Fundamentals of Transport Processes II
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Lecture - 3
Vectors and Tensors

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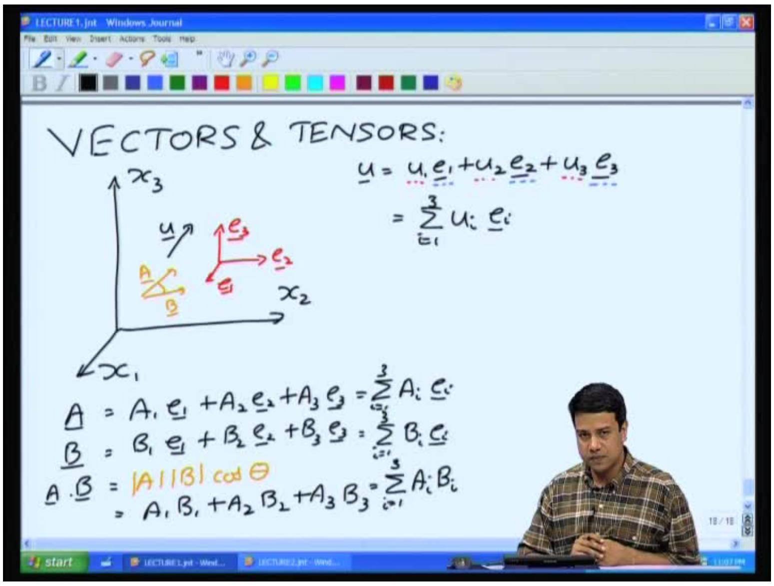

Welcome to this lecture 3 of fundamentals of transport processes, we get down to the hard work now. We start with vectors and tensors, a brief introduction in this particular lecture. We will or in this particular course, we will be dealing only with what are called orthogonal coordinate systems; that means coordinate systems in which the three axis are perpendicular to each other. So in three-dimensional spaces, you can have only three perpendicular axis. My first part of the discussion will be on the Cartesian coordinate system, and then later on, we will go back and look at how deal with a spherical and cylindrical coordinate systems. A lot of what we do for Cartesian coordinate systems will carry through to spherical, and cylindrical coordinate systems with the only modification; that the operators, the integral and differential operators will have slightly different forms.

So, Cartesian coordinate system, where I will use 3 orthogonal axis, x 1, x 2, x 3 rather than using x y and z. We will use the axis x 1, x 2 and x 3. It will become a little clearer little later why we are choosing to it this way. The unit vectors in this coordinate system are e 1, e 2 and e 3. These are the 3 unit vectors perpendicular to each other. We will use e 1, e 2, e 3 instead of i j k or e x, e y, e z. So, any vector can be written in terms of its components, if I had the velocity for example, the velocity vector representing the fluid velocity at one particular location.

We can write this as u vector is equal to u 1 e 1 plus u 2 e 2 plus u 3 e 3, so that is the velocity vector. It consists of the components plus the unit vectors; the unit vectors and the three components. So, any vector can be written in this form for example, if I had some other vectors say I call it a vector, this is A 1 e 1 plus A 2 e 2 plus A 3 e 3. Some other vector B is B 1 e 1 plus B 2 e 2 plus B 3 e 3. I can take the dot product of these two; the dot product of two vectors is a scalar. It is equal to A dot B, you can write it in various ways. So, if I had two vectors, A vector and B vector. It is written in various ways for one is in terms the angle between these two. It is written as the modules of A the modules of B times cos of the angle between them. For the present course, you will write things just in terms of the components A 1 B 1 plus A 2 B 2 plus A 3 B 3.

Now rather than write these vectors out in long notation, I can just write it simply in a simplified form. I write this as summation, i is equal to 1 to 3 of u i times e i vector. So, if i is equal to 1, it is u 1 times e 1 vector; i is equal to 2, it is u 2 times 2 vector, and i is equal to 3 it is u 3 times e 3 vector so that gives us the norm form that we had above. Similarly, for these vectors, I can write them as summation i is equal to 1 to 3 of A i e i vector. So this has two parts, one is of course, the unit vector and the other is the component B i and e i are the components e i is of the unit vectors. The dot product of course, is a scalar. So, this is just equal to summation i is equal to 1 to 3 of A i times B i. In the last lecture, we had also looked at tensors; one tensor that we are looked at for example, was the stress tensor.

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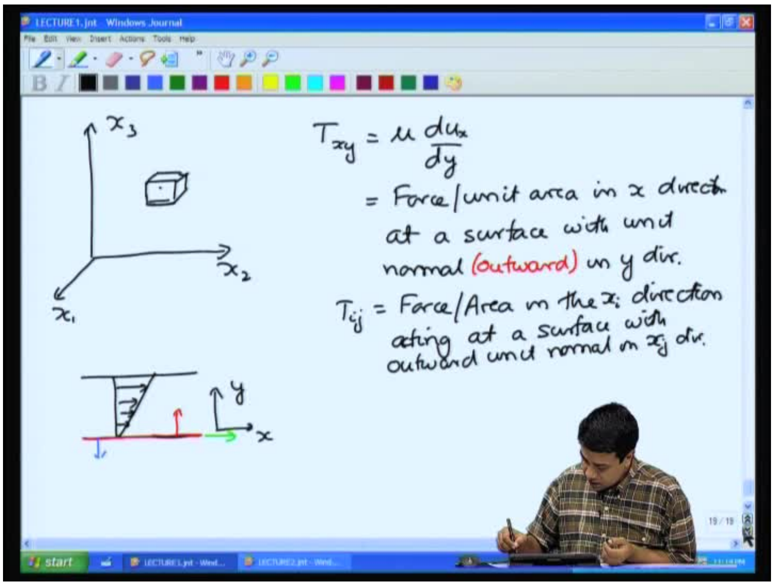

So, let us look at that the stress tensor is defined as I will use t here instead of tau not to be confused with temperature, I will use capital T for this stress tensor T x y. So, the simplest case, you can consider as for example, the flow between two flat plates, where the flow direction is x, the normal direction is y then I have some kind of a linear velocity profile. The shear stress acting on the surface in given in Newton's law of viscosity by mu d u x by d y, if there is a linear velocity profile, and if there is only one directional flow. We will see a little later that this is not the most general form. This is only when you have one directional flow for a linear velocity profile ok.

What is the definition of the stress? This is equal to force per unit area in x direction at a surface with unit normal; note that this is outward unit normal in y direction. So, there are two directions here, one is the direction of the force; the force is in the x direction, the outward unit normal of the surface, the surface any surface with in this three dimensional system can be completely defined by the line perpendicular to that surface; the vector perpendicular to that surface.

