Fundamentals of Transport Process II
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Lecture - 29
Flow around a Cylinder

This is lecture number 29 of our course on Fundamentals of Transport processes. We were discussing in the last lecture two-dimensional potential flows, flows in which there is variation in only two directions, and there is no variation in the third direction. We will see a little later why these kinds of flows are significant in practical applications, but let us continue our discussion today on two-dimensional potential flows.

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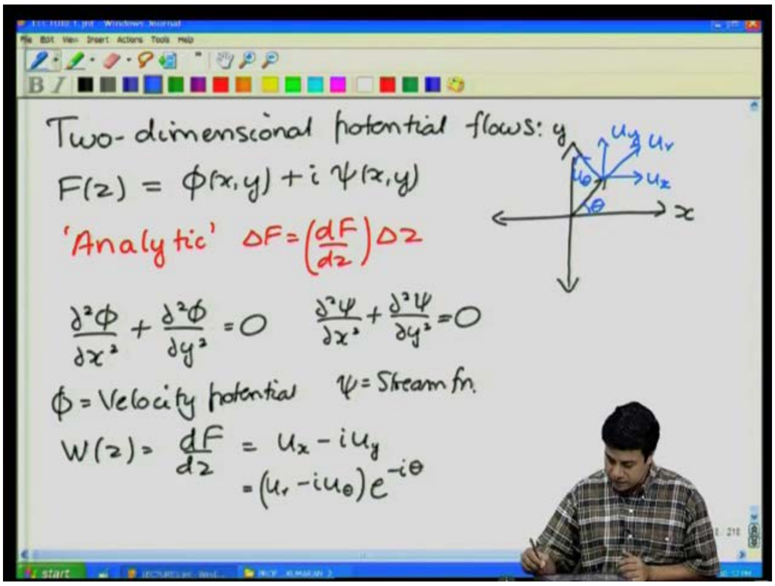

So, the procedure that we were adopting was to analyze these in the complex plane, that is we have a two-dimensional coordinate system x and y. Rather than look at the equations in this x y coordinate system, we rather consider the equations in the complex plane where any position location is given by x plus i y, where z is the complex number, so that is the location in the complex plane. And as I showed you in the previous lecture, if any function F of z this is in general a complex function, it has a real part and an imaginary part can be written as pi of x y plus i times psi x y, where pi and psi are the real and imaginary parts of this function F.

If this function is analytic if this function is analytic if this function is analytic, that is the change in F when you go a small distance delta z can be written as d f by d z times delta z, that is at a given location if you are at given location z and you move a small distance delta z to a new location. The change in f between the new location and the original location is equal to something the derivative times delta z.

So, if this is true then it can be shown that both the real and imaginary parts satisfy the Laplace equation that is partial square pi by plus partial square pi by partial y square is equal to 0. And similarly for since the since pi satisfies the Laplace equation we can without loss of generality we consider this real part pi as the potential for a potential flow, because for a potential flow we require that pi has to satisfy the Laplace equation.

So, if we assign the physical interpretation of the velocity potential to this function pi then the from the (( )) conditions that we had in the previous lectures the function psi is a stream function, so I have pi is the potential and psi is the stream function. So, these are the two functions which are the real and imaginary parts of the complex function F. So, F is a complex potential whose real part is the velocity potential, imaginary part is the stream function. Now, for this function F I can define for this complex potential F, I can define a complex velocity is equal to d f by d z and this complex velocity is related to the real components of the velocity by u x minus i u y will also get in as u r minus i u theta e power minus i theta. Where u x and u y are the components of the velocities in the x and y directions at given location.

So, at a given locations this is u x, this is u y, u r is along the radius vector along the displacement from the origin. And u theta is along the direction of increase in theta u theta is along the direction of increase in theta. So, theta increases in the anti-clockwise direction theta is equal to 0 on the x axis and increases in the anti-clock wise direction, therefore u theta has to be in in this direction as well good.

So, we reverse the question around rather than asking what is the solution for the potential flow appropriate for a certain problem with well-defined boundary conditions. We reverse the question around and the ask what is the potential flow that corresponds to specific forms of this potential function F.

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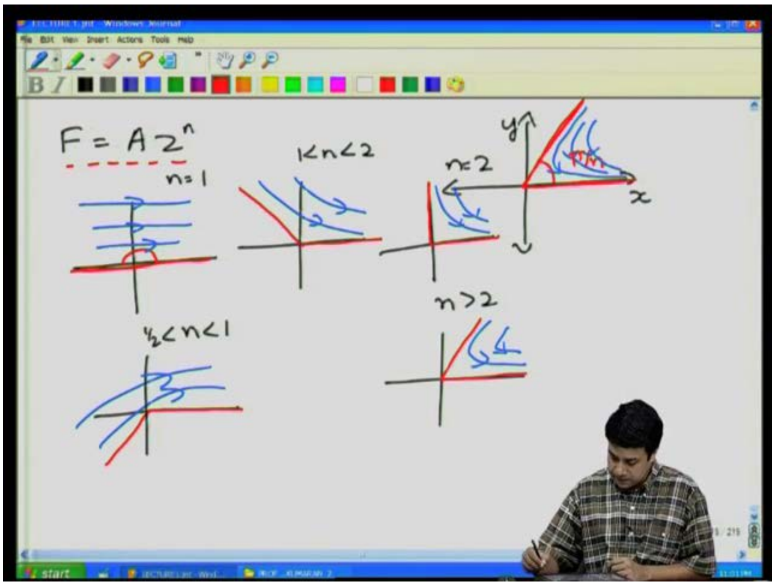

The simplest function that we could consider was the function of the form F is equal to A times Z power n and we saw that this potential and this potential corresponds to a velocity field within a domain within a corner of angle pi by n this velocity field within a corner of angle pi by n and the velocity field looks something like this, within this corner. So, this is the solution for the velocity field for a function of the form A Z power n the the velocity is the fluid flows flow in a corner of angle pi by n.

Note, that this two sides the two walls of the corner or locations at which the normal velocity is equal to 0. So, I require for my potential flow boundary conditions a normal velocity has to be 0 at the bounding surfaces, since the normal velocity is 0 at these two surfaces these act as bounding surfaces for the potential flow. Recall we cannot satisfy tangential velocity boundary conditions at surfaces in potential flow, therefore the velocity field the tangential velocity is non zero at these surfaces.

