinity for the surface s itself is the outward unit normal to the fluid the outward unit normal to the fluid is the inward unit normal to the object; however, I will define my unit normal n as the outward unit normal to the object because that is what I require for getting the force. So, this is equal to min plus integral over the surface s of d s times the outward unit normal to the fluid shown in blue times half u j square minus u j times u j.

The outward unit normal to the fluid is the inward unit normal to the object, because the unit normal to the object whereas the force is defined with respect to, in this case the unit normal is the outward unit normal to the object. So therefore, this can be written as integral over the surface at infinity d s n i into half u j square minus u j u j minus the force. I should put a rho here; there is a density factor everywhere. So, this is just an expression for the divergence theorem for the force; of course, I have two unknowns here. I have one is the volume integral, the other is the integral over the surface at infinity.

(Refer Slide Time: 11:52)

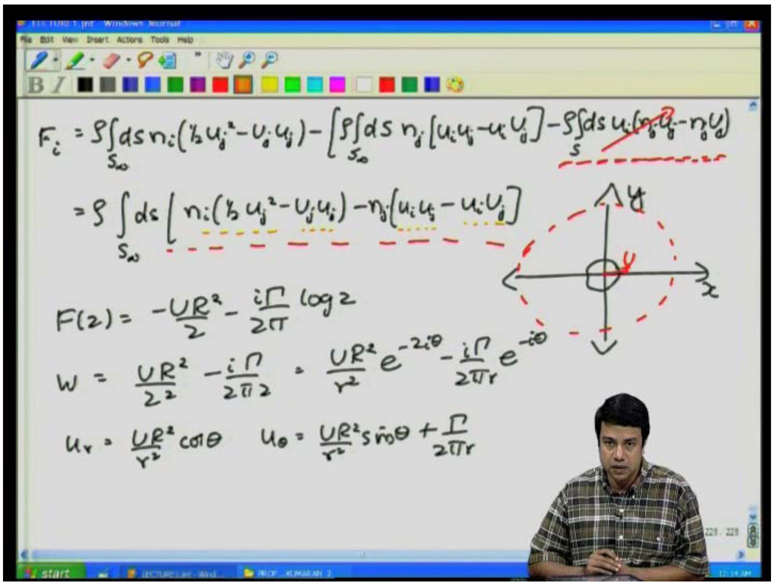


So therefore, I can write this force F i is equal to rho times integral over the surface at infinity d s n i half u j square minus u j u j minus integral of the volume d v. Let us simplify out the second term in this expression. Chain rule for differentiation u j partial u j by partial x i minus u j partial u j by partial x i. Note that capital U is the velocity of the objects. So, that is a constant. So, I have used that while doing the differentiation. I know that the rate of deformation tensor has to be symmetric for a potential flow because the anti-symmetric part is equal to 0. The vorticity is 0 everywhere in the fluid; therefore, the anti-symmetric part of the rate of deformation tensor is equal to 0 everywhere in the fluid. So therefore, I can write replace partial u j by partial x i the derivate of u j with respect to x i by its transpose partial u i by partial x j.

I have just replaced partial u j by partial x i by its transpose, because I know that the rate of deformation tensor is symmetric in a potential flow. So, this I can write as minus integral d v; I do differentiation using chain role here. I can write the first term as partial by partial x j of u I u j minus u i partial u j by partial x j. In the second term since capital u j is a constant I can write this as partial by partial x j of u i times capital u j. And of course, the term partial u j by partial x j is the divergence of the velocity; the divergence of the velocity is identically equal to 0. The divergence of the velocity is identically equal to 0 because the potential flows incompressible.

So, this term is identically equal to 0, and the equation for the force becomes F i is equal to, we should have a density here rho integral over the surface at infinity minus rho times integral over the volume. Note that this term is also the integral over a volume of the divergence of something. So, this is also the integral over the volume of the divergence of something; that can be reduced to a surface integral as usual.