So, surface in three-dimensional spaces completely defined by that vector perpendicular to the surface. It is preferable to use unit vector just to make sure that there is no ambiguity, so with as outward unit normal unit vector normal to the surface in the y direction. So, for example, the force exerted on this bottom plate here, for this bottom surface, the outward unit normal to this bottom surface that is the unit normal that faces into the fluid, for this bottom surface, the outward unit normal is upward; for the fluid volume itself, the outward unit normal is downwards.

So, the force exerted by the fluid on the surfaces opposite to the force exerted by the surface on the fluid. So, the force in the x direction that is; the force in this x direction acting at a surface whose outward unit normal is in the y direction the force per unit area, that is the stress tensor. So, there are two unit vectors here or two directions associated with this quantity called the stress. One is the direction of the force, the force that is the exerted on the surface; the force of course, depends upon the orientation of the surface, and so there is a surface orientation, which is the second direction.

So, there are two directions here. So, this is the stress one particular component of the stress but, if am sitting at some particular location, if am sitting at some particular location in the fluid, the stress is defined at that particular location. So, I can construct a small box around that location, and if I expand out this box; if I expand out this box, this differential volume. This differential volume, this fluid element sitting inside the fluid has forces exerted on it in all directions.

So, the force on the top surface, the top surface has a unit normal in the plus x 3 direction. The top surface has a unit normal in the plus x 3 direction. The bottom surface has unit in the minus x 3 direction. So, at this top surface there is some force some force that force of course, decreases as the area decreases, because the force is propositional to the area in the limit as the area goes to 0. The component of this force in the x 1 direction, this force will have three components once again, the component of this force in the x 1 direction is t 1 3; force direction is x 1 unit normal direction is x 3, so t 1 3, the component in the x 2 direction is t 2 3 and so on.

So, the first index is associated with the force. The second index is associated with the direction of the unit normal. So, for any general for any general component can be represented as t i j is equal to force per area in the i direction. Let me write that as x i direction to avoid confusion, x i direction acting at a surface with outward unit normal in x j direction. The force made in unit area in the x i direction acting a surface with outward unit normal in the x j direction. So, that is the definition of the stress, so rather than so there are, now total of 9 components. I can go from 1 2 3. So, the force can have 3 different directions. At that particular location, the unit normal to the surface can also have 1 of 3 directions. So, you have for the unit normal therefore; total 9 directions for that total stress. Similarly to 3 directions for the velocity, 3 components of the velocity in this particular case since there are two fundamental directions associated to the stress. There are now nine components of the stress.

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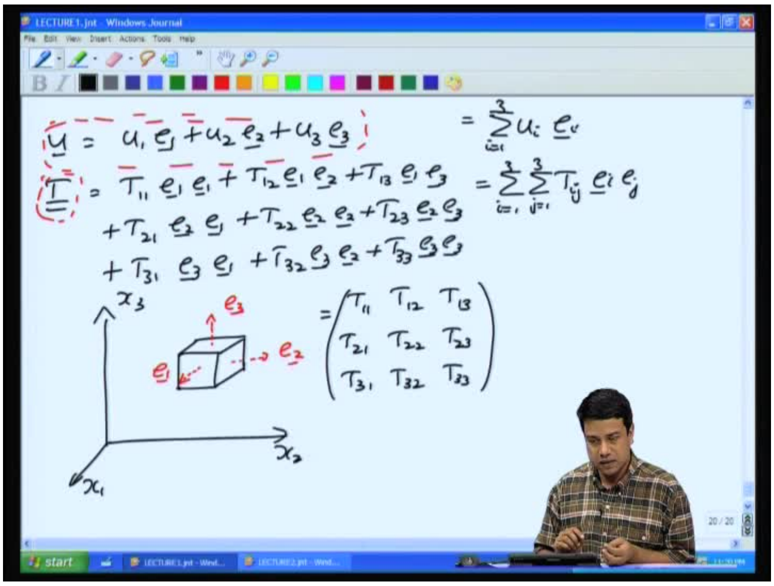

I can write this in short form just as, just as I had written for the velocity vector, u vector is equal to u 1 e 1 plus u 2 e 2 plus u 3 e 3. Similarly, I can write the stress, it now has 2 fundamental directions associated with it. Each direction has 3 components, direction of force, direction of unit normal. So, I can write the stress as T 1 1 e 1 e 1 plus T 1 2 e 1 e 2 plus T 1 3 e 1 e 3 plus T 2 1 e 2 e 1 plus T 2 2, then I have this plus other plus T. So, it has total of 9 components just as I had written this as summation i is equal to 1 to 3 of u i e i. I can write this as summation i is equal to 1 to 3 summation j is equal to 1 to 3 of T i j e i e j. So, what do each of these mean when i have, when I have my differential volume; I have 3 directions for the unit normal. So, the first is this is in the x 3 direction; this is in the x 2 direction. So, this unit normal is e 2 and coming out of the board for you is the unit normal e 1.

So, the front and back surfaces of this volume have unit normals in the plus e 1 and minus e 1 direction respectively. The right and left have unit normal's in the plus e 2 and minus e 2 directions. Top and bottom have unit normals in the plus e 3 and minus e 3 directions. At each of these surfaces, there is a force acting due to either fluid pressure or due to fluid flow, and the component of the force acting on this front surface in the one direction is T 1 1 per unit area, and the 2 direction is T 2 1; force direction is 2; unit normal direction is 1 etcetera.

So this just as this velocity vector contains all the information about the velocity at a given location. This vector contains all the information about the velocity at a given position in space. It has 3 components and you can also get the vector direction out of these 3 components. Similarly, this object the stress tensor which has 9 components; in it contains all the information about the force per unit area at a given location for the force direction as well as the direction of orientation of the surface; contains all of that information in these 9 components, and I can write it out this way. We will see a little later in some applications. It is also convenient to write this in a matrix form. The stress tensor is sometimes conveniently written in a matrix form. There are certain advantages to writing in this form. We will see that as we go along in this course.

So, this is the notation for the stress tensor and of course, you can have tensors with more fundamental directions, velocity has one fundamental direction. It is a first order tensor temperature field has 0 directions. It is a scalar so it is a 0 th order tensor. The stress have 2 directions, it is a second order tensor. You could in general have tensors with 3 fundamental directions, third order, fourth order, fifth order and so on. The second first tensor which is a vector has 3 components 3 power 1. The second order tensor has 2 fundamental directions. Each of those directions has 3 components, so we get total of 9, 3 power 2; the second order will have 3 power 3, I am sorry, the third order will have 3 power 3; fourth order will have 3 power 4 and so on.