So, obviously, for n is equal to 2 the angle is pi by 2, so it is right angled n is equal to 1 it is just flat. So, for n is equal to n is equal to 1 you get something that is just flat, the angle is just pi and velocity stream lines looks straight. For n is equal to 2 you get an angle that is a right angle looks like this and the velocity field looks something like this between 1 and 2 this is n is equal to 2, 1 less than n less than 2 corresponds to an angle that looks like this.

For n greater than 2 you get an acute angle, which looks something like this and one can have n less than 1 is as well is possible to have n less than 1 as well. In the case of n less than 1 you get corner whose angle is greater than pi and velocity field looks something like this and the minimum value of n is half. So, this is n less than 1 the minimum value of n is a half which the angle is perfectly pi, I should note that these solutions a Z power n are exactly the same the equivalent of the growing harmonics that we calculated for three dimensional potential flows.

In that case we have to calculate it either in terms of some legendary polynomials or in terms of vector notations. If you recall we got r power n PNM of cos theta, these solutions are identical to the growing harmonics the equivalent of that in two dimensions.

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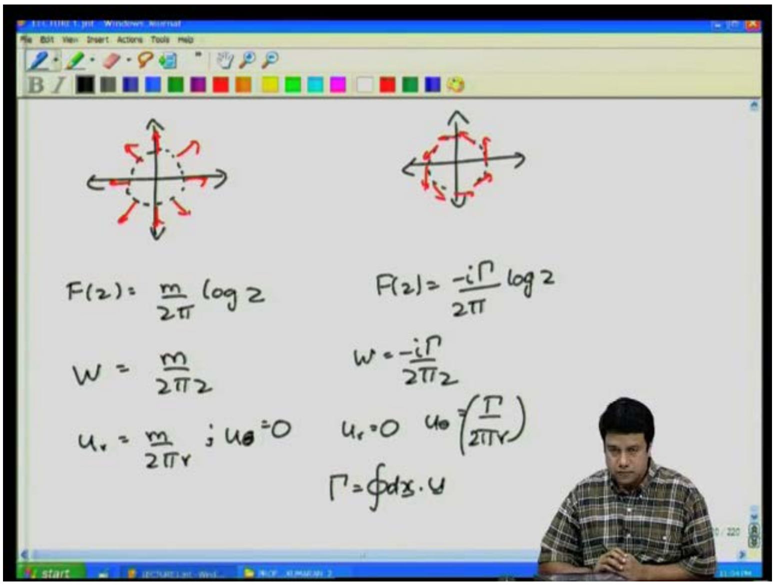

Next we looked at two specific forms one was F of z is equal to m by 2 pi log z in which case w is equal to m by 2 pi z. And that corresponded to a flow that was radially outward this is corresponded to a flow that was radially outward and the source strength was m the amount of fluid coming out per unit length perpendicular to the plane of the flow. As I said this is two dimensions and therefore, we assume that it is infinite perpendicular to the plane of the flow and all quantities whether it is the mask coming out the force etcetera or per unit length perpendicular to the flow.

So, this was a radially outward flow m by 2 pi log z in which case the velocity was m by 2 pi z, this corresponded to u r is equal to m by 2 pi r and u theta is equal to 0. So, this was the the solution for m u r is equal to m by 2 pi r u theta is equal to 0. The other special flow that we had analyzed was was F of z is equal to minus i gamma by 2 pi log z and if you work out this this w is equal to minus i gamma by 2 pi z which means that u r is equal to 0 u theta is equal to gamma by 2 pi. So, this corresponded to a circulating flow, circulating around the origin. And the integral of the the value the parameter gamma is just a integral over any closed loop d x dot u over a closed surface integral of d x dot u over a closed surface. Now, this of course, this this relationship emerges from the stokes theorem, for for the curl of the vector, let me just retrieve that one more time before we proceed to a more complicated velocity profile.

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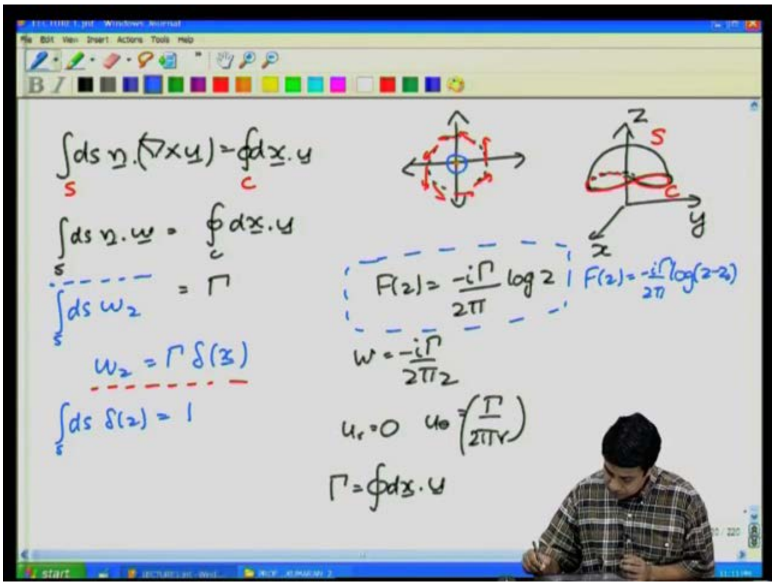

The stokes theorem states that integral over a surface of n dot delta cross u is equal to integral over a line d x dot u this line is over a closed loop. The figure for that is as follows, if I have some surface, so this is the surface s, this is the perimeter of the surface going around this is the perimeter of the surface c. So, what this says is that the integral over the surface s of n dot del cross u is equal to integral of d x dot u over the perimeter c. In this case we have a two dimensional system; that means, that there is no variation in the direction perpendicular to the plane.

In this case the plane x y is this is the plane surface the z is third coordinate and I have no variation in the third z direction, since I have an x y coordinate system the velocity is the function only of x and y, the curl of the velocity has to be in the z direction. The curl of the velocity is perpendicular to the direction to the plane of the velocity as well as the plane in which there are gradients of the velocity. So, therefore, the curl of the velocity is perpendicular to the plane.