(Refer Slide Time: 15:47)



So, this is equal to rho integral of the surface at infinity minus. Now this is integral s infinity d s n j minus the integral over the surface of the object. Once again for this term that I have shown here this is integral of the divergence of something over this volume v that can be written as an integral of the unit normal times that thing over the bounding surfaces. There are two surfaces; one surface at infinity and one surface on the surface of the object. So, I have just written this as the divergence of something over this volume is equal to integral of the unit normal times that same thing on the two surfaces. This is an integral over the surface of the object as you can see, integral of n j small u j minus n j capital U j times u i integral over the surface. This is equal to F i.

On the surface of the object itself I have the known normal velocity boundary condition. So, on the surface of the object itself I have the known normal velocity boundary conditions that is that u dot n is equal to u dot n; that is the normal velocity of the fluid, the fluid velocity normal to the surface is equal to the velocity object in that direction. We are not enforcing tangential velocity boundary condition, so there can be a slip in the transitional direction; however, the normal direction the normal velocity of the fluid perpendicular to the object surface is equal to the velocity of the surface itself. So, there is no penetration, and because of this normal velocity boundary condition we can see this term is identically equal to 0. The term that I have shown in red here is u i times n j u j minus n j times capital U j; that is identically equal to zero.

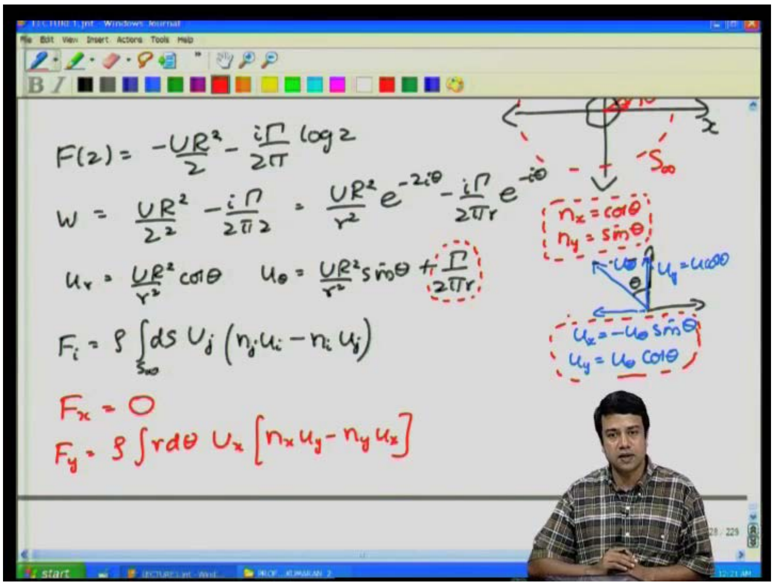
So, the net force on this object has been reduced to two surface integrals over the surface at infinity. So, I have shown you this is equal to rho times integral over the surface at infinity d s of n i into half u j square minus u j u j minus n j into u i u j minus u i times capital U j. Now this is over a surface that is far away. In two dimensions for the surface that is far away in two dimensions, x and y in two dimensions if I have an object at the center the fluid velocity of course has to decay as r goes to infinity, because the object is moving with some velocity u; however, the fluid is stationary far away. Therefore, the fluid velocity has to decrease as you go far away. How does the velocity decrease? If you recall if there is circulation we had got the velocity decrease for the flow around a cylinder as the complex potential is equal to minus u r square by z minus gamma i gamma by 2 pi log z which means the w is equal to u r square by z square minus i gamma by 2 pi z is equal to.

So, these are the two components of the velocity due to circulation as well as due to the mean flow; due to the flow of the object with velocity u as well as due to the circulation around this object. You can see that the velocity decay due to circulation due to the line vortex is proportional to 1 over r. Due to this constant velocity is proportional to 1 over r square. The surface area of this object itself increases as r in the limit as r goes to infinity. So, if I want to draw out u theta for this case I get u r is equal to u r square by r square into cos theta u theta is equal to u r square by r square sin theta plus gamma by 2 pi r. So, u r decreases one over r square far away whereas u theta decreases one over r.

