Fundamentals of Transport Processes II
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Lecture - 28
Two-Dimensional Potential Flow- Part 2

This is lecture number 28 of our course on Fundamentals of Transport Processes. Welcome. As we were discussing in the last lecture, Two Dimensional Potential Flows...

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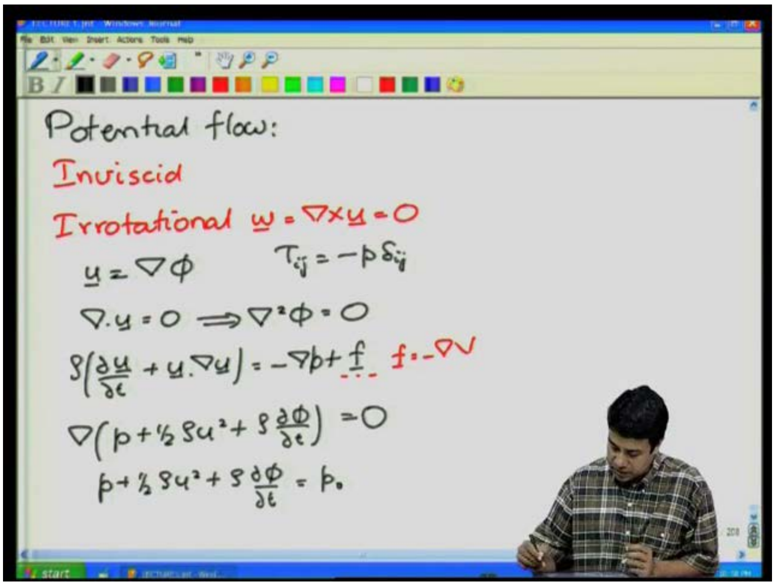

So, just to recap potential flows in the limit of high Reynolds number in viscid viscosity is set equal to 0 and irrotational. What is meant by irrotational is that the verticity which is equal to del cross u this is equal to 0. And if the curl of a vector is equal to 0, it can always be expressed as the gradient of a potential, that is the reason for the name potential flow. So, therefore, I can always write velocity as the gradient of a potential.

And this expression can be substituted into the mass and momentum Navier stokes equation, the mass conservation equation requires that the divergence of the velocity is equal to 0, and what that straight away implies is that the laplacian of the potential has to be equal to 0 that is del square pi is equal to 0, where pi is the velocity potential. The momentum conservation equation in the absence of viscosity minus grad p plus to mu the viscosity is equal to 0. So, that can at most be a pressure force a pressure plus a body force we have neglected the viscose term in the conservation equation.

And we have seen that this can be reduced to an equation of the form gradient of p plus half rho u square plus rho d pi by d t this is equal to 0. In this particular case I am assuming that this pressure can be written as minus of the gradient of potential in other words it is a conservative force, the body force is a conservative face. So, it can be written as minus grad pi gradient of the potential, and that potential term can be incorporated into the definition of the pressure itself.

So, even though the momentum conservation equation is a vector equation for 3 components this can be reduced to any statement that it is just the gradient of a scalar function is equal to 0; that means, that scalar function has to be a constant is equal to some constant. So, these are the mass and momentum conservation equations, the mass conservation is a laplacian equation solved for the velocity potential and the momentum conservation equation can be solved to obtain the pressure.

The the stress tensor in this case just contains only the pressure alone minus p times delta i j. So, it is isotropic it contains only the pressure a consequence 2 consequences that we discussed in the previous lectures. Since, we have neglected the viscose terms it is not possible to satisfy both the tangential and normal velocity of stress conditions for a potential flow one can satisfy only the normal velocity or the normal stress continuity conditions, one cannot satisfy the tangential velocity or tangential stress continuity conditions.

So, it is seen how to solve this potential flow problem for a sphere 3 dimensions and we manage to get the result for this sphere to go at constant velocity, there need be no force applied on this sphere in other words there is no drag force, due to the sphere moving at constant velocity. In the last class we have also showed that there is no drag forces object of any shape, moving at constant velocity. The reason is because we neglected viscose dissipation in the potential flow equations.

And therefore, there is no change in energy, the energy is not dissipated by viscosity, energy is conserved, if energy is conserved at constant velocity there is no need to put an energy in order to keep the object in a state of motion. And therefore, you need no net force in order to move the object at a constant velocity. When the object is accelerating of course, you do need a net force and that force is equal to the added mass times the acceleration, for a sphere we saw that the added mass is equal to half the mass of the fluid displaced by this sphere.

For a shape objects of other shape one can still calculate an added mass, either from the kinetic energy or from the force. Either from the total kinetic energy at the flow field, so in that case the kinetic energy will be equal to half added mass times u square or from the force require, and in general that we some other fraction of the mass displaced by this way. So, in the last lecture we had restricted our discussion to 2 dimensional potential flows.

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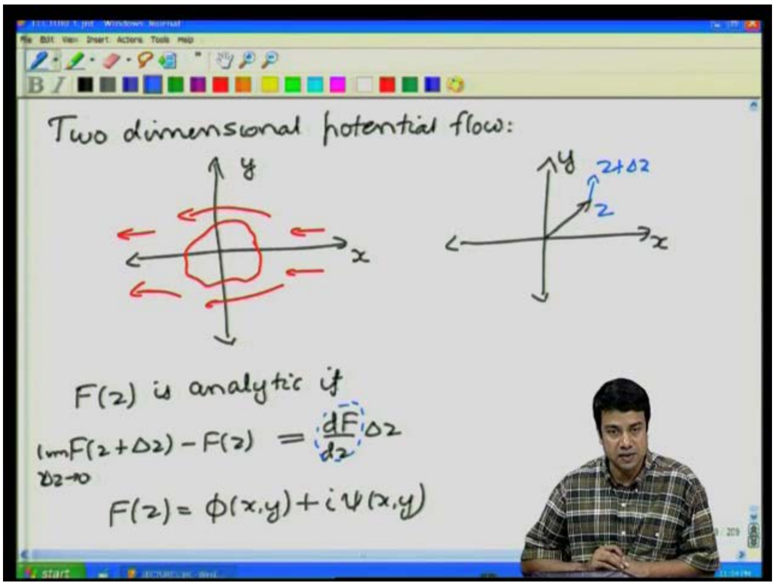

It is an important class of flows where you have variations only in 2 directions the x and y direction. We could have for example, some object around which there, is flow that is flowing in 2 dimensions and we were looking at the potential flow equations for flows of this time. All objects are of course, three dimensional what is meant by a 2 dimensional object is that there is no variation in the third dimension, in other words in the direction perpendicular to the plane of the board there is no variation in either the shape of the object or the velocity field.