So, in this particular figure the curl of the velocity is perpendicular to this plane coming outward at u coming outward from this plane at u. So, therefore, the curl of the velocity is perpendicular to this plane the stokes theorem for this velocity is basically integral s d s of n dot omega is equal to integral over c d x dot u, this as we showed was equal to gamma the circulation this was equal to gamma the circulation. So, what this says is that since the circulation is exactly the same on each and every contour around the origin, we can see that I have u theta is equal to gamma by 2 pi r.

Therefore, if I take integral of this theta around a circle of radius r, I get a result that is independent of r is equal to gamma for each and every plane and for each and every circle that I take around the origin. That means, that this is exactly equal to gamma for each and every surface, that cuts the origin n is the unit normal to the surface. So, n is in the z direction so; obviously, the result will be non-zero only if omega is in the z direction.

So, what this is saying is that integral over the surface d s of omega z is equal to gamma, omega is perpendicular to the plane of the flow therefore, it has to be only in the z direction perpendicular to the plane of the flow. Now, integral d s omega z is going to be equal to gamma for each and every surface only if gamma z delta function at the origin remember that I require that integral d s gamma has to be the same for each and every circle that I take around this origin regard less the radius.

Refer to v a constant value for each and every circle even though the limit of the circle going to 0, only if omega is a delta function exactly at the origin only if omega z is equal to gamma times delta of x. This delta is a two dimensional delta function, note that we are using a two dimensional coordinate system and therefore, this delta is a two dimensional delta function, that is integral over the surface d s delta of z this has to be equal to 1.

So, therefore, this delta function has to have units of 1 over length square if you recall we did delta functions previously, when we looked at heat and mass transfer if you are not familiar with that kindly go through that once again in one dimension delta. Function has dimensional of inversed length, two dimensions it has dimension of inverse area and three dimensions it has dimension of inverse volume. So, this is the two dimensional delta function. So, this corresponds to what is called a line vertex, the verticity is the delta function in the plane and the direction of the verticity vectors perpendicular to the plane, that is the rotation is in the plane.

Therefore, the verticity vectors perpendicular to the plane of rotation. So, therefore, this potential function f of z is equal to minus i gamma by 2 pi log z corresponds to a line vertex at the origin. Of course, you can have a line vortex at other locations as well; in that case the F of z would just be written as F of z is equal to minus i gamma by 2 pi log of z minus z naught where z naught is the location of the verticity. So, you just shift the origin to that particular location.

So, this is a two dimensional a line vertex I told you that the potential flow is irrotational, this flow is also irrotational everywhere except at the origin only at the location of the line vertex there is a verticity is non zero. In fact, this is a delta function delta function integrable. So, the the intensity goes to infinity, the thickness goes to 0 or the area goes to 0 in such way that the product is finite pi. Everywhere else it is completely irrotational because we have we’ve we know that that this the real part of f of z does satisfy Laplace equation everywhere except at the origin.

Where f of z itself is singular log of z goes to infinity as z goes to 0. Everywhere else it is well behaved and it satisfies the potential flow conditions.

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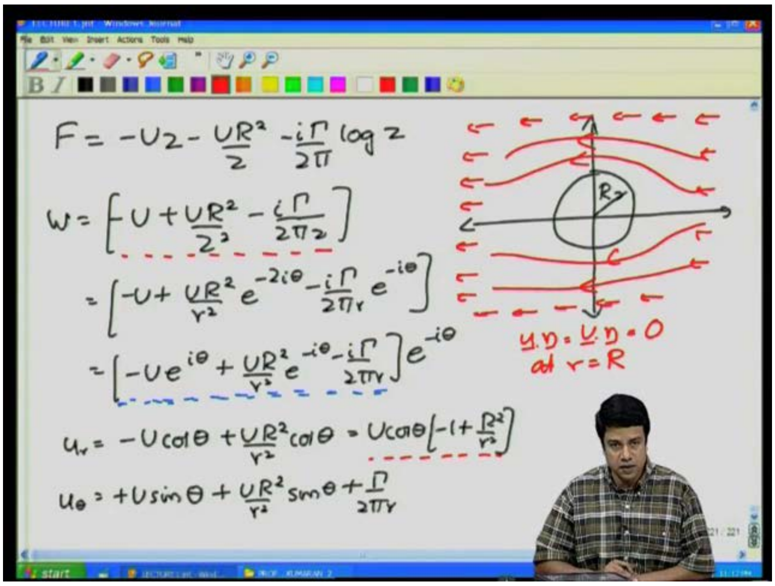

Let us look at lightly more complicated form this function, I will just write down the function first and then we can discuss, what it exactly means. So, this is the function that we would like to discuss, this the complex potential the complex velocity of course, is is given by minus U plus U R square by Z square minus i gamma by 2 pi Z. I can write it in a polar coordinate system as minus U plus U R square by R square e power minus 2 i theta minus i gamma by 2 pi R.

And I would like to take e power minus i theta out because I know that this has to be equal to U R minus i u theta e power minus i theta i know that it has to be of this form. So, I prefer to take e power minus i theta out and write this as minus U e power i theta plus U R square by r square e power minus i theta. So, once it is expressed in this form the radial component of the velocity is just the real part of everything that is within brackets, U theta is equal to the negative of the imaginary part, I will just write this down and then we will discuss it.

So, this implies that U R is equal to minus u cos theta plus U R square by R square cos theta. So, this is just equal to U cos theta into minus 1 plus R square by R square and U theta will be equal to minus U I am sorry plus U sin theta. Note that the u theta is the negative of the imaginary part of everything that is within bracket u theta is the negative of the imaginary part of this. So, is equal to plus U sin theta plus U R square by r square sin theta plus gamma by 2 pi r, what kind of a flow does this represent.

So, let us just briefly take a look at that, if you look at this expression, if you look at this expression in the limit as z goes to infinity w is equal to just minus u in the limit as z goes to infinity w is equal to just minus u. This is for all values of z anywhere if z goes to infinity if the magnitude of z goes to infinity w is equal to minus u; that means, that u x is equal to minus u and u y is equal to 0. So, far away from this in the limit as z goes to infinity we just have a uni directional flow.

So, far away we just have a unidirectional flow with magnitude minus U the equation for U R the equation for U R tells you that U R is equal to 0 at r is equal to capital R I have sign yeah now this is correct. So, at r is equal to capital R U R is identically equal to zero; that means, that you have a surface of radius capital R within the flow at which the radial component of the velocity is identically equal to 0, for this particular circle the radial component of the velocity is perpendicular to the surface.