We would not go much further than second order in this particular course. But, it is important to realize that this stress tensor as defined is a second order tensor which has information about all components of the force per unit area at a given location. As I briefly introduced in the previous lecture the force per unit area is essentially the momentum flux. So, the flux direction, there are 3 flux directions. There are 3 force directions therefore; you have a total of 9 components.

Now, we had introduced the dot product in the previous slide, the dot product of 2 vectors is just is a scalar. So, you take the dot product of the 2 the components multiply the components with each other and I written in terms of the unit vector. I can rewrite the dot product as follows.

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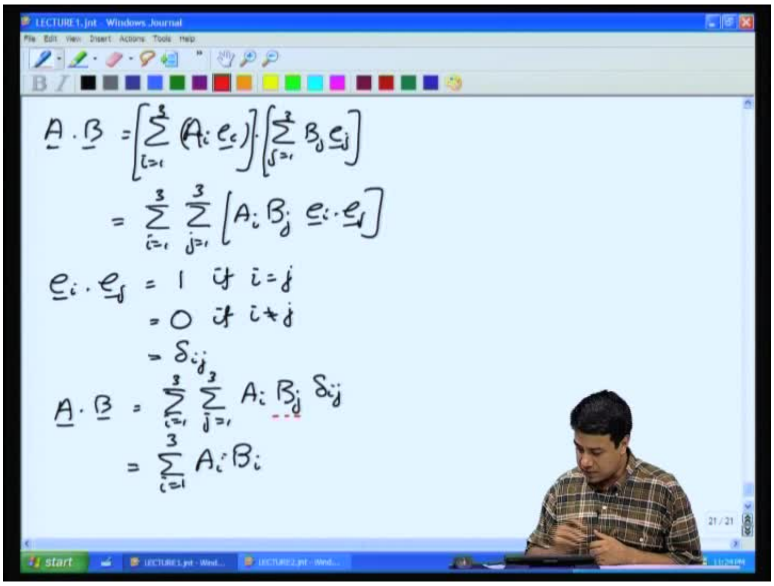

A dot B is equal to A i i is equal to 1 to 3 dotted with the second vector, so in dotting 2 vectors. Note here that I have used 2 different indices for the 2 vectors; in the first case, I have used I; second case, I have used j. The reason will be become clearer in a little bit. so when you take the dot product of 2 components and unit vectors. You multiply the components dot the unit vectors, 1 to 3 of multiply the components and dot the unit vectors; multiply the components dot the unit vectors. Now, what is e i dotted with e j; i is 1 2 or 3, j is also 1 2 or 3.

So if i and j are equal, then e i dot e j is equal to 1, because the 2 are both unit vectors. If i and j are not equal then e i dot e j is equal to 0; because the vectors are orthogonal. So, e 1 dot e 1 is equal to 0, i am sorry e 1 dot e 1 is 1; e 1 dot e 2 is 0; e 1 dot e 3 is 0 and so on. So, this dot product e i dot e j is also written as this is equal to 1 if i is equal to j, and this equal to 0, if i is not equal to j. So, it is either 1 or 0, i is equal to j, it is equal to 1. If i is not equal to j, it is equal to 0.

It is written by the simple delta i j. Delta i j often called as the chronicle delta function. Delta i j is 1, if i is equal to j; delta i j is 0, if i is not equal to j. Therefore, i can write A dot B as summation i is equal to 1 to 3, summation j is equal to 1 to 3 of A i B j delta i j. Because, e i dot e j was equal to delta i j, which was 1 if i is equal to j, and it was 0 if i is not equal to j. Now, I have summation i is equal to 1 to 3 A i B j times delta i j, I have two summations; i is equal to 1 to 3 and j is equal to 1 to 3. Of course, delta i j is non-zero only, if i is equal to j. So, when i take the summation over j is equal to 1 to 3, I will get A non-zero result only, when i is equal to j.

Therefore, I can rewrite this as summation i is equal to 1 to 3 of A i times B i. Because, in the second summation only when j is equal to I, you get a non-zero result, In the second summation only when j is equal to I, do you get a nonzero result. Therefore, I can replace j by i and remove that summation I get summation of A i B i. This as you will recall is exactly what we had when we discussed the dot product earlier. This is exactly what we had when we discussed the dot product. One important point to remember, here is that the indices that I use for these two have to be different. Because, one index is getting some lower from 1 to 3; the other is also getting some lower from 1 to 3. It is not the same as both of them simultaneously getting some lower from 1 to 3. So, that is the dot product. The other vector product that you are used to is what is called the cross product.

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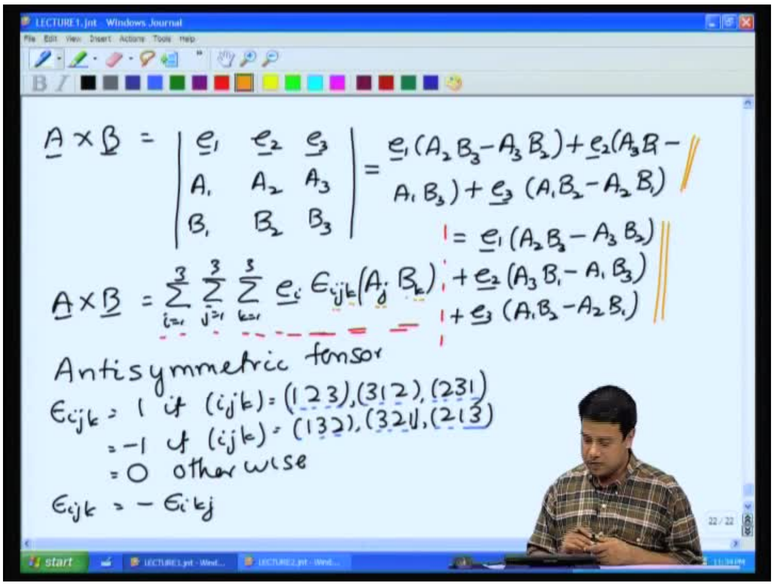

A vector cross B vector, there are many ways to write it, one is to write it as a magnitude of A magnitude of B sin theta. The other way that it is usually written is to take the determinant of the matrix which consists of e 1 e 2 e 3 A 1 A 2 A 3 B 1, you take the determinant of that matrix and you will get the vector. You will get a vector, which is e 1 into A 2 B 3 minus A 3 B 3 plus, so that is the cross product to written it in long form. It is a vector, which is perpendicular to both vectors A vector and B vector, and it is equal to 0 if both vectors are collinear.