So, if you take a cross section through this object there is no variation in the third direction perpendicular to this cross section therefore, we can analyze the flow in this 2 dimensional plane alone. All long and slender objects can be approximated by 2 dimensional objects, and air plane wing for example, the variation of that wing along the third direction is actually, small compare to the shape in the 2 dimensions perpendicular to the way.

So, I can analyze the flow in a cross section, perpendicular to the direction of the taxes of the way. So, it is understood that there are 3 dimensions, but we are analyzing only the 2 dimensional flow in the plane perpendicular to the access of the object that access is now, perpendicular to the plane of the board and therefore, there is no variation in that direction. So, all quantities that we will calculate such as, the force on the object or the energy of the flow are all per unit length in the direction perpendicular to the flow.

So, they we have what dimensions length inverse times what you would normally expect. So, in this case for the force on this object it will be a force per unit length in the plane perpendicular to the flow is, so for example, if you had a long cylinder we cannot calculate and if you approximate that cylinder as being of infinite length, we cannot calculate the total force in that entire cylinder we can only calculate the force per unit length that multiplied by the length will be with the total force.

The length is long enough you would expect that some where, away from the ends the flows going to be 2 dimensional of course, there will be a variation at the ends, but provided that is small it makes only a small contribution to the net force in that cylinder. It is good approximation to consider to the cylinder to be in 2 dimensional calculate the force per unit length then multiply it by the length in the third direction. So, that is the kind of philosophy that we use for analyzing 2 dimensional flows.

So, we are looking for flows that satisfy the potential flows equations in this 2 dimensional flow. And if you recall we started the discussion in the previous lecture, rather than work separately in the x and y coordinate, we can consider some point in this plane as simply representing a number in the complex plane, a complex number in the complex plane. So, any location will have one position in the complex plane, which has real part plus and imaginary part.

So, we will work in terms of the complex number z itself rather than individually in terms of x and y. The reason we can do that is as follows, we have any function of z this can be any function z square, z cubed, cos z, exponent of z and so on. This function is considered analytic if I can write F at if F at z plus delta z minus F at z in the limit as said delta z goes to 0 can be written as d F by d z times delta z. In other words I can define a derivative of this function in the complex plane.

A complete derivative or total derivative with respect to the complex variable z, rather than partial derivatives with respect to x and y. This complex function f is in general a complex function because; z square for example, will have a real part and imaginary part. So, if this function F of z will in general have a real part plus an imaginary part. And these 2 real part and imaginary part have to be related or that partial derivative with respect to x and y have to be relative if the function F is analytic in other words if I require that if I go from a location z to another location z plus delta z.

And if I can write delta F which is F of z plus delta z minus F of z as this times d F by d z derivative times delta z, then there is a relationship between the real and imaginary parts of this complex function z.

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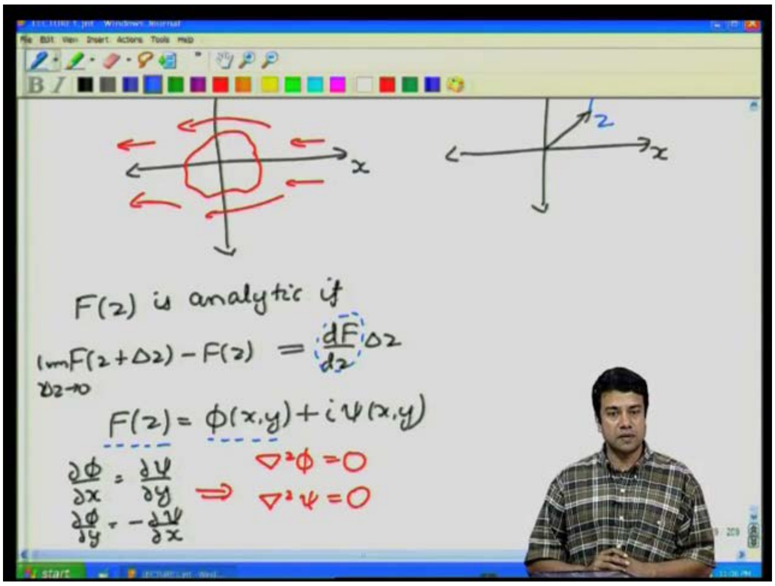

The relationship is partial pi by partial x is equal to partial psi by partial y partial pi by partial y is equal to minus by partial x. And quazi-remon conditions they are called, these conditions have to be satisfied for the partial derivatives of pi and psi if these if the function f is an analytic function. And just for a simple algebraic manipulation you can show that if these conditions are valid then del square pi has to be is equal to 0 and del square psi has to be equal to 0. So, for the complex function f of z the requirement of analyticity requires that both the real and imaginary parts of this function satisfy the Laplace equation.

For solving for potential flow we require that the potential field satisfies Laplace equation. Every real part of every analytic function satisfies Laplace equation; that means, that the real part of every analytic function represents some potential flow provided it is satisfies the boundary conditions that are prescribed on the surfaces in 2 dimensions. Note that in 2 dimensions the surface reduces to just a line in the plane because it taking you assuming that infinite in the third direction.

So, we can identify the complex function f as a complex potential, we can identify the complex function f as a complex potential. Then the real part of f is what is identified as the velocity potential pi. So, it is no coincident that I wrote f in terms of this function pi here. So, the real part of every complex function represents the velocity potential of some flow.