You have to multiply this by insert this into this expression in order to find out how the force varies. Clearly the surface area far away in two dimension to the surface as I told you it is just a line; the length of that that perimeter far away increases proportional to r. Therefore, I will get a nonzero contribution to the force only due to velocity contributions which go as one over r. If the contribution due to the velocity goes as one over r square the surface area increases proportional to r whereas the velocity is decreasing as one over r square. The net integral will go as one over r which goes to 0 as r goes to infinity.

A non-zero result can be obtained only due to the contribution to the integrand which goes to one over r. If you look at this expression here I have u j square which has to decrease as one over r square, because the slowest decaying component of the velocity decreases as one over r. Therefore, u square has to decrease as one over r square. Similarly I have this small u i times u j here; that also decreases as one over r square because it is proportional to the square of the velocity. So, these two terms will give me a zero result when it is integrated over the surface far away; however, if you look at these two terms u j times u i u j minus n j u i u j, capital U in this expression is a constant is the velocity with which the object is moving. Therefore, these contributions are linear in the velocity u i and therefore, this could give a contribution which goes as one over r when integrated over the surface will in general give me a nonzero value for the force.

(Refer Slide Time: 24:02)




Therefore over the surface at infinity the only nonzero value that I am going to get to the force is due to these two blue terms here, clearly nonzero contribution I will get is due to the blue terms. The orange terms are proportional to u square which goes to zero as one over r square; therefore, when I integrate that over the surface I will end up with the zero result where F i is equal to rho integral d s over the surface at infinity. There is a common factor of u j here, and you can see that this u j is common between this term and this term. Then the other term is of the form n j u i minus n i u j. So, this is the final result for the force. F i is equal to integral rho integral d s of capital U j times n j u i minus n i u j.

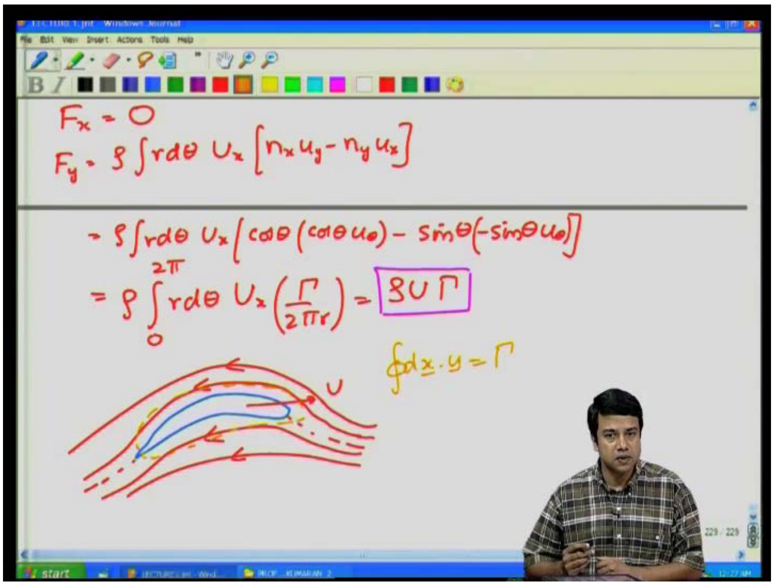
So, I have to take the velocity and multiply it by the unit normal. The only contribution that I am going to get to the velocity which will give me a nonzero force is going to be due to the term that is decreasing as one over r here, the u theta contribution which is decreasing as one over r. There is this other contribution which is decreasing as one over r square that will give me a zero value, because when I integrate over the surface the velocities goes as one over r square and the surface area increases proportional to r itself. So, over this surface I have the unit normal. This is the unit normal to the surface and just two components n x and n y. I can locate them using the polar coordinate system r n theta point on this surface as infinity, and clearly n x is equal to cos theta, and n y is equal to sin theta. What about the velocity at this point? U theta is equal to gamma by 2 pi r.