For our proposes rather than write it this way, we will write the cross product in terms of a dot product using what is called the anti symmetric tensor. So, for example, if i have the cross product A cross B, what I am going to write this is as summation i is equal to 1 to 3 e i epsilon i j k A j B k. Note this is now a vector, this is a vector because it has 1 unit vector.

Whenever there is a vector, there is a free index and there is a unit vector, and when the you have a dot product, the index is repeated as you can see when you have a dot product here. You have the same index repeated two times. So, if i write it this way what is this epsilon? Epsilon is what is called the anti symmetric tensor is equal to 1, if i j k is equal to 1 2 3, 3 1 2, if it is equal to minus 1 if is equal to 0, if any two well, let us just say is equal to 0 otherwise is equal to 0 otherwise.

So, epsilon i j k, if i j and k are 1 2 3, 3 1 2 or 2 3 1, then epsilon i j k is 1; if i j k are 1 3 2, 3 2 1 or 2 1 3 then is equal to minus 1 and 0 otherwise; otherwise means that any 2 of these indices i j and k are repeated, because there is only 6 combinations, in which all 3 are not repeated. In all other combination, there had to be at least two indices that are repeated, and if 2 indices are repeated then this is identically equal to 0.

Notice one thing about this tensor, if i inter change any two indices, I get the negative of its value; that is epsilon i j k is equal to minus epsilon i k j that is why it is called an anti symmetric tensor. That is epsilon i k j and epsilon i j k are go from one to the other by multiplying by minus 1. So, in this case, I have inter change 3 and 2, and I get minus 1. In this case, I have interchanged 1 and 2 and I get minus 1 and vice versa. So, that is why it is called the anti symmetric tensor. How was this useful for cross products? I have here my formula for the anti symmetric tensor you is with cross products.

So, let us just put this in and use this formula keeping in mind that epsilon i j k is non-zero only when all 3 indices are different, if non-zero only when all 3 indices are different, and it is 0 if any 2 of the these indices are the same. So, let just implement this formula the first is when i is equal to 1, in that case the unit vector e i is just e 1, unit vector e i is just e 1. Once i is equal to 1 then j and k have to be either 2 and 3 or 3 and 2.

Because they cannot repeat, so if j and k are 2 and 3 then I have epsilon i j k is 1, if i j k is 1 2 3, if i j k is 1 2 3 then epsilon i j k is 1. Therefore, I have plus 1 into A 2 into B, so that is my epsilon i j k, j and k are 2 and 3 i is 1. There is one other combination, which is non- zero that is when j and k are 3 and 2, and when j and k are 3 and 2 epsilon i j k as you can see from here is minus 1; when j and k are 3 and 2 in that case I get minus 1 into A 3 into B 2. So, these are the only results left over, when i is equal to 1, because all other cases epsilon i j k is equal to 0 then I have for i is equal to 2 in which case, the unit vector has e 2; i is equal to 2 that means j and k have to be either 3 and 1 or 1 and 3. If j and k are 3 and 1 then I get plus 1. If j and k are 3 and 1, i is 2 j and k are 3 and 1 then I get plus 1 so j is 3 that means i have A 3 k is 1 B 1 ok.

The other combination when i is 2 is when j and k are 1 and 3 in that case epsilon is equal to minus 1. So, I get minus A 1 B 3. Similarly, you can do it for i is equal to 3 in which case you will get e 3 vector into you can do it quite easily. The first is when i is 1 and j is equal to 2 in that case epsilon is equal to plus 1. So, you will get A 1 B 2, and the other cases, where i is 2 and j is 1, you will get minus 1 there. So, applying this particular formula, we see that the result that we get for the cross product is identical to what we had in this case.

So, this also gives me the result for the cross product. So, in this I have one unit vector and I have two dot products. Because, there are these two indices which are repeated, j is repeated and k is repeated; so that cross product itself can be considered as the dot product of this epsilon i j k, which is actually a third order tensor, I thus 3 indices with 2 vectors. So, the cross product once, you get used to second order, third order tensors; the cross product is no different from the dot product.

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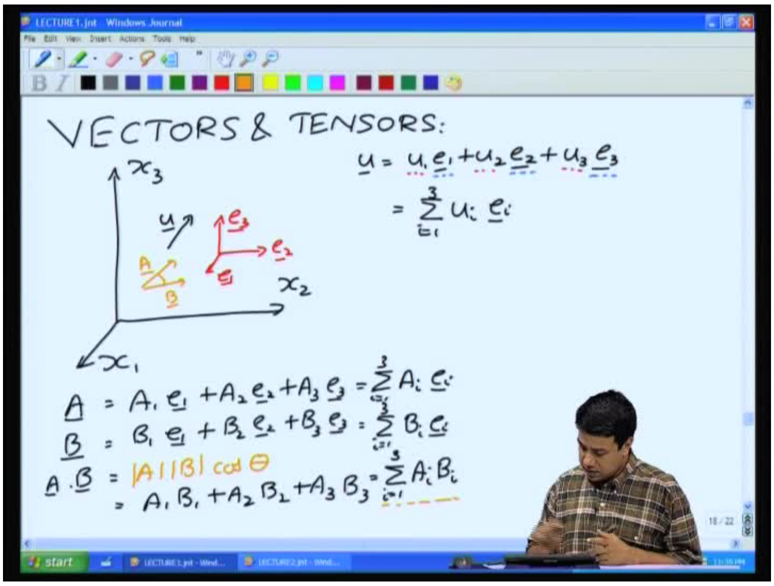

Now, let us make a notational simplification yeah. So, in this case I have written down u vector as u i times e i. There was one index i that was not repeated and that index was summed over, one is to 1 to 3 and there was a unit vector next to it. Similarly for A and B for the dot product; now, there is 1 index but that appears two times A i B i. It appears two times, i appear 2 times and there is 1 summation; for the second order tensor on the other hand that I wrote down for you.

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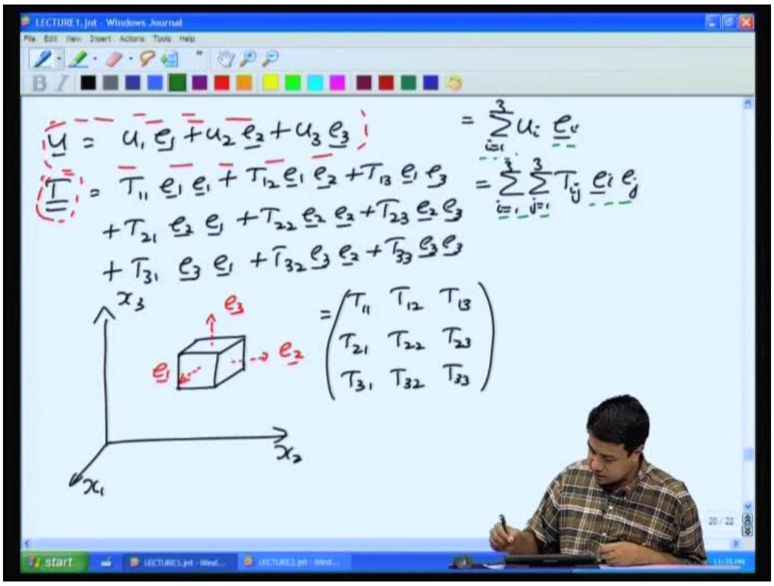

There are Summations over two indices T i j. Each one of those appears one time. There is a summation and a unit vector. Without loss of generality without any notational ambiguity, one can remove the summations and the unit vectors.