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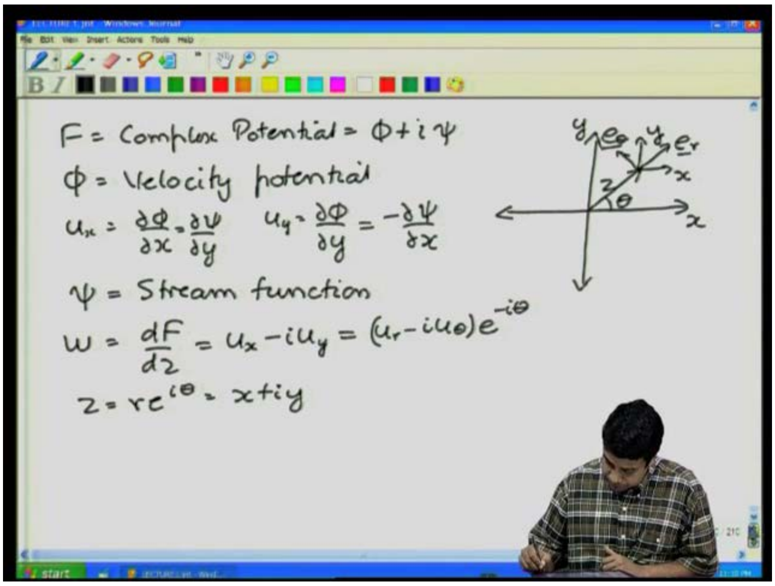

So, we have f is equal to the complex potential, which is pi plus i psi. We can identify pi is equal to the velocity potential, which means that u x is equal to partial pi by partial x and u y is equal to partial pi by partial y. And from the quazi-remon condition, we know that partial pi by partial x is also equal to partial psi by partial y and partial pi by partial y is minus partial psi by partial x.

What; that means, is that the imaginary part of the function f is just the stream function, the imaginary part of the function f is just the stream function. I can define the complex velocity w is equal to d f by d z. And we saw that in the last class when this is expressed in terms of the 2 component of the velocities u x and u y, this can written as u x minus I u y and one can also express this in terms of the velocities in a r theta coordinate system in a polar coordinate system.

This is some x location z, this is x, this is y, this is theta, and we know that the complex number is z is equal to r e power i theta is equal to x plus i y, where x is r cos theta and y is r sin theta. So, the radial direction the unit vector in the radial direction is in this direction and the theta vector is long the direction of increasing theta, which is the anti clock wise direction in this plane, and these are x and y.

So, I can also express w in terms of u r and u theta, the components of the velocity along the radial and the theta direction. And express in terms of this it turns out to be equal to u r minus i u theta into e power minus i theta. So, that is how the complex potential and the complex velocity are related to the potential in the velocity in the two-dimensional coordinate system x y coordinate system.

So, now, we showed that every potential function or every analytical function in the complex plane the real part of that function represents the potential for some velocity field, the imaginary part represents the stream function for that same velocity field. So, one can ask the question what do common forms of this functions, what velocity fields do they represents. So, we look at some common, some simple functional forms and ask the question, what is the kind of velocity field, that is represented by this particular form.

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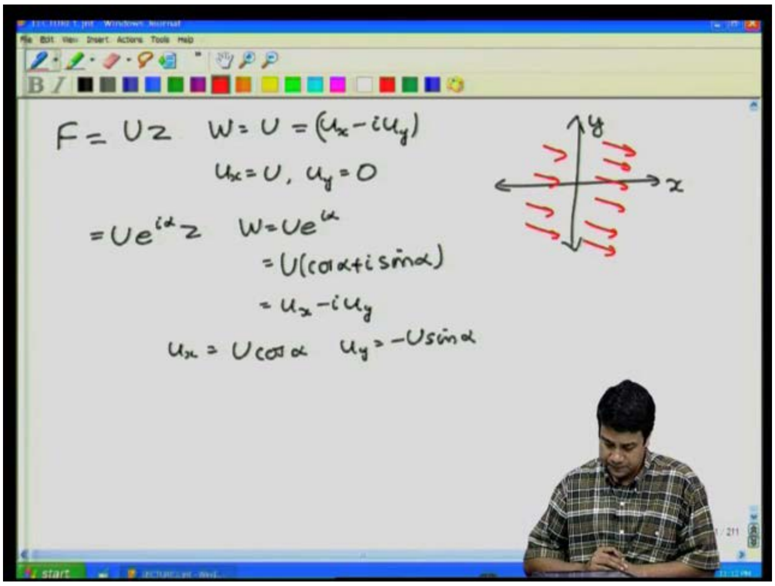

So, the simplest form to take of course, the simplest form is the constant, but if you take the derivative of that tensor being 0, so the velocity is 0. So, this just represents a fluid at rest, but the next simplest form that one can take is a function of the form u times z, so; that means, that the complex velocity for this case w is equal to just u. And we know that the velocity field for this has to be can be written as u x minus i u y. So, what this means as that the velocity u x is equal to capital u and u y is equal to 0.

So, u x is a constant is equal to capital U and u y is equal to 0; that means, that you have a constant velocity u everywhere, the velocity is just a constant everywhere independent of position. So, potential that is linear in z of course, gives you a constant velocity that is no surprise. You could have taken in this particular case I took U as a real number I need not have done that.

The next simplest thing I could consider is to take U e power i alpha, z which means the w is equal to u e power i alpha which will be u into cos alpha minus i sin alpha is equal to sorry u x minus i u y. Now, what; that means, is that u x is equal to u cos alpha and u y is equal to minus u sin alpha. So, this is the flow which is at some angle it is not along the x axis, but rather it is in client at an angle to the x axis. So, it is a flow that looks something like this, there is an angle alpha with respect to the x axis and u i is negative. So, it is actually, going down at the angle minus alpha with respect to the x axis.