So, let us just calculate what is the velocity at this point? The only nonzero contribution I will get is due to the velocity u theta at this point. U theta is along the theta direction; this is u theta, u theta which is equal to gamma by 2 pi r. So, that is the direction of u theta; I will just remove the y here. So, that is the direction of u theta, it has two components u x and u y. You can see clearly that u y is in this direction. Let me plot it separately here to make it clear. So, I am having a velocity u theta which is going in this direction u theta with respect to the coordinate axis; clearly the angle theta with respect to the y coordinate and with respect to x coordinate the angle is pi by 2 plus theta. So, with x coordinate the angle is pi by 2 plus theta. So, if u theta is in this direction; that means that the component u i the component u i is equal to u cos theta.

The component u x is actually in the minus x direction, because u theta is positive; it is going in the anticlockwise direction, u y will be positive for that, u x will be negative. So, u x is equal to minus u; u x is equal to minus u theta sin theta, and u y is equal to u theta times cos theta. So, using the two values n x n y u, x u y I can calculate what is the force. First take the force F x. The velocity we have assumed without loss of generality is only in the x direction. So therefore, j is equal to x because the velocity u j is 0 for the y direction, the velocity of the object itself is only in the x direction. This we can choose without loss of generality, right, if the object is moving in some direction I can always align my x axis along that direction. So, if j is x if I want the force in the x direction then I have i is also x and j is also x, because the direction of the force is I, direction of the velocity is j.

If I want the component of the force along the velocity vector then i is x and j is x; therefore, n j u i minus n i u j is just n x u x minus n x u x, it turns out to be equal to zero. So, this tells me that the force along the x direction along the velocity direction, the force along the velocity direction F x has to be equal to 0; that is the force along the velocity direction, drag force that is identically equal to 0. What about the force along the y direction? The force along the y direction is equal to rho integral r d theta of this force u x. d s is of course the line in two dimensions it is r d theta u x times n x u y minus n y u x and of course, for n x and n y I have to use the cos theta and sin theta here because it is a normalized unit vectors, and I had calculated u x and u y for a velocity in the theta direction because that is the only one that gives me a contribution to the velocity proportional to 1 over r.

(Refer Slide Time: 30:38)



So, this will give me rho times integral r d theta u x, n x is cos theta into u y is cos theta u theta plus n y is sin theta, I am sorry minus sin theta into u x is minus sin theta u theta. So, you can see cos square plus sin square is equal to 1, and I get rho times integral r d theta capital u x into u theta the only non-zero contribution to u theta comes out of the circulation term here which is gamma by 2 pi r into gamma by 2 pi r. The integral over r goes from 0 to 2 pi; it goes all the way around the object. So, it goes all the way around the surface at infinity. It goes from 0 to 2 pi, and you can easily see that this is equal to rho u times gamma. So, this tells me that for an object that is moving at a constant velocity in a fluid provided there is circulation around the object in potential flow the net drag force itself is 0.

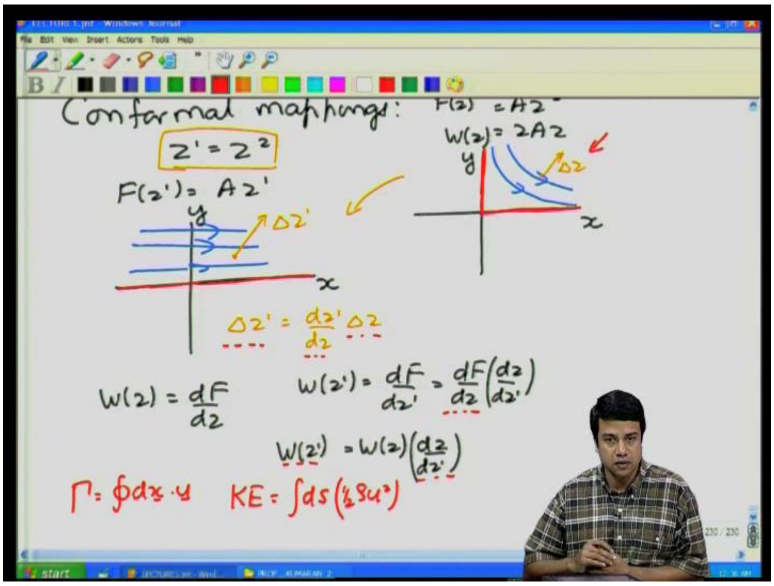
There is no force acting along the direction of motion; however, there is a net lift force perpendicular to the direction of flow depends upon the direction of circulation. The circulation is clockwise around the object; the lift force is acting upwards. If it is anticlockwise it will act in the opposite downward direction, and this forms the basis of all aerodynamics. This result as I showed you holds only for two dimensional flows; that is the reason that aircraft wings are long and slender object, because you require that the flow should be approximately two dimensional at every point along the aircraft along the wing. Otherwise you cannot generate a lift force; three dimensional object moving in a fluid will not generate a lift force, because in that case the velocity perturbation becomes one over r cubed. The potential goes as one over r square, and the velocities goes as one over r cubed.