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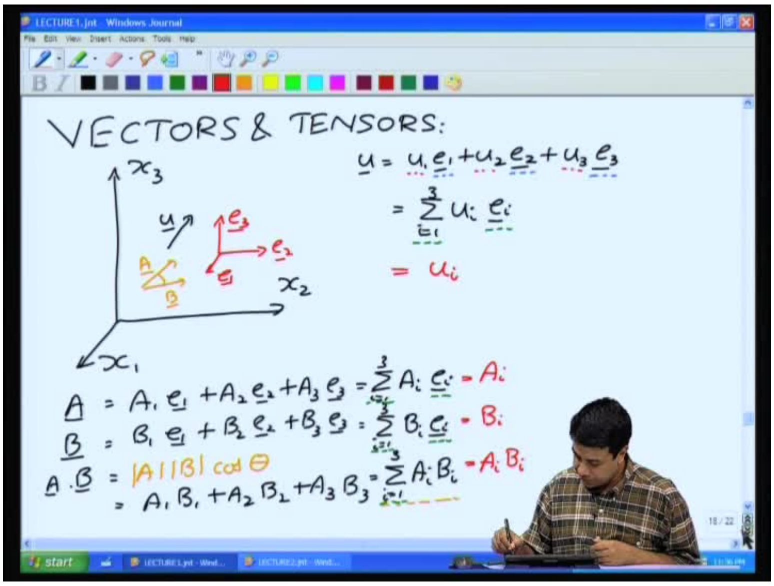

The unit vector as well as the summation can in general be removed. This unit vectors the summation, and so I can just write this vector. This I can write it as just u i. There is one index, which is not repeated that means that one the index is not repeated; there is a summation and there is a unit vector that index represents the one fundamental direction in this case the direction of the third velocity itself. Similarly, in this case A i; this is B i; this dot product, I can write it as A i times B i. One index i that means there is a summation but, that index is repeated two times that means, there is a dot product; it is summed over, so this is dot product.

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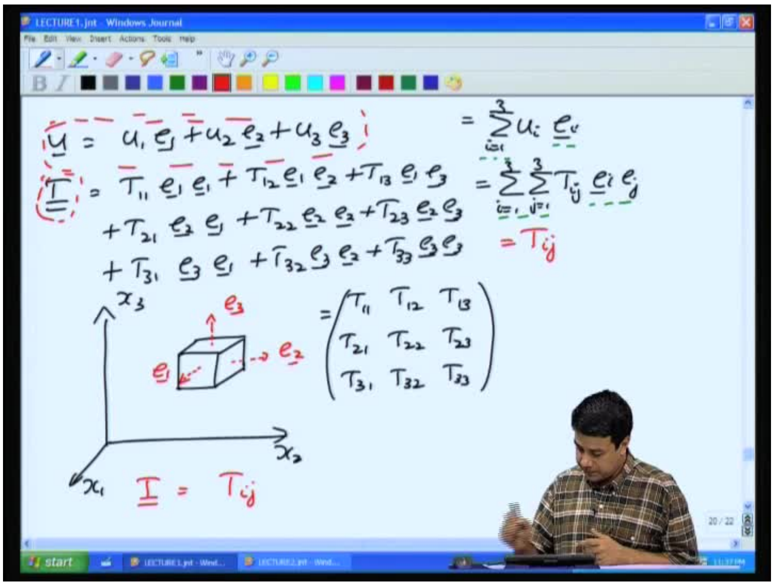

My notation the stress tensor, I will just write it as T i j; it has 2 indices which are not repeated that means there are two unit vectors and two summations just as I have here. There are two unit vectors just as I have here, so the index is not repeated; whenever there is an index; that is not repeated that means, there is a unit vector associated with it and a summation; that means that the order of the tensor is equal to the number of un repeated indices that are present in this tensor.

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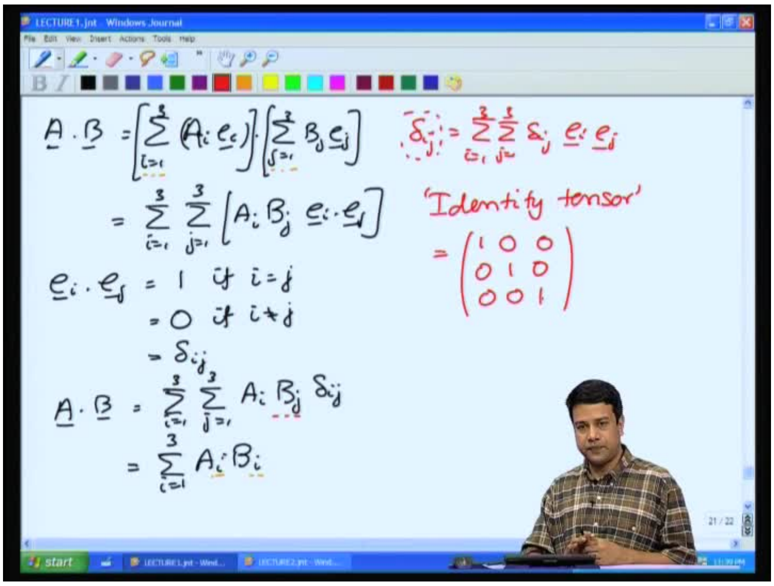

How about this? So, I have here A i B j times delta i j with two summations. You can see that i is repeated two times that means; there is a summation but, no unit vector. It is a dot product; j is repeated two times that means there is a summation but, no unit vector so there is a dot product. So, both of these A and B vector are dotted separately with delta; delta i j has two indices that is a second tensor. So, delta i j, if expand it out will be delta delta i j e i e j. So, in my short hand notation, I just wrote this as delta i j, whereas in my long hand, I have delta i j with two unit vectors sitting next to it.