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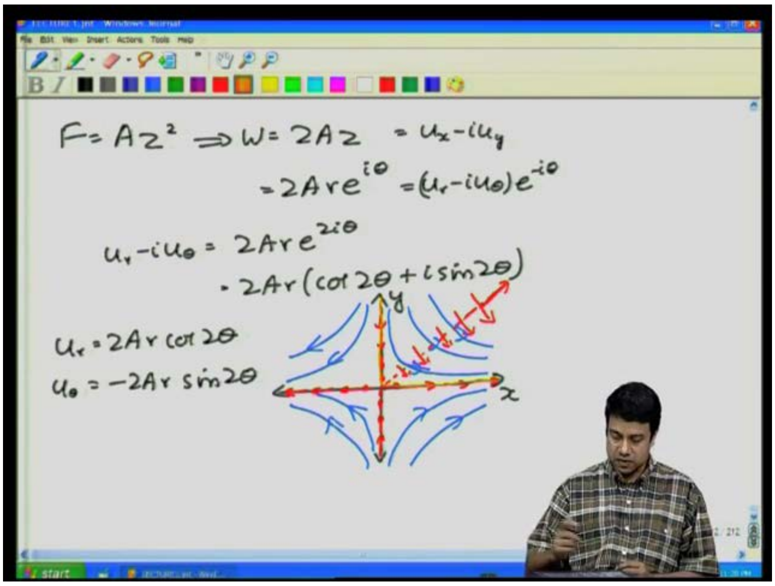

The next more complicated form f is equal to let us call it as A z square, which implies that w is equal to 2 A z d f by d z. So, this can be written as, 2 A r e power i theta. Now, this I could write it as u A r 2 A r into cos theta plus i sin theta, and then write x and y in terms of u x and u y in terms of w using the relationship that is equal to u x minus i u y. But, for our physical understanding it is more convenient to write this in a radial polar coordinate system as u r minus i u theta e power minus i theta.

So, I will use the expression that w is equal to u r minus i u theta e power minus i theta, and then use this in order to find out what is the expression for u r and u theta. So, therefore, I find that u r minus i u theta is equal to 2 A r e power 2 i theta is equal to 2 A r into cos 2 theta plus i sin 2 theta . So, therefore, the velocity field satisfies u r is equal to 2 A r cos 2 theta u theta is equal to minus sin 2 theta. So, now, let us try to plot this velocity field.

As you know r is the radial coordinate and theta is the polar coordinate, the angle from the x axis, along the x axis theta is equal to 0 along the x axis theta is equal to 0; that means, that u r is equal to 2 A r, because theta is equal to 0 along the x axis therefore, u r is equal to 2 A times r. So, it is a velocity there is increasing as you go outward, the velocity is proportional to 2 A r. So, it is increases with radius as you go outward, along the y axis theta is equal to pi by 2; that means, that cos 2 theta is cos of pi which is minus 1 sin 2 theta sin of pi which is 0.

Therefore, u r is equal to minus 2 A r and u theta is equal to 0. So, the radial velocity is coming inwards it is in the negative r direction is equal to minus 2 A r because cos 2 theta is minus 1. So, therefore, the velocity is coming inward and once again the magnitude is decreasing as you come in. So, the magnitude of the velocity is still proportional to r and; however, it is coming inwards it is coming towards the origin along the y axis it is going away from the from the origin along the x axis.

What about in between, if you take at an angle of pi by 4 if for example, take it an angle of pi by four for example, you find that at theta is equal to pi by 4 2 theta is pi by 2. So, cos of 2 theta is equal to 0; that means, that there is no radial component of the velocity sin 2 theta is 1; that means, that u theta is minus 2 A r. So, once again the radial direction is along the line, the dotted line that I shown here at pi by 4 it is radially outward.

The component of the velocity along this radial outward direction is equal to 0; that means, that there is no component along the radial direction the only the component is along the theta direction perpendicular to that one. Note the direction of increasing theta is anti clock wise where as, u theta is negative in this case because sin 2 theta is 1. So, u theta is minus 2 A r. So, therefore, the velocity is in the clock wise direction along this.

So, velocity is in this direction, once again u theta increases proportional to r as r increases. So, we are going to get a velocity that looks something like this. So, it is inward along the y axis, outward along the x axis, hence perpendicular to the to the to the position vector along the pi by 4 line. And if you plot the velocity at various intermediate points you will get a velocity field that looks something like this.

We will get a velocity field that looks something like this. So, this is only in the first quadrant you could of course, plot it for other quadrants. And I will leave it as an exercise its quite simple to do we will get a velocity that is radially outward along both the positive and negative x axis, It is inward of both positive and negative y axis and end up with the velocity that looks something like this, end up with the velocity.

So, I said that this represents some potential flow, which satisfies the no normal velocity boundary conditions at the bounding surfaces. One can turn the argument around and say that the bounding surfaces are those surfaces along which the normal velocity is equal to 0, for this particular flow valid boundary surfaces, are those surfaces for which the normal velocity perpendicular to that surface is equal to 0. And clearly you can see that there are 4 bounding surfaces here, both the x axis plus and minus the velocity is radially outwards it is along the axis there is no velocity perpendicular to the axis.

So, that constitutes a boundary surface for this flow similarly, for the y axis, the velocity is inward, it is along the axis, parallel to the axis, there is no normal velocity to the access. So, that constitutes as a valid boundary surface. So, therefore, this complex function a z square represents a flow in a corner whose, axis are 90 degrees apart, it represents a flow in a corner in which the angle between, the 2 surfaces at that corner is equal to pi by 2. So, that is for a z square one can of course, consider any higher power of z, any polynomial approximation for z.

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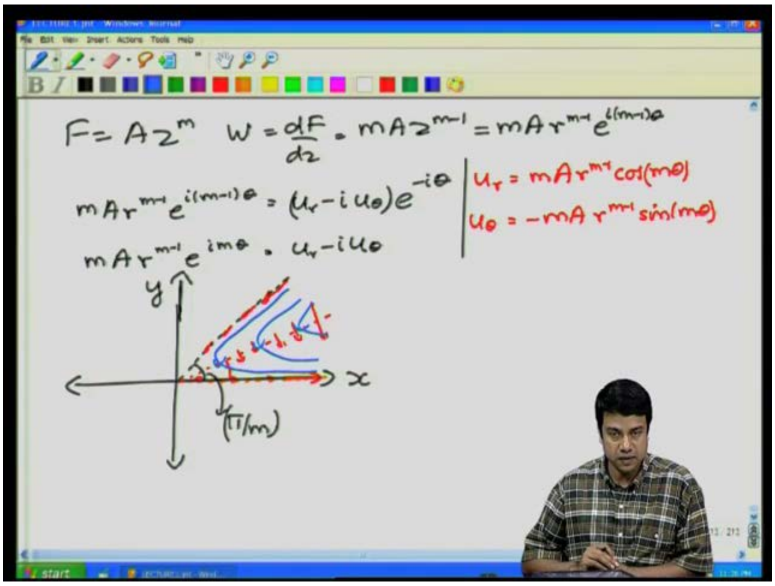

So, if I consider a general function of the form f is equal to A times z power m, the complex velocity is equal to d f by d z is equal to m A z power m minus 1. And I can write this as in the polar coordinate system as m A r power m minus 1 e power i into m minus 1 theta. This complex velocity is equal to u r minus i u theta e power minus i theta. So, therefore, I have m A r power m minus 1 e power i into m minus 1 theta is equal to u r minus i u theta e power minus i theta.