So, at a circle of radius r or three dimensional cylinder of radius r of infinite extent, the radial velocity U R is identically equal to 0. So, that means, that there is no normal velocity at this surface. So, what this flow represents is a flow in which the normal velocity is equal to 0 at that surface, which corresponds to a flow around this cylinder because if I have a surface on which the normal velocity is equal to zero; that means, that boundary condition that is U dot n is equal to capital U dot n is equal to 0 at is equal to capital R.

Therefore, I have 0 normal velocity at this particular surface therefore, this represents the flow around a cylinder in which I have zero normal velocity on the cylinder, and I have a flow which is coming in far away from the cylinder I have a flow that is coming in far away from the cylinder this satisfies the zero normal velocity of boundary condition on the cylinder. So, U R is equal to 0 on the surface of the cylinder what about U theta, clearly U theta is not equal to 0.

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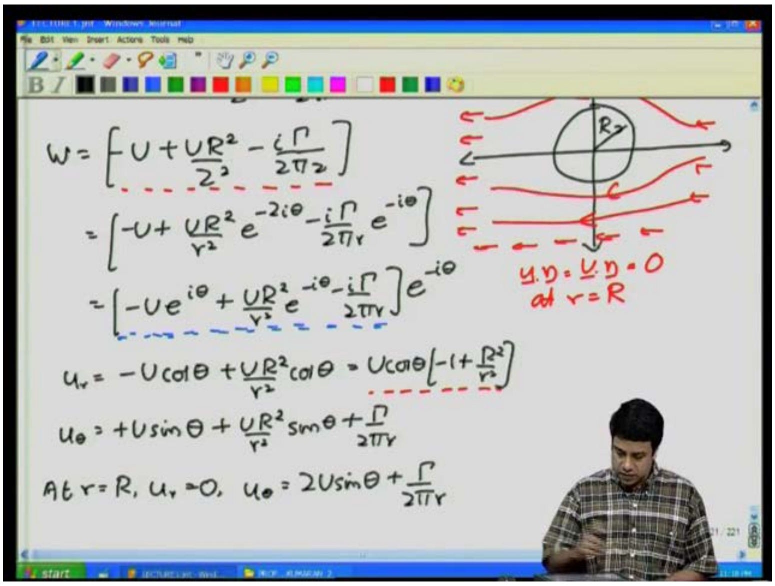

So, on the surface of the cylinder at r is equal to capital R, U r is equal to 0 theta, clearly is not equal to 0 is equal to 0, U sin theta plus gamma by 2 pi r. So, let us try to look physically at what kind of a velocity field this represents.

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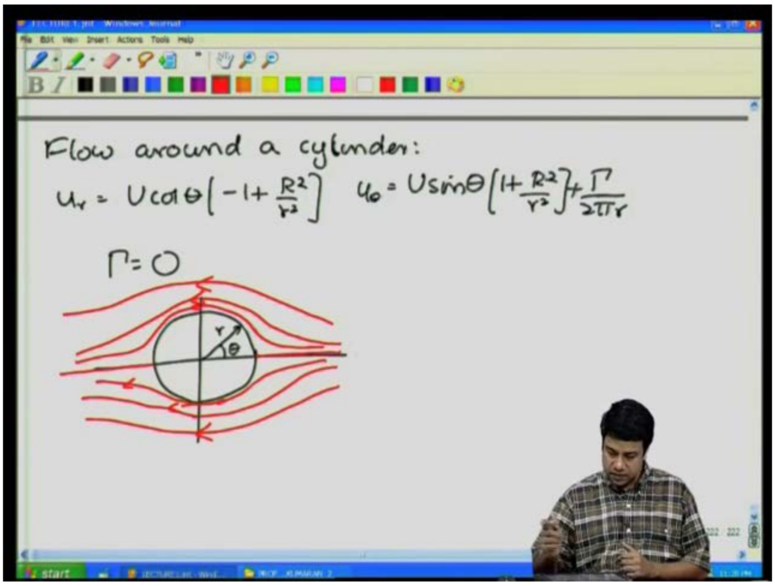

So, this as I said is the flow around a cylinder, so that is the flow around the cylinder. Now, as I said gamma is the circulation, let us first take the case of the circulation is identically equal to 0. So, let us first take the case where gamma is equal to zero at the flow around the cylinder and the flow in the case the where the circulation is equal to 0, so gamma is equal to 0, so u theta is first equal to u sin theta into 1 plus R square by r square. So, what I will have is a flow that is looks something like this it is a symmetric velocity it is a symmetric velocity and on this half plane.

So, velocity profile look something like this and as you know this we have a coordinate system where I have this is r and this is theta at theta is equal to 0 and at theta is equal to pi, u theta is also equal to 0, because u theta is proportional to sin theta. As I said everywhere on the surface of the cylinder u r is equal to 0, because r equal to capital R and at theta is equal to 0 and pi that is on the front and the back what are called the stagnation points, u theta is also equal to 0, because theta is equal to either zero or theta is equal to pi.

And therefore, very close to this point I have a flow that separates out between the front and the rear separates out between going above and going below this cylinder. Now, what happens when gamma is not equal to 0. So, let us first look at what happens when gamma is not equal to 0 where are the stagnation points on the surface of the cylinder, when gamma is not equal to 0.