So, this second order tensor is dotted with both the vectors A and B. To get the dot product of the two vectors which is A i times B i and what I have here is once again one repeated index and one summation, no unit vectors because it is a scalar. This thing is what is also called as the identity tensor. In long hand notation, if i write it down, it will be delta 1 1 e 1 e 1 plus delta 1 2 e 1 e 2 plus etcetera. However, delta is 1 1 is 1; delta 1 2 is 0; delta 1 3 is 0 and so on. So, if you expand this out in a matrix form, you should write it you just get an identity matrix that is why it is called the identity tensor written in matrix form. It is the identity matrix and this tensor is actually independent of coordinate systems just as the anti-symmetric tensor.

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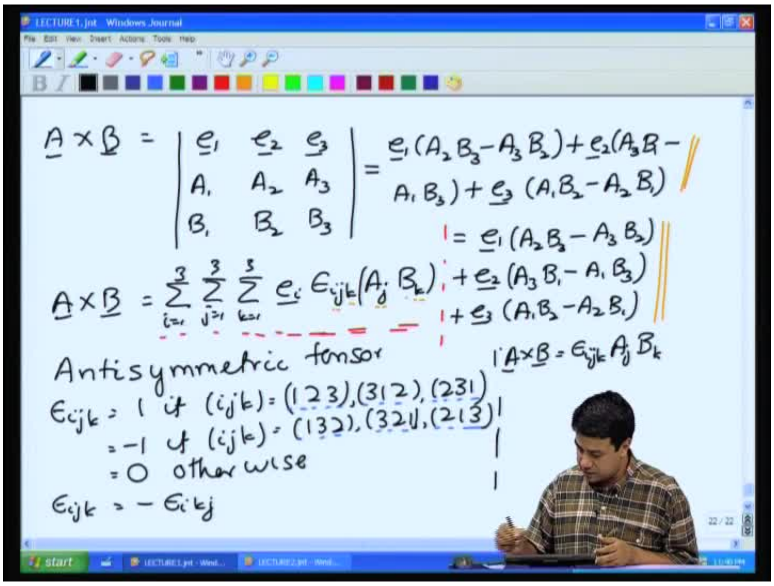

So, back to my cross products if I have to write this in my indicial notation, I will just write A cross B is equal to epsilon i j k A j B k. So, this if you look at the right hand side, there are three indices i j and k; only one of these is not repeated that is i. So that gives you one direction one unit normal and the summation over i; j is repeated two times implies a dot product; k is repeated 2 times it also implies a dot product. So, there are no unit vectors, there are three summations and one unit vector corresponding to i. So, it is a vector what you get in the and this is a vector so that cross product is no different from a dot product may be can reduce it to a dot product by dotting, this anti symmetric tensor with the 2 vectors that we are considering.

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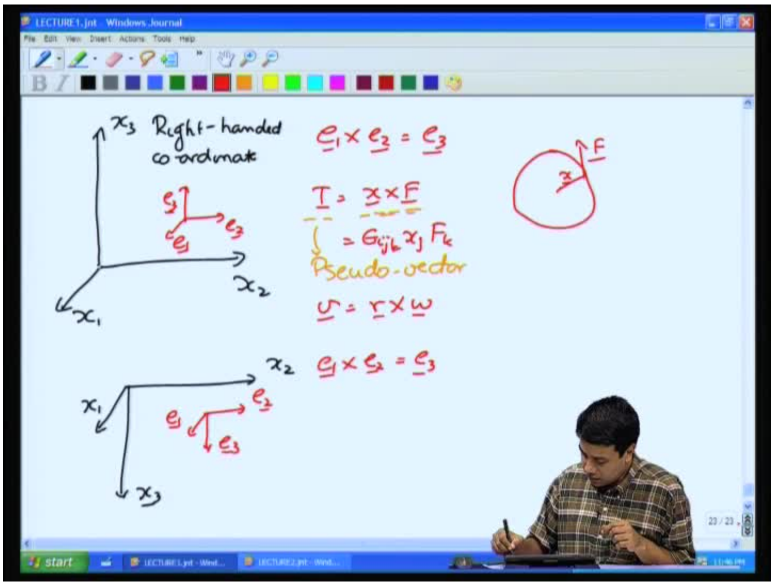

Now, 1 has to be careful when using cross products, because the result could depend upon the coordinate system that you are using. So, let me just try to explain that and I have three unit vectors. So, you know that e 1 cross e 2 is equal to e 3. I take the cross product of two vectors e 1 and e 2 space; it is perpendicular as per right hand rule. It has to be in the plus e 3 direction; so this is for a right-handed coordinate system but, there is no reason why we should use only a right handed coordinate system.

One could as well do the analysis in a coordinate system that is left handed. So, if i show is a left-handed coordinate system then my unit vectors would be e 1 e 2 and e 3. This is little longer in this case as well we know that e 1 cross e 2 is equal to e 3. So, it appears that if i have two vectors e 1 and e 2, the cross product of those two is upward in a right handed coordinate system and downward in a left hand coordinate system. It seems like a trick question, but it is not. In fact in a right handed coordinate system, the cross product that you get is opposite to what you get in a left hand coordinate system. The cross product depends upon the convention that you use for calculating that cross product. If you go back to fundamental mechanics courses, you know that the torque is defined often as x cross f. So, if I have some, some object and you have a force exerted on that object at a certain distance from the center of mass, the torque vector is often written as the cross product of these two.

In a right handed coordinate system, the torque will be one way. In left handed coordinate system will be the opposite way that is consistent, there is no inconsistency. there the reason is because the torque is a derived quantity x and f are measureable quantities. For example, the distance can actually be measured; the distance is the vector pointing from the initial to the final location that displacement is actually a measurable quantity. The force is a measurable quantity. Basically, the force is equal to the mass times acceleration.

The acceleration is rate of change of velocity, so that is a vector that is a real vector; the position is a real vector. If you take the cross product of these two, the result depends upon coordinate system, points in one direction for one coordinate system and in the opposite direction for the other coordinate system. So, because of that the torque is not a real quantity. It is often what is called as a pseudo vector for a given configuration. You can measure distance, you can measure force but the direction of torque depends upon the coordinate system that you use ok.

So if I have one cross product, so this is basically, equal to epsilon i j k x j f k. If you have one cross product then you get a pseudo vector. Because the sign of this epsilon, the direction in which this epsilon vector points depends upon the coordinate system; there is being used; so one has to keep this in mind when interpreting results. If you take the cross product of two real vectors as in this case x and f, you will get a pseudo vector whose direction will change. If you should go from a right handed to a left handed coordinate system, on the other hand the linear velocity for example, is written as r cross omega, the linear velocity is often written as u is equal to when displacement cross to the angular velocity. In this case, the linear velocity is a real quantity; displacement is a real quantity.