Divide both sides by e power minus i theta to obtain is equal to u r minus i theta. What this means is that the velocity is defined by u r is equal to m A r power m minus 1 cos m theta is equal to minus m A sin of m theta. So, those are the u r and u theta velocities. Now, we can plug these once again, on an x y plane, along theta is equal to 0 cos of m theta is equal to 1 this theta is equal to 0 sin of m theta is equal to 0 therefore, u r is equal to M a times r power m minus 1 and u theta is equal to 0.

So; that means, that along the x axis you have a velocity that is only in the radial direction, radially outward. We have a radially outward velocity the velocity magnitude increases as you go outward proportional to r power m minus 1. So, the velocity magnitude is increasing as you go outward. Since, the velocity is along the the axis along the line tangential to be axis there is no normal velocity. So, this constitutes a valid boundary for the flow, because the whenever, the velocity is radially outward it is along the line from the origin and therefore, that is a boundary.

There is one other access along which the velocity is radially outward, that is when cos of m theta is equal to minus 1 and sin of m theta is equal to 0. So, along a boundary access the cos function has to be plus or minus 1 and sin function has to be is equal to 0. Now, that is going to happen along an axis, whose angle subtended is equal to pi by m, this angel is equal to pi by m because if this angle is equal to pi by m; that means, that cos of m theta is cos pi which is minus 1 sin of m theta is sin pi which is 0.

So, for an angle of pi by m the velocity is u r is equal to minus m A r power m minus 1, and u theta has to be equal to 0. So, along this axis once again, you have velocity that is radial to the axis, but it is radially inward because when the angle is pi by m cos theta is equal to minus 1. So, I have a velocity that is radially inward. Once again, the magnitude of that velocity increases as you go further out as r increases the velocity magnitude increases as r power m minus 1.

So, along the axis pi by m you have radially inward and along the axis 0 you have radially outward, you would expect that in between these 2 half way in between these 2 the velocity is actually, in the theta direction. So, if in half way between these 2 and the angle is pi by 2 m m theta is a pi by 2 when the angle is pi by 2 m about a half way between, these 2 pi by 2 m m theta is equal to pi by 2 and therefore, the velocity u r cos of pi by 2 is 0.

So, there is no radial component u theta cos of sin of pi by 2 is equal to 1 theta is equal to pi by 2 m, therefore sin of m theta is 1 and u theta is minus m A r power m minus 1. So, you will get a theta velocity that comes something like this, and this represents the flow in a corner once again, but the angle of that corner is now, pi by m. Previously, for a z square the angle was pi by 2 for a z power m the angle is pi by m. So, when m is greater than 2 of course, pi by m is less than pi by 2. So, you get an angle flow in a corner of in an acute angled corner.

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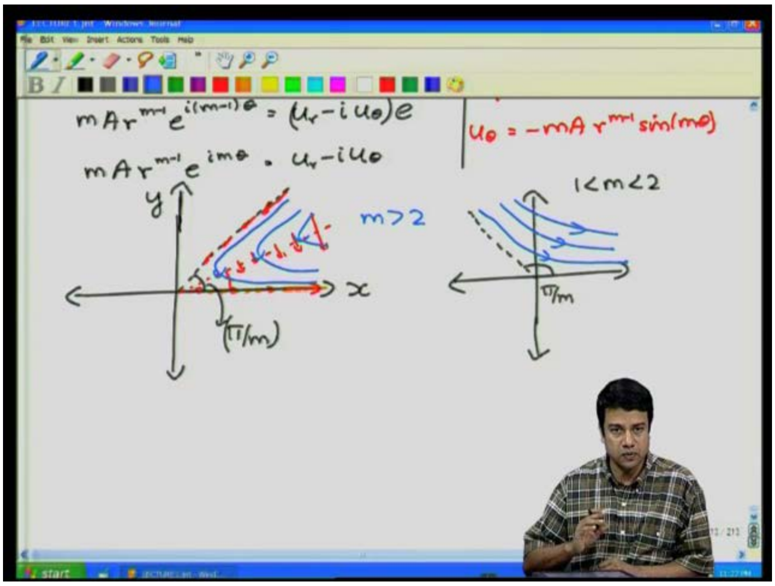

On the other hand is, so this is for m greater than 2, if m is less than 2 of course, pi by m is between, if… So, this if I consider 1 less than m less than 2, if m is between 1 and 2, the angle pi by m is between pi by 2 and pi. So, this is the angle pi by m for m between 1 and 2 pi by m for one m between one and two the angle is between pi by 2 and pi and therefore, you get a flow in a corner that looks something like this, in a angled corner, you get a flow and angled corner if the angel is between pi by 2 and pi r m is between 1 and 2. This is for m greater than 2 for m is equal to 2 the angle w is equal to a z square. So, the angle is pi by 2 you to get a flow in a right angle corner.