Only if the object is two dimensional and if there is a net circulation around the object there could be a net lift force due to the circulation around the object. The total force exerted is just equal to the density times the velocity with which the object is moving times the circulation around the object and the circulation around the object is generated by the shape of the object. If you have an object that is shaped like this for example, and this is moving with the constant velocity within a fluid, this is moving with the constant velocity within the fluid then I have this front and the back stagnation points, and I have a net flow that goes around this object. And if I calculate the net calculation along any contour that is going around the object, take some contour going around the object, and I calculate integral d x dot u over this closed contour.

This will give me a net nonzero circulation, because this object is no longer symmetric. This object does not have up-down symmetry, and because of that I will get a nonzero circulation around this object. And the net force exerted upward on this object is just equal to the density times the velocity with which the object is moving times the circulation. In air craft of course, this is required in order to generate a lift force that it can fly; of course, there is a drag force as well, but we do not get that drag force in potential flow because we neglected viscosity and therefore, we have neglected the viscosity mechanism, but there is net lift force which is purely a potential flow effect. Similarly submarines have this the opposite way; they have wings which are up transfers it they can go downwards.

And similarly whenever you want to generate a lift on an object you need to have a long two dimensional flow locally around that object; of course, wings are three dimensional objects. So, the net circulation around this object has to be reflected in net vorticity at the tips of the wings, and for that reason you have what is called vortex shedding at the tips of the wings. So, that is beyond the scope of this course. The only thing I want to emphasize here is that for an object in two dimensions provided it is moving with the constant velocity, and there is a net circulation around this object is going to generate a lift force which forms the bases for why aero planes fly. So, before I close potential flow I would just like to say that the framework that we have developed here for two dimensional flows is more general.

(Refer Slide Time: 36:15)



It can be used for what are called conformal mappings as follows. So, I showed you that the flow in a corner for example, if the corner had a flow in a corner we had done that flow, the flow itself look something like this. The complex potential is given by F is equal to a z square and w the complex velocity is equal to 2 A times z. If I want to define a new complex variable z prime is equal to z square. Since F of z is equal to a z square; that means that F of z prime is equal to A times z prime. This thing I had defined with my independent variable as F of z is equal to A times z square; that means this was w of z. I am just changing variables; I am writing z square as z prime. Therefore, I have z prime is equal to A times z prime. How does this flow look in the z prime plane? In the z prime plane since F is linear in z prime I get a constant velocity. So, this of course is a much simpler flow.

So, let me put my axis here x and y. This of course is a much simpler flow; it is just a straight line. So, this is just a constant velocity everywhere. So, what I have done is by my change of variables I have defined that I have transformed the flow in a corner to a constant flow. So, if you are given the task of finding the velocity field in a flow that is the flow in the corner is shown here. If you have given the task of finding the velocity field in this flow; one way of course would be to actually solve and find out what the complex potential is. The other way would be to see if there is some way to convert this domain into a more simpler domain.