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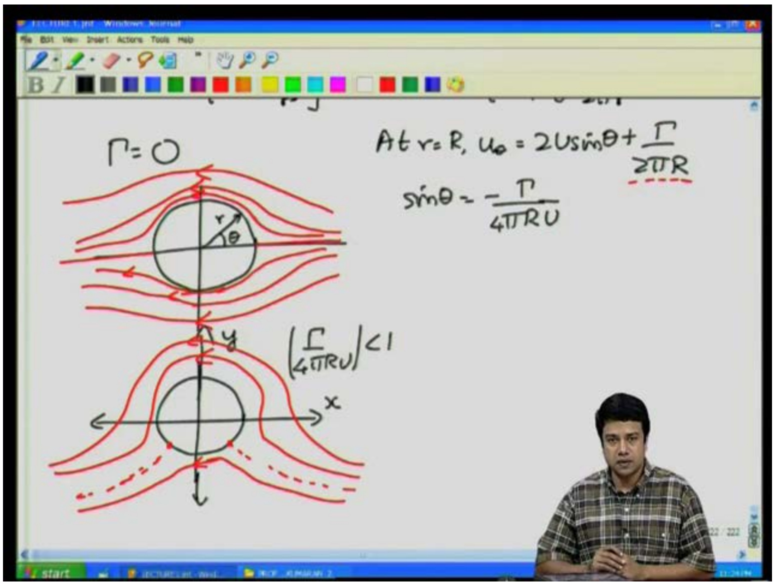

So, at the surface of the cylinder at r is equal to capital R u theta is equal to 2 u sin theta plus gamma by 2 pi r. Now, clearly if gamma is not equal to 0 u theta will be 0 only when 2 u sin theta plus gamma by 2 pi r is equal to 0. Alternatively when sin theta is equal to minus gamma by 4 pi r u clearly there will be the the u theta will be equal to 0 only when when sin theta is equal to gamma by 4 pi r u. Sin theta is negative; that means, that it u theta is equal to 0 either in the third or the fourth quadrants. Because recall that I have my cylinder here at the center, this is x and y.

Recall that sin theta is negative only in the third and fourth quadrants, sin theta is positive in the first and the quadrants, sin theta is positive in the first and second quadrants and sin theta is negative in the third and fourth quadrants. That means, that the location where u theta is equal to 0 has to be somewhere in the third and the fourth quadrants. So, on the surface of the cylinder this is the location at which u theta is identically equal to 0. And if if you plot the stream lines coming to these locations you will get a stream line a stagnation streamline that looks something like this and the flow around the cylinder will now look something that looks like this.

So, this is no longer symmetric like the one that we had for gamma is equal to 0 it loses its up down symmetry. So, the flow is no longer symmetric around the x axis, that is because this term here this term here represents its net circulation. So, in this case there is a net circulation around this cylinder represented by this gamma and because of this the flow around the cylinder is no longer symmetric. Of course, this can this kind of a flow profile can happen only, so long as minus gamma by 4 pi r u is less than 1 because sin theta has to be between minus 1 and plus 1.

However, if I keep increasing the circulation, so that gamma by 4 pi r u is greater than 1 at sin theta is equal this first this is for gamma by 4 pi R u less than 1. So, this is for gamma by 4 pi r u less than 1, what happens when gamma by 4 pi R u is exactly equal to 1.

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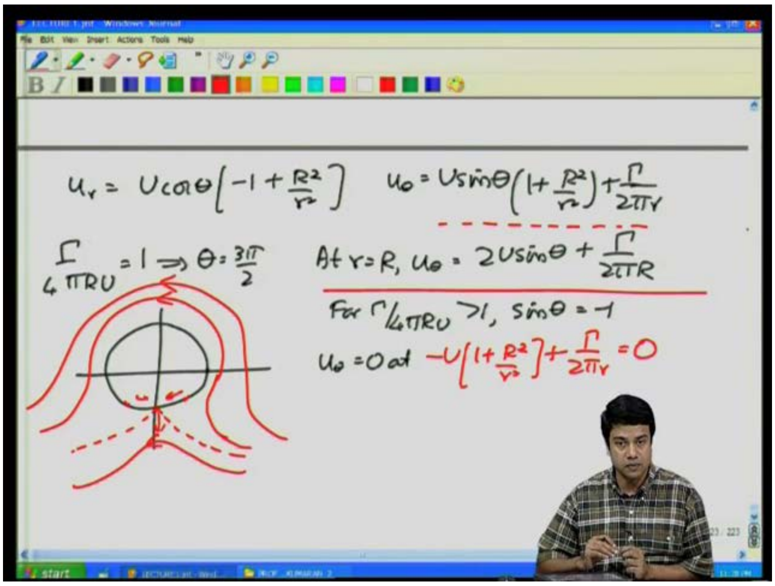

I have once again U r is equal to u cos theta into minus 1 plus R square by r square and u theta is equal to u sin theta into 1 plus R square by r square plus gamma by 2 pi r. And at r is equal to capital R, u theta is equal to 2 u sin theta plus gamma by 2 pi r. Now, exactly at sin theta is gamma by 2 pi r u is equal to 1, implies that u theta is equal to 0 when sin theta is equal to minus 1 that is that theta is equal to 3 pi by 2. U theta is identically equal to 0, if gamma by 2 pi I am sorry 4 pi r u is equal to 1, u theta is equal to 0 when theta is equal to pi by 2, 3 pi by 2.

So, in this particular case this will correspond to only one stagnation streamline you recall that for gamma by 4 pi r u less than 1, I had two stagnations streamlines which was symmetrically located about the 3 pi by 2 angle as it approach as one these two will come together and merged to get just one stagnation streamline. Because there is only one solution at which u theta is equal to 0 for gamma by four pi r u is equal to 1. So, therefore I will get stagnation stream line that comes something like this and the flow around the cylinder will look something like this.

Of course, I could still further increase the circulation of course, I could keep increasing gamma relative to r times u, but sin theta cannot become greater than 1. So, the minimum value I can have is for sin theta is equal to minus 1, we cannot go below minus 1. So, in that case what happens the maximum the the the minimum value of sin theta is minus 1.

So, the only thing that can happen here is for this stagnation streamline to leave the surface of the cylinder and move downwards.

So, for gamma by 4 pi r u greater than 1 greater than 1 I have sin theta is equal to minus 1 therefore, u theta is equal to 0 at u into 1 plus r square by r square plus. So, if sin theta is equal to minus 1 u theta is equal to 0 at in this equation, I insert sin theta is equal to minus 1and I get the value of minus u into 1 plus r square by r square plus gamma by 2 pi r is equal to 0. So, I solve this equation to find out the value of r at which u theta is equal to 0 and therefore, the value of r at which u theta is equal to 0 is greater than capital R in this case because sin theta is equal minus 1.