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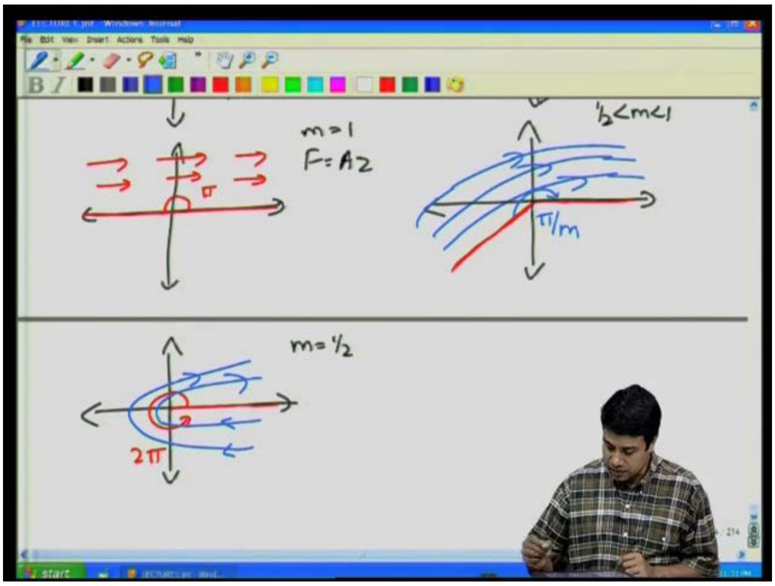

For m exactly equal to 1 m exactly equal to 1 or f is equal to A times z to the first part that is what we had solved first when, f is equal to a times z w is equal to A. So, the velocity is a constant. So, that represents a flow in a corner it is not a corner really, but it is just a straight flat surface, this subtended angle is just pi the velocity field looks something like this, and m can of course, be less than 1 m can be less than 1 because the angle is pi by m theta can go anywhere, from 0 to 2 pi; that means, m can be less than 1 we can go down to a half.

So, that pi by m is equal to 2 pi. So, if m is less than 1 what you get is an angle that looks like this, hmm you get a corner that look something like this, half less than m less than 1 and the flow look something like that once again, these 2 lines 0 and pi by m 0 and pi by m are lines along which there is no velocity, normal to the lines and therefore, these constitute valid bounding surfaces for the flow. So, this angle now, is pi by m this angle now is pi by m m is less than 1, but greater than a half. So, the angle is greater than pi, but less than 2 pi.

And one can have a flow solution for which the angle is actually, 2 pi 1 can have a flow solution for which the angle is actually 2 pi that happens for m is equal to half. So, that the angles pi by m is pi by a half which is 2 times pi. So, therefore, I have a surface that looks like this, the angle around is 2 pi and the flow field will look, something like this the flow field comes in on the lower half plane and goes out on the upper half plane the.

So, these are the kinds of velocity fields that you get if you consider simple functions which are some powers of the complex number.

The lowest power that you can have is z power half m is equal to half, in that case the angle is equal to 2 pi between, half and 1 you have an expanding flow at m is equal to 1 it is just a constant velocity, m is equal to 2 is in right angle corner and greater than 2 is in acute angle corner and less than 2 is corner with angle greater than pi by 2. So, these are simple forms of the. So, these are power forms of the complex function.

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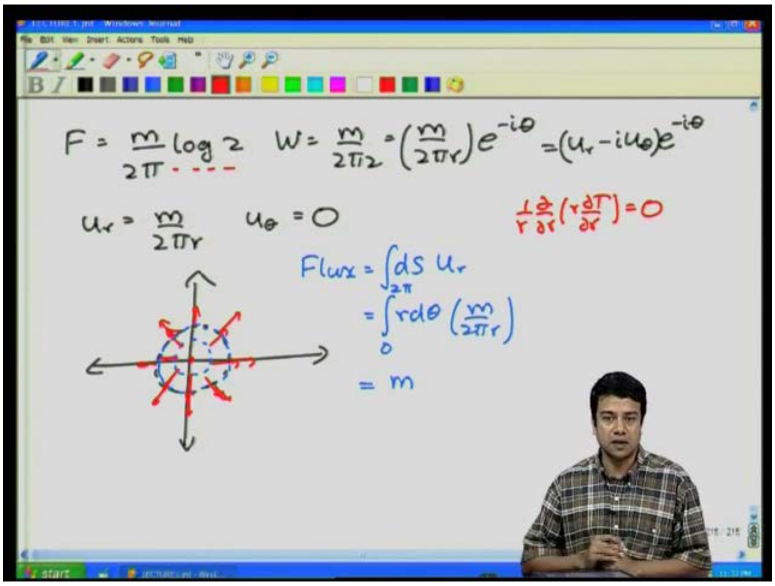

Now, after considering simple powers the next thing to consider is logarithmic function. So, f is equal to m by 2 pi log z . So, therefore, the complex velocity w is equal to m by 2 pi z is equal to m by 2 pi r into e power minus i theta. So, that is the complex velocity, we know that this has to be equal to u r minus i u theta e power minus i theta. So, therefore, we find that u r is equal to m by 2 pi r u theta is equal to 0. So, this represents a radial velocity profile, the velocity field everywhere is always radially outward. So, at some location the velocity field is always radially outward.

Note that the magnitude of the radial velocity decreases as r increases. The magnitude of the velocity goes to infinity at r is equal to 0. So, at the origin itself the velocity is infinite, as you go outward from the origin the magnitude of the velocity decreases as we go further and further outward. However, if you calculate the flux, total mass flux, coming out the mass flux coming out of surface such as this 1 the mass flux coming out of a surface is such of this 1 can be calculated as flux is equal to integral over the surface of u dot n the unit normal to the surface since I have taken a circular surface the unit normal is along the radial direction.

Therefore, u dot m is just equal to u r itself, integral d s u r, note that as I discussed at the beginning of this lecture, this flux in two dimensions has to be per unit length in the third direction. So, this is a flux in two dimensions, that is rather than mass per unit area as it is in 3 dimensions this will be a mass per unit length, per unit length perpendicular to the third direction. So, in this particular case d s the surface is just a line it is the line along that circle because I am taking a per unit length in direction perpendicular to the plane.

So, that d s is just equal to integral r d theta, because along this line r is a constant it is theta that is varying all the all around from 0 to pi along this line r is constant. So, theta goes from 0 to pi into m by 2 pi r. So, even though the velocity is increasing this flux you can easily do this integral r and r will cancel out m d theta from 0 to 2 pi divided by 2 pi will just give you back m, even though the velocity is increasing as you go towards the origin, the amount of mass coming out per unit length in the third direction is remaining the same.