So, both of these remain the same, when you interchange from right handed to left handed coordinate system; that means that this angular velocity has to be a pseudo vector whose direction will change depending upon whether you use a right handed or a left hand coordinate system. So, that is something that is to be keep in mind when you take a cross product you should. If there is one cross product between real quantities, you get a pseudo quantity who direction changes depending upon the right or left handed coordinate system used.

On the other hand, if you have a cross product involving one real and one pseudo vector, we will get end up with a real vector. Because, omega changes in sign that cross that epsilon also changes direction therefore, the velocity will be independent of the coordinate system used. So, this cross product makes a distinction between two different types of quantities the real quantity and the pseudo quantity.

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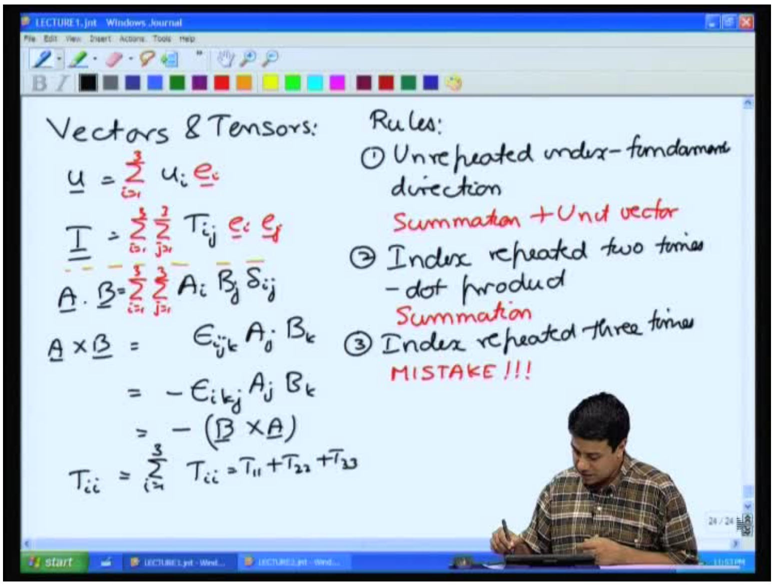

So, let us briefly review, what we done so far in vectors and tensors. The velocity u vector is just written as u i; implicit in this notation is that there is a unit vector there and there is a summation there, whenever 1 has a doubt about this, we go from the short hand notation u i to the complete long hand including all the other e i as well as summation. Second order tensor is written as T i j implicit that there is one two summations for each of the unrepeated indices and two unit vectors. The dot product is equal to A i B j.

These are two vectors and the dot product has the identity. Second order tensor in it A i B j delta i j. In this case, there are two indices i and j which are repeated two times each that means, there is only two summations and there are no unit vectors. There are no unit vectors, because each index is repeated two times. The cross product is written as epsilon i j k A j B k.

Note that when i interchange any two indices, epsilon gets reversed in sign. So, when i interchange any two indices, I can write this as epsilon i k j. Because i interchange j and k, I get minus sign and this is equal to minus B cross A, because whenever I take the cross product, I take the one that the index that appears that second here; i is the first index that is the free index gives you the direction of the resulting cross product vector. So, i is the first index, which gives you the direction of the vector; j is the second index and k is the third index epsilon, and i j k A j times B k is equal to A cross B; therefore, epsilon i k j. That means that I have to take the second index B crossed with A and because the tensor is anti-symmetric, I get the familiar result that A cross B is equal to minus B cross A.

The rules to be followed, represents some fundamental direction and for this there is summation plus unit vector make it clear, and index is repeated two times within the same expression that means there is a dot product. And with this there is only a summation with this, there is only a summation. There is no unit vector, because the dot product is a scalar, and just for consistency index repeated three times index repeated, three times is is there is some mistake. Because once it has been repeated two times, it has already been summed over. It is already become scalar, you cannot have that same index repeated the third time. So, with these brief rules, we can go ahead and work with vectors tensors of any order without having to worry about the distinction between dot products, cross products and tensors of different order.

For example, i said that the that the tensor t i j was equal to summation i is equal to 1 to 3, summation j is equal to 1 to 3 of T i j e i dotted with e j. What is T i i? In this case, there is one index that is repeated within the same tensor two times. There is one index that is repeated within the same tensor two times. Therefore, if I expand this out what I am going to get is, for this particular index, I am going to have one summation and no unit vectors. Because, there is no unrepeated indices summations i is equal to 1 to 3, there are no unrepeated indices. So, there is no unit vector; this is a scalar one summation, this is equal to T 1 1 plus T 2 2 plus T 3 3. So, for those of you that that that know that recall your matrices, this is the trace of that stress tensor matrix. So for example, I had written the stress tensor as in this matrix form T 1 1 plus T 2 2 plus T 3 3 is the sum of the diagonal elements. So, it is also called the trace of this matrix.

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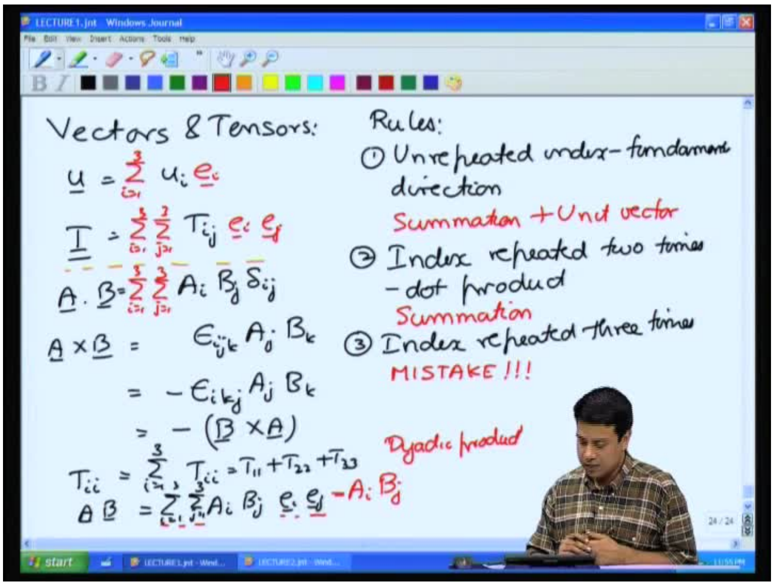

When I take products, I do not have to take the cross product or dot product alone. I could for example, defined the product as A B without any dot or cross setting inside. So, this if i just take this product A i B j e i e j with two summations, with two summations A i B j e i e j. In my short hand notation, I do not write down the summations in the unit vectors; so this just becomes equal to A i B j; this has two unrepeated indices. Therefore, it has two fundamental directions; there are two unit vectors; two summations that means that this product is a second order tensor is called the dyadic product of, it is often called the dyadic product of two vectors. So, I do not have to take just the dot or cross product dot product will give me a scalar, cross will give me a vector. A dyadic product will give me a second order tensor. In this case, so I can take the dyadic product of two vectors to get a second order tensor.