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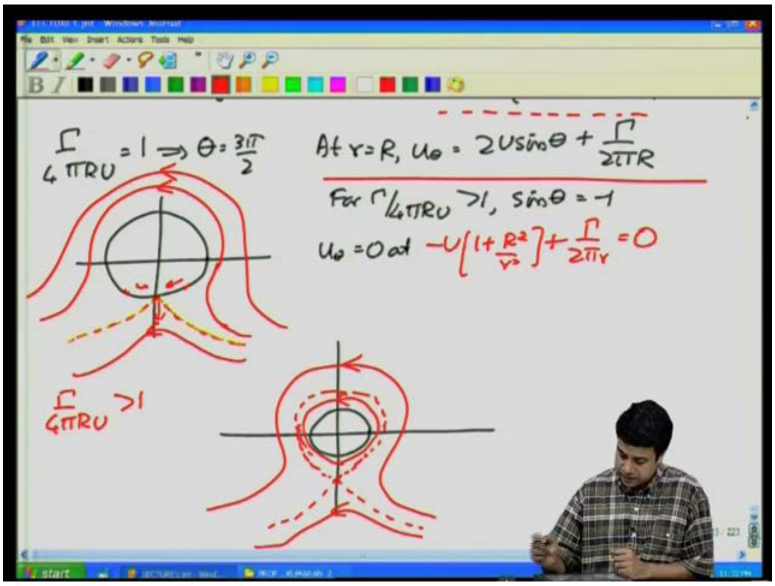

And therefore for gamma by 4 pi R u greater than 1 theta at which u theta is equal to 0 will still be pi by 2 I am sorry 3 pi by 2. However, the stagnation point will be not be at the surface of the cylinder will be somewhere below. So, this stagnation point will be somewhere below the cylinder and I will have closed stream line a closed stagnation stream line I will just draw that carefully I will draw it bigger for you, so that it is easier to see just to make the point correctly.

So, this is my plane, this is the cylinder the stagnation point is at some distance that is greater than capital R, the stagnation point is at some distance is greater than capital r and I have stream lines coming in to the stagnation streamline it is just the same way as I had these stream lines over here. Similar manner to the streamline that I have over here.

So, I have this streamlines coming in there is actually a closed region around the cylinder where there is basically just a circulating flow is a closed region around the cylinder within which there is just a circulating flow, because my stagnation stream line has shifted downwards away from the surface of the cylinder.

So, there is one more surface at which the normal velocity is equal to 0 within that there is a close circulation within the cylinder and outside I have a flow that looks something like this. So, this basically represents the flow around the cylinder one might be tempted to think that this kind of a flow can be achieved if we had a rotating rotating cylinder that is not quite true, because in potential flow we have a 0 tangential velocity that is we have zero normal velocity boundary condition, where the tangential velocity in general is non-zero.

So, just by rotating the cylinder we cannot generate a flow in the fluid, because we are satisfying the tangential velocity boundary condition. We will come back a little later and see how one can achieve flows. But the point here is that if there is in addition to the useful potential flow velocity if there is circulation as well, there is going to be an a symmetry in the velocity profile around the cylinder. The stagnation points shift from the upstream and downstream ends of the cylinder downwards, if the circulation is positive because if the circulation is positive is is negative they will shift up wards due to circulations clockwise gamma can be positive or negative.

In this case if gamma is positive it means that circulation is anti-clockwise. So, in that case you get a ah an a symmetric velocity profile and if gamma by 4 pi R u is less than 1 you have stagnation points on the surface of the cylinder will becomes greater than 1 this stagnation points shift into the fluid along the angle 3 pi by 2 downwards. And you have closed stream lines within a region around the cylinder, so this gave us the velocity profiles for the flow around cylinder.

The next thing to do is try and calculate what is the force exerted on this due to the flow around the cylinder just to re iterate the cylinder is of infinite extent if the plane perpendicular the direction perpendicular to the flow. Therefore, any force that we calculate will be per unit length in that perpendicular direction.

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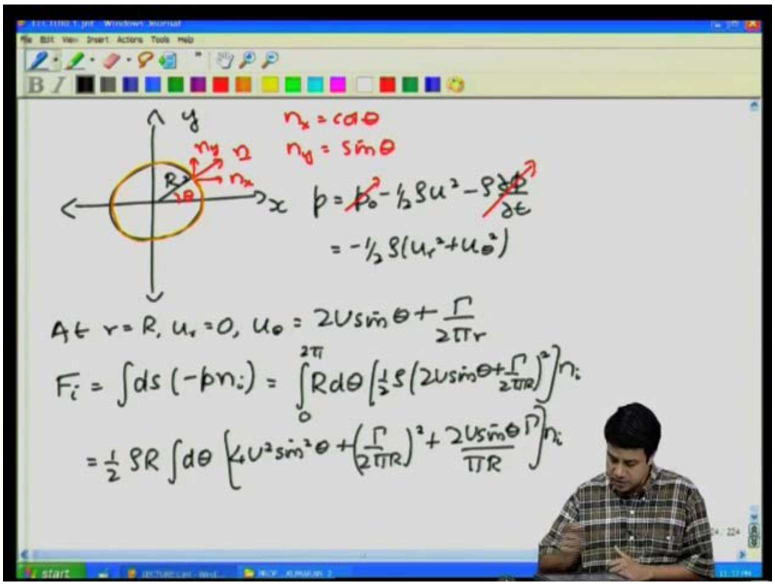

So, we want to calculate the force on the cylinder cylinder with radius R x and y the unit normal to the cylinder at the surface n has two components n x and n y. The angle theta the angle theta if I have n x will be equal to cos theta, n y is equal to sin theta how do I calculate the force, first I calculate the pressure from that I integrate over the surface to get the force. So, the pressure is given by p is equal to p naught minus half rho u square minus rho partial pi by partial d.

This is a flow where the flow is coming in far away from the cylinder and on the cylinder itself it is stationary the cylinder itself is stationary. Therefore, this is actually a study state flow this is a this is not a flow where we have a cylinder moving at constant velocity with respect to the fluid, in this case as we defined it the cylinder is stationary and the fluid is coming in and for that reason this is equal to 0. So, p is just equal to p naught minus half rho u square of course, p naught is just a constant pressure is going to have a constant exert equal and opposite forces in opposite surfaces.

So, that itself does not affect the net force therefore, I can, so just write p as minus half rho u square, because p naught is just a constant value which exerts an an equal force gradient area on all points on their surface. So, this is equal to minus half rho into u r square plus u theta square on the surface of the cylinder itself we know that u r is equal to 0 at r is equal to capital R U r is equal to 0 U theta is equal to 2 U sin theta plus gamma by 2 pi r. So, since U r is equal to 0, the only non zero component of the velocity is just U theta.