That is if I can convert if I can do this mapping which converts z to z prime then I know what the flow in the simpler domain is, I know what the potential is, I know what the velocity is; that velocity is just got be expressed back into the more complex domain. We will find out what the velocity in that complex domain is. So, you do not really have to solve the problem. You just have to find a way of mapping this complex geometry onto a simpler geometry, and I already know what the flow in the simpler geometry is and therefore, I can go back and find out what the flow in the complex geometry is. Note that I have kept the potential the same between the two; I have kept the potential between the simplex and the complex geometry to be exactly the same. So, if I take some aligned segment in this more complex in the simpler geometry here delta z and that maps onto some other line segment in the complex geometry delta z prime.

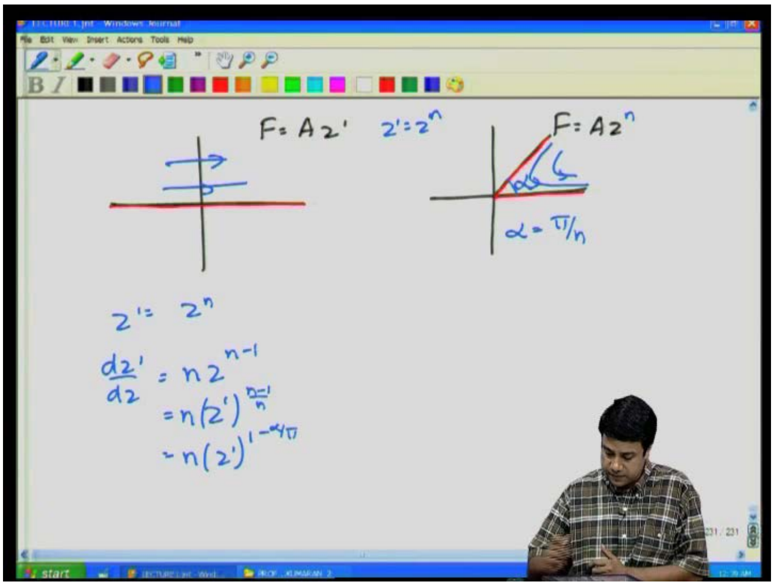
What I have is I have z primer is equal to z square. So, I take the initial location of this line and transform it into z square is equal to z prime in this one. Take the final location, transform it here, and that basically gives me the line segment in these two locations, and just by differentiation you know that delta z prime is equal to d z prime by d z times delta z. So, when you do the transformation the line lens transform as the derivative of z prime with respect to z. How about the velocities? We know that the potentials are exactly the same. So, I know that w of z is equal to d F by d z; w of z prime is equal to d F by d z prime. I can use chain rule for differentiation, and write this as d F by d z times d z by d z prime. I can use change rule of differentiation to write this as d F by d z times d z by d z prime.

So, this is equal to; this term here is of course w of z. So, this gives me basically w of z d z by d z prime. So, note that delta z prime their line segments in the z prime coordinate the delta z prime, the line segment in the z prime coordinate transformed as d z by d z prime time's delta z. The velocity w of z prime is now transforming as the inverse of that d z prime by d z times, I am sorry the velocity is transforming as the inverse of that d z by d z prime time's w of z. So, the velocity transforms as the inverse of the integral of a line segment. So, that is the property of conformal mappings. The potential in the z and the z prime are both the same except that you have expressed z prime in terms of z. Line segments will transform as d z prime by d z times delta z is equal to delta z prime the new coordinate system.

Velocities will transform as the inverse of that consequences of that. Firstly, circulation in both remain a constant, because circulation is defined as gamma is equal to integral d x dot u. Line segments are transforming as delta z d z prime by d z ,velocities are transforming as d z by d z prime. Therefore, velocity time distance remains the same in both of these coordinate systems; that means that the circulation in both of these is identically the same. If there is a circulation in the transformed coordinate system there is an equal circulation in the original coordinate system. Secondly, the kinetic energy of the flow is equal to integral over the surface area of half rho u square. The total kinetic energy of the flows integral over surface area half rho times u square. Distances are transforming as d z prime by d z; that means velocities will transform as the square of that. Velocities transform as d z prime I am sorry distances are transforming as d z prime by d z implies that surface area is transforming as the square of that is transforming as d z prime by d z the whole square.