So; that means, that if you take any surface here, any surface here the amount of mass coming out of the inner surface, if the velocity at the inner surface is larger, the surface area is smaller the, amount of mass coming out is the same. That is equal to m at each and every surface, that means that there is no source anywhere, in the flow except right at the origin there is no source anywhere in the flow except right at the origin. And because of that the velocity turns out to be 2 pi r, if I calculated the complex potential for this case I would find that it went as log i of r.

In 2 dimensions a source of mass, a source of heat, the temperature of the potential goes as log of r, in 3 dimensions is proportional to one over r in 2 dimensions its proportional to log of r. That is obtained by solving the Laplace equation in 2 dimensions either, for the temperate or for the potential you can easily, see that the solution of this is log of r. So, when I to this form log z for the potential that corresponded to a point source, right at the origin in 2 dimensions the velocity decreases as one over r, as r goes to infinity the surface area increases proportional to r.

Therefore, the mass coming out of any surface is the same as delta function source at the origin and that is generating radially outward flow in this 2 dimensional plane.

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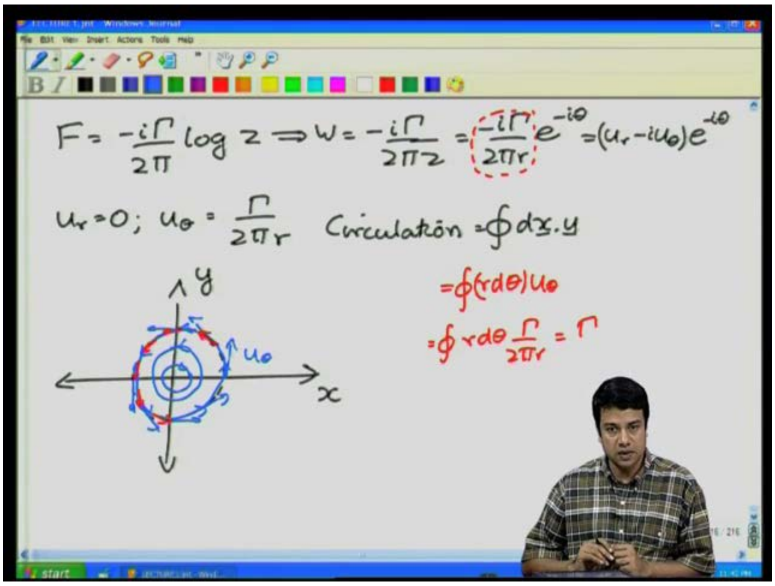

There is another important logarithmic dependence that can give you an important flow let us look at the, f is equal to minus i gamma, which implies the w is equal to w is equal to d f by d z is equal to minus i gamma by 2 pi z expressed in terms of r and theta this turns out to be equal to minus i gamma by 2 pi r e power minus i theta. And I can write this as usual as u r minus i u theta e power minus i theta. So, clearly u r is equal to 0 because the coefficient of e power minus i theta is purely imaginary.

So, therefore, for this flow I have u r is equal to 0 and u theta is equal to gamma by 2 pi r. So, therefore, along any particular circle over here, u r is equal to 0 there is no radial component of the velocity u theta is equal to gamma by 2 pi r. So, therefore the velocity u theta, note that u theta is defined to be positive in the anti clock wise direction in the direction of increasing theta therefore, u theta is constant everywhere, along this, and this represents a flow that is circulating around this origin.

So, this represents a flow that is a circulating around this origin. So, the flow is circulating around the center of the coordinate system. What is gamma, in a similar manner to we found out the mass flux for point source the origin, gamma is what is defined as the circulation and that is obtained as follows, the circulation is defined as integral over the surface of sorry integral over a closed contour of d x dot u, integral over the close contour d x dot u, where d x is the tangent vector along the contour. So, if I have a contour at each point we have a tangent vector along that contour.

So, we can see that for this circle if I take d x dot u for the r component of the velocity d x dot u will be 0, is non 0 only for the theta component, the component that is tangential to the surface. So, this circulation around the circle is equal to integral d x dot u and this in this case this d x is given by r d theta times u theta d x is r d theta along the surface times u theta, because the theta vector is along the direction of the displacement along that contour.

So, this will be equal to integral of r d theta times gamma by 2 pi r, which is nothing, but gamma itself. So, therefore, the u theta velocity the theta velocity goes to infinity as you go towards the origin, u theta goes as is equal to gamma by 2 pi r, but the circulation along any contour is exactly the same. The circulation along the any contour is exactly the same. So, this is what is called a line vertex, a line vertex in which the circulation is exactly the same on each and every contour.

Note that I have got a velocity field which seems to be rotating when I started off, I said potential flow irrotational, there should be no rotation. I have defined here for you a circulation, which is basically, integral over the surface of d x dot u. So, apparently it appears that the velocity field is rotating, but that is actually not, so. The reason is because local rotation depends upon the local anti-symmetric part of the rate of deformation tensor.

Now, a velocity field would be rotational only if locally at at points within the flow there is non-zero anti-symmetric part of the rate of deformation tensor or there is a verticity somewhere, within the flow. This...what we have is what is called a line vortex.

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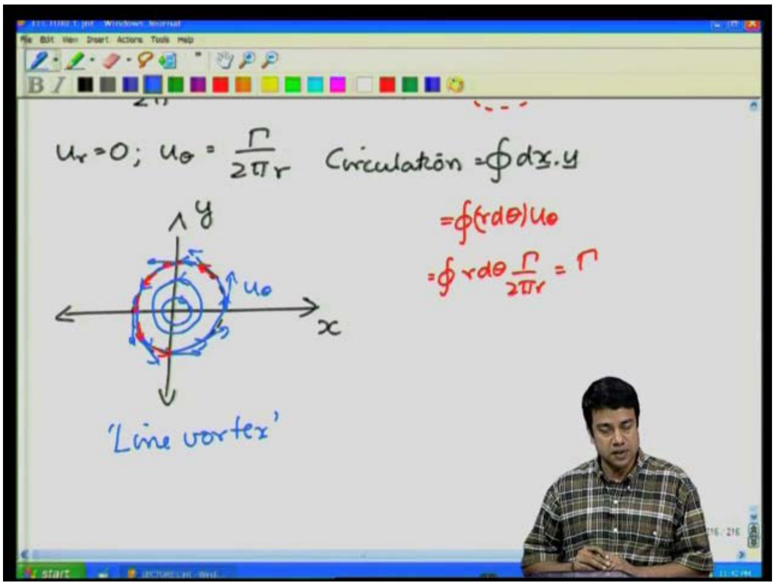

And, note that we have done the stokes theorem earlier when we discussed a vector theorems.