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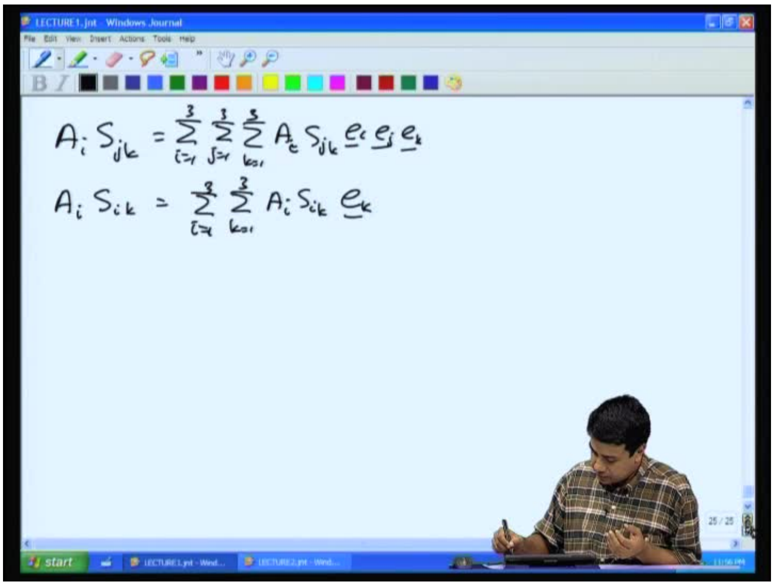

Similarly, you can take the product of anything so for example, you can write down A i s j k. This basically is the direct product of one vector with the second order tensor; this obviously gives me a third order tensor; write it out in long notations summation i is equal to 1 to 3 A i s j k e i e je k, it has three unit vectors. Therefore, it is a third order tensor, the dyadic product where A and S are a first and the second second order tensor respectively. I could also define another one A i s i k.

In this case, now there are two indices. So, there are two summations, there are two indices i and k; summation i is equal to 1 to 3, summation k is equal to 1 to 3 of A i s i k, and there is only one unit vector left. Because, there is only one unrepeated index and that is e k. So, this is a dot product between a first order vector, the first order tensor and a second order tensor. The result is first order, because one dot product reduces the order by two, because you are dotting two directions. So it reduces it by two, so whereas I have a first order vector and a second order tensor when I take the dot product, I get A vector. And similarly, these can be done for for tensors of all orders, and the other important thing to remember is that, I told you these rule to keep in mind, these are for just for the vectors and tensors themselves.

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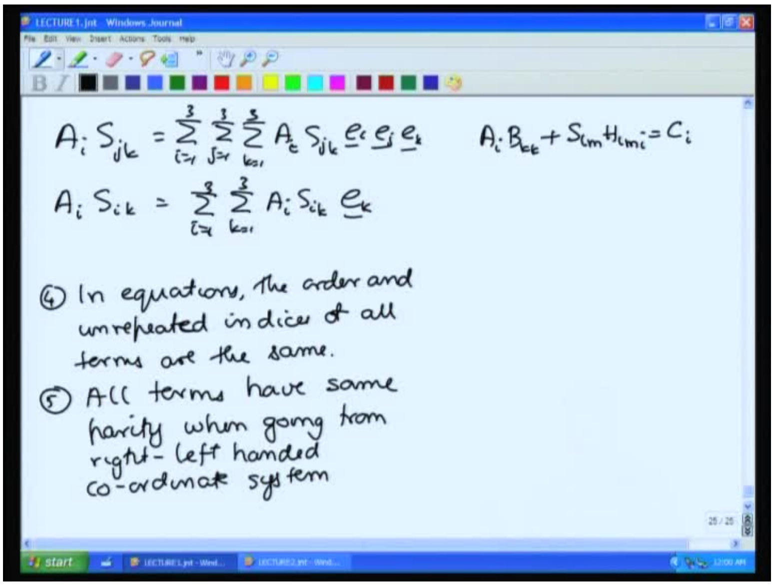

I can also write equations equating different vectors and tensors and one has to remember that the forth rule which, which I have not put up there was that in equations, the order and unrepeated indices of all terms are the same. The order of all terms have to the same for example, I cannot equate a vector to a third order tensor. The vector has three components. The third order tensor has three times three times three that is 27. So, I cannot equate a vector to a second order tensor. I cannot equate a vector to a scalar. It make no sense, all terms when you write an equation have to have scalars vectors tensors of the same order.

The other thing is that all the unrepeated indices have to be the same, because each unrepeated index represents one particular direction, and the equation is some kind of a balance for that particular direction. So, all the unrepeated indices on all sides of the equation have to be the same. For example, I can write an equation as form A i B k k plus s s l m h l m i is equal to c i. This is a valid equation because each of the these terms is a vector. The first term A i times B k k k is repeated. So it is a scalar, so there is one vector. Second term s l m times h l m i l and m are repeated. So, once again one gets a vector and the right hand side also there is a vector.

So, the unrepeated indices as well as the order of each term in the equation have to be exactly the same when you write an equation. And the other rule to remember is that all terms have same parity when going from right to left handed coordinate system; that is if one term changes sign, when you go from right to left; all other terms also have to be change sign. If one term remains unchanged, then all the terms have to remain unchanged; otherwise what will happened is that, if the equation is valid under right handed coordinate system may go to left handed coordinate system; some terms have changed sign; some have not and the equation is no longer valid.

So, valid equation can be written only, if all terms have the same parity under going from right to left handed coordinate system that is; all terms either have to be pseudo vectors or tensors or they have to real vectors or tensors either all have to change sign or none has to change sign. So, with this brief background on vectors and tensors mostly, what we did was notational simplifications along with simpler ways of writing things like dot and cross products. As we will see these simplifications will have a significant consequences, when we write down the equations later on.

So, after having done basic vectors and tensors in this lecture, in the next lecture I will do vector calculus, how do you translate differentiation and integration, when you are working with vectors in three-dimensional spaces. So, we will continue that in the next lecture, please go through this again before, you come to the next lecture. So that you are clear about the fundamentals here and we will continue in the next class so we will see you then.