So, therefore, force the force f i is equal to integral over the surface d s of minus p times the unit normal, the surface of the cylinder is this one, this is the surface of the cylinder just circle. So, it is a surface area in three dimensions, in two dimensions it will just be a line along which we are integrating and if you integrate along the line we will of course, get a force per unit length in the direction perpendicular. So, therefore, d s along this, this d s has to be equal to integral R d theta, theta goes from 0 to 2 pi along the surface minus p n i p is equal to minus half rho U theta square.

So, I will get half rho into 2 u sin theta plus gamma by 2 pi R the whole square times the unit normal n i. So, I can expand this out as half rho r integral d theta of 2 u sin theta plus gamma by 2 pi R the whole square times n i. So, the terms in the brackets can be expanded out quite easily to give 4 u square sin square theta plus gamma by 2 pi R the whole square plus 2 u sin theta gamma by pi R times n i. So, for the force the x direction n x is equal to cos theta for the force the y direction and y is equal to sin theta.

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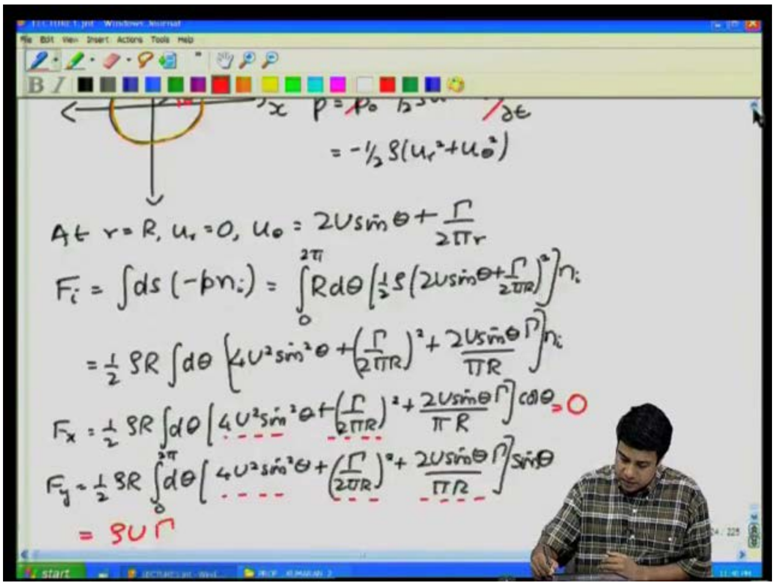

So, therefore, f x is equal to half rho R integral rho theta of into cos theta you can easily verify that these two terms are actually when you multiply these I am sorry, when you multiply these two the first two terms, when you take the first two terms and multiply them by cos theta they end up being odd functions of theta. And therefore, they will integral out to 2, 0 exactly the last term is of the form sin theta cos theta which can be written as sin 2 theta integral over the surface that is also equal to 0.

So, this just shows you that the force in the x direction is identically equal to 0 the force in the x direction acting on the cylinder is identically equal to 0. What about the force in the y direction f y is equal to half rho R integral d theta of 4 u square sin square theta plus gamma by 2 pi R the hole square plus 2 u sin theta gamma by pi R into sin theta. Once again these two terms will integral to 0, because sin square theta times sin theta is an odd function therefore, it integrates to 0.

Similarly, sin theta times gamma by gamma by 2 pi R the whole square. Once again integrates out to 0 the only term that we are left with is 2 u sin theta gamma by pi R times sin theta, sin square theta, integrate from 0 to 2 pi sin square theta integrate from zero two pi is equal to half of 1 minus cos 2 theta integrates from 0 to pi. And therefore, if you carry out the integral finally, you will find the result is rho u times gamma it is very easy to verify that I just take sin square theta integrate from 0 to 2 pi.

So, I get 2 pi times a half which just gives me a pi and the r cancels out to finally, give me rho u times gamma, I leave that as exercise for you is quite easy to do. So, what I found is that the force in the x direction is equal to 0, the force in the y direction is equal to rho u times gamma, where u is the velocity far away and gamma is the circulation.

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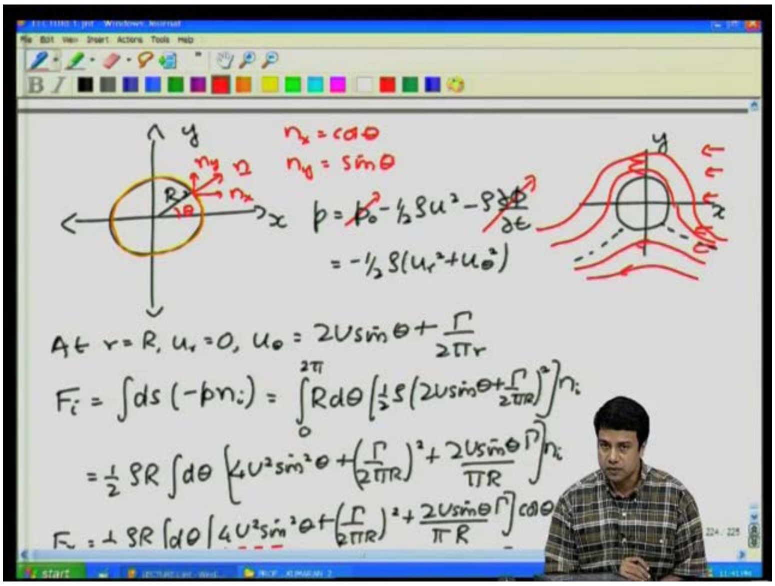

So, to put in this in prospective I have a cylinder in which there is a velocity coming in far away. So, this is x and this is y and because there is a red circulation this is actually the velocity field if you look at it look something like this, the velocity field faraway look something like this. The force in the x direction is identically equal to 0, the force in the x direction is what is called the drag force, because it is opposite to the direction of the velocity.

That drag force is identically equal to 0, force in the y direction is non-zero. If there is a circulation only if gamma is non-zero will the force in the y direction be non-zero, the force in the y direction in this case is what is called the lift force?