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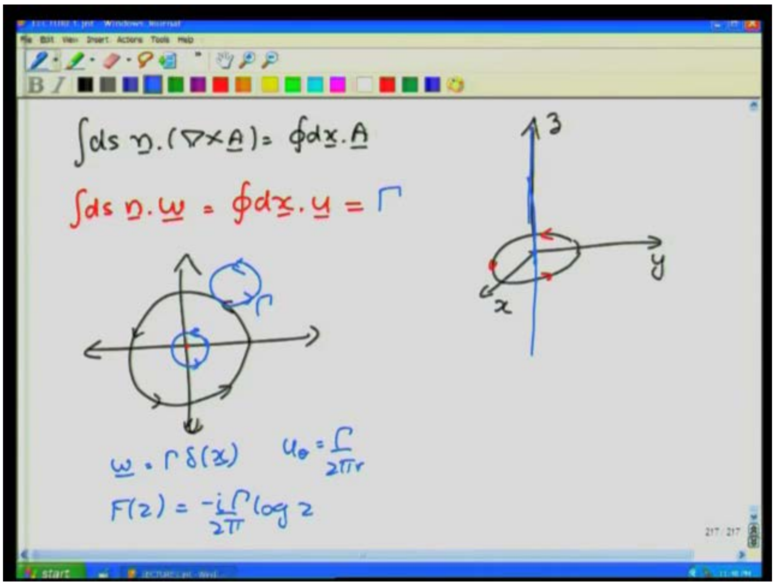

And that stokes theorem what it told us was that integral d s of n dot del cross a equal to integral over the contour of d x dot A. So, the idea was as follows I have some surface, with some bounding contour on it and the vector a is defined everywhere, on the surface and what the stokes theorem said was that integral over the surface of n dot del cross A.

That is everywhere, over the surface if I take n dot del cross a and integration over the entire surface is actually, equal to the integral over the perimeter of the surface.

Taken in the anti-clock wise direction as appropriate for the curl the perimeter of the surface, of d x dotted with A. So, that is what the stokes theorem said, consider that we are considering now, a two dimensional system. So, that the surface is now, in the x y plane. So, that is our two dimensional flow the surface itself is the surface that is in the x y plane, the surface itself is the surface in the x y plane the unit normal to the surface is in the z direction the unit normal to the surface is in the z direction.

And my contour goes in the anti-clock wise direction in the x y plane in contour goes in the anti-clock wise direction in the x y plane then for this particular case I will have integral over the surface that is the surface d s n dot I consider this vector a to be the velocity. So, del cross a is just the verticity is equal to integral d x dot u. Now, what this implies is that integral d x dot u which was the circulation I just calculated this as the circulation. So, this integral d x dot u is non 0 for this particular case, note that it is a 2 dimensional flow.

I would assume that the potential flow was irrotational therefore, n dot omega has to be equal to 0, can I have a situation, where the left hand side n dot omega is equal to 0 whereas, the right hand side is non 0 clearly it seems is not true. The answer to the paradox is as follows, the verticity itself is a delta function along the origin here along the z axis. So, for this 2 dimensional flow, one can consider the verticity in 2 dimensions to be a delta function right at the origin; that means, that there is a verticity at the origin itself, but there is no verticity anywhere, else in the flow.

So, therefore, what will happen is if I will take a 2 dimensional plane and I go all the way around the origin. Since, the verticity is a delta function at the origin the circulation here around the surface will in general be non 0 because only at the origin I have a delta function verticity because of that the circulation is non 0 in any contour that includes the origin; however, if I take some contour somewhere, within the flow that does not include the origin there the circulation will be 0 as because the verticity is 0 everywhere.

The fact of the verticity is 0 everywhere. It is reflected in the fact, that the circulation along any contour is exactly the same. So, if my surface cuts the z axis cuts the origin the verticity is a delta function there and therefore, I get a non 0 result, if it does not cut the origin then the verticity is 0 because the verticity is a delta function omega is equal to gamma delta function in this 2 dimensional plane. Note that the delta function in 2 dimensional planes has dimensions of one over length square.

The circulation is velocity times length gamma as the verticity goes as velocity divided by length. So, for this flow with omega is equal to gamma times delta of x the solution is u theta is equal to gamma by 2 pi r, just as in the case of the mass which we just considered, there is a source at the origin, but everywhere, else the flow is incompressible. Similarly, in this particular case there is a delta function verticity at the origin, but the flow is irrotational everywhere, else. So, this is the flow around a line vortex in for which the function f of z is equal to minus i gamma by 2 pi log z.

So, this is a line vortex m by 2 pi log z is a line is a point source, line source at the origin a point source in 2 dimensions and line source in 3 dimensions. This is a line vortex, in the next lecture will look at some more complicated forms, in particular the flow around a cylinder and our objective will be t calculate the net force that is exerted on the for the flow around a cylinder in 2 dimensions, in a manner similar to the force that we calculated for a spherical object in the last class.

In that case we found that the net force on sphere has to be 0, because there is no viscose dissipation, in this case we will see that it does not have to be 0 the net drag force along the direction of velocity has to be 0, but the net force perpendicular what is called the lift force does not necessarily, have to be 0. So, in the next lecture, we will look at the velocity field around cylinder in 2 dimensions rather a circle in two dimensions or a cylinder infinite cylinder in 3 dimensions. And we will calculate the net force on that after calculating the velocity field. So, we will continue this in the next lecture we will see you them.