Velocity is transforming as the inverse one over d z prime by d z. Therefore, velocity square goes as one over d z prime by d z the whole square; that means that between the original and the transformed coordinates the kinetic energy has to be exactly the same, because the velocity is transforming as the inverse of the distances in the coordinate system. So, these kinds of transforms basically preserve circulation, they also preserve kinetic energy. The velocity in the new coordinate system is a simple one; therefore, I can transform it back in terms of the original coordinate system to get the velocity back in the original coordinate system. That is the big advantage of conformal mappings. You do not really have to solve the problem; you just have to find a way to transform from one coordinate system to the other, and of course, the subject of conformal mapping itself is beyond the scope of this course, but I will just take a little time here to layout two commonly used coordinate systems which are often used for mappings.

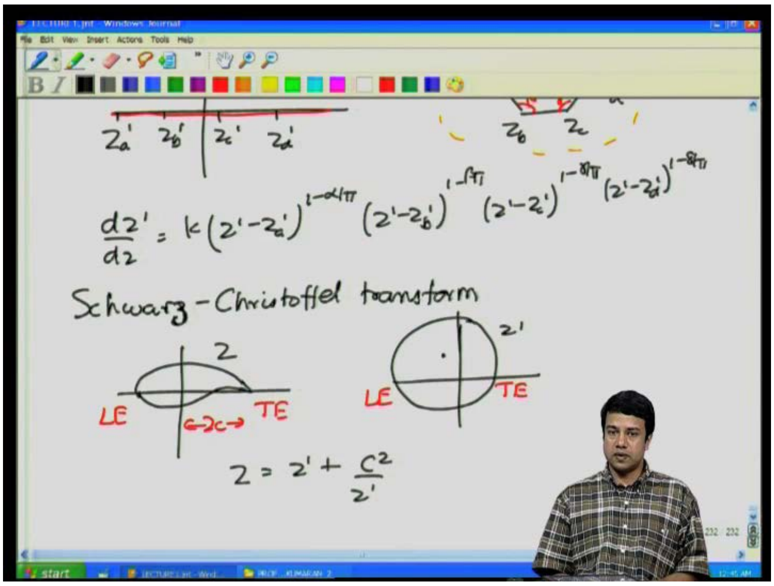
(Refer Slide Time: 45:28)




So, the first thing I showed you was that if I want to go from this one to this one this angle was pi by 2; therefore, F was equal to a z square and here I got F is equal to a z prime. And I did that by going from by defining z prime is equals to z square; it does not have to be just square it can be any power of z again. So, you had a general power of z. The general power of z is at power n that would in general represent some other angle; that would in general represent some other angle with the flow that looks something like this. That can be transformed into this flow just by doing this transformation z prime is equal to z power n. If this angle is alpha we know that alpha is equal to pi by n.

Therefore I have z prime is equal to z power n which is equal to. So therefore, the derivative d z prime by d z is equal to n z power n minus 1. If I express this z power n minus one back in terms of z prime, I will get n into z prime power n minus 1 by n. It is equal to n into z prime power 1 minus alpha by pi, because I know that alpha is equal to pi by n; therefore, alpha by pi is equal to 1 over n. So, this basically gives me a transform for the corner. If I define my function d z prime by d z as n time z prime power 1 minus alpha by pi that transforms one corner into just a flat flow. However, this transformation which is called the Schwarz-Christoffel transformation is more general than that. It can be used to transform a more complex flow.

(Refer Slide Time: 48:49)




So, if I had a complex flow in the domain that look liked this, if I had one angle here one angle, if I had a flow that is going around in this domain, if I had a complex flow that is going around at this domain and the angles were defined as this is alpha, this is beta, this is gamma and so on. We had multiple angles there. This can still be transformed into the flow in just the upper half plane. This can still be transformed into this flow by this thing; this is let us call it as z a, z b, z c, z d. These points z a, z b, z c and z d will be transformed into points over here, and you have just a constant flow, and this transformation the way it is done is to define d z prime by d z is equal to some constant into, all this is primes, and so on. You can have more and more corners. All you need to do is put this transformation in, and this very complex flow transforms into a flow in a flat way in a linear flow in a plaster flat surface.