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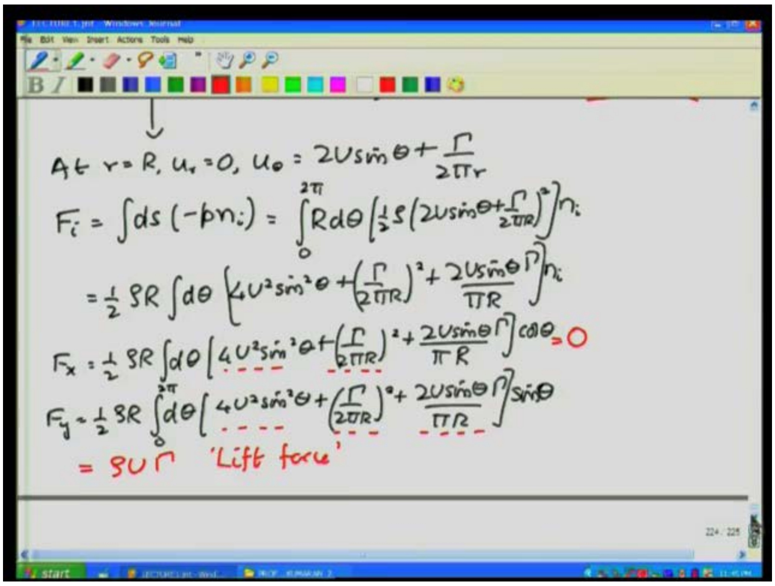

It is acting in the direction perpendicular to the cylinder it is acting in the direction perpendicular to the cylinder as well as the direction perpendicular to the direction of the flow. Recall when we did looked at the force acting on an object in three dimensions we found out that the force was identically equal to 0, in all directions, we needed no net force in order to move the cylinder at a constant velocity within the fluid. That I had justified for you saying that that is because we have neglected viscose dissipation in the absence of viscose dissipation there is no energy dissipation in the fluid.

Therefore, there is no need for energy source in order to move the object at constant velocity, in this case for the flow around a cylinder we find that the drag force is still equal to 0, recall that the work done, the work required for moving the cylinder at constant velocity. Or alternatively moving the fluid at constant velocity related to the cylinder work done is equal to the force times the velocity the dot product of the force in the velocity.

So, it is equal to f x times u x that is the force in the x direction times the velocity in the x direction. In this particular case the velocity is only in the x direction, we have assumed that without loss of generality that is far away the velocity is only in the x direction. Therefore, the work required is the force in the x direction times the velocity in the x direction you are finding that the force in the x direction is identically equal to 0, that is the drag force is identically equal to 0.

So, we still do not have any net force require to move the cylinder at a constant velocity; however, when there is circulation the force perpendicular to the direction of flow is non-zero that is what we are finding here, the force is equal to the density times the velocity times the circulation per unit length perpendicular to the plane of the flow. So, therefore even though there is no drag force if there is a cylinder which is moving with respect to the fluid far away or alternatively if the fluid is coming in related to the cylinder far away.

And there is a net circulation around the cylinder, there is going to be a net force in the direction perpendicular to the plane of the perpendicular to the velocity and that net force is just equal to the density times the mean velocity times the circulation, that we did for the specific case for the flow around a cylinder its rather simple object. However, it can be shown that this is the general result this results holds for object of any shape all you require that is in there should be the object should be moving with respect to the fluid far away and there should be a net circulation around the object.

Now, where is this applicable the place where it is applicable is for in aero dynamics where you need to generate a lift around an object.

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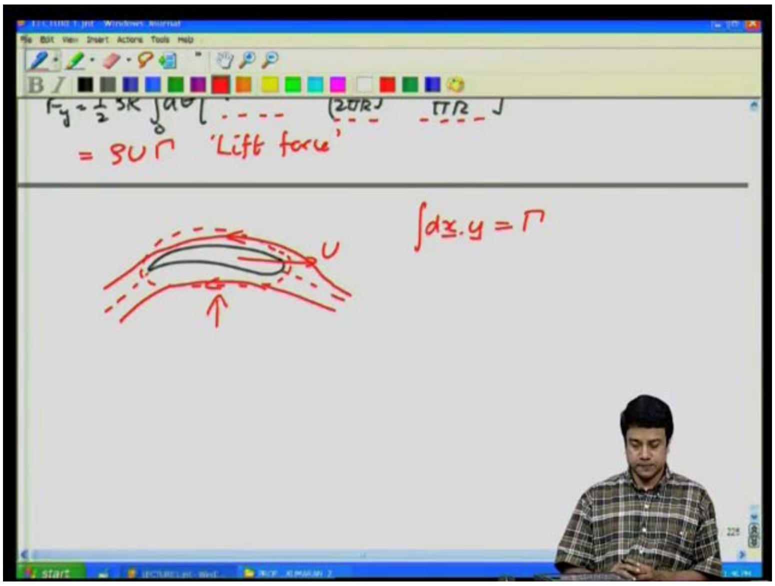

I said that just rotating cylinder cannot in general generate a lift what you need is circulation around the cylinder and that is why if you see the cross section of aircrafts wings. For example, they have a shape that look something like this the they do not have circular shape, but rather they have a shape that look something like this and when they are moving you will find that the velocity field around this looks something.

Like this the shape of this object is optimized in such a way that if I were to calculate integral d x dot u around this object that will take any contour never to calculate d x dot u around this object this would be a non-zero value gamma this would be a non-zero in general. So, I have the I have the the object itself moving this direction to the velocity u, and I have a net circulation because of the shape of the object these objects are, so shaped in such a way that there is a net circulation around them in potential flow.

Therefore, if I have a an object moving with a constant velocity with a net circulation around it there is going to be a net force acting perpendicular and that is what keeps aircrafts. So, in the next lecture I will continue this and I will show you that this result is more general the result rho u gamma is in general result it is holds for object of any shape the only thing you require is that there has to be a net velocity of the of that object with respect to the fluid far away.

And there has to be a net circulation around that object, if these two are true you will generate a net lift force though the drag force will still be zero the drag force is 0, because we have neglected viscose dissipation. Our next series of lectures will look at how do we include the effects of viscose dissipation very near objects using modular theory. That will be our next lecture, but before that we will complete the rest of this to derive the relationship between the lift force and the circulation, we will continue this in the next lecture will see you them.