So, this is a big advantage. If I have a very complicated shape object all I need to do is unfold it using this transformation. Then I know what the velocity in the simple configuration is; then use this transformation once again to transform the velocity and the potential back to the complex geometry that I have. So, this was called the. There is another transform which is often used in aerodynamics which actually transforms something that looks like an aero plane wing into a circle. So, if you have an object that looks something like this. So, this here is what is called the leading edge, and this is what they call the trailing edge, and this is often called 2 c where c is what is called the cord length.

So, if I have a particular specific object that looks like this I can transform it onto a circle whose centre is of axis in the complex plain. So, I can transfer it into circle that looks something like this whose center is not at the origin. It is off the origin in the complex plain, and the leading edge gets transformed down here, and the trailing edge gets transformed here and the transformation for that. So, if this is z and this is z prime the transformation for that is equal to z prime plus c square by z prime, and so this is very useful because it transforms something that looks like an aero plane wing on to a circle. And we know what the velocity field around a circle is; we just solved that for the flow around a cylinder the potential flow around a cylinder with a net circulation and a net velocity. From that straight away you can find out what is the velocity field in this complex object.

So, I just mentioned these two just to give you some idea of how powerful these complex these techniques in complex plain are in two dimensions. We do not have unfortunately equivalent techniques in three dimensions; if we had those then many of the problems that we deal would have been much simpler. So, this completes our discussion of potential flow. I first derived the equations for you for an inviscid rotational flow, and I showed you that you get two scalar equations. One is for the velocity potential itself, because the vorticity is equal to 0; therefore, the velocity can always be expressed as the gradient of the potential. So, one equation we get the Laplacian of the potential is equal to 0, and then the Bernoulli equation for the pressure.

Then we solved it for simple cases. One is this sphere moving at constant velocity we got the velocity field around the sphere. We found that for constant velocity the net force is 0. If this sphere is accelerating the net force has to be equal to the added mass times the acceleration. For a sphere added mass is equal to one-half of the mass of fluid displaced by this sphere. At constant velocity all components of the force are 0. Then we went on to 2 dimensional potential flows where we used techniques from the complex variables. I showed you that any analytic complex function the real and the imaginary parts satisfy the Laplace equation. So, we identified the velocity potential as the real part without loss of generality, and the imaginary part turns out to be the stream function.

Then we looked at simple flows which are generated by common simple functions. For example power loss functions generate flow in a corner; log function generates either a point source of fluid or a line vortex in two dimensions. And we also looked at the flow around a sphere with cylinder with circulation around that cylinder found that the lift force is 0 at constant velocity; however, I am sorry the drag force is 0 at constant velocity. So, you need no net to do no work in order to move this object at constant velocity. The lift force is in general nonzero. If the circulation is nonzero the lift force is by rho u times gamma. So, you require a constant velocity as well as the circulation around the object in order to generate a lift force and finally, I showed you that that result that the lift forces rho u times gamma is general, not restricted to cylinders alone.

For any object moving at a constant velocity in two dimensional object moving at constant velocity if there is a net circulation around that object there will be a net lift force; there is still no drag force. So, the work done is still zero, but there is a lift force. I discussed for you a few simple things about conformal mappings. So, this completes our discussion of potential flow; of course, in potential flow we have neglected to discuss F x completely. We have neglected diffusion of momentum; therefore, we cannot satisfy transactional velocity boundary conditions. However in the real system the transactional velocity does have to come to 0 at the surface of the object itself.

The rest be movement diffusion which is restricted to a thin layer near the surface. That we will see in the next few lectures boundary layer theory similar to what we had done for heat and mass transfer, except that this is boundary layer from momentum transfer where the connective terms are non-linear in the velocity. So, we will solve boundary layer problems in the next lecture. Kindly go back and revise the boundary layer solutions that were done for heat and mass transfer, because many of the scaling arguments that we will use will be very similar to what we used previously. We will start boundary layer theory in the next lecture, will see